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Developing an Accurate Model of Thermo-Electric Generator for Energy Harvesting

A Thesis submitted in Partial Fulfilment of the Requirements for the Master
Degree in Applied Physics

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Abstract

A model based on one-dimension steady-state heat transfer was developed, to calculate the generated current, voltage and power of the TEG system. Using the Finite Element Method (FEM) and the temperature dependent conductivity, resistivity, Seebeck coefficient, and Thomson coefficient, the output current and voltage were compared for the FEM and the Average Method (the average value between the hot and cold surface). The FEM and the Average Method showed different values, especially at high temperature differences ΔT above 80 K for the current and above 130 K for the voltage.

The effect of changing the load resistor was studied, proving that the maximum current can be obtained when the load resistor is equal to the internal resistance of the TEG. In addition, the model is used to find the optimum combination of size and length of the p-n materials (thermocouple). The result shows that larger area and the shorter length for the design of the thermocouple, gives the maximum power.

Parallel to the modelling, the effect of the size of the applied heat system on the current and voltage was investigated experimentally, showing the effect of the size of the heated area on the current linear fitting. It is found that the larger the size the better linear fitting.

Dedication

To my parents,
To everyone who has supported me in my educational journey

Acknowledgement

I would like to thank my supervisors Dr. Hanan Albuflasa and Dr. Hessa Aljunaid for their encouragement and patience. A special thanks to my colleague Mr. Ahmed Saif for his technical support. Finally, I would like to thank my family and friends for all the support and courage.

Nomenclature

Symbol	Parameter	Unit
A	Cross-sectional Area	m ²
C _p	Specific Heat Capacity	J/Kg.K
H	Hight	m
I	Electrical Current	A
J	Current Density	A/ m ²
k	Thermal Conductivity	W/m.K
K	Thermal Conductance	W/K
L, L _n , L _p	Length, n-type Length, p-type Length	m
N	Number of Thermocouples	-
P	Output Power	W
Q	Heat Flow	W
Q _p	Peltier Heat	W
Q _T	Thomson Heat	W
R	Internal Electrical Resistance	Ω
R _L	Electrical Load Resistance	Ω
T	Absolute Temperature	K
V	Voltage	V
W	Width	m
ZT	Figure of Merit	Dimensionless
α	Seebeck Coefficient	V/K
η	Efficiency	%
ρ	Electrical Resistivity	Ωm

σ	Electrical Conductivity	$1/\Omega.m$
τ	Thomson Coefficient	V/K
π	Peltier Coefficient	V/K

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Chapter One

Introduction to the Study

- 1.1 Waste heat recovery
- 1.2 Research Problem
- 1.3 Research Questions
- 1.4 Research Objectives
- 1.5 Research Significance
- 1.6 Thesis Outline
- 1.7 Research Terminology

Chapter One

Introduction to the Study

1.1 Waste heat recovery

Waste heat is not well defined in the literature, but it is generally considered as the unused heat in the atmosphere that generated due to the flow of energy between two systems with different temperatures (Mahmoudi et al., 2018).

In our surroundings, waste heat is in large numbers and increasing rapidly because of the expansion of the industrial processes in addition to the heat produced from natural sources.

The process of burning the fuel, the radiated heat of the ventilation systems, vehicles and other machines, the radiated heat from solar energy, geothermal and human bodies are examples of the waste heat sources. Figure 1, shows the different sources and their contribution to the heat waste in the industry (Mahmoudi et al., 2018).

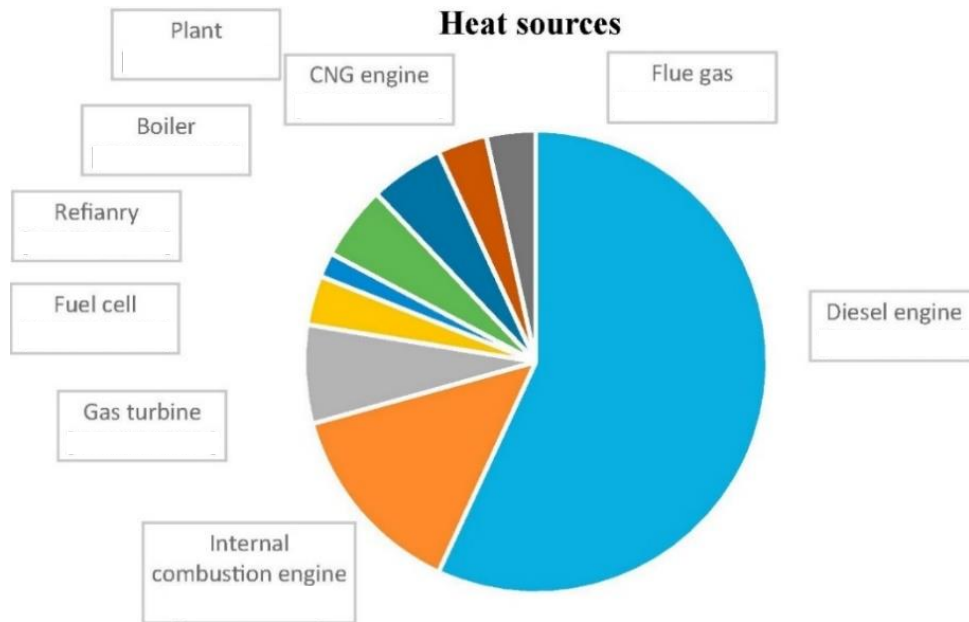


Figure 1: Heat sources (Mahmoudi, Fazli, & Morad, 2018).

This heat can be classified into three grade categories according to its temperature: Low grade ($T < 100^{\circ}\text{C}$), Medium grade ($T: 100 - 400^{\circ}\text{C}$) and High grade ($T > 400^{\circ}\text{C}$) (Jouhara et al., 2018).

Waste heat recovery systems are one of the solutions used to improve the instrument's efficiency and can be used as a useful alternative source of energy. For example, the heat can be harnessed and converted to a useful electric power using the thermoelectric effect where the thermal energy (heat, temperature difference, and temperature gradient) is converted into electrical energy and the electrical energy is converted into thermal energy.

1.2 Research Problem

The main limitation found in all work done to model the thermoelectric generator (TEG) is that most of the models studied the effect of one or two parameters. However, all the parameters are interdependent, which will affect the performance at the same time and give inaccurate results as the number of assumptions are increased.

The main aim of this research is to accurately model the thermoelectric generator based on its physical specification considering all the parameters, such as Thomson coefficient and understanding the relationship between different parameters using experimental methods that help to improve the performance of TEG.

This research will underline different parameters regarding the performance and output power of thermoelectric generators: the approximated Thomson coefficient, the one-dimension heat transfer assumption and the Seebeck coefficient as a function of temperature.

1.3 Research Questions

1. What are the equations used to model the thermoelectric generator?
2. What is the current research done in the field to improve modelling the system?
3. Are the assumptions used in the modelling valid and can be generalised?
4. What are the main parameters affecting the performance of the TEG?
5. Which parameter can be theoretically investigated to the modelling?
6. How to improve the performance of the TEG?

1.4 Research Objectives

The objectives of this project are:

1. Develop a TEG model taking into account different physical effects, which are neglected or approximated.
2. Find optimum parameters that give better performance.

1.5 Research Significance

TEG are used in various applications that do not need frequent maintenance ranging from small scale in wearable devices to the large scale in industries. However, they still have low efficiencies and give low power and electricity compared to the system cost.

The research is important to study and understand the behaviour of the thermoelectric generator under different conditions such as the ratio of the heated area to the TEG surface and the coordinate of the heated area. Also, this project helps in designing the optimal device for different operating conditions based on the heat generated and other parameters which is important for performance optimisation.

This research can give an overview of TEG behaviour that will help in developing more accurate model for steady state in this work and transient state in future and also to find how can TEG be used in different applications.

1.6 Thesis Outline

The thesis includes five chapters; chapter one gives a brief introduction about the waste heat recovery, the objectives, and significance of the project. Chapter two shows the physics of thermoelectric generator and the literature review of the other researches. In chapter three, the finite element method is described and implemented in MATLAB to find the TEG output.

The results of the experiments and the implementation are discussed in chapter four and the work's conclusion in chapter five.

1.7 Research Terminology

- TEG: Thermoelectric Generator is a device that converts temperature difference into electricity.
- TEC: Thermoelectric Cooler is a device for cooling and heating.
- FEM: Finite Element Method, a method used in finding the solution of a partial differential equation by dividing it into small nodes and solve each node separately.
- FVM: Finite Volume Method, a method used in finding the solution of a partial differential equation by dividing it into controlled volume cell with boundary conditions.

Chapter Two

Theoretical Framework and Literature Review

2.1 Theoretical Framework

2.1.1 Thermoelectric Effect

2.1.2 Thermoelectric Generator (TEG)

2.1.2 Thermoelectric Generator Heat Equation

2.2 Literature Review

Chapter Two

Theoretical Framework and Literature Review

2.1 Theoretical Framework

Thermoelectric Generator devices (TEG) has various applications because it does not need frequent maintenance, and it can range from small scale in wearable devices (Siddique, Mahmud, and Heyst, 2017) to large scale in industries (Champier, 2017).

One of the greatest merits for the thermoelectric power generation is its lightweight, high reliability, environmental friendliness, no moving parts, and no fluids needed. Therefore, thermoelectric generators are used as waste heat recovery from power plants and automotive vehicles. In addition, it can provide reliable power for rural or remote areas, such as on top of mountains for telecommunication systems or in space probes.

In addition, thermoelectric technology can contribute to power generation as an alternative to the conventional systems. It can be used in applications where renewable technologies cannot be used, or it can be combined with renewable energy to improve the efficiency and the performance of conventional or solar thermal power generation (Twaha, Zhu, Yan, & Li, 2016).

On the other hand, Thermoelectric Cooler (TEC) systems are implemented in refrigeration and temperature control, which are mostly used in medical instruments and many electronic packages.

2.1.1 Thermoelectric Effect

Thermoelectric effect is explained in three main effects:

a. Seebeck effect: The production of an electromotive force (EMF) from two dissimilar materials with different temperature as shown in Figure 2, it was discovered in 1821 by Thomas Seebeck when the junction of two dissimilar metals was heated, and a voltage difference (V) was produced.

The voltage difference between the two metals can be calculated using Eq.1, where ΔT is the temperature difference between the hot and cold sides ($T_H - T_C$), and α is the Seebeck coefficient, and it is defined as the difference of the Seebeck coefficient of the two metals ($\alpha = \alpha_H - \alpha_C$)

$$V = \alpha \Delta T = (\alpha_H - \alpha_C) \Delta T \quad (1)$$

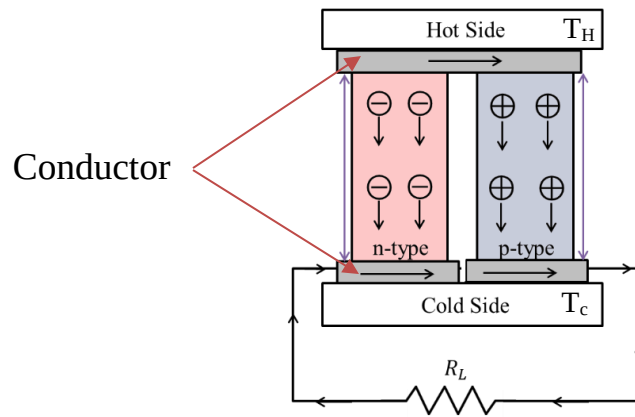


Figure 2: Seebeck effect

b. Peltier effect: The heat absorption or generation when the current flows through the junction. In 1834, Jean Peltier discovered that as the electric

current passes through the junction of the two dissimilar metals, heat absorption or generation appears.

Eq. 2 shows the relation between the rate of heat generated/absorbed (Q_p) and the current (I) passing through the junction where π is the Peltier coefficient, and it causes the heating flow when two materials are used with different coefficient values ($\pi = \pi_H - \pi_C$)

$$Q_P = \pi I = (\pi_H - \pi_C)I \quad (2)$$

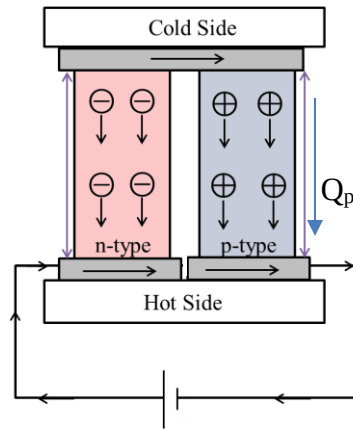


Figure 3: Peltier effect

c. Thomson effect: The emission or absorption of heat in a single metal when it has a temperature difference between two points, and there is a flow of electric current through it. It was discovered by Willian Thomson in 1854 when he discovered that temperature difference occurs in a conductor when a voltage difference is applied between two points.

The rate of heat Q_T can be calculated using Eq. 3 where τ is Thomson coefficient, J is the current density, and dT/dx is the temperature gradient. The negative sign represents the flow direction.

$$Q_T = -\tau JT = -\tau I \frac{dT}{dx} \quad (3)$$

$$\tau = T \frac{d\alpha}{dT} \quad (4)$$

However, for single material having a temperature gradient, the effect of Joule heating appears: As the current flows through the conductor having resistance (R) the Joule effect is internally appear (Dziurdzia, 2011); this heat rate (Q_J) can be calculated using:

$$Q_J = I^2 R \quad (5)$$

Combining both Thomson and Joule effects gives the net heat rate Q_{net} as follows:

$$Q_{net} = Q_J - Q_T = I^2 R - \tau I \frac{dT}{dx} \quad (6)$$

The electrical Seebeck effect can be related to the thermal Peltier and Thomson effect using “The Kelvin relationship” which combine the three effects, thermodynamically, as follows:

$$\frac{d\pi}{dT} = \alpha + \tau \quad (7)$$

$$\pi = \alpha T \quad (8)$$

2.1.2 Thermoelectric Generator (TEG)

The thermoelectric generator, illustrated in Figure 4, is a semiconductor device consists of a number of thermocouples (p-n materials) that are connected thermally in parallel and electrically in series, to give the maximum output voltage (Montecucco, Siviter & Knox, 2014).

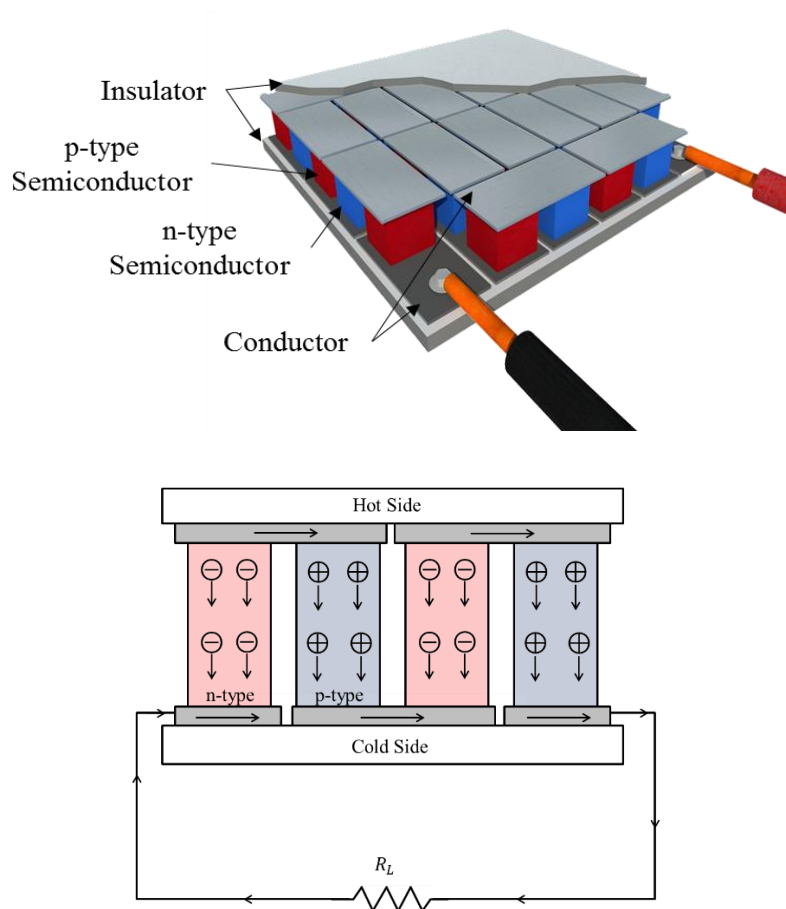


Figure 4: Thermoelectric Generator Module

Those p-n junctions are connected by conductors and is sandwiched between two ceramic plates that work on isolating the module and supporting it.

The optimum size of the TEG depends on the application's conditions and the required output. For example, in the hearing aid, the area of TEG should be in the range of mm² while in the industry it can reach the range of cm².

The TEG working mechanism based on thermoelectric effect generally and Seebeck effect especially, where the voltage is produced due to the excitation of the electrons and the flow of current when there is a temperature difference in the thermocouples.

The efficiency (η) of the TEG is calculated using Eq. 11:

$$\eta = \frac{\text{Power output}}{\text{Power absorbed by hot junction}} \quad (9)$$

The materials used in the TEG has an impact on the efficiency of the device since the physical parameters like conductivity, resistivity, Seebeck's and Thomson's coefficients are temperature dependent.

The performance of the thermoelectric material is calculated using the figure of merit (z) that relates all the mentioned coefficients. It is represented in the following equation:

$$zT = \frac{\alpha^2}{\rho\kappa_{tc}} T \quad (10)$$

The figure of merit can be found if the efficiency is maximum and the temperatures and the ratio of the load resistor to the internal resistor ($m = R_L/R$) are fixed. Using the steady-state heat equation, the figure of merit can be found:

$$\begin{aligned}
\eta &= \frac{P}{q_h} = \frac{I^2 R_L}{\kappa \Delta T + \alpha I T_h - \frac{1}{2} I^2 R} \\
&= \frac{\left(\frac{\alpha \Delta T}{R + R_L} \right)^2 R_L}{\kappa \Delta T + \alpha \left(\frac{\alpha \Delta T}{R + R_L} \right) T_h - \frac{1}{2} \left(\frac{\alpha \Delta T}{R + R_L} \right)^2 R} \\
&= \frac{\left(\frac{\alpha \Delta T}{R + (mR)} \right)^2 (mR)}{\kappa \Delta T + \alpha \left(\frac{\alpha \Delta T}{R + (mR)} \right) T_h - \frac{1}{2} \left(\frac{\alpha \Delta T}{R + (mR)} \right)^2 R} \\
&= \frac{m \Delta T}{\frac{\kappa R}{\alpha^2} (1 + m) + T_h - \frac{1}{2} \frac{\Delta T}{(1 + m)}} \\
\eta &= \frac{m \Delta T}{\frac{1}{Z} (1 + m) + T_h - \frac{1}{2} \frac{\Delta T}{(1 + m)}}
\end{aligned}$$

From the steps above and from Eq. 10, the efficiency (η) of the TEG increases as the value of the figure of merit (z) get larger, and this can be reached by choosing materials with high Seebeck coefficient, low thermal conductivity, and low electrical resistivity (for single material).

The development of nanotechnology in material science has improved the efficiency and performance of the thermoelectric generators. However, they still have low efficiencies and give low power and current compared with the system's cost. The performance of the generators under different conditions (high and low temperature difference and gradient) has been investigated by introducing new materials and alloys, such as, BiSb and Bi₂Te₃ or thin films like p-Sb₂Te₃ and n-Bi₂Te₃, that are characterized by high electrical conductivity and low thermal conductivity (Twaha, Zhu, Yan, & Li, 2016).

Materials used in the TEG should go under financial investigation by considering different costs (raw material, components, and fabrication) and coupled thermal and electrical transport factors. (LeBlanc, Yee, Scullin, Dames & Goodson, 2014)

Figure 5 shows the value of the figure of merit for different thermoelectric materials, for most materials the value is increasing as the temperature increases.

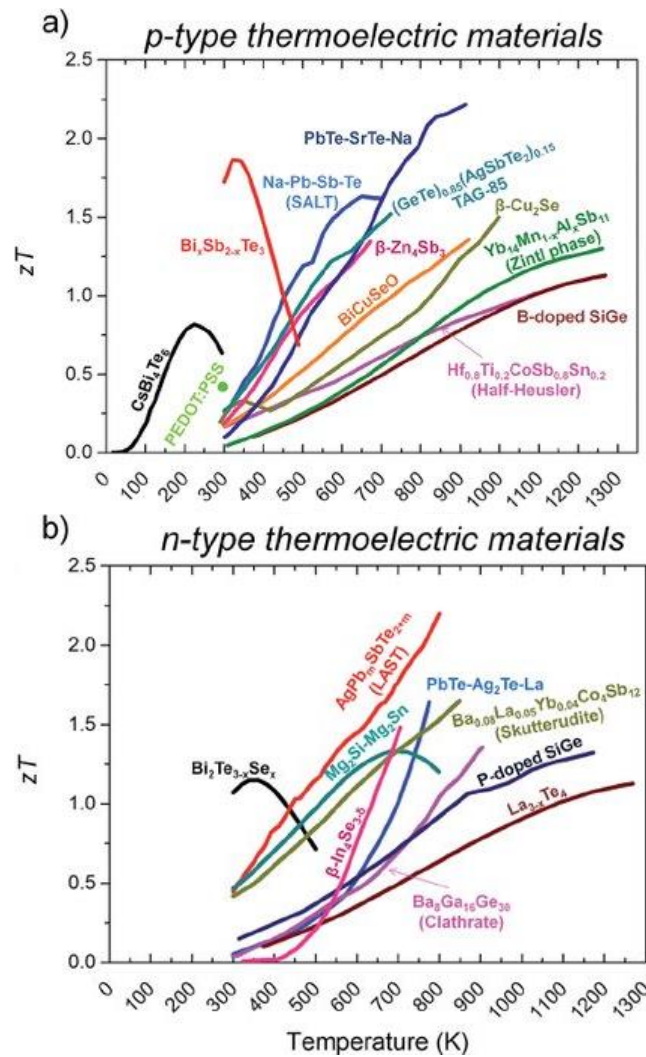


Figure 5: Figure of merit for different thermoelectric materials
(Rull, Moure, Fernández, & Martín-González, 2015)

2.1.3 Thermoelectric Generator Heat Equation

The heat equation of the thermoelectric generator, in general, is the sum of the heat sources and losses ($Q_{net} = Q_{Peltier} + Q_{conduction} + Q_{Joule} + Q_{Thomson}$).

The radiation heat loss from the thermocouples can be neglectable since it has a small value as well as the convection heat loss for small modules (Bjørk, Christensen, Eriksen, & Pryds, 2014). Eq. 11 represents the heat equation of the TEG:

$$\begin{aligned} \rho_d C_p \left(\frac{\partial T}{\partial t} \right) &= \vec{\nabla} \cdot (k \vec{\nabla} T) + TJ(\vec{\nabla} \alpha)_T + \frac{J^2}{\sigma} - \tau \vec{j} \cdot \vec{\nabla} T \\ &= \vec{\nabla} \cdot (k \vec{\nabla} T) + T \frac{I}{A} (\vec{\nabla} \alpha)_T + \frac{I^2}{A^2 \sigma} - T \frac{I}{A} \frac{d\alpha}{dT} \cdot \vec{\nabla} T \end{aligned} \quad (11)$$

Where the first term on the left represents the transient heat rate, while on the right, the first term is the conduction heat, the second is Joule heating, and the third is Thomson heat.

Since the thermal conductivity, electric conductivity and Seebeck coefficient are temperature dependent then for a single material each parameter can be calculated as a function of temperature using the following equations:

$$k = k_o * (1 + F(T - T_o) + G(T - T_o)^2) \quad (12)$$

$$\sigma = \sigma_o * (1 + F(T - T_o) + G(T - T_o)^2) \quad (13)$$

$$\alpha = \alpha_o * (1 + F(T - T_o) + G(T - T_o)^2), \quad (14)$$

where F and G are the temperature coefficients of the material.

For each thermocouple, the thermal conductance and the resistance should be calculated by finding the average conductivity and electric conductivity of the p-n materials as follows:

$$\bar{k}_{p,n} = \frac{\int_{T_C}^{T_H} k_{p,n}(T) dT}{T_H - T_C}, \quad (15)$$

$$K = \left(\frac{\bar{k}_p A_p}{L_p} + \frac{\bar{k}_n A_n}{L_n} \right), \quad (16)$$

$$\bar{\sigma}_{p,n} = \frac{\int_{T_C}^{T_H} \sigma_{p,n}(T) dT}{T_H - T_C}, \quad (17)$$

$$R = \left(\frac{L_p}{\bar{\sigma}_p A_p} + \frac{L_n}{\bar{\sigma}_n A_n} \right), \quad (18)$$

$$\bar{\alpha}_{p,n} = \frac{\int_{T_C}^{T_H} \alpha_{p,n}(T) dT}{T_H - T_C}, \quad (19)$$

$$\alpha = (\bar{\alpha}_p - \bar{\alpha}_n), \quad (20)$$

As a result, the heat equation for the thermocouple will become as follows:

$$Q_H = K\Delta T - I^2 R + \alpha_h T_h I - I\tau\Delta T \quad (21)$$

$$Q_C = K\Delta T + I^2 R + \alpha_c T_c I + I\tau\Delta T \quad (22)$$

where Q_H and Q_C are the heat rate at the hot and cold sides of the TEG.

The current generated from the TEG can be calculated using the following equation:

$$I = \frac{\alpha_h T_h - \alpha_c T_c - \tau\Delta T}{R + R_L} \quad (23)$$

The voltage will be equal to:

$$V = N(\alpha_h T_h - \alpha_c T_c - IR - \tau \Delta T) \quad (24)$$

where N is the number of thermocouples in the module.

The output power will be equal to:

$$P = Q_H - Q_C = N(\alpha_h T_h - \alpha_c T_c)I - I^2 R - I\tau \Delta T = I^2 R_L \quad (25)$$

Therefore, the efficiency will be:

$$\eta = \frac{(\alpha_h T_h - \alpha_c T_c)I - I^2 R - I\tau \Delta T}{K\Delta T - I^2 R + \alpha_h T_h I - I\tau \Delta T} = \frac{I^2 R_L}{K\Delta T - I^2 R + \alpha_h T_h I - I\tau \Delta T} \quad (26)$$

The maximum output can be calculated when the derivation of the power with respect to the load resistor is equal to zero, which can be obtained when the load resistor equals the internal resistor ($R_L = R$) as shown in Eq.28 and Eq.29.

$$\frac{dP}{dR} = 0$$

$$\frac{d}{dR} ((\alpha_h T_h - \alpha_c T_c)I - I^2 R - I\tau \Delta T) = \frac{d}{dR} (I^2 R_L) = 0 \quad (27)$$

Then,

$$I_{max} = N \frac{\alpha_h T_h - \alpha_c T_c - \tau \Delta T}{2R}, \quad (28)$$

$$P_{max} = N^2 \frac{(\alpha_h T_h - \alpha_c T_c - \tau \Delta T)^2}{4R}, \quad (29)$$

The maximum voltage can be found when the current equal to zero (open circuit) as follows.

$$V_{max} = N(\alpha_h T_h - \alpha_c T_c - \tau \Delta T) \quad (30)$$

By including the effect of Thomson, the figure of merit becomes:

$$\begin{aligned} \eta &= \frac{P}{q_h} = \frac{I^2 R_L}{\kappa \Delta T + \alpha_h I T_h - \frac{1}{2} I^2 R - \frac{1}{2} \tau I \Delta T} \\ &= \frac{\left(\frac{\alpha \Delta T - \tau \Delta T}{R + R_L} \right)^2 R_L}{\kappa \Delta T + \alpha_h \left(\frac{\alpha \Delta T - \tau \Delta T}{R + R_L} \right) T_h - \frac{1}{2} \left(\frac{\alpha \Delta T - \tau \Delta T}{R + R_L} \right)^2 R - \frac{1}{2} \tau \left(\frac{\alpha \Delta T - \tau \Delta T}{R + R_L} \right) \Delta T} \\ &= \frac{\left(\frac{\alpha \Delta T - \tau \Delta T}{R + (mR)} \right)^2 (mR)}{\kappa \Delta T + \alpha_h \left(\frac{\alpha \Delta T - \tau \Delta T}{R + (mR)} \right) T_h - \frac{1}{2} \left(\frac{\alpha \Delta T - \tau \Delta T}{R + (mR)} \right)^2 R - \frac{1}{2} \tau \left(\frac{\alpha \Delta T - \tau \Delta T}{R + (mR)} \right) \Delta T} \\ &= \frac{m \Delta T}{\frac{\kappa R}{(\alpha - \tau)^2} (1 + m) + (\alpha - \tau) [\alpha_h T_h - \frac{1}{2} \tau \Delta T] - \frac{1}{2} \frac{\Delta T}{(1 + m)}} \\ zT &= \frac{(\alpha_h T_h - \alpha_c T_c - \tau \Delta T) (\alpha_h - \alpha_c \frac{T_c}{T_h}) \frac{\sigma_n}{k_n}}{2 \left(1 + \sqrt{\frac{k_p \sigma_n}{k_n \sigma_p}} \right)^2 \frac{\Delta T}{T_h}} \end{aligned} \quad (31)$$

Figure 6 shows the parameters that go in the heat (Eq.11) and in calculating the output of the model.

2.2 Literature Review

Modelling the TEG devices to investigate its performance and to find the optimum design requires a comprehensive understanding of the thermodynamics and analysis of the thermal/electrical behaviour under different operating conditions. (Goupil, Seifert, Zabrocki, Müller & Snyder, 2011) and (Twaha, Zhu, Yan, & Li, 2016).

In the literature, the TEG was modelled under different conditions, different analysis methods (Finite element method, Finite volume method...etc) in addition to several validation techniques (experimental or theoretical).

In general, the literature can be divided into three categories, as follows:

1. Studying the TEG parameters such as the thermal contact resistance, Thomson coefficient, heat losses ... etc.

- The contact thermal resistance was investigated, and the constant heat gradient and constant heat flow were implemented (Siouane, Jovanović, & Poure, 2016a).
- Studying the influence of the contact layer resistance, Thomson effect, Joule heat and thermo-pellet gap heat leakage. However, the Thomson coefficient was assumed to be constant (Wu, Zuo, Chen, & Klein, 2016a).
- The equivalent electrical circuits of thermoelectric generators under both constant temperature gradient and constant heat flow conditions (Siouane, Jovanović, & Poure, 2016b) and (Siouane, Jovanović, & Poure, 2017).

- Coupling the temperature field and electric potential field by developing a three-dimensional numerical model (Wang, Huang, Cheng, Ta-Wei Lin, & Kang, 2012).
- The conduction, convection, and radiation heat losses were considered, and they found that the convective heat loss can be neglectable -only for the small size- and they suggested some solution in manufacturing the legs to maximise the efficiency (Björk et al., 2014).
- The convection heat loss has an impact on the performance by studying the behaviour analytically including different parameters such as ohmic heating, Peltier, Seebeck and Thomson coefficients exposed to convective boundary conditions for the one-dimension heat transfer and then developed it numerically to two dimensions heat transfer (Rabari, Mahmud, Dutta & Biglarbegan, 2015).
- The temperature dependence of Seebeck coefficient, electrical resistance, thermal conductance, and figure-of-merit experimentally was studied by calculating the average values of a group of direct measurements acquired every second (Mitrani et al., 2005).
- A correlation between Seebeck coefficient and its impact on the system's performance (Liu et al., 2017). However, the correlation was validated for one system and a new definition “effective Seebeck coefficient” was proposed (Hsu, Huang, Chu, Yu, & Yao, 2011).
- An analytical model was derived to find the effective temperature difference as a function of the load current and propose the “effective Seebeck coefficient” (Kim, 2013).

- A study modelled the system with a three-dimensional numerical model including nonlinear fluid-thermal-electric multi-physics coupled effects (M. Chen, Rosendahl, & Condra, 2011).
- Theoretically analysing two parallel TEGs based on thermodynamics theory, semiconductor thermoelectric theory and law of conservation of energy and tested the model experimentally (Liang, Zhou, & Huang, 2011).
- Thermal-electric coupled multi-physics analysis technique was used to find the thermal and electrical performance numerically with considering the effect of thermal and electric contact (Wenguang Li et al., 2015).

2. The impact of TEG structure and design on the efficiency.

- Developing a maximum energy conversion model mathematically. A new compact TEG is designed by changing the internal layout and arrangement of the p-n materials to be alternative inline, and that has improved the performance compared to the traditional TEG (Ming et al., 2017).
- Two numerical models were developed work by solving the conservation equations of Seebeck and Ohmic potentials using the implementation of finite volume method in computational dynamic fluid software (FLUENT) without including the contact resistance effect in the simulation to understand the heat and electricity transfer characteristics, in addition to the investigation of the shape of the semiconductor (Niu et al., 2015).

It was found that the power increases when the cross-sectional area of the semiconductor is varying because it will affect the temperature gradient.

- The influence of the pin material configuration on the performance was studied (Ali & Yilbas, 2016) and the relation between the leg sizing and spacing and the performance was derived (Erturun, Erermis, & Mossi, 2015).
- Studying analytically the performance of the TEG including the influence of Thomson effect and the effect of the leg geometry by introducing the dimensionless geometry/shape parameter for various operating temperature ratio (Lamba & Kaushik, 2017).

3. The analysis method used to model the TEG (FEM, FVM)

- Using the finite time thermodynamics method to model the internal heat generation inside the TEG, taking into consideration the effect of Joule, Thomson, and Peltier effects, and the external heat transfer which was found that the last one has no effect on the characterization of the TEG performance (L. Chen, Meng, & Sun, 2012).

Also, optimizing the maximum power and efficiency for different linear and nonlinear heat transfer laws such as the Newtonian heat transfer, radiative heat transfer law and generalized convective heat transfer in addition to others using Euler-Lagrange functions. It was found that the Newtonian heat transfer had the maximum power and efficiency compared to the other heat transfer laws (Figure 7).

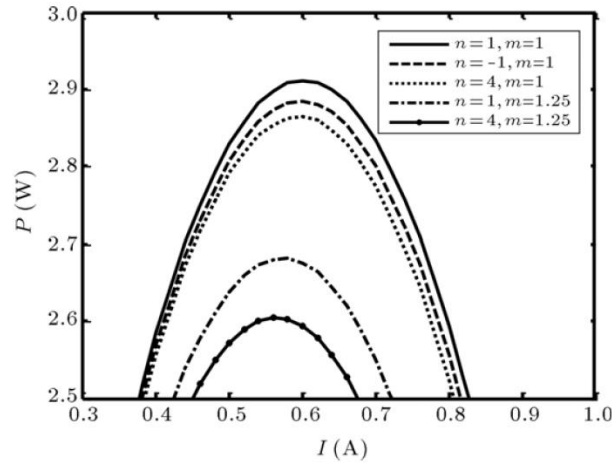


Figure 7: IV curve for different heat transfer laws
(L. Chen, Meng, & Sun, 2012).

- Using the finite element method (as shown in Figure 8) in addition to the integral averaged Seebeck coefficient to study the effects of the temperature dependence of thermoelectric materials and Thomson effect (Shen, Wu, Xiao, & Yin, 2016).

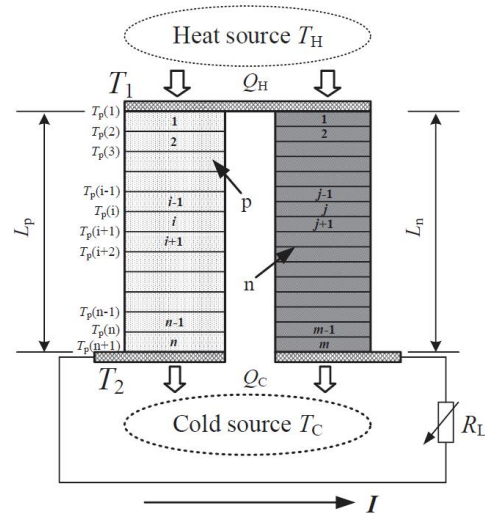


Figure 8: Models schematic view
(Shen, Wu, Xiao, & Yin, 2016).

The model takes the segments and initial temperature distributions of p-type and n-type legs, calculate the physical properties of each segment base on the temperature distributions that has been improved and then the electric current (I) is found. However, they assumed that the p-type and n-type legs have the same segments. It was found that the relative errors of the output power and efficiency increases if the side surface of the legs is increased, also the degree of nonlinear temperature distributions along the legs increases when the side surface heat transfer coefficient increases, whereas both the junction temperatures and the temperature difference between two junctions decrease.

- Using the temperature-dependent coefficients in three subdomains: the high and low sides heat exchanger and the thermoelectric material (Figure 9) which was based on the finite volume method (Yan, Dawson, Pugh, & El-Deib, 2014). The results of the simulated and measured data were almost the same; however, the contact resistance was not considered.

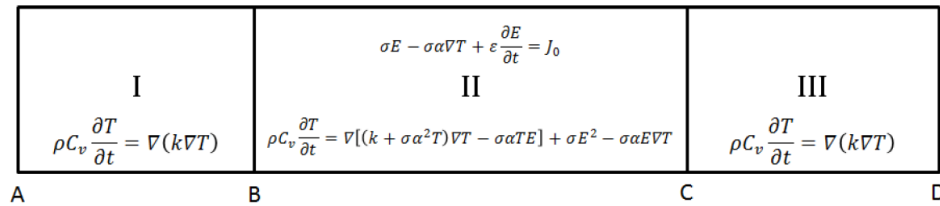


Figure 9: Model's subdomain
(Yan, Dawson, Pugh, & El-Deib, 2014).

Table 1 and Table 2 summarise the surveyed TEG models with different assumptions and methods.

The finite element method is applied to steady-state heat equations including the influence of the Thomson effect, Joule resistance, Seebeck effect, and heat conduction. Also, the effect of current on the temperature distribution and the effect of applying different heat sources will be studied in order to understand the behaviour of temperature on the TEG output. Finally, this model will studies the effect of thermocouple size on the performance of the TEG.

Table 1: A summary of TEG models in the literature for parameters used and different analysis methods

Authors	Parameters					Analysis Method	Comments
	α	τ	Thermal contact Resistance	Heat Losses	Heat Transfer		
(Md Yusop, Mohamed, & Ayob, 2013)	√						Transient heat sources
(M. Chen et al., 2011)	√	√				FVM	
(Cheng, 2016)	√	√	√				Insulated
(W. Li et al., 2017)	√		√			FEM	
(Liang et al., 2011)			√				With heat sink
(Singh, 2003)		√				FEM	
(Siouane et al., 2017)			√				Electrical Modeling
(Yan et al., 2014)	√				√	FVM	Transient + steady state analysis
(Bjørk et al., 2014)				All			Radiation, Conduction, Convection
(Marchenko, 2018)	√	√			1D		Perturbation method
(Zou, Kanimba, Diller, Tian, & He, 2018)							
(Wu et al., 2016a)	√	√	√		1D		Steady state analysis
(Ali & Yilbas, 2016)		√					
(Lazard, 2009)	√	√				FEM	Assumption: Thermophysical properties are constant + Steady state analysis

Table 2: A summary of TEG mathematical models in the literature

Paper	Assumption and methods	Equations	Conclusion
Effect of Convection Heat Transfer on Performance of Waste Heat Thermoelectric Generator. (Rabari, Mahmud, Dutta & Biglarbegian, 2015)	• Steady state • Neglect contact resistance • With convection heat • 1D Heat Transfer	$\frac{d^2T}{dx^2} + \frac{I^2R}{kA^2} - \frac{hp}{kA}(T - T_a) - \frac{\tau I}{kA} \frac{dT}{dx} = 0$ $T_{(x)} = C_1 e^{D_1 x} + C_2 e^{D_2 x} + \frac{I^2 R}{Ah p} + T_a$ $I = \frac{(\alpha_p - \alpha_n)(T_1 - T_2)}{R + R_L}$ $\eta = \frac{I \Delta V}{q_1} = \frac{I((\alpha_p - \alpha_n)(T_1 - T_2) - IR)}{q_1}$	the convection heat transfer coefficient affects the performance of TEG.
A model to analyze the device level performance of the thermoelectric generator (Wu, Zuo, Chen, & Klein, 2016b)	• Steady state • With contact resistance • Constant Thomson coefficient • 1D Heat Transfer	$\frac{d^2T}{dx^2} + \frac{I^2 \rho}{\lambda A^2} - \frac{\tau I}{\lambda A} \frac{dT}{dx} = 0$ $T_{(x)} = \frac{T_h - T_c - \frac{I \rho L}{\sigma \tau}}{e^{\tau I L / \lambda A} - 1} (e^{\tau I x / \lambda A} - 1) - \frac{I \rho x}{\tau A} + T_c$ $I = \frac{T_h(\alpha_{h,p} - \alpha_{h,n}) - T_c(\alpha_{c,p} - \alpha_{c,n}) - \tau \Delta T}{R_G - R_L}$ $\eta = \frac{T_h I \alpha_{h,pn} - \Delta T I \tau_{pn} - I^2 R_{pn} - I^2 R_c}{T_h I \alpha_{h,pn} + (K_p^* - K_n^*) \Delta T + I^2 (R_p^* + R_n^* - R_{pn}) - \Delta T I \tau_{pn} - I^2 R_{ch}}$	• Thomson effect has a big impact on the higher temperature gradient. • The thermal resistance of the contact layer reduces the temperature drop and the electrical resistance lower the power output. • Minimising the gap between the pallets makes the performance better.

Theoretical modelling of a thermoelectric generator with particular emphasis on the effect of side surface heat transfer (Shen et al., 2016)	• Steady state • With side surface heat transfer • Thomson coefficient • 1D Heat Transfer • Finite Element Method	$\frac{d^2T}{dx^2} + \frac{I^2}{\sigma k S^2} - \frac{hp}{kS}(T - T_\infty) = 0$ $T_{(x)} = \frac{-Bx^2}{2} + \frac{\bar{T}_2 - \bar{T}_1 + 0.5BL^2}{L}x + \bar{T}_1$ $I = \frac{(\alpha_p - \alpha_n)(T_1 - T_2)}{R + R_L}$ $P = \left(\frac{(\alpha_p - \alpha_n)(T_1 - T_2)}{R + R_L} \right)^2 R_L$	• As the SSHT coefficient increases, the behaviour of temperature distributions changed to nonlinear. • The effect of the coefficient on the output power can be insignificant for small legs.
Thermodynamic analysis of thermoelectric generator including the influence of Thomson effect and leg geometry configuration (Lamba & Kaushik, 2017)	• Steady state • Purpose new leg geometry configuration (trapezoidal) • Thomson coefficient equally distributed. • 1D Heat Transfer • Neglect radiative and convective heat losses. • Neglect metal and contact electrical and thermal resistance	$\frac{d^2T}{dx^2} + \frac{1}{A_{(x)}} \left[\frac{2A_0}{L} \left(\frac{R_A - 1}{R_A + 1} \right) - \frac{\tau I}{k} \right] \frac{dT}{dx} + \frac{\rho I^2}{\lambda A_{(x)}^2} = 0$ $T_{(x)} = C_1 + \frac{C_2(L(1-a)+2ax)^{\left(1-\frac{Lb}{2a}\right)}}{(2a-Lb)} + \frac{cL^2 \log(L(1-a)+2ax)}{(4a^2-2Lab)}$ $- \frac{cL^2(L(1-a)+2ax)}{(2a-Lb)(Lb(1-a)(Lb-2a)-(4a^2x+2Labx))}$ $I = \frac{(\alpha_h T_h - \alpha_c T_c) - \tau \Delta T}{R + R_L}$ $P = (\alpha_h T_h - \alpha_c T_c)I - I^2 R - \tau I \Delta T$	• The Thomson coefficient has an impact on power output. • The optimum values of the shape parameter and external load ratio for maximum power output were found.

Chapter Three

Methods and Procedure

3.1 Research Study's Methodology

3.1.1 Modelling Thermoelectric Generator using Finite Element Method

3.2 Experimental Setup

3.2.1 Previous Work's Setup

3.2.2 Experimental Setup

Chapter Three

Methods and Procedure

3.1 Research Study's Methodology

Since it is difficult to model the TEG with considering all the parameters without knowing the effect of varying each parameter individually, the impact of Thomson coefficient is considered in this work.

In addition, all the detailed properties and dimensions in the model will make it more complicated, as well as, ignoring and simplifying the equations used in the model will lead to an inaccurate result.

To improve the modelling of the TEG, the research methodology was divided into two main stages:

- Developing a theoretical model for TEG.
- Investigating the optimum size of the element for a specific temperature difference.

Furthermore, to understand the behaviour of the TEG voltage and current with temperature and size. This work has included an experimental part which will be discussed in the following section.

The model was built in the MATLAB, where for each thermocouple, the length of the element was divided into intervals, and the finite element method was applied by finding the temperature distributions for each part and then calculate the average properties of that part. Following this, the average properties are calculated, and the output current, voltage power, and efficiency is found. The following flowchart shows the steps of the model:

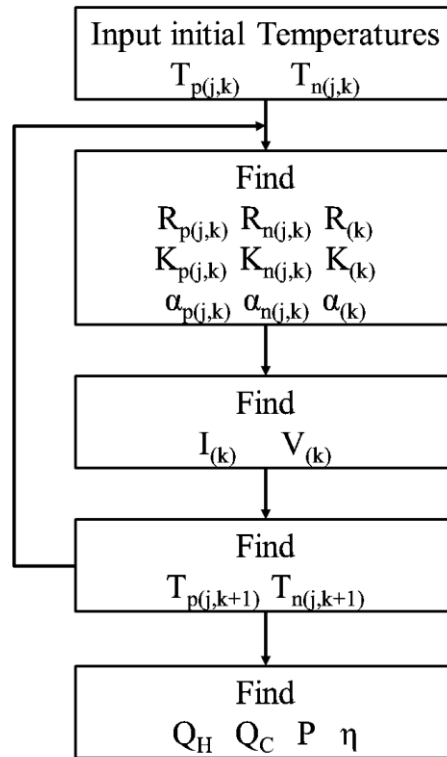


Figure 10: Matlab flowchart

The main objectives of this model are:

1. To compare the average value of conductivity, resistivity, Seebeck and Thomson coefficients using the average of the cold and hot sides and using the finite element method.

2. To find the effect of hot temperature variation on the current generated, voltage and power output.
3. To find the optimum value for the load resistor that gives the maximum power output.
4. To find the effect of the length and area on the output power and efficiency.
5. To study the effect of temperature variation along the length of the material.

3.1.1 Modelling Thermoelectric Generator using Finite Element Method

The main limitation found in all the works done to model the TEG is that most of the models studied the effect of one or two parameters. However, all the parameters are interdependent, which will affect the performance at the same time.

This section will underline some of the main parameters and assumptions that affect the performance and output power such as the approximated Thomson coefficient and Seebeck coefficient as a function of temperature.

The second part of the model is to find the matched load resistor for a specific temperature difference and to find the relation between the length and the area of the thermocouples on the TEG performance.

The Finite Element Method (FEM) is used in this work since there is a temperature gradient along each thermocouple which will affect the properties of the material.

Figure 11 shows the scheme of applying the FEM to the model.

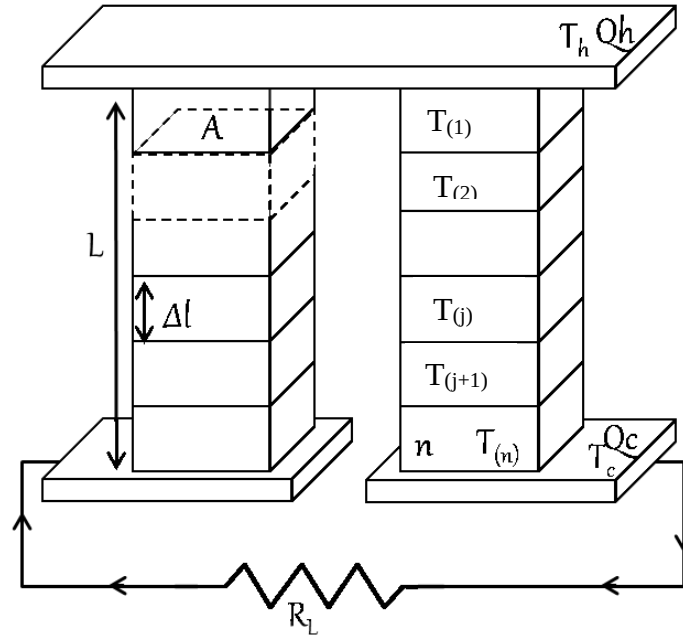


Figure 11: Model's scheme

In this model, the module is assumed to be perfectly insulated, where there is no heat on the side surfaces, and the hot and cold sides temperature equals the ambient temperature.

The temperature distribution function of the material over its length is assumed to be decreasing linearly from the hot to the cold temperatures, and to validate this assumption, the temperature function including the effect the current generated Eq.32 and Eq. 33 (Wu, Zuo, Chen, & Klein, 2016b) was used to find the temperature distribution behaviour:

$$T_{n(j)} = \frac{T_{n(1)} - T_{n(n)} - \frac{I}{A_n \bar{\sigma}_n \bar{\tau}} L_n}{e^{\bar{\tau} I L_n / \bar{k}_n A_n} - 1} (e^{\bar{\tau} I(j) / \bar{k}_n A_n} - 1) - \frac{I}{A_n \bar{\sigma}_n \bar{\tau}} (j) + T_{n(n)} \quad (32)$$

$$T_{p(j)} = \frac{T_{p(1)} - T_{p(n)} + \frac{I}{A_p \bar{\sigma}_p \bar{\tau}} L_p}{e^{\bar{\tau} I L_p / \bar{k}_p A_p} - 1} (e^{\bar{\tau} I(j) / \bar{k}_p A_p} - 1) - \frac{I}{A_p \bar{\sigma}_p \bar{\tau}} (j) + T_{p(n)} \quad (33)$$

The following code was used to find the effect of the temperature distribution as the current flows in the p-n material using Eq. 32 and Eq. 33

```

for t=1:tend
    for j = 2:loops
        Tni = Txn(j-1,:);
        Tpi = Txp(j-1,:);

        conductivity_T_nTypeUnit = ...
        conductivity_T_pTypeUnit = ...
        Conductance = ...

        resistivity_T_nTypeUnit = ...
        resistivity_T_pTypeUnit = ...
        Resistance = ...

        seebeck_coef_T_nTypeUnit = ...
        seebeck_coef_T_pTypeUnit = ...
        Seebeck_n_avg = ...
        Seebeck_p_avg = ...

        thomson_coef_T_nTypeUnit = ...
        thomson_coef_T_pTypeUnit = ...
        Thomson_voltage = ...

        ...
        ...
        ...
        ...

        Current=((Th*Seebeck_H)-(Tc*Seebeck_C)-(Thomson_voltage))/
        (Rtotal + RL);
        Voltage=((Th*Seebeck_H)-(Tc*Seebeck_C)-(Thomson_voltage))-
        (Current*(Rtotal));

```

```

    Tn_n = (Th-Tc-Current*mean(resistivity_T_nTypeUnit)*L/
    (Thomson_n_avg*A))*(exp(Thomson_n_avg*Current*dl*(x)./(mean(con
ductivity_T_nTypeUnit)*A))-1)./(exp(Thomson_n_avg*Current*L/
(mean(conductivity_T_nTypeUnit)*A))-1)+ Current
*mean(resistivity_T_nTypeUnit)*dl*(x)./(Thomson_n_avg*A) + Tc;

    Tn_p = (Th-Tc+Current*mean(resistivity_T_pTypeUnit)*L/
    (Thomson_p_avg*A))*(exp(Thomson_p_avg*Current*dl*(x)./(mean(con
ductivity_T_pTypeUnit)*A))-1)./(exp(Thomson_p_avg*Current*L/
(mean(conductivity_T_pTypeUnit)*A))-1)+Current
*mean(resistivity_T_pTypeUnit)*dl*(x)./(Thomson_p_avg*A) + Tc;

    T_diff_n = -diff(Tn_n);
    T_diff_p = -diff(Tn_p);

    Txn(j,:) = (Tn_n(1:end-1)+Tn_n(2:end))/2;
    Txp(j,:) = (Tn_p(1:end-1)+Tn_p(2:end))/2;

End

```

Eq. 15 to Eq. 26 listed in the previous chapter have been modified using FEM. The equations for calculating the conductivity, resistivity and Seebeck coefficient as a function of temperature were derived from fitting the graphs provided in the data sheets.

Since the average conductivity, resistivity, and Thomson are volume properties so they are calculated for each $\Delta\ell$, then the conductance (K), resistance(R), and average Thomson coefficient (τ) are calculated as follow:

$$\bar{k}_{n(j+1)} = \frac{k_{n(j)} + k_{n(j+2)}}{2} \quad (34)$$

$$\bar{k}_{p(j+1)} = \frac{k_{p(j)} + k_{p(j+2)}}{2} \quad (35)$$

$$K_{(t)} = \left(\frac{\bar{k}_p A_p}{L_p} + \frac{\bar{k}_n A_n}{L_n} \right) \quad (36)$$

$$\bar{\rho}_{n(j+1)} = \frac{\rho_{n(j)} + \rho_{n(j+2)}}{2} \quad (37)$$

$$\bar{\rho}_{p(j+1)} = \frac{\rho_{p(j)} + \rho_{p(j+2)}}{2} \quad (38)$$

$$R_{(t)} = \left(\frac{\bar{\rho}_p L_p}{A_p} + \frac{\bar{\rho}_n L_n}{A_n} \right) \quad (39)$$

$$\bar{\tau}_{n(j)} = \frac{\tau_{p(j)} + \tau_{p(j+2)}}{2} \quad (40)$$

$$\bar{\tau}_{p(j)} = \frac{\tau_{p(j)} + \tau_{p(j+2)}}{2} \quad (41)$$

$$\tau_{(t)} = (\bar{\tau}_p - \bar{\tau}_n) \quad (42)$$

However, the Seebeck effect occurs only between the junction of two materials which mean that it affects only the surfaces of the material and the generated current as follow:

$$\bar{\alpha}_{n(j)} = \frac{\alpha_{n(1)} + \alpha_{n(n)}}{2} \quad (43)$$

$$\bar{\alpha}_{p(j)} = \frac{\alpha_{p(1)} + \alpha_{p(n)}}{2} \quad (44)$$

$$\alpha_{(t)} = (\bar{\alpha}_p - \bar{\alpha}_n) \quad (45)$$

$$I_{(t)} = \frac{(\alpha_{(1)} - \alpha_{(n)})(T_{(1)} - T_{(n)}) - \tau_{(t)}(T_{(1)} - T_{(n)})}{R_{(t)} + R_L} \quad (46)$$

$$V_{(t)} = (\alpha_{(1)} - \alpha_{(n)})(T_{(1)} - T_{(n)}) - \tau_{(t)}(T_{(1)} - T_{(n)}) - I_{(t)}^2 R_{(t)} \quad (47)$$

To compare between the average method and the finite element method the following code was used:

```

...
...
...

%----Finite Element Method
for i=1:n

conductivity_T_nTypeUnit = 1.098018*(10^-13)*(Ti^5) - 3.359249*(10^-
10)*(Ti^4) + 4.194853*(10^-7)*(Ti^3) - 2.596344*(10^-4)*(Ti^2) +
7.41657*(10^-2)*Ti - 4.678652;

```

```

conductivity_T_pTypeUnit = -6.020716 *(10^-14)*(Ti^5) +
1.685395*(10^-10)*(Ti^4) - 1.851765*(10^-7)*(Ti^3)+1.026943*(10^-
4)*(Ti^2) - 3.144958 *(10^-2)*Ti + 6.002589;
conductivity_n_avg = (conductivity_n_avg +
conductivity_T_nTypeUnit/n);
conductivity_p_avg = (conductivity_p_avg +
conductivity_T_pTypeUnit/n);

resistivity_T_nTypeUnit = 1/(7.795516 *(10^-9)*(Ti^5) - 1.970185
*(10^-5)*(Ti^4) + 1.774757 *(10^-2)*(Ti^3) - 5.836094*(Ti^2) -
3.849606*(10^2)*Ti + 4.877652*(10^5));
resistivity_T_pTypeUnit = 1/(-1.021713*(10^-8)*(Ti^5) + 2.610834
*(10^-5)*(Ti^4) - 2.497777 *(10^-2)*(Ti^3) + 1.133756*(10)*(Ti^2) -
2.728566 *(10^3)*Ti + 3.977257*(10^5));
resistivity_n_avg = (resistivity_n_avg + resistivity_T_nTypeUnit/n);
resistivity_p_avg = (resistivity_p_avg + resistivity_T_pTypeUnit/n);

seebeck_coef_T_nTypeUnit = 2.800502 *(10^-18)*(Ti^5) -
7.003341*(10^-15)*(Ti^4) + 8.500587*(10^-12)*(Ti^3) - 5.754601*(10^-
9)*(Ti^2) + 1.629824*(10^-6)*Ti + 2.786129*(10^-4);
seebeck_coef_T_pTypeUnit = 3.821203 *(10^-17)*(Ti^5) - 9.49083*(10^-
14)*(Ti^4) + 8.648769 *(10^-11)*(Ti^3) - 3.556152*(10^-8)*(Ti^2) +
6.80366*(10^-6)*Ti - 4.40752*(10^-4);
Seebeck_n_avg = (Seebeck_n_avg + seebeck_coef_T_nTypeUnit/n);
Seebeck_p_avg = (Seebeck_p_avg + seebeck_coef_T_pTypeUnit/n);

thomson_coef_T_nTypeUnit = Ti*(5*2.800502 *(10^-18)*(Ti^4) -
4*7.003341*(10^-15)*(Ti^3) + 3*8.500587*(10^-12)*(Ti^2) -
2*5.754601*(10^-9)*(Ti) + 1.629824*(10^-6));
thomson_coef_T_pTypeUnit = Ti*(5*3.821203 *(10^-17)*(Ti^4) -
4*9.49083*(10^-14)*(Ti^3) + 3*8.648769 *(10^-11)*(Ti^2) -
2*3.556152*(10^-8)*(Ti)+ 6.80366*(10^-6));
Thomson_n_avg = (Thomson_n_avg + thomson_coef_T_nTypeUnit/n);
Thomson_p_avg = (Thomson_p_avg + thomson_coef_T_pTypeUnit/n);

if i == n
    conductivity_n_C = conductivity_T_nTypeUnit;
    conductivity_p_C = conductivity_T_pTypeUnit;
    resistivity_n_C = resistivity_T_nTypeUnit;
    resistivity_p_C = resistivity_T_pTypeUnit;
    seebeck_coef_n_C = seebeck_coef_T_nTypeUnit;
    seebeck_coef_p_C = seebeck_coef_T_pTypeUnit;
    thomson_coef_n_C = thomson_coef_T_nTypeUnit;
    thomson_coef_p_C = thomson_coef_T_pTypeUnit;
end
if i == 1
    conductivity_n_H = conductivity_T_nTypeUnit;
    conductivity_p_H = conductivity_T_pTypeUnit;
    resistivity_n_H = resistivity_T_nTypeUnit;
    resistivity_p_H = resistivity_T_pTypeUnit;
    seebeck_coef_n_H = seebeck_coef_T_nTypeUnit;

```

```

seebeck_coef_p_H = seebeck_coef_T_pTypeUnit;
thomson_coef_n_H = thomson_coef_T_nTypeUnit;
thomson_coef_p_H = thomson_coef_T_pTypeUnit;
end
Ti = Ti - Tn;
end
%----Average Method
sum_cond_n_TcTh = (conductivity_n_H + conductivity_n_C)/2;
sum_cond_p_TcTh = (conductivity_p_H + conductivity_p_C)/2;
sum_resis_n_TcTh = (resistivity_n_H + resistivity_n_C)/2;
sum_resis_p_TcTh = (resistivity_p_H + resistivity_p_C)/2;
sum_Seebeck_n_TcTh = (seebeck_coef_n_H+ seebeck_coef_n_C)/2;
sum_Seebeck_p_TcTh = (seebeck_coef_p_H+ seebeck_coef_p_C)/2;
sum_Thomson_n_TcTh = (thomson_coef_n_H + thomson_coef_n_C)/2;
sum_Thomson_p_TcTh = (thomson_coef_p_H + thomson_coef_p_C)/2;

```

Also, the passing current will affect the amount of heat transfer by changing the value of total conductance and resistance of the module; the new values are calculated using the equations below:

$$K_{n(t)}^* = \frac{\bar{\tau}_n I}{1 - e^{-\bar{\tau}_n I L_n / \bar{k}_n A_n}} \quad (48)$$

$$K_{p(t)}^* = \frac{\bar{\tau}_p I}{e^{\bar{\tau}_p I L_p / \bar{k}_p A_p} - 1} \quad (49)$$

$$R_{n(t)}^* = R_{n(t)} \left(\frac{\bar{k}_n A_n}{\bar{\tau}_n I L_n} - \frac{1}{1 - e^{-\bar{\tau}_n I L_n / \bar{k}_n A_n}} \right) \quad (50)$$

$$R_{p(t)}^* = R_{p(t)} \left(\frac{\bar{k}_p A_p}{\bar{\tau}_p I L_p} - \frac{1}{e^{\bar{\tau}_p I L_p / \bar{k}_p A_p} - 1} \right) \quad (51)$$

The generated heat on both sides will be calculated as follow:

$$Q_{H(t)} = \alpha_{(1)} T_{(1)} I_{(t)} + \left(K_{p(t)}^* + K_{n(t)}^* \right) (T_{(1)} - T_{(n)}) + I_{(t)}^2 (R_{p(t)}^* + R_{n(t)}^* - R_{(t)}) - \tau_{(t)} (T_{(1)} - T_{(n)}) I_{(t)} \quad (52)$$

$$Q_{C(t)} = \alpha_{(n)} T_{(n)} I_{(t)} + \left(K_{p(t)}^* - K_{n(t)}^* \right) (T_{(1)} - T_{(n)}) + I_{(t)}^2 R_{(t)}^* \quad (53)$$

However, there is no significant impact on the output due to the above equations at the steady state.

```

...
...
...

Current(t) = ((Th*Seebeck_H) - (Tc*Seebeck_C) - (Thomson_voltage)) /
(Rtotal + RL);

Conductance_star_n(t) = Thomson_n_avg*Current(t) / ((1-exp(-
1*Thomson_n_avg*Current(t)*L / (A*conductivity_n_avg)))));

Conductance_star_p(t) = Thomson_p_avg*Current(t) /
((exp(Thomson_p_avg*Current(t)*L / (A*conductivity_p_avg)) - 1));

Resistance_star_n(t) = Resistance_n(t) *
((A*conductivity_n_avg / (Thomson_n_avg*Current(t)*L)) - (1 / (1-
exp(-1*Thomson_n_avg*Current(t)*L / (conductivity_n_avg*A)))));

Resistance_star_p(t) = Resistance_p(t) * ((A*conductivity_p_avg
/ (Thomson_p_avg*Current(t)*L)) - (1 / (exp(Thomson_p_avg*Current(t)
*L / (conductivity_p_avg*A)) - 1)));

Resistance_star(t) = Resistance_star_n(t) + Resistance_star_p(t);
RstarTotal = Resistance_star(t);

Q_Hot(t) = Seebeck_H*Current(t)*Th + (Conductance_star_n(t) +
Conductance_star_p(t)) * (Th - Tc) + ((Current(t)^2) *
(Resistance_Thomson_p(t) + Resistance_star_n(t) - Rtotal) -
(Thomson_n_avg*Current(t)));

Q_Cold(t) = Seebeck_C*Current(t)*Tc + (Conductance_star_n(t) +
Conductance_star_p(t)) * (Th - Tc) + ((Current(t)^2)
* (Resistance_Thomson_p(t) - Resistance_star_n(t)));

```

The output power was calculated using two equations:

$$P_{(t)} = Q_{H(t)} - Q_{C(t)} \quad (54)$$

$$P_{(t)} = I^2 R_L \quad (55)$$

and the efficiency is found by:

$$\eta = \frac{P_{(t)}}{Q_{H(t)}} \quad (56)$$

```
%      1
Power(t)= Q_Hot(t)- Q_Cold(t);
%      2
Power1(t)= Seebeck_H*Current(t)*th(t) - Seebeck_C*Current(t)*tc
- Ttotal*Current(t)*dt - (Current(t)^2)*Rtotal;
%      3
Power2(t)= (Current(t)^2).*Rls(t);
Effeciency(t)= Power(t)./Q_Hot(t);
```

To find the current versus power curve the following code was used:

```
Rls = 0:0.001:1;

for i = 1:steps;

    ...
    ...
    ...

    RL = Rls(i);

    Stotal = Seebeck_p_avg - Seebeck_n_avg;
    Ttotal = Thomson_p_avg - Thomson_n_avg;

    Current(i) = ((Th*Seebeck_H) - (Tc*Seebeck_C) -
    (Thomson_voltage)) / (Rtotal + RL);
    Voltage(i) = (Th*Seebeck_H) - (Tc*Seebeck_C) - (Thomson_voltage) -
    (Current(i) * (Rtotal));

    Q_Cold(i) = Seebeck_C*Current(i)*Tc +
    (Ktotal)*dt + 0.5*((Current(i)^2)*Rtotal);
    Q_Hot(i) = Seebeck_H*Current(i)*Th + (Ktotal)*dt -
    0.5*((Current(i)^2)*Rtotal);

    Power(i) = Q_Hot(i) - Q_Cold(i);
    Power2(i) = (Current(i)^2) .* RL;

    Efficiency(i) = 100*Power(i) ./ Q_Hot(i);
end

hold on
subplot(2,2,1)
plot(Current, Power)
hold off

hold on
subplot(2,2,2)
plot(Current, Voltage)
hold off

hold on
subplot(2,2,3)
plot(Rls, Efficiency)
hold off

hold on
subplot(2,2,4)
plot(Rls, Power)
hold off
```

The variation of temperature with different parameters was found using:

```

Th = 320:500;

for Tc = [280,300,320]
for i = 1:steps;
    dt = Th(i) - Tc;
    Tn_n = linspace(Th(i),Tc,n+1);
    Tn_p = Tn_n;

                                ...
                                ...
                                ...

    RL = Rtotal ;
    Rls(i) = RL;

    Current(i) = ((Th(i)*Seebeck_H) - (Tc*Seebeck_C) -
(Thomson_voltage)) / (Rtotal + RL);

    Voltage(i) = (Th(i)*Seebeck_H) - (Tc*Seebeck_C) -
(Thomson_voltage) - (Current(i)*(Rtotal));

    Q_Cold(i) = Seebeck_C*Current(i)*Tc +
(Ktotal)*dt + 0.5*((Current(i)^2)*Rtotal);
    Q_Hot(i) = Seebeck_H*Current(i)*Th(i) + (Ktotal)*dt -
0.5*((Current(i)^2)*Rtotal);

    Power(i) = Q_Hot(i) - Q_Cold(i);
    Power2(i) = (Current(i)^2).*Rls(i);

    Efficiency(i) = 100*Power(i)./Q_Hot(i);
end

hold on
subplot(2,2,1)
plot(Th,Voltage)
xlabel('T_H_o_t (K)')
ylabel('?' (V/K^2)')
hold off

hold on
subplot(2,2,2)
plot(Th,Current)
xlabel('T_H_o_t (K)')
ylabel('?' (V/K^2)')
hold off

```

```

hold on
subplot(2,2,3)
plot(Th,Power)
xlabel('T_H_o_t (K)')
ylabel('?' (V/K^2)')
hold off

hold on
subplot(2,2,3)
plot(Th,Power)
xlabel('T_H_o_t (K)')
ylabel('?' (V/K^2)')
hold off

end

```

The last part was to find the optimum length and area for specific efficiency and output:

```

for k=1:1:A_steps
    A=Area(k);
    for j=1:1:L_steps
        l=L(j);
        for t=1:1:steps
            Ti = th(t) - 300;
            dt = th(t) - tc;
            Tt(t)= th(t);
            Tn = dt / n;
            dl = l / n;

            ...
            ...
            ...

%----%
            for i=1:n

                ...
                ...
                ...

                Ti = Ti - Tn;

            end

%----%
            RL = Rtotal ;
            Rls(t,j,k)=RL;
            det(t,j,k)= dt;

```

```

        K_total(t,j,k) = Ktotal;
        R_total(t,j,k) = Rtotal;
        S_total(t,j,k) = Stotal;
        T_total(t,j,k) = Ttotal;

        Current(t,j,k)=((th(t)*Seebeck_H)-(tc*Seebeck_C)-(Thomson_voltage))/(Rtotal + Rls(t,j,k));

        Voltage(t,j,k)=((th(t)*Seebeck_H)-(tc*Seebeck_C)-(Thomson_voltage))-(Current(t,j,k)*(Rtotal));

        Q_Cold(t,j,k)=Seebeck_C*Current(t,j,k)*tc+(Ktotal)*dt+0.5*(Current(t,j,k)^2)*Rtotal)+ Ttotal*Current(t,j,k)*dt/2;

        Q_Hot(t,j,k)= Seebeck_H*Current(t,j,k)*th(t)+(Ktotal)*dt-0.5*(Current(t,j,k)^2)*Rtotal) - Ttotal*Current(t,j,k)*dt/2;;

        Power(t,j,k)= Q_Hot(t,j,k)- Q_Cold(t,j,k);
        Efficiency(t,j,k)= 100*Power(t,j,k)./Q_Hot(t,j,k);
        end
    end
end

subplot(2,2,1)
surf(Area,L,reshape(Current(1,:,:),[L_steps,A_steps]));
xlabel('Area (m^2)')
ylabel('Length (m)')
zlabel('Current (A)');
view(3)

subplot(2,2,2)
surf(Area,L,reshape(Voltage(1,:,:),[L_steps,A_steps]));
xlabel('Area (m^2)')
ylabel('Length (m)')
zlabel('Voltage (V)');
view(3)

subplot(2,2,3)
surf(Area,L,reshape(Power(1,:,:),[L_steps,A_steps]));
xlabel('Area (m^2)')
ylabel('Length (m)')
zlabel('Power (W)');
view(3)

subplot(2,2,4)
surf(Area,L,reshape(Efficiency(1,:,:),[L_steps,A_steps]));
xlabel('Area (m^2)')
ylabel('Length (m)')
zlabel('Efficiency (%)');
view(3)

```

3.2 Experimental Setup

The main aim of the experiments done in this work was to study the behaviour of the temperature distribution on the surface of the TEG with time and to investigate the effect of the coordinate of the heated area on the output.

3.2.1 Previous Work's Setup

The investigation of the behaviour of the TEG, experimentally, looking at the effect of the thermal contact on the thermal, electrical properties was conducted by applying two different clamping forces as shown in Figure 12 (Jouhara et al., 2018).

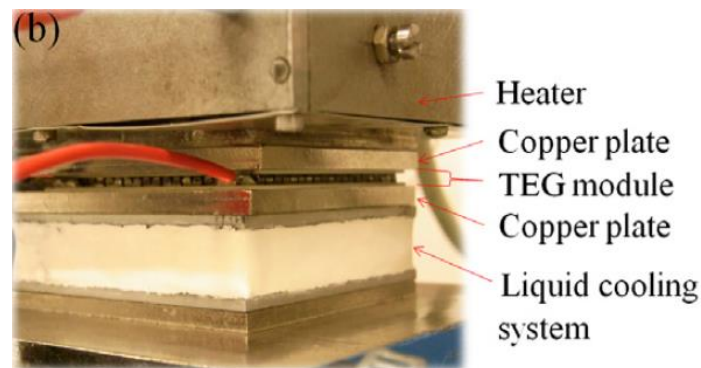


Figure 12: Experimental setup (Jouhara et al., 2018).

To ensure getting uniform thermal field, two copper plates on both sides of the TEG were used, and had used two thermocouples to measure the temperature difference.

In other work (Kwan, Wu, & Yao, 2018), a liquid cooling system and electrical heating system with temperature sensors and variable resistor is used (Figure 13).

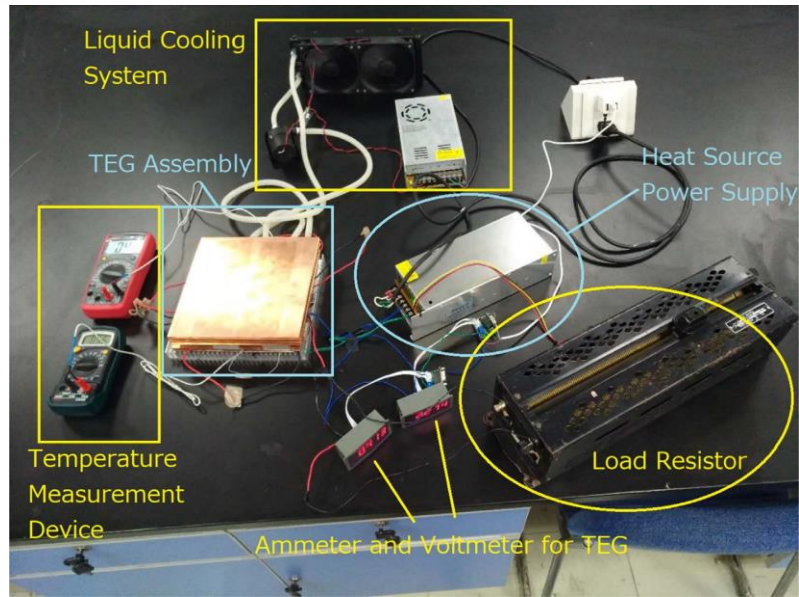


Figure 13: Experimental setup (Kwan, Wu, & Yao, 2018).

3.2.2 Experimental Setup

Using a combination of different experimental setups mentioned in the previous section, the setup of this experiment is illustrated in Figure 14. The TEG was installed in a foam box to ensure that there is no air flow on the sides, the two fans were used for the cooling systems, the wilder with the square area metals were used for the heating system. Pasco Capstone interface was used for the data acquisition, and two temperature sensors were used to measure the temperature on both surfaces (hot and cold). The voltage/current sensor to measure the output of the TEG and the resistor was used to fix the load resistance.

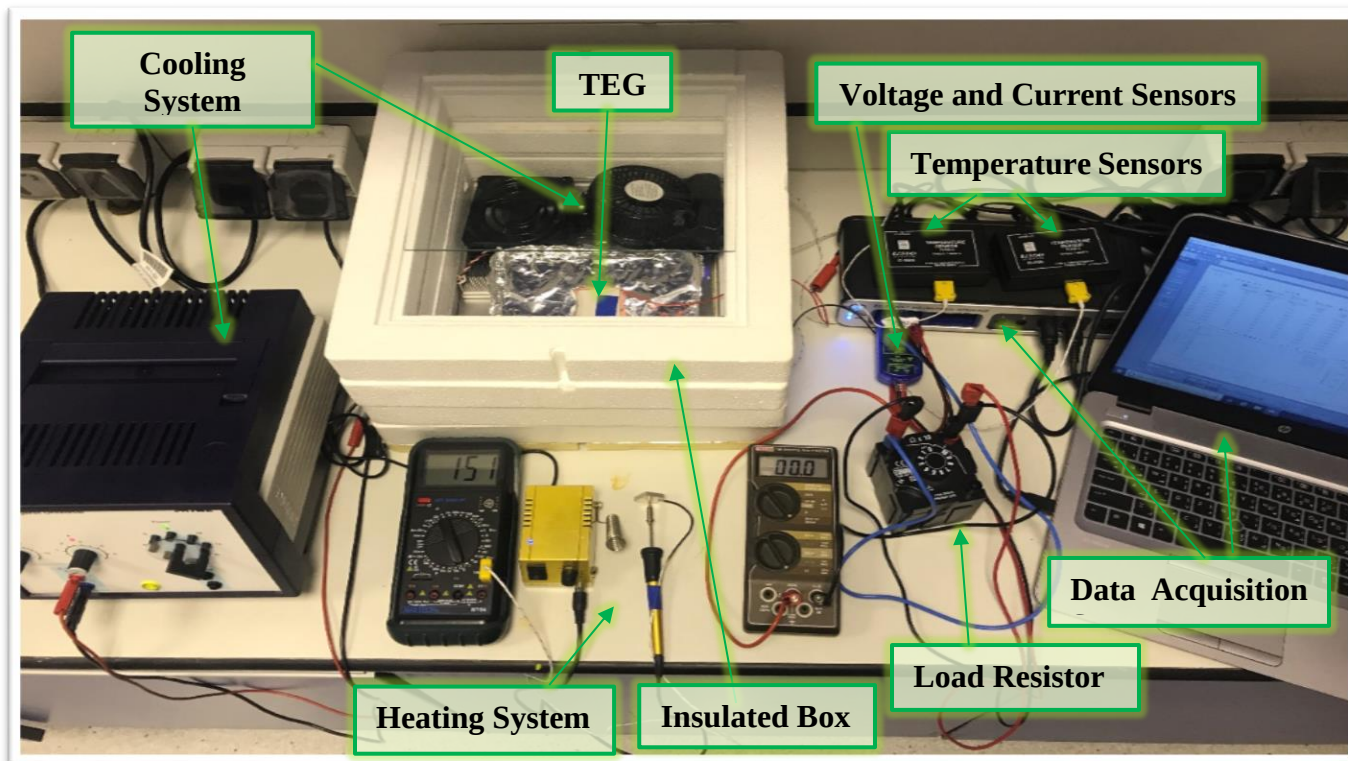


Figure 14: Experimental setup

The electrical circuit is shown in the following Figure 15:

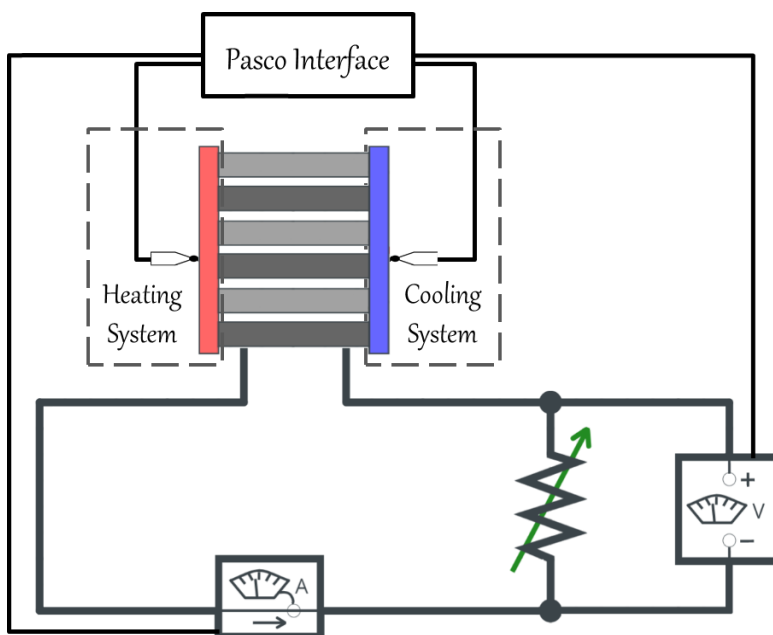


Figure 15: Experimental circuit diagram

Chapter Four

Results and Discussion

- 4.1 Experimental Result
- 4.2 Effect of using Finite Element method
- 4.3 Effect of current on the temperature distribution
- 4.4 Temperature with Current, Voltage, and Power
- 4.5 Load resistor versus Current, Voltage, and Power
- 4.6 Area and length versus different parameters

Chapter Four

Results and Discussion

As mentioned previously, it is important to understand the behaviour of the TEG to know the relevance of the different parameters in the modelling. The results of the experimental part and the output of the modelling will be discussed herein.

The experimental results were measured for TEC1-12706 module, and the modelled data were calculated using the properties of PbSnTe n-type and p-type materials.

4.1 Experimental Result

Metals of different sizes (0.2mm^2 , 0.4mm^2 , 1cm^2 , 2cm^2) were used to find the effect of the heated area on the output of the TEG.

The effect of changing the heating size on the current and temperature is shown in Figure 16.

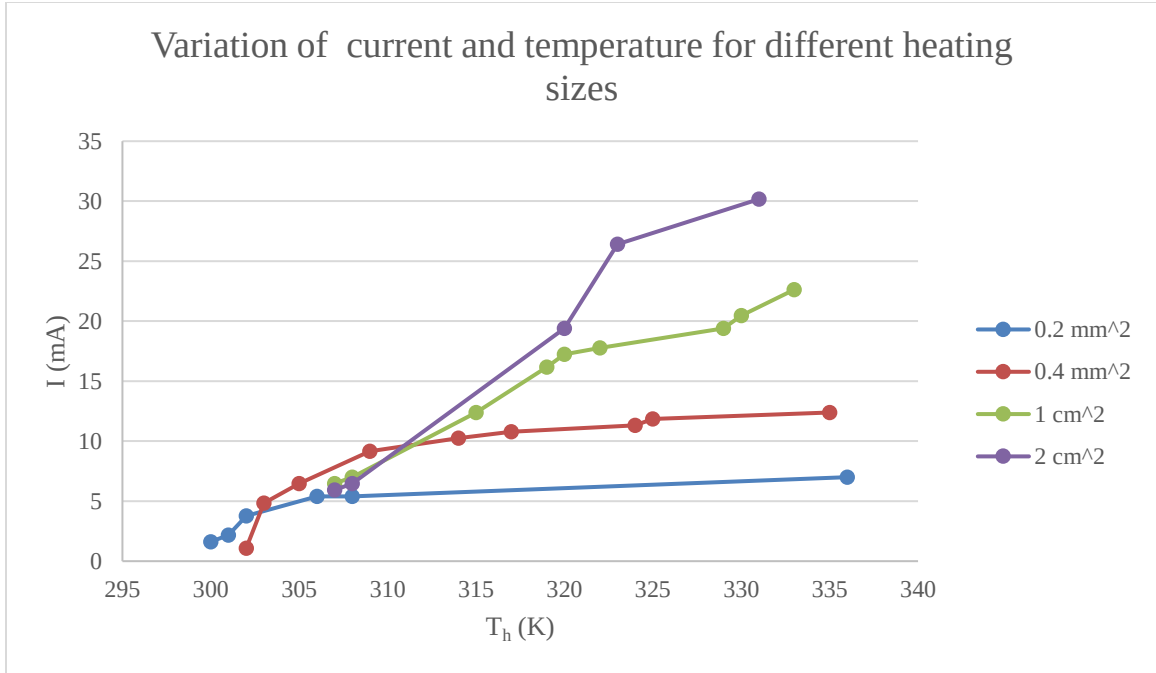


Figure 16: Variation of Current and Temperature

Table 3: Linear Fitting details

Size	0.2 mm ²	0.4 mm ²	1 cm ²	2 cm ²
Fitting equation	0.1216x - 33.326	0.2862x - 81.442	0.6016x - 177.21	1.0763x - 324.37
R ²	0.6362	0.7364	0.9528	0.9728

The graphs show the relation between the hot side temperature with the output current for different heating system sizes and Table 3 summarises the fitting data. It is clear that the generated current is higher as the heating surface is increased. Also the linearity of the fitting shows that the temperature is varying faster with the larger surfaces.

The relation between the current and temperature is linear (Eq. 46) and the R^2 for the different sizes show that the larger the surface, the better the fitting, i.e. the sensitivity of the modelling accuracy with the area.

Figure 17 shows the variation of hot and cold side's temperature with time when applying a heat source to the TEG. The upper plot was by covering the whole surface, and the lower one was by covering quarter the surface.

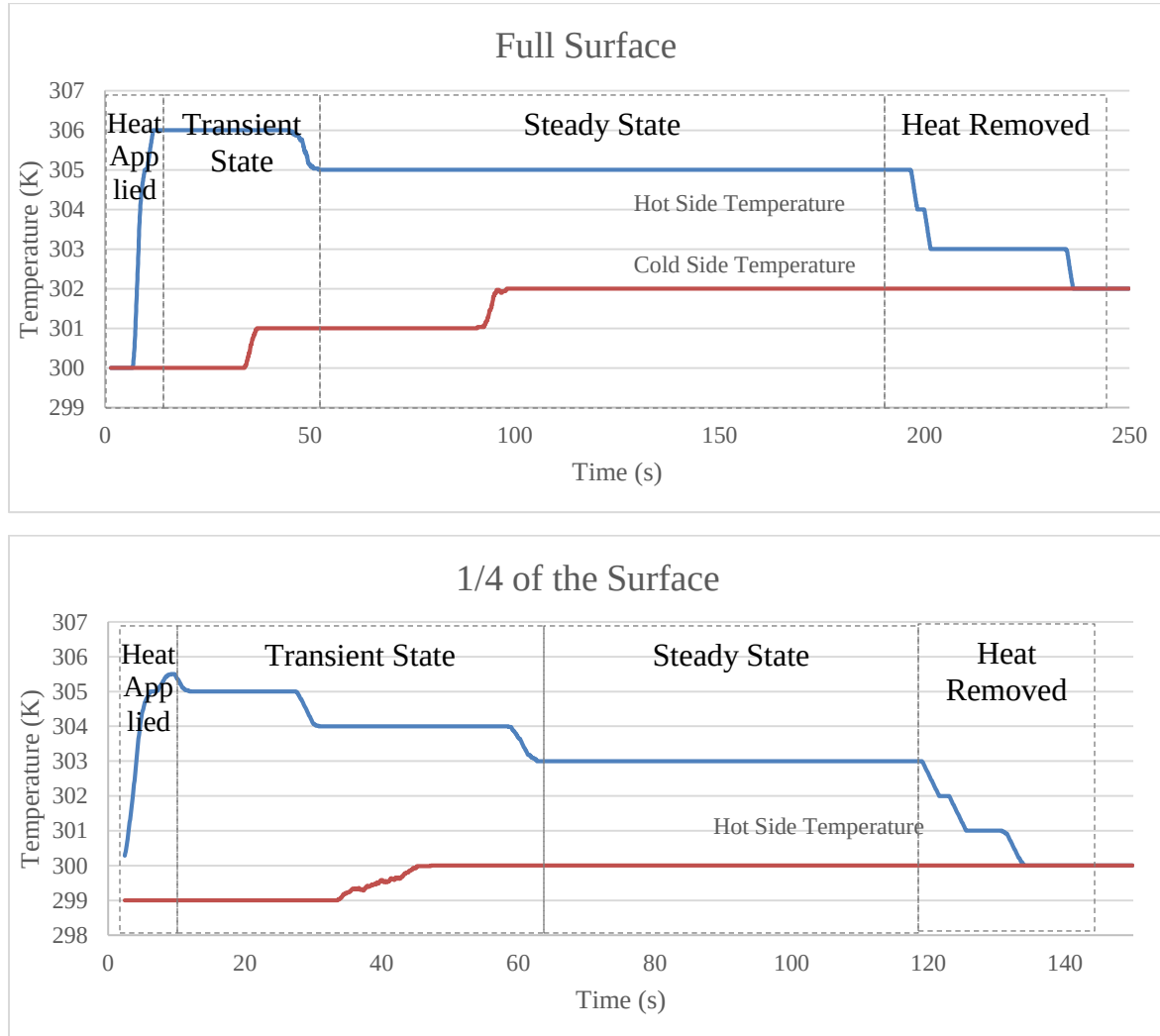


Figure 17: Variation of Temperature with Time

The hot side temperature reaches the maximum by covering the whole surface, and it takes more time to reach the steady-state temperature, however covering smaller surface allows the cold side to keep its temperature for a longer time with less variation. The graphs show the temperature behaviour in four periods: when the heat source is applied, the transient state, the steady state and when the heat source is removed.

4.2 Effect of using Finite Element method

The result of the TEG modelling using the modified equations for calculating the parameters using FEM is compared with the same equations without the modification. The equations 15 to 26 considered an average value for the calculation of temperature difference between hot and cold sides whereas, FEM divides the length of the thermocouples to small nodes.

The results of using the finite element and averaging methods are shown in Figure 18, Figure 19 and Figure 20 for the conductivity, resistivity, Seebeck coefficient and Thomson coefficient, respectively.

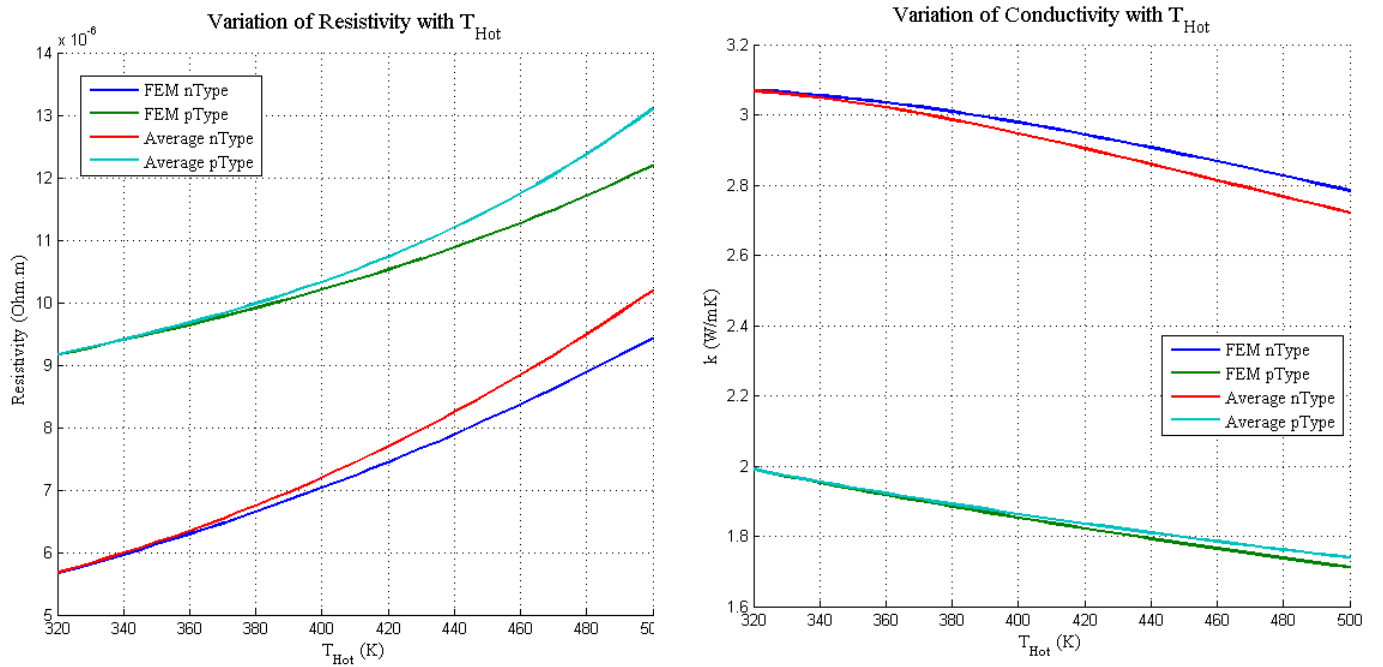


Figure 18: Variation of hot side temperature with conductivity and resistivity using FEM and average values

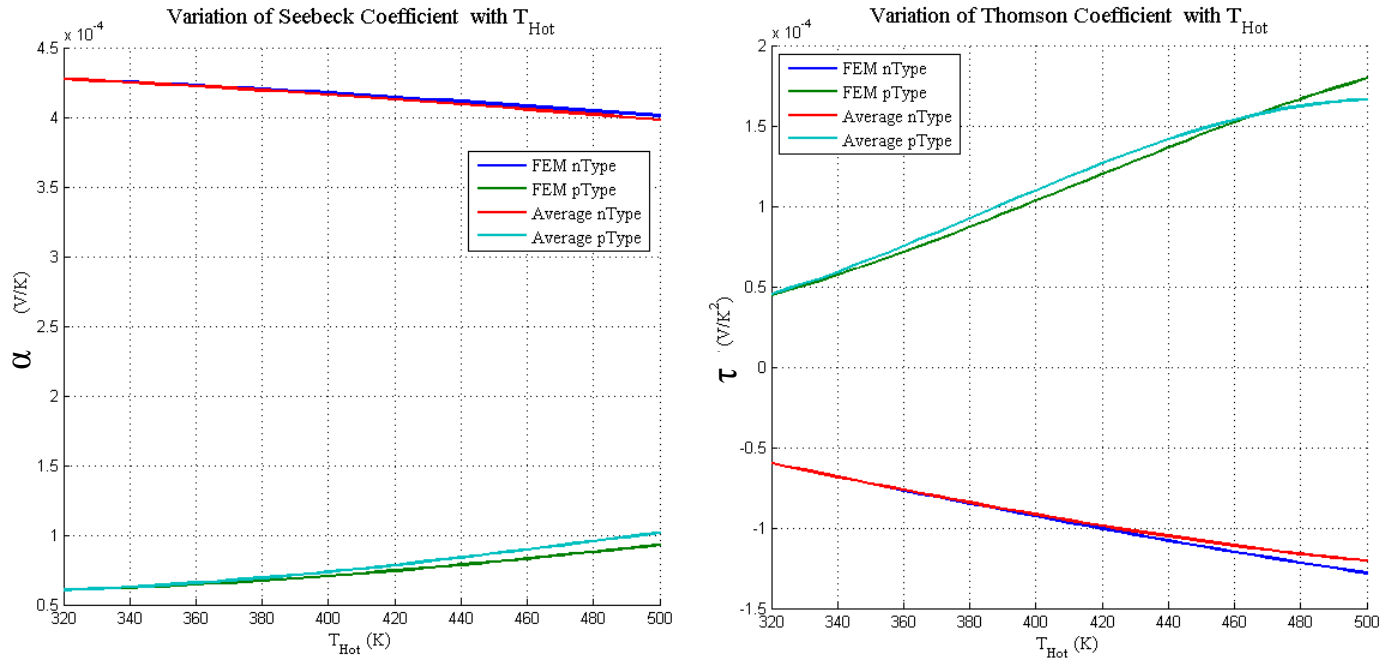


Figure 19: Variation of hot side temperature with Seebeck and Thomson's coefficients using FEM and average values

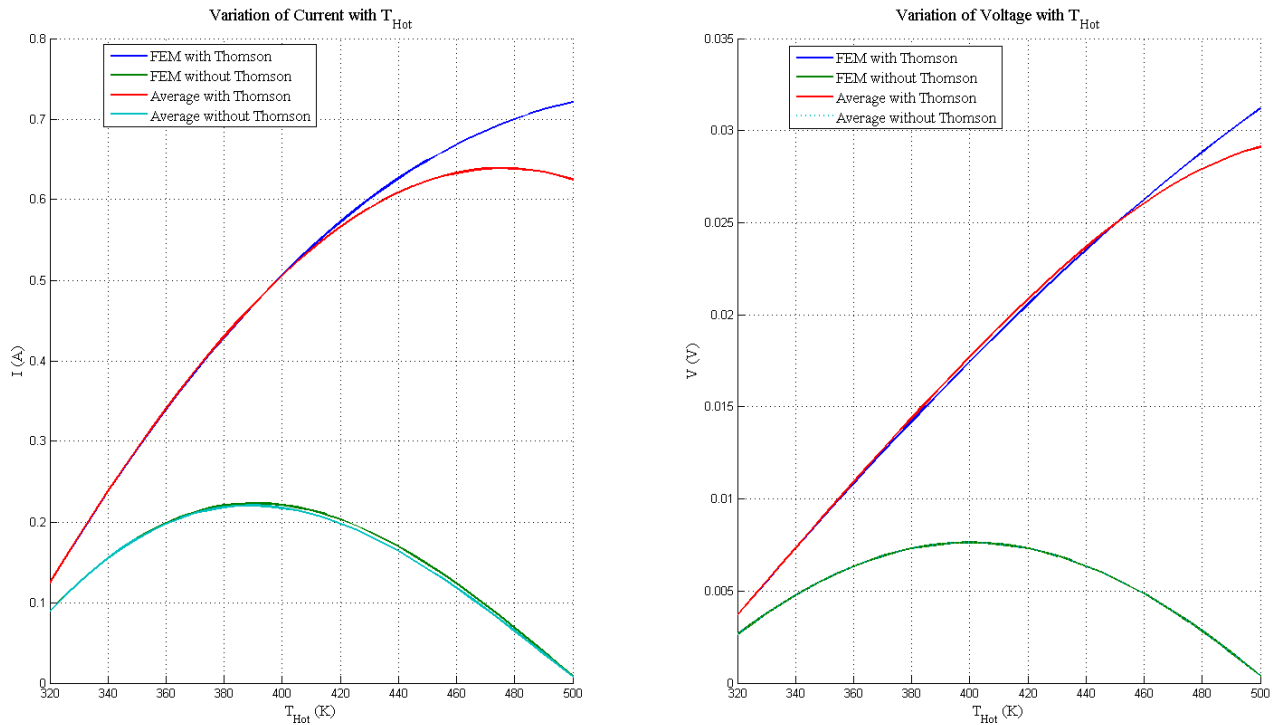


Figure 20: Comparison between FEM and Average values on the current and voltage

As mentioned in the previous chapters, all the parameters except the Seebeck coefficient depend on the gradient of the temperature along the material, and it is clear from the plots that using the difference between the hot and cold temperatures as one block gives different values than using the finite element method especially if the temperature gets higher.

However, because the Seebeck coefficient is affected by the temperature difference of the junction of two materials ($i = 1$ and $i = n$), which has the hot and cold temperatures in this situation, the values of the two methods are almost the same.

The output voltage and current using the FEM method and average values including the influence of Thomson coefficient were studied (Figure 20). The difference between the two calculation methods appears more as the temperature increases, while they gave almost the same results if Thomson coefficient was neglected.

4.3 Effect of current on the temperature distribution

Figure 21 shows the impact of flowing the current on the p-n material temperature distribution. From the figure, the effect is insignificant in the steady-state and can be approximated as a linear variation decreasing from the hot temperature side to the cold temperatures side.

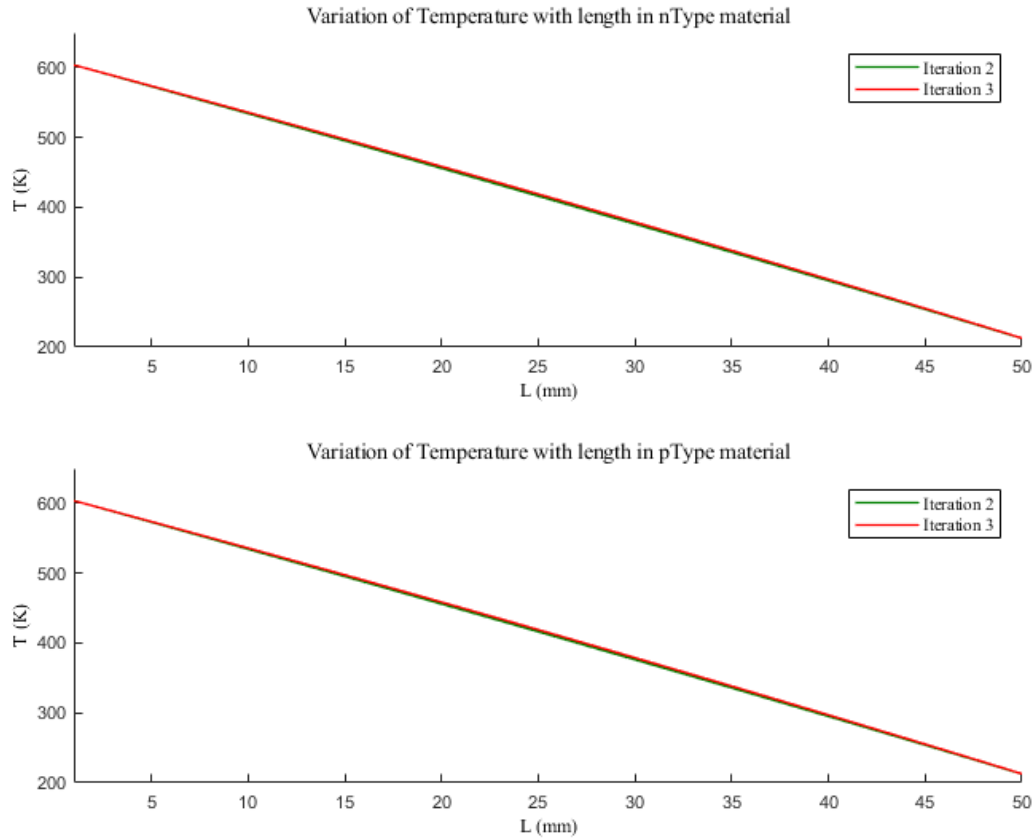


Figure 21: Variation of Temperature distribution over the length

4.4 Temperature with Current, Voltage, and Power

The effect of different temperature on the steady-state output is shown in Figure 22.

The load resistor is set to equal the internal resistance of the module to get the maximum output, and the effect of temperature on the steady-state output was investigated as shown in Figure 23.

The figures show that as the hot side's temperature increases, the voltage, current, and power output are increasing. Also, it is clear that decreasing the cold side's temperature gives more output since the temperature difference is larger.

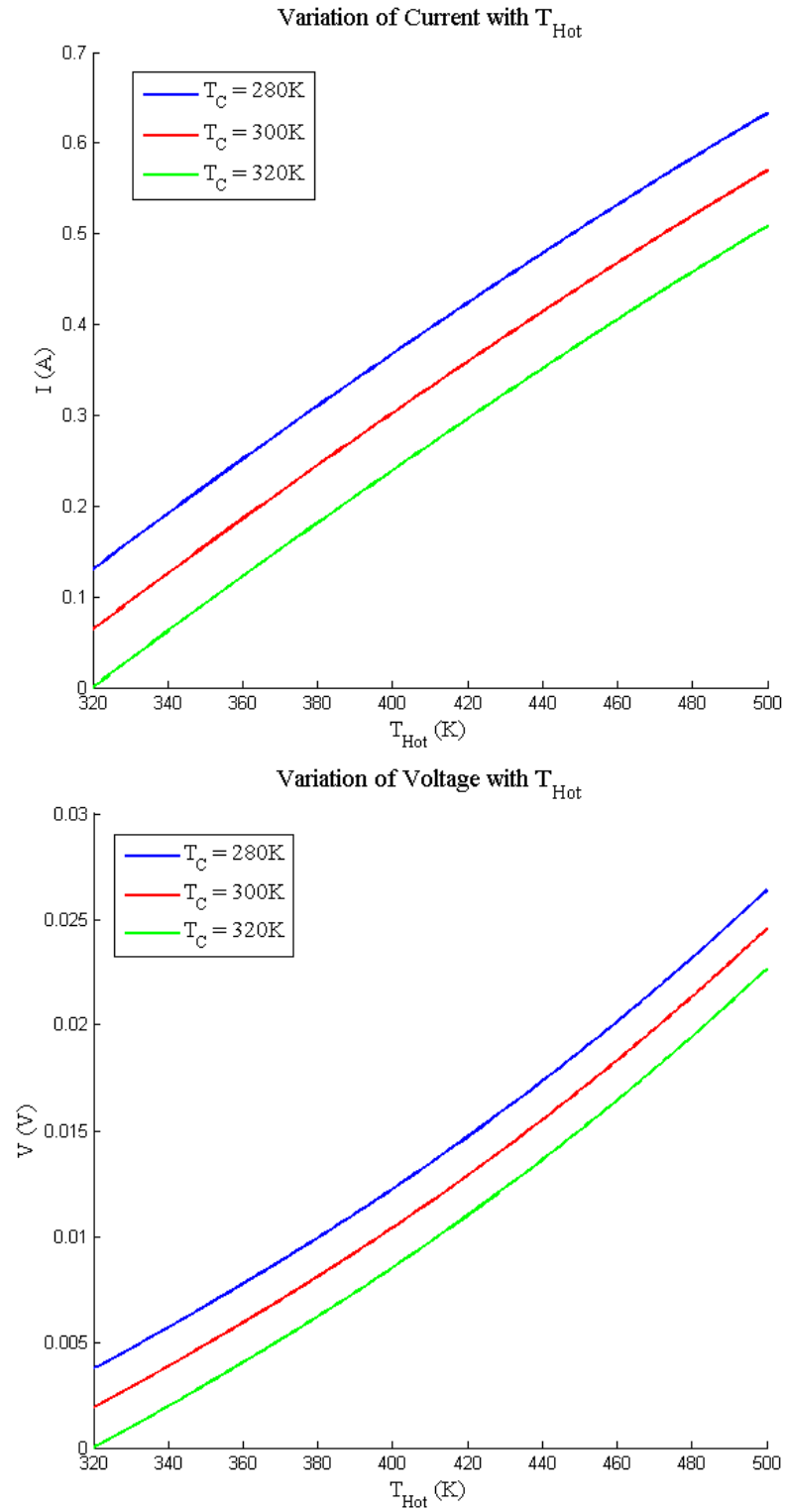


Figure 22: Current and Voltage variation with temperature

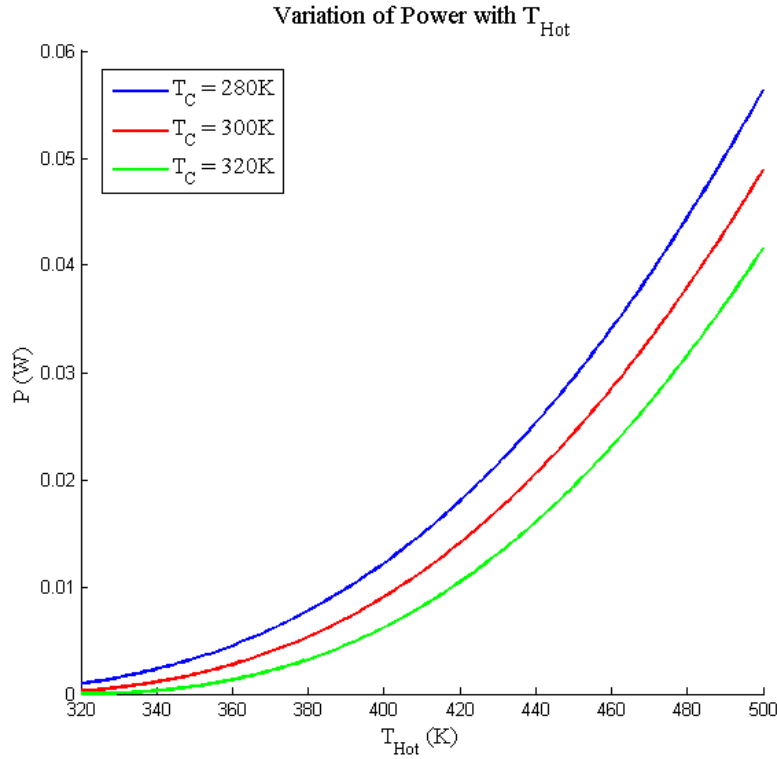


Figure 23: Power variation with temperature

4.5 Load resistor versus Current, Voltage, and Power

In this section, the hot side temperature is fixed, and the load resistor is varied. Figures 24 to 27 show the impact of changing the load resistor on the power current curve, the voltage-current curve the resistor power curve and the resistor efficiency curve, respectively.

From the graphs, the maximum power and efficiency can be found at the point where the load resistor equals the internal resistance.

Also, the maximum power can be found when the current is half the maximum current of the module, while the voltage is maximum when the current is zero (open circuit).

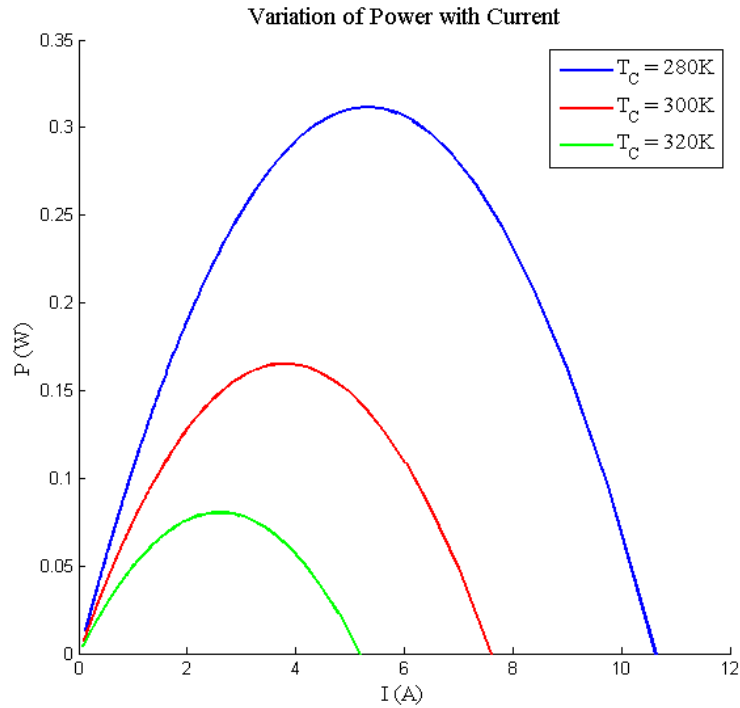


Figure 25: Variation of current with power

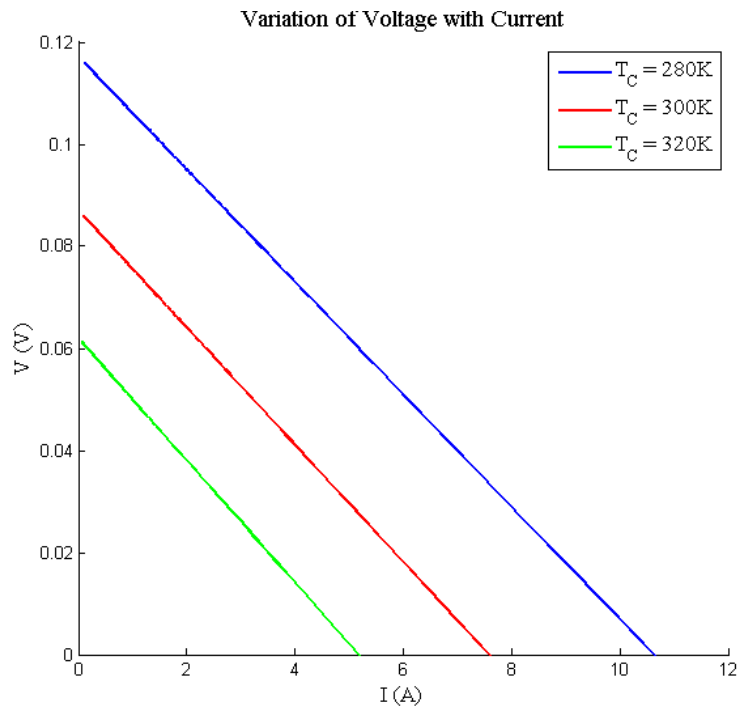


Figure 24: Variation of current with voltage

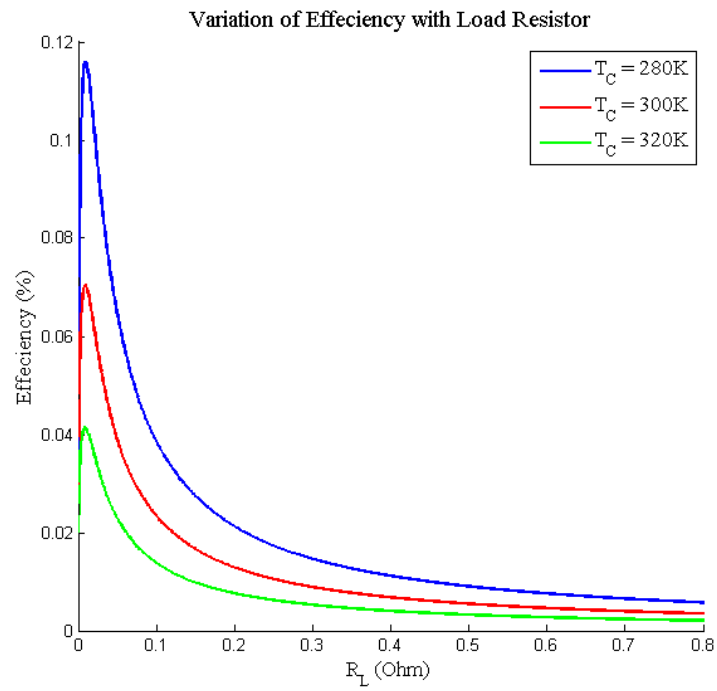


Figure 27: Efficiency variation with load resistor

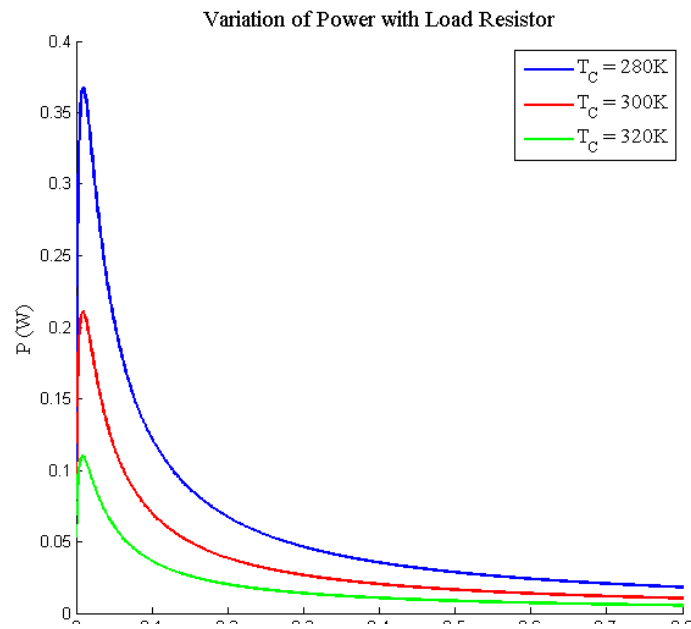


Figure 26: Power variation with load resistor

4.6 Area and length versus different parameters

From the experimental result, the size and area were important for the overall performance of the TEG device. So, in this section and using the modelling, the size, and length of thermocouples is found to have an impact on current, voltage and power. The results are shown in Figures 28 – 30.

The maximum power and current can be found as the area get larger, and the length is shorter because as the length increases the resistivity of the materials increases too which resist the flow of current. As the area increases, the thermal conductivity of the surface increases which allows more heat to transfer. While the voltage stays constant and depends only on the properties of the material and not the size and the area.

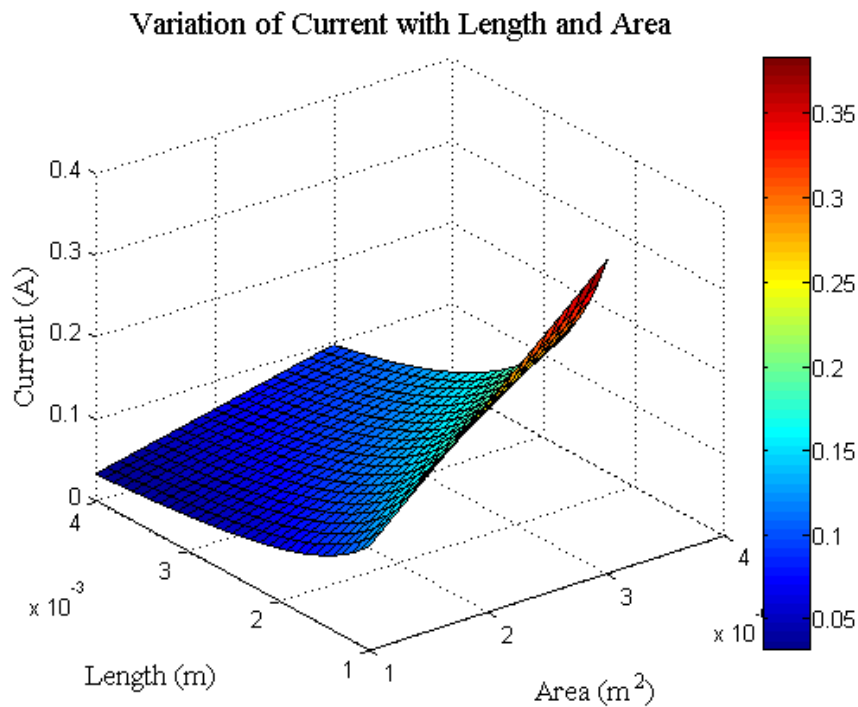


Figure 28: Current variation with area and length

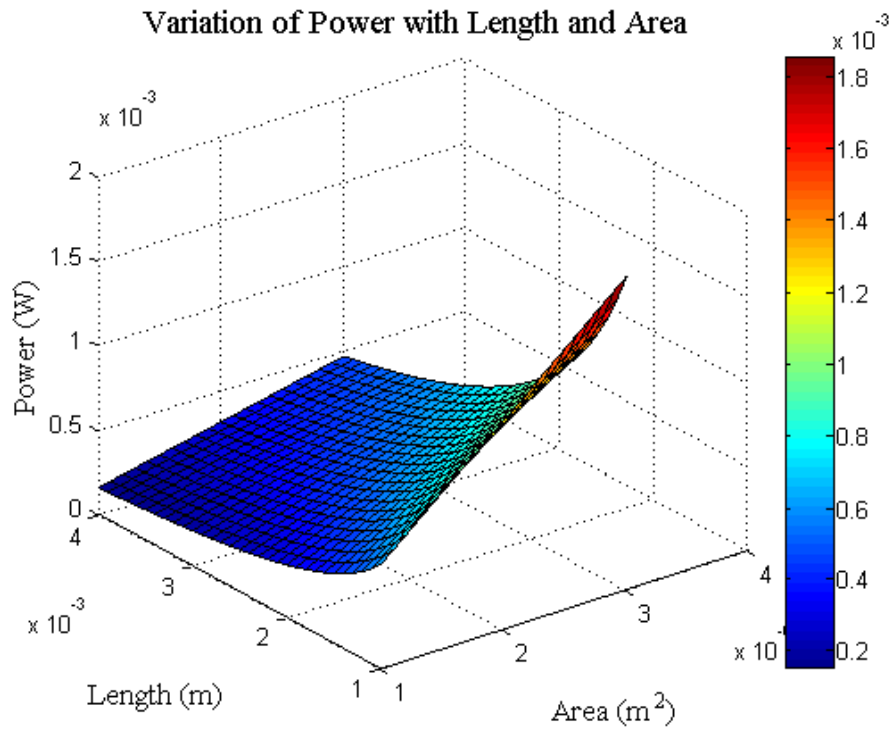


Figure 29: Power variation with area and length

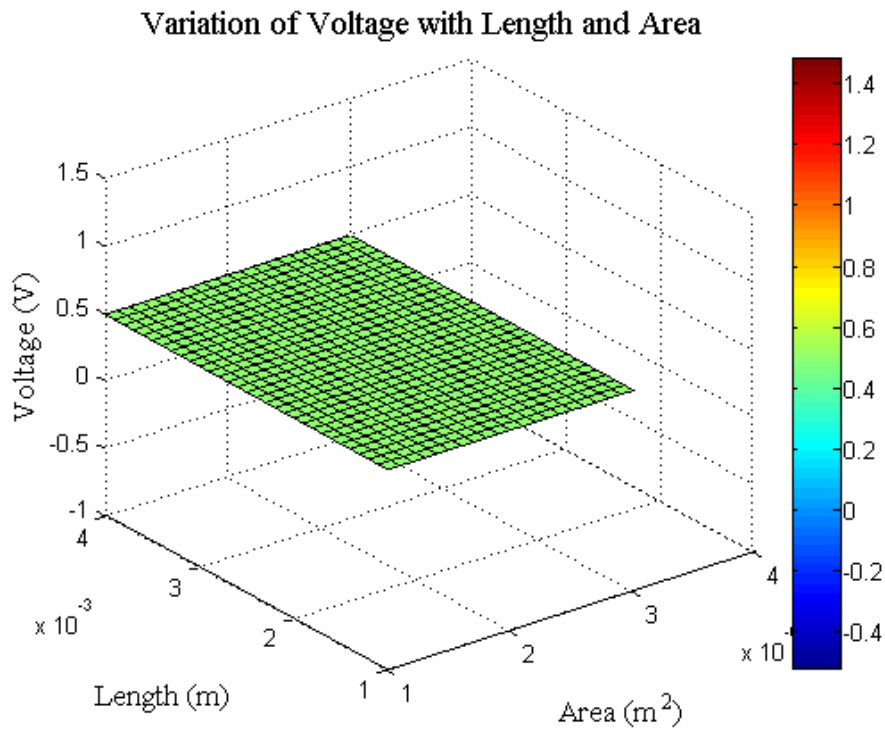


Figure 30: Power and voltage variation with area and length

Chapter Five

Conclusion and Recommendations

5.1 Conclusion

5.2 Recommendations

Chapter Five

Conclusion and Recommendations

5.1 Conclusion

Thermoelectric Generators (TEGs) are widely used in industry for different applications as a form of waste heat harvesting and energy production. The need to have an accurate model was the aim of many researchers. Different modelling methods and techniques have been used. However, these methods did include a different assumptions for the TEG parameters, one of which was the Thomson effect. Also it neglects the impact of the averaging values for the parameters, especially the calculation of the temperature difference.

In this work, it was important to understand the behaviour of the TEG device experimentally, however, the devices used for the experimental part did not have a complete data sheets which will help in modelling the same devices. The experiment was done to study the impact of the size of the heated area on the TEG output. It showed that the larger the size the better linear fitting for the current readings.

The model developed for one-dimensional heat transfer in the thermoelectric generator is made using the finite element method. It included the influence of temperature dependent coefficients. Although, the output curves from the model are similar to the curves that the TEG should give, the FEM and the Average Method showed different values, especially at high temperature differences; ΔT above 80 K for the current and above 130 K for the voltage.

The use of the FEM will help to improve the modeling of the TEG devices, especially with a high-temperature difference, and less impact on the low-temperature difference. Consequently, the Thomson effect is very important to be included in the modeling, which has shown a huge difference in the current and voltage calculations.

The model also showed the impact of temperature, dimensions and load resistor on different output variables and investigated the optimum size of the p-n material that gives the best power. It was shown that the voltage remains constant while the power, current and efficiency increase where the area increases and the length decreases, which helps in designing TEG module with minimum size.

5.2 Recommendations

To further improve the TEG modelling, the following need to be considered:

- 1- Investigating more parameters and their impact on the module performance such as including the conductor and ceramic layers in the model.
- 2- Solve the time-dependent equations and find the temperature behaviour at transient-state.
- 3- Developing the model to includes the effect of side heat losses and the convections losses.
- 4- Evaluating the developed model using experimental methods, providing full information about the TEG material parameters.
- 5- To, thoroughly, understand the TEG behaviour, experimenting more type of TEGs using advance setup (perfect insulation, different methods for cooling and heating, and calibrated sensors) to get more accurate results and to control as much as variables, which helps in modelling the temperature distribution and variation with time.

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الملخص

تم تطوير نموذج لحساب التيار، والجهد الكهربائي، والقوة، والكفاءة للمولد الحراري الكهربائي في حالة انتقال الحرارة المستقر في اتجاه واحد باستخدام طريقة العناصر المحدودة لإيجاد خواص المادة المستخدمة في التصنيع كالموصلية، والمقاومة، ومعامل سيببيك ومعامل طومسون، عند فرق درجات حرارة مختلفة، واستخدامه في التطبيقات المتعلقة بحفظ الطاقة وتقدير الطاقة الناتجة من المولد عند معطيات محددة.

وقد تمت مقارنة النتائج عند استخدام القيم المتوسطة بين السطح الحار والبارد، واستخدام طريقة العناصر المحدودة والتي اعطت نتائج أفضل، وأكثر دقة، خصوصاً عند ازدياد الفرق في درجة الحرارة (أعلى من 80 كلفن في حساب التيار وأعلى من 180 كلفن في حساب فرق الجهد). بالإضافة إلى ذلك، فقد تم اضافة تأثير معامل طومسون إلى النموذج والذي أدى إلى تغيير كبير في شكل المنحنى الخاص بالتيار وفرق الجهد.

أيضاً، تم استخدام النموذج لدراسة تأثير المقاومة والتي أظهرت أن أقصى حد للتيار يكون عندما تكون قيمة المقاومة الخارجية مساوية لقيمة المقاومة الداخلية للمولد، كما استخدم النموذج لإيجاد الحجم الأمثل لأشباه الموصلات المستخدمة في التصنيع حيث تم تغيير مساحة وطول المواد مع تثبيت فرق الحرارة المستخدم لدراسة تأثيره على الطاقة الناتجة، وقد وجد أن المساحة الأكبر ذات الطول الأصغر تعطي أكبر طاقة، وهذا يساعد في اختيار الحجم المناسب عند التصميم.

بالإضافة إلى ذلك، تم دراسة تأثير حجم النظام الحراري المطبق على التيار والجهد بشكل تجريبي من خلال تغيير مساحة التسخين حيث وجد أن الحجم الأكبر للحرارة يعطي نتائج خطية أفضل وإنتاجاً أعلى وذلك لأن السطح يسخن بسرعة أكبر.

جامعة البحرين

قسم الفيزياء

كلية العلوم



تطوير نموذج دقيق للمولدات الحرارية الكهربائية لحصاد الطاقة

أطروحة مقدمة كجزء من متطلبات الحصول على درجة الماجستير في برنامج الفيزياء التطبيقية

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