UNIVERSITY OF BAHRAIN
COLLEGE OF SCIENCE
DEPARTMENT OF MATHEMATICS

SHOCK WAVES IN LIQUID WITH SOLUBLE GAS BUBBLES

BY

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ABSTRACT

In this thesis we present a study of growth of spherical soluble gas bubble in liquid controlled by diffusion by using basic equations that describing the behaviour of a soluble gas bubble in a liquid. And we find analytical self-similar solution by transformation of partial differential diffusion equation to ordinary differential equation. This is possible when $R\dot{R}$= constant, where R is the bubble radius and $\dot{R}$ is the time derivative of the radius.

Also we find expressions for the bubble growth, the parameter h which controlling the radial motion, and curves are presented for parameter h and $\frac{C_o - C_s}{\rho_g}$ for different values of non-dimensional parameter $\varepsilon = \frac{\rho_g}{\rho_L}$.

Where C is the mass concentration of gas in the liquid, the subscript o and s refers to the value at remote from the bubble and saturation respectively.

ρ is the density, the subscript g and L refers to the parameters of gas and liquid.

Structure of stationary shock waves in a liquid with soluble gas bubbles are discussed by using the equations for a stationary one-dimensional flow. And we find the solution for shock-wave structure near critical point by investigating the solution behaviour before the shock wave . To investigate the asymptotic behaviour of the solution of system of equations for a one - dimensional stationary flow of liquid with soluble gas bubbles in the neighbourhood of the initial equilibrium state, these equations need to be linearized with respect to the values of the parameters at the point 0 .

Their solution is sought in the form of an exponential damped as $\bar{x} \to \infty$

$$\Phi=\Phi_o \left(1+A^{(\Phi)} \exp k_o \bar{x} \right), \text{Re } k_o < 0 \qquad (*)$$

where Φ is any parameter (V(x), P(x), R(x), C(r,x)) where V is the velocity, P is the pressure, R is the radius and C is the concentration.

After linearization we obtain from system of equations for a one-dimensional

stationary flow of liquid with soluble gas bubbles a system of linear homogeneous equations in the amplitudes $A^{(\Phi)}$.

The condition of the existence of non-trivial solution to this system leads to an algebraic equation of sixth order with respect to $K=\sqrt{k_o \bar{v}_o}$, where k_o is exponention coefficient and $\bar{v}_o$ is the dimensionless wave velocity. It has been proved that the desired solution exists and is unique. This proves that in the presence of diffusion there exists unique solution of (*), and the required root exists only if $v^2_o > C^2_o$,where v_o is the wave velocity and C_o is the sound speed in bubbly liquid.

The states before ($x \to +\infty$) and behind the shock wave ($x \to -\infty$) are equilibrium ones, which is why an analysis of the asymptotic behaviour at $x \to +\infty$ is necessary to get away from the critical point, The integral curve is connecting these two states and determins the structure of the steady shock wave.

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INTRODUCTION

The study of oscillations of gas bubbles in a liquid and of shock waves in a liquid containing bubbles is of considerable practical interest specifically in regard to the question of the possible use of bubble screen for damping of shock waves.

In chapter I, we study the growth of soluble gas bubble controlled by diffusion by using basic equations that describing the behaviour of a soluble gas bubbles in a liquid. We found analytical self-similar solution by transforming partial differential equation to ordinary differential equation. This is possible only when $R\dot{R}$= constant, where R is the radius of the bubble and $\dot{R}$ is the time derivative of the radius. This self-similar solution describing the bubble growth. And presented curves show the relationship between the parameter h controlling the radial motion and $\frac{C_o - C_s}{\rho_g}$ at various ratios of phase densities $\varepsilon = \frac{\rho_g}{\rho_L}$, where C is the mass concentration of gas in the liquid, the subscript o and S refers to the value at remote from the bubble and saturation respectively. ρ is the density, the subscripts g and L refers to the parameters of gas and liquid.

In chapter II, we present the structure of stationary shock waves in a liquid with soluble gas bubbles by using the equations for a stationary one-dimensional flow. The structure of wave is integral curve connecting two critical points of the system of differential equations. Also we investigated the solution behaviour before the shock wave and proved the existence of steady shock - wave structure.

CHAPTER I

The growth of soluble gas bubbles in a liquid

1.1 Introduction

Dynamics of gas bubbles in liquid has been investigated in many papers (see for example the book of Leigton 1993), the review (Plesset & Prosperetti 1977, Feng & Leal 1997). Many papers are divoted to experimantel investigation of bubble dynamics (see for example Lauterborn & Holzfuss 1990). Dynamics of soluble gas bubbles was investigated in (Epstein & Plesset 1950, Nagiev & Khabeev 1988).

In this chapter we will study the growth of a spherically symmetric soluble gas bubble controlled by diffusion.

Diffusion is the process by which matter is transported from one part of system to another as a result of random molecular motions. (Crank,J.)

The diffusion process is similar to the conduction of heat through a rod. The conduction of heat take place in response to a temperature gradient, diffusion in a gas (or in a liquid) takes place in response to a concentration gradient. (Jerry et al 1985).

The mathematical theory of diffusion in isotropic substances is therefore based on the hypothesis that the rate of transfer of diffusing substance through unit area of a section is proportional to the concentration gradient measured normal to the section i. e(Fick's law).

$$j = - D \partial C / \partial X \qquad (1.1.1)$$

where j is the rate of transfer per unit area of section,C the concentration of diffusion substance, x the space coordinate measured normal to the section, and D is called the diffusion coefficient. (Crank,J.)

The mass - diffusion processes taking place across the bubble - liquid interface play a significant role in the behaviour of gas bubbles because they may ultimately determine the presence or absence of bubbles in a liquid. The key to these processes is furnished by Henry's Law, which established a connection between the partial pressure of a gas acting on a liquid surface P, and the equilibrium (or saturation) concentration of gas in the liquid which we denote by C_s:

$$C_s = \Gamma P_g \qquad (1.1.2)$$

Here Γ is a constant characteristic of the particular gas - liquid combination and is primarily a function of temperature.

If we consider first a situation in which the ambient pressure is fixed and equal to P_o, then it is clear that unless the gas concentration C_s at the bubble surface satisfies equation (1.1.2) (where we now interpret P_g as the gas pressure in the bubble), the bubble will not be in equilibrium, and it will either grow or shrink according to whether $C_o > C_s$ or $C_o < C_s$ (Plesset & Prosperetti 1977)

where C is the mass concentration of gas in the liquid, the subscripts o and S refers to the value at remote from the bubble and saturation respectively.

1.2 Basic Equations

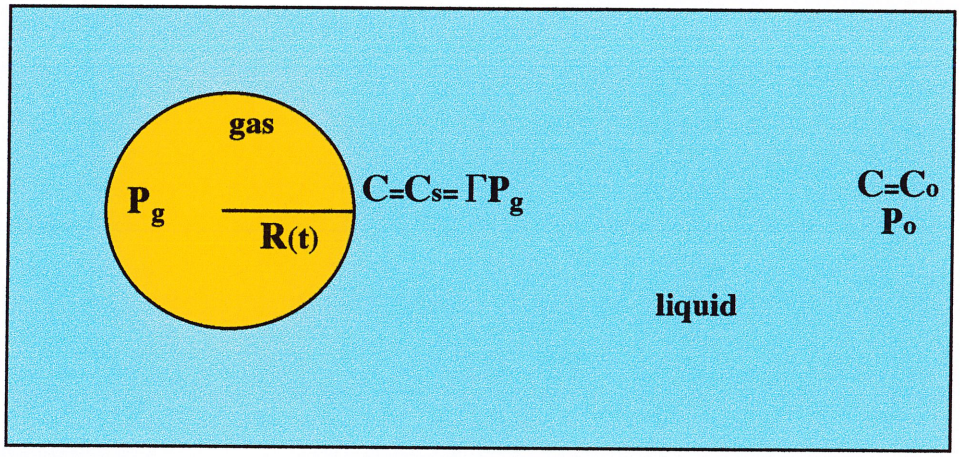


Fig.(1)

We suppose that the process is very slow. If the dynamic inertial effects are neglected, $P_g = P_o + \frac{2\sigma}{R}$

For large bubbles ($R \geqslant 10^{-3}$ cm) the cappillar effects is very small, so $P_g = P_o$.

We consider in this section a spherical bubble located in incompressible liquid (ρ_L = const.) saturated by a gas. We assume that $C_s < C_o$, so the gas will go inside the bubble and the bubble will grow so the radius will increase. P_o is the pressure in the liquid at a large distance from the bubble, it is fixed and P_g is the pressure within the bubble. So the parameters within the bubble become uniform and time independent.

It follows from incompressibility of the liquid that the liquid velocity at a distance r from the bubble center is $u(r,t)= \frac{R^2}{r^2}\ \omega$. (Plesset & Prosperetti 1977) where ω is the radial velocity of the liquid at the bubble surface. Within the frame- work of these assumptions the mathematical formulation for this process clearly consists of the diffusion equation for the gas in the liquid in the spherical Euler coordinates (r,t) takes the form:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial r} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right), \quad r > R \qquad (1.2.1)$$

where $u = \frac{\epsilon R^2 \dot{R}}{r^2}$, $\epsilon = 1 - \frac{\rho_g}{\rho_L}$

Where ρ is the density, the subscripts g and L refers to the parameters of gas and liquid, C is the concentration of the gas in the liquid, D is the diffusion coefficient and R is the radius of the bubble.

The boundary conditions at the surface of and remote from the bubble are as follows:

$$r = R(t) : C = C_s = \Gamma P_g \quad , \quad r = \infty : C = C_o \qquad (1.2.2)$$

Where C_s is the saturation concentration and Γ is Henry's coefficient.

The velocity of the bubble surface $\dot{R}$ and the mass velocity of the gas at that surface v are related by the expressions:

$$\dot{R} = v + \frac{j}{\rho_g(R)} \quad , \quad j = D\left.\frac{\partial C}{\partial r}\right|_R \qquad (1.2.3)$$

$$\dot{R} = \omega + \frac{j}{\rho_L}$$

if $v = 0$

$$j = \rho_g \dot{R}$$

$$\omega = \dot{R} - \frac{j}{\rho_L} = \dot{R} - \dot{R}\frac{\rho_g}{\rho_L} = \dot{R}\left(1 - \frac{\rho_g}{\rho_L}\right)$$

where j is the rate of mass transfer per unit interphase surface.

The problem can be substantially simplified by observing that the bubble - wall velocities v that are induced by mass diffusion alone are usually quite small. (Plesset & Prosperetti 1977).

System of equations (1.2.1) - (1.2.3) is describing the behaviour of a soluble gas bubble in a liquid.

1.3 Growth controlled by diffusion.

Growth is governed by equations (1.2.1), (1.2.2), (1.2.3) we will use the similar procedure which was used in (Scriven 1959) for the growth of vapour bubble in superheated liquid.

Here, the solution of diffusion equation depends on r and time because of R= R(t) sometimes solution of partial differential equation doesnot depend on two variables separately, but depends on some combination of this independent variables i.e. depends on one parameter only.

In such case partial differential equation can be transformed to ordinary differential equation. Such solution called self-similar solution (Barenblatt, 1979). Let us try to find the self-similar solution to this problem. We will suppose that this solution depends only on the dimensionless variable $\eta = \frac{r}{R(t)}$, i.e. we find a

solution of the form $C(r,t) = C(\eta)$ Then chain rules

$$\frac{\partial}{\partial r} = \frac{1}{R}\frac{d}{d\eta} \quad \text{and} \quad \frac{\partial}{\partial t} = \frac{d}{d\eta}\cdot\frac{\partial \eta}{\partial t} = \frac{-r}{R^2}\dot{R}\frac{d}{d\eta}$$

introducing the dimensionless concentration $\bar{C} = \frac{C}{\Gamma P_g}$, the equation (1.2.1)

may be written in the form:

$$-\frac{r}{R^2}\dot{R}\frac{dC}{d\eta} + \frac{\epsilon \dot{R} R^2}{r^2}\cdot\frac{1}{R}\frac{dC}{d\eta} = \frac{D}{r^2}\cdot\frac{1}{R}\frac{d}{d\eta}\left(\frac{r^2}{R}\frac{dC}{d\eta}\right)$$

$$\left(\frac{-r\dot{R}}{R^2} + \frac{\epsilon \dot{R} R}{r^2} \right) \frac{dC}{d\eta} = \frac{D}{r^2} \frac{d}{d\eta} \left(\frac{r^2}{R^2} \frac{dC}{d\eta} \right)$$

$$\left(\frac{-r^3\dot{R}}{R^2} + \epsilon \dot{R} R \right) \frac{dC}{d\eta} = D \frac{d}{d\eta} \left(\eta^2 \frac{dC}{d\eta} \right)$$

$$R\dot{R} \left(\frac{-r^3}{R^3} + \epsilon \right) \frac{dC}{d\eta} = D \frac{d}{d\eta} \left(\eta^2 \frac{dC}{d\eta} \right)$$

$$R\dot{R} \left(-\eta^3 + 1 - \frac{\rho_g}{\rho_L} \right) \frac{dC}{d\eta} = D \frac{d}{d\eta} \left(\eta^2 \frac{dC}{d\eta} \right)$$

$$R\dot{R} \left(1 - \frac{\rho_g}{\rho_L} - \eta^3 \right) \Gamma P_g \frac{d\bar{C}}{d\eta} = D \frac{d}{d\eta} \left(\eta^2 \Gamma P_g \frac{d\bar{C}}{d\eta} \right)$$

$$R\dot{R} \left(1 - \frac{\rho_g}{\rho_L} - \eta^3 \right) \Gamma P_g \frac{d\bar{C}}{d\eta} = D \Gamma P_g \frac{d}{d\eta} \left(\eta^2 \frac{d\bar{C}}{d\eta} \right)$$

$$R\dot{R} \left(1 - \frac{\rho_g}{\rho_L} - \eta^3 \right) \frac{d\bar{C}}{d\eta} = D \frac{d}{d\eta} \left(\eta^2 \frac{d\bar{C}}{d\eta} \right) \qquad (1.3.1)$$

From (1.3.1) we can make conclusion that the self similar solution of the type $\bar{C}(\eta)$ exists only if $\dot{R}(t)\, R(t) = \text{const}$.

Only in such case partial differential equation can be transformed to ordinary differential equation.

As follows from

$$\dot{R} = \frac{j}{\rho_{g(R)}} \quad , \quad j = D \left.\frac{\partial C}{\partial r}\right|_R$$

$$R\dot{R} = \frac{Rj}{\rho_{g(R)}} = \frac{R}{\rho_g} D \left(\frac{\partial C}{\partial r}\right)_R$$

$$\left(\frac{dC}{dr}\right)_R = \frac{\Gamma P_g}{R} \left(\frac{d\bar{C}}{d\eta}\right)_1$$

$$R\dot{R} = \frac{R}{\rho_g} D \frac{\Gamma P_g}{R} \left(\frac{d\bar{C}}{d\eta}\right)_1 = \frac{D}{\rho_g} \Gamma P_g \left(\frac{d\bar{C}}{d\eta}\right)_1 = \text{const.}$$

let us introduce the following dimensionless parameters.

$$h = \frac{R\dot{R}}{D} = \frac{\Gamma P_g}{\rho_g} \left(\frac{d\bar{C}}{d\eta}\right)_1 \quad , \quad \varepsilon = \frac{\rho_g}{\rho_L} << 1 \qquad (1.3.2)$$

Then equation (1.3.1) became

$$h\left(1 - \frac{\rho_g}{\rho_L} - \eta^3\right) \frac{d\bar{C}}{d\eta} = \eta^2 \frac{d^2\bar{C}}{d\eta^2} + 2\eta \frac{d\bar{C}}{d\eta}$$

$$\left(h - h\frac{\rho_g}{\rho_L} - h\eta^3 - 2\eta\right) \frac{d\bar{C}}{d\eta} = \eta^2 \frac{d^2\bar{C}}{d\eta^2}$$

$$\eta^2 \frac{d^2\bar{C}}{d\eta^2} + \left(2\eta + h\eta^3 + h\frac{\rho_g}{\rho_L} - h \right) \frac{d\bar{C}}{d\eta} = 0$$

$$\frac{d^2\bar{C}}{d\eta^2} + \left(\frac{2}{\eta} + h\eta + \frac{h}{\eta^2}\frac{\rho_g}{\rho_L} - \frac{h}{\eta^2} \right) \frac{d\bar{C}}{d\eta} = 0$$

$$\frac{d^2\bar{C}}{d\eta^2} + \left(\frac{2}{\eta} + h\eta + \frac{h}{\eta^2}\left(\frac{\rho_g}{\rho_L} - 1 \right) \right) \frac{d\bar{C}}{d\eta} = 0$$

$$\frac{d^2\bar{C}}{d\eta^2} + \left(\frac{2}{\eta} + h\eta - \frac{h}{\eta^2}(1-\varepsilon) \right) \frac{d\bar{C}}{d\eta} = 0 \qquad (1.3.3)$$

Substitute for $\frac{d\bar{C}}{d\eta} = P$, equation (1.3.3) becomes the first-order differential equation

$$\hat{P} + \left(\frac{2}{\eta} + h\eta - \frac{h}{\eta^2}(1-\varepsilon) \right) P = 0 \qquad (1.3.4)$$

by using integrating factors, we notes that

$$\int_e \left(h\eta + \frac{2}{\eta} - \frac{h}{\eta^2}(1-\varepsilon) \right) d\eta = \exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon) \right)$$

multiplied equation (1.3.4) by $\exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon) \right)$, we have

$$\exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon)\right) P' + \exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon)\right) \left(\frac{2}{\eta} + h\eta - \frac{h}{\eta^2}(1-\varepsilon)\right) P = 0$$

$$\left[\exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon)\right) P\right]' = 0$$

It follows that a solution P is given by

$$\exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon)\right) P = \frac{1}{C_1}$$

or

$$P = \frac{1}{C_1 \exp\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon)\right)}$$

$$\frac{d\bar{C}}{d\eta} = \frac{1}{C_1} \exp -\left(\frac{h\eta^2}{2} + 2\ln\eta + \frac{h}{\eta}(1-\varepsilon)\right)$$

$$= \frac{1}{C_1} \exp\left[-h\left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta}\right)\right].\exp(-\ln\eta^2)$$

$$= \frac{1}{C_1} \exp\left[-h\left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta}\right)\right].\exp \ln\eta^{-2}$$

$$= \frac{1}{C_1} \exp\left[- h \left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta} \right) \right] . \eta^{-2}$$

$$\frac{d\bar{C}}{d\eta} = \frac{1}{C_1 \eta^2} \exp\left[- h \left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta} \right) \right] \qquad (1.3.5)$$

Boundary conditions (1.2.2) assume the form

$$\eta = 1 : \bar{C} = \frac{C}{\Gamma P_g} = \frac{\Gamma P_g}{\Gamma P_g} = 1 \quad , \eta \longrightarrow \infty : \bar{C} = \bar{C}_\infty = \frac{C_o}{C_s}$$

integrating equation (1.3.5) we get

$$\int_1^\eta \frac{d\bar{C}}{d\eta} d\eta = \frac{1}{C_1} \int_1^\eta \frac{1}{\eta^2} \exp\left[- h \left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta} \right) \right] d\eta$$

$$\bar{C}\Big|_1^\eta = \frac{1}{C_1} \int_1^\eta \frac{1}{\eta^2} \exp\left[- h \left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta} \right) \right] d\eta$$

$$\bar{C}_\infty - 1 = \frac{1}{C_1} \int_1^\infty \frac{1}{\eta^2} \exp\left[- h \left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta} \right) \right] d\eta$$

Then the general solution of equation (1.3.5), together with its constants as defined by the boundary conditions are written in the form:

$$\bar{C}(\eta) = \frac{1}{C_1} \int_1^\eta \frac{1}{\eta^2} \exp\left[- h \left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta} \right) \right] d\eta + C_2$$

$$C_1 = \frac{1}{\bar{C}_\infty - 1} \int_1^\infty \frac{1}{\eta^2} \exp\left[-h\left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta}\right)\right] d\eta \qquad (1.3.6)$$

We introduce an independent variable instead of η

$$\zeta = \frac{1}{\eta} = \frac{R(t)}{r} \quad , \quad \frac{d}{d\eta} = -\zeta^2 \frac{d}{d\zeta}$$

Then equation (1.3.6) became

$$C_1 = \frac{1}{\bar{C}_\infty - 1} \int_1^0 \zeta^2 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] \cdot \left(\frac{-1}{\zeta^2}\right) d\zeta$$

$$= \frac{1}{\bar{C}_\infty - 1} \int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] d\zeta \qquad (1.3.7)$$

The self-similar solution exists only for the bubble - growth case when

$h \geq 0$, $C_0 > C_s$

The self- similar solution does not exists when $h < 0$ because

$\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right]$ became positive when $h < 0$ and at $\zeta = 0$,

$\exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right]$ will be infinity then we get unlimited integral in (1.3.7).

From equation (1.3.5) and (1.3.2). Substitute for C_1 in equation (1.3.5), we get

$$\frac{d\bar{C}}{d\eta} = \frac{(\bar{C}_\infty - 1)\ \exp\left[-h\left(\frac{\eta^2}{2} + \frac{(1-\varepsilon)}{\eta}\right)\right]}{\eta^2 \int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] d\zeta}$$

$$\left(\frac{d\bar{C}}{d\eta}\right)_1 = \frac{(\bar{C}_\infty - 1)\ \exp\left[-h\left(\frac{1}{2} + (1-\varepsilon)\right)\right]}{\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] d\zeta}$$

$$= \frac{(\bar{C}_\infty - 1)\ \exp\left[-h\left(\frac{(3-2\varepsilon)}{2}\right)\right]}{\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] d\zeta}$$

Substitute for $\left(\frac{d\bar{C}}{d\eta}\right)_1$ in equation (1.3.2), we get

$$h = \frac{\Gamma p_g}{\rho_g} \cdot \frac{(\bar{C}_\infty - 1)\ \exp\left[-h\left(\frac{3-2\varepsilon}{2}\right)\right]}{\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] d\zeta}$$

$$h \int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2} + (1-\varepsilon)\zeta\right)\right] d\zeta = \frac{\Gamma p_g}{\rho_g} (\bar{C}_\infty - 1) \exp\left[-h\left(\frac{3-2\varepsilon}{2}\right)\right]$$

$$h\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2}+(1-\varepsilon)\zeta\right)\right]d\zeta=\frac{C_s(\bar{C}_\infty-1)}{\rho_g}\exp\left[-h\left(\frac{3-2\varepsilon}{2}\right)\right]$$

$$h\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2}+(1-\varepsilon)\zeta\right)\right]d\zeta=\frac{(C_s\bar{C}_\infty-C_s)}{\rho_g}\exp\left[-h\left(\frac{3-2\varepsilon}{2}\right)\right]$$

$$h\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2}+(1-\varepsilon)\zeta\right)\right]d\zeta=\frac{(C_o-C_s)}{\rho_g}\exp\left[-h\left(\frac{3-2\varepsilon}{2}\right)\right]$$

It follows that the parameter h, controlling the radial motion or R is calculated from equation

$$h\ \exp\left(\frac{3-2\varepsilon}{2}\right)\ h\int_0^1 \exp\left[-h\left(\frac{1}{2\zeta^2}+(1-\varepsilon)\zeta\right)\right]d\zeta=\frac{(C_o-C_s)}{\rho_g}$$

The bubble growth in accordance with (1.3.2)

$$R\dot{R}=Dh$$

$$R\frac{dR}{dt}=Dh$$

integrating both sides, we get

$$\frac{R^2}{2}=Dht+\text{Const.}$$

$$R^2=2Dht+A \qquad \text{Where A : Const.}$$

$$\therefore R=\sqrt{A+2Dht}$$

So the bubble growth law is parabolic as shown in Fig.(2)

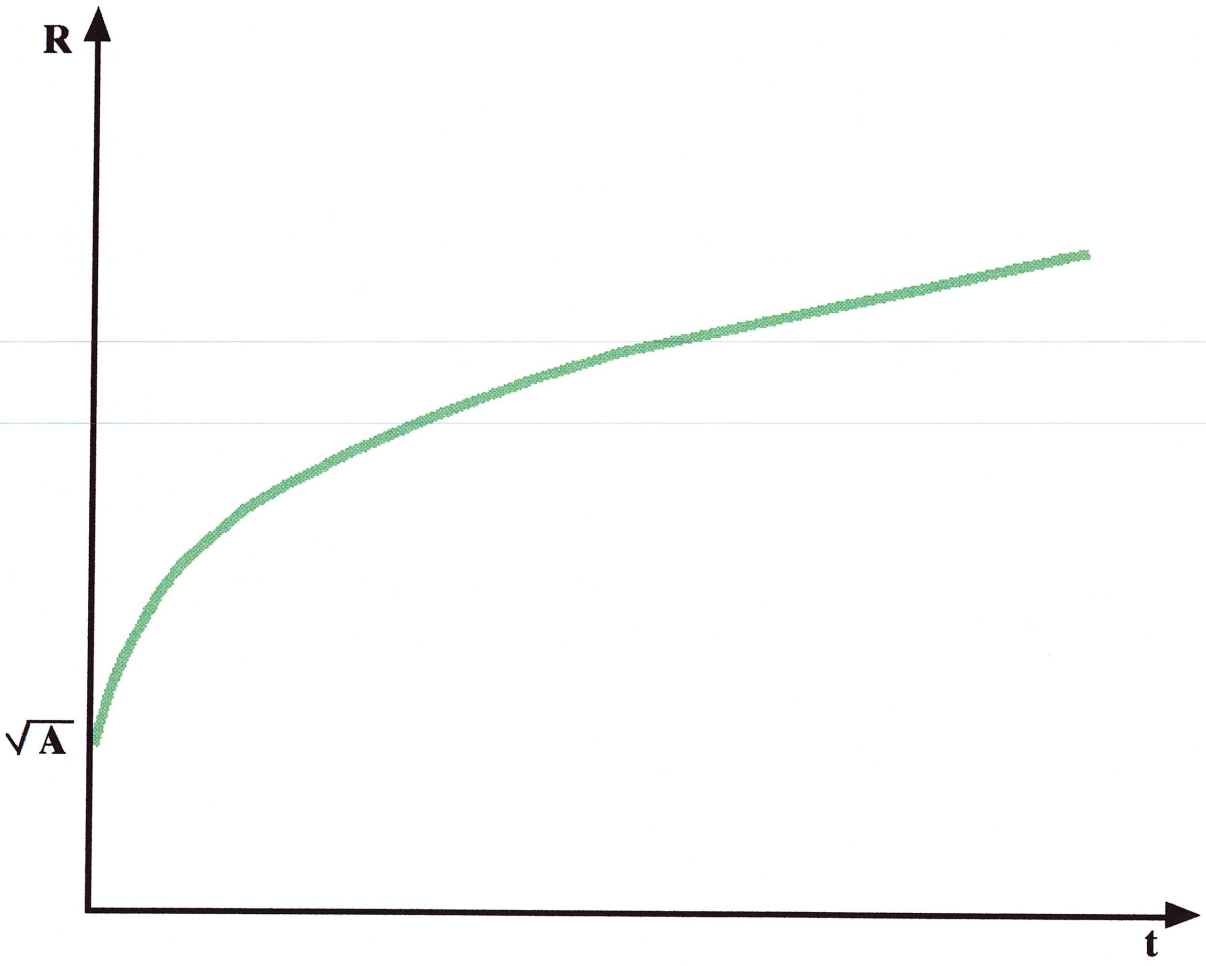


Fig. (2)

The relationship between parameter h which controls the radial motion and $\frac{(C_o - C_s)}{\rho_g}$ at various ratios of phase densities $\varepsilon = \frac{\rho_g}{\rho_L}$ is depicted in Fig.(3)

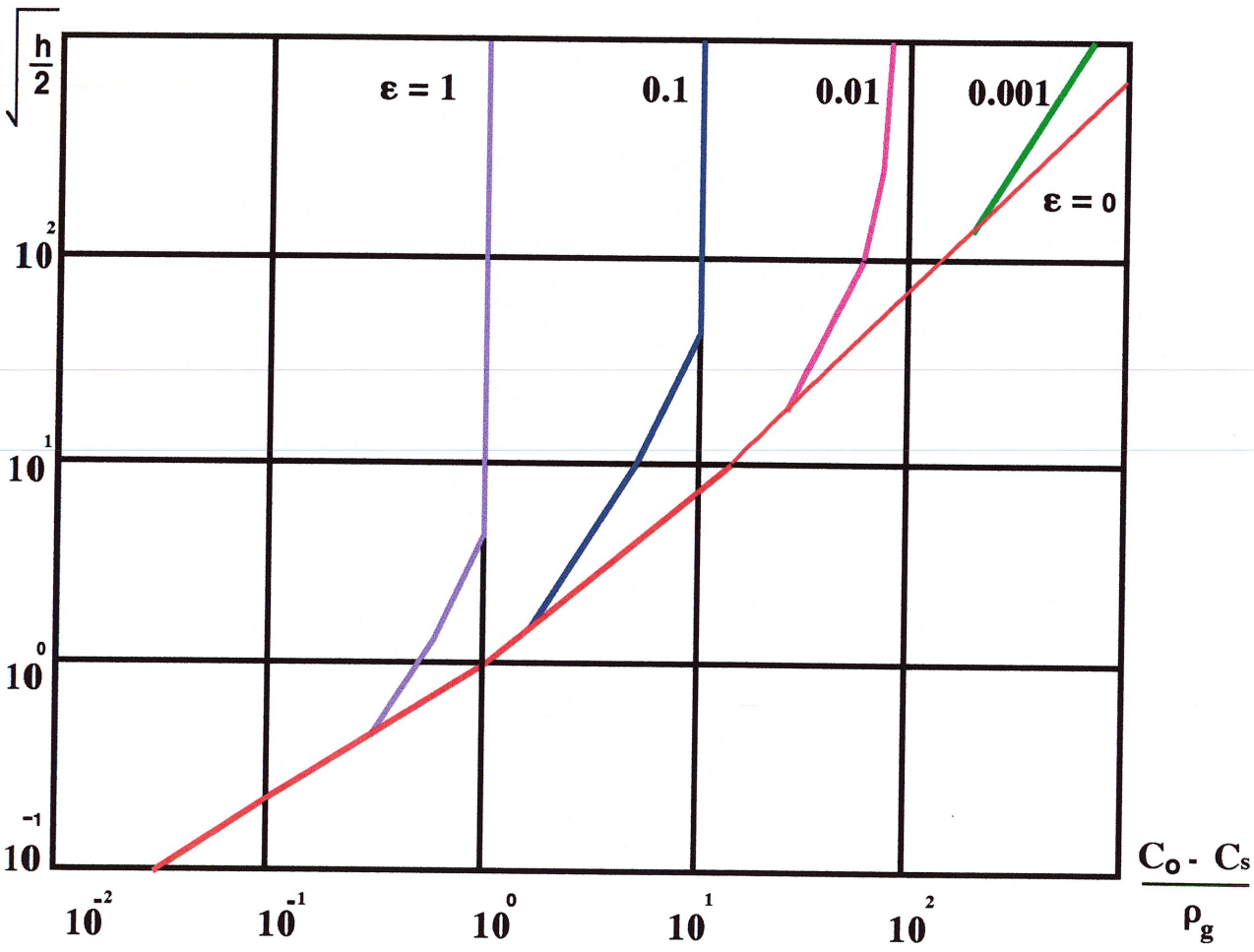


Fig. (3)

CHAPTER II

Shock waves in a liquid with soluble gas bubbles

2.1 Introduction

Shock waves in liquids with bubbles of an insoluble and non-condensable (inert) gas have been investigated theoretically and experimentally (Batchelor 1969, Wijngaarden 1970, 1972, Kutateladze et al. 1972, Gel' fand et al. 1973, Noordzij 1973, Noordzij & Wijngaarden 1974, Nigmatulin, Khabeev & Shagapov 1974, Gubaidulin, Ivandaev & Nigmatulin 1976, Kuznetsov et al. 1978, Drumheller, Kipp & Bedford 1982, Tan & Bankoff 1984).

An analysis has been undertaken (Nigmatulin et al. 1974) that allows for non-equilibrium effects caused by the inertia and viscosity of the liquid in its radial motion around the bubbles and by the translational motion of the bubbles relative to the carrier liquid, as well as by a finite thermal conductivity of the gas (thermal non-equilibrium).

It is shown that the structure of the steady wave in a gas-liquid mixture with bubbles of constant mass has an appreciable thickness (for bubbles with a radius of $\sim$ 1mm this thickness is of the order of 1m), and that for not too viscous liquid the two-velocity inertial and viscous effects due to bubble longitudinal motion relative to the liquid are hardly noticeable a gainst the back ground of the interphase heat-transfer effects determined by the gas thermal diffusivity. An investigation of transient waves has revealed the effect of shock-wave enhancement, when a shock pulse is divided in to solitary waves the amplitudes of which may greatly exceed the initial pulse amplitude (Nigmatulin 1982). The effect results from the previously mentioned local deformation inertia of a bubbly liquid which can compress even though the Loading has already been removed.

This effect depends strongly on the signal duration, volume concentration of the bubbles and, strange as it may seen, on the properties of the gas.

The propagation of waves in a liquid with vapour bubbles has been investigated by Trammell (1962), Nakoryakov et al. (1984), Nigmatulin et al. (1982) and Zuong Ngok Hai, Nigmatulin & Khabeev (1982, 1984). In work by Borisov et al. (1977); Gel'fand et al. (1978) and Borisov et al. (1982) the effect of a strong local increase in the intensity of the waves propagation through the mixture was experimentally observed and it was discovered that waves in these media may have a peaky structure. Borisov et al. (1977, 1982) showed that the anomalous growth of pressure in a bubbly vapour-liquid medium during wave propagation is connected with fragmentation of large bubbles in to numerous small ones and a rapid condensational collapse of the latter, As a result, the kinetic energy of the liquid radial motion around the bubbles is transformed in to the energy of compression of the liquid.

Nigmatulin, Khabeev & Nagiev (1979), Zuong Ngok Hai et al. (1982, 1984), Nigmatulin et al. (1982) proved theoretically that in these media, owing to radial motion around bubbles, the waves may have a peaky structure, and the effect of a pressure increase in the wave was estimated in terms of the cell model.

Acoustics of liquids with soluble gas bubbles was investigated by Khabeev & Shagapov (1988). Areview of modern problems connected with shock-wave propagation in bubbly gas-liquid media is given elsewhere (Nigmatulin 1982, Gubaidullin et al 1982, Nigmatulin et al 1988).

The processes of the propagation of waves in bubbly liquids are experimentally investigated in vertical shock tubes (Noordzij 1973, Kuznetsov 1978). Their generic layout is shown in Fig. (1). The shock tube consists of a high-pressure chamber (1) (HPch) and an operating part, or a law-pressure chamber (2) (LPch), which are separated by a diaphragm (3). The gas is pumped in to the HPch, where a high pressure is built up, and the LPch is filled with liquid to a

level slightly lower than the diaphragm, bubbles of a predetermined radius are dispersed in the liquid so that a mixture resembling a monodisperse system is produced. The bubble radii R varied in separate experiments within a range 0.2 - 2mm, and their volume concentration α_g, measured by the rise of the column of liquid, ranged from 0.01-0.1. Following the diaphragm rupture affected by gas contained in the HPch, a shock wave which propagates through the bubbly mixture downward is produced in the LPch. The disturbance duration is defined by the size of HPch, and its pressure in the HPch.

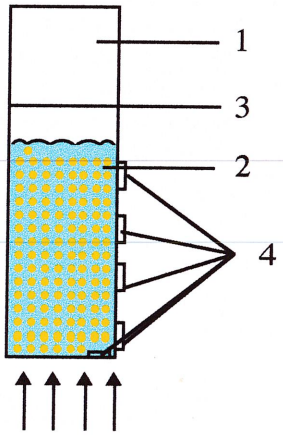


Fig.(1)

The pressure evolution in the mixture is recorded by quick-response pressure indicators installed in several locations throughout the liquid column in the LPch wall.

The primary property of such shock waves in bubbly liquid (with either gas or vapour) is that at some conditions they are characterized by an oscillatory structure associated with the bubble-volume pulsations due to both liquid inertia and gas elasticity (G. Batchelor (1967), L. Wijngaarden (1972), V. E Nakoryakov et al. (1983), R. Nigmatulin (1982), Nigmatulin 1990) .

Shock waves propagating in a liquid with soluble gas bubbles were discussed in this chapter. An investigation of the structure of steady shock waves in a mixture of a liquid with soluble gas bubbles is presented.

The existence of the steady structure of shock waves is proved.

2.2 Equations for stationary one-dimensional flow

In this section we will consider a particular case of nonsoluble gas bubbles. Following to (Nigmatulin 1990) let us analyse of a stationary wave structure in a simplified formulation which is of special interest for further studies.

In this case, one assumes that: the carrying liquid is incompressible, there is no fragmentation and no coalescence of bubbles, there is no capillary effects, the bubble volumetric concentration is small, and the most important, the gas with in bubbles is polytropic. In this case, the effective viscosity in the radial - motion equations is used to take in to a ccount the thermal dissipation during oscillations. (Nigmatulin 1990).

Wave processes in a bubbly liquid are considered here using continuum - mechanics methods under the following basic assumptions:

(i) the distances over which the flow parameters (for example, oscillatory wavelengths) very significantly are much larger than the distances between the bubbles, which are themselves much larger than the bubble diameters (i.e the volume fraction of the vapour phase is small enough, $\alpha_V \lesssim 0.1$.)

(ii) the mixture is locally monodispersed, i.e in each materialvolume all the bubbles are spherical and of the same radius.

(iii) Viscosity and thermal conduction are important only in the processes of interphase interaction and , in particular, in bubble pulsations.

(iV) nucleation, fragmentation, interaction and coagulation of the bubbles are absent.

(V) the velocities of the macroscopic motion of the phases coincide .

The last assumption allows us to describe bubble volume changes, concentration distribution around the bubbles in terms of the spherically symmetrical model.

The indicated simplifications lead to a system of equations, these equations may be written in a stationary wave-related frame system (parameters in which are time-independent) in the following form.

$$\frac{d}{dx}\left(\alpha_L V\right) = 0 \ , \ \frac{d}{dx}\left(\rho_g^o \alpha_g v\right) = 0 \ , \ \frac{d}{dx}\left(nv\right) = 0$$

$$v\frac{dR}{dx} = \omega \ , \ Rv\frac{d\omega}{dx} = \frac{P_g - P_L}{\rho_L^o} - \frac{4\mu_{eff}\omega}{R\rho_L^o} - \frac{3\omega^2}{2}$$

$$\rho v\frac{dv}{dx} + \frac{dp}{dx} = 0 \ , \frac{\rho_g^o}{\rho_{g_o}^o} = \left(\frac{R_o}{R}\right)^3 \ , \ \frac{P_g}{P_{g_o}} = \left(\frac{R_o}{R}\right)^{3\chi} \qquad (2.2.1)$$

$$\rho = \rho_L^o \alpha_L + \rho_g^o \alpha_g \approx \rho_L^o \alpha_L \ , \ \alpha_g = \frac{4}{3}\Pi R^3 n$$

Where $\frac{d}{dx}\left(\alpha_L V\right) = 0$, $\frac{d}{dx}\left(\rho_g^o \alpha_g v\right) = 0$ represent the

Continuity equations for liquid and gas respectively,

$Rv\frac{d\omega}{dx} = \frac{P_g - P_L}{\rho_L^o} - \frac{4\,\mu_{eff}\omega}{R\rho_L^o} - \frac{3\omega^2}{2}$ represent Rayleigh equation,

$\rho v\frac{dv}{dx} + \frac{dp}{dx} = 0$ represent Euler equation , $\frac{\rho_g^o}{\rho_{g_o}^o} = \left(\frac{R_o}{R}\right)^3$

represent mass conservation equation for gas and $\frac{d}{dx}\left(nv\right) = 0$

represent the equation for the number of bubbles.

The subscripts L , g show that the parameters pertain to liquid and gas, respectively ; while the subscript o denotes that the values of the parameters in the initial state, ρ_L^o, ρ_g^o are the true densities of phases, α_L , α_g are the volume contents of phases, v is the macroscopic velocity along the x - axis, n is the number of bubbles in a unit volume of the mixture, ω is the radial velocity of the liquid at the bubble wall, P_L , P_g are the average pressures of phases, μ_{eff} is the effective viscosity of the liquid, R is the bubble radius. χ is the polytropic expo-

nent of the gas.

The equilibrium parameters before the wave ($x \to + \infty$) , (they must be predetermined), are denoted by a subscript o, and those behind the wave ($x \to - \infty$) (where equilibrium is also established) by a subscript e. Therefore, the boundary conditions are

$$x \to + \infty : \; V = V_o = - D_o < 0 \quad , \quad P = P_g = P_o \quad , \quad \omega_o = 0$$

$$\alpha_g = \alpha_{go} \quad , \quad \rho_g^o = \rho_{go}^o \quad , \; R = R_o$$

Where D_o is the wave velocity relative to the undisturbed medium

$$x \to - \infty : \quad P = P_g = P_e \quad , \; \omega_e = 0 \qquad (2.2.2)$$

The system of equations (2 . 2 . 1) has the following four first integrals

$$\alpha_L V = \alpha_{Lo} V_o \quad , \quad \alpha_g \rho_g^o V = \alpha_{go} \rho_{go}^o V_o \quad , \quad nv = n_o V_o$$

$$\rho_L^o \; \alpha_{L_o} V_o V \; + \; P = \rho_L^o \alpha_{L_o} V_o^2 \; + \; P_o \qquad (2.2.3)$$

These integrals enable us to determine the equilibrium parameters behind the wave based on parameter before the wave

$$\alpha_{Le} Ve = \alpha_{Lo} V_o \; , \alpha_{ge} Ve \left(\frac{P_e}{P_o} \right)^{1/\chi} = \alpha_{go} V_o \; , V_o - Ve = \frac{P_e - P_o}{\rho_L^o \alpha_{L_o} V_o} \qquad (2.2.4)$$

Let us introduce dimensionless Variables denoted by bars

$$\overline{P_i} = \frac{P_i}{P_o}, \; \overline{R} = \frac{R}{R_o}, \; \overline{V} = \frac{V}{C_\alpha} \quad , \quad \overline{\omega} = \frac{\omega}{C_*} \quad , \; \overline{x} = \frac{x}{L_\alpha} \qquad (2.2.5)$$

$$\left(\; C_* = \sqrt{\frac{P_o}{\rho_L^o}} \; , \; C\alpha = \frac{C_*}{\sqrt{\alpha_{L_o} \alpha_{g_o}}} \quad , \quad L_\alpha = \frac{R_o}{\sqrt{\alpha_{L_o} \alpha_{g_o}}} \right) \quad , \quad \left(i = L \, , g \right)$$

Then, we obtain the following equation for dimensionless pressure $\bar{P}_e$ behind the wave, which may be referred to as the shock adiabat of the medium under consideration in coordinates (Nigmatulin 1990).

From (2 . 2 . 4) divide by P_o , We obtain

$$\bar{P}_e - 1 = \frac{\rho_L^o \alpha_{L_o} v_o}{P_o} (v_o - v_e) \qquad (2.2.6)$$

Since $v_o \alpha_{Lo} = v_e \alpha_{Le}$, $\alpha_L + \alpha_g = 1$, we get

$$v_o - v_e = v_o \alpha_{go} \left[1 - \frac{v_e \alpha_{ge}}{v_o \alpha_{go}} \right]$$

Since $\bar{P}_e^{-1/\chi} = \frac{v_e \alpha_{ge}}{v_o \alpha_{go}}$

Hence $v_o - v_e = v_o \alpha_{go} \left[1 - \bar{P}_e^{-1/\chi} \right]$

Substitute for $v_o - v_e$ in (2 . 2 . 6) , we obtain

$$\bar{P}_e - 1 = \bar{D}_o^2 \left[1 - \bar{P}_e^{-1/\chi} \right] \qquad \left(\bar{D}_o = -\bar{v}_o = \frac{D_o}{C_\alpha} \right) \qquad (2.2.7)$$

Where $\bar{D}_o$ is the dimensionless wave velocity relative to the undisturbed medium.

The minimum value $D_o = C_o$ of the velocity, which corresponds to the stationary compression wave ($\bar{P}_e > 1$) partially defines the equilibrium, polytropic for gas , speed of sound in the mixture,

$$\bar{C}_o = \sqrt{\chi} \;, C_o = C_\alpha \sqrt{\chi} = \sqrt{\frac{\chi P_{L_o}}{\rho_L^o \alpha_{L_o} \alpha_{g_o}}} \qquad (2.2.8)$$

For an isothermal gas ($\chi = 1$) and strong waves ($\bar{P}_e >> 1$, the pressure behind the wave is independent of the gas polytrope) , we have (Nigmatulin 1990).

$$\bar{P}_e = \bar{v}_o^2 = \bar{D}_o^2 \qquad (2.2.9)$$

From the mass conservation equation for liquid, we have

$$\alpha_L \frac{dV}{dx} + V \frac{d\alpha_L}{dx} = 0$$

From $\frac{d\alpha_L}{dx} = - \frac{d\alpha_g}{dx}$, we get

$$\alpha_L \frac{dv}{dx} = v \frac{d\alpha_g}{dx} \qquad (2.2.10)$$

and in the dimensionless form it became

$$\frac{d\bar{V}}{d\bar{x}} = \frac{\bar{V}}{\alpha_L} \frac{d\alpha_g}{d\bar{x}} \qquad (2.2.11)$$

From (2 . 2 . 10) and $\alpha_g = 4/3\ \Pi R^3 n$, we get

$$\frac{dv}{dx} = \frac{V}{\alpha_L} \left(4\Pi R^2 \frac{dR}{dx} n + \frac{4}{3} \Pi R^3 \frac{dn}{dx} \right) \qquad (2.2.12)$$

From $V \frac{dn}{dx} + n \frac{dv}{dx} = 0$ and $V \frac{dR}{dx} = \omega$, substitute in (2 . 2 . 12)

$$\frac{dv}{dx} = \frac{4\Pi R^2}{\alpha_L} \omega n - \frac{\alpha_g}{n\alpha_L} n \frac{dv}{dx}$$

Then $\frac{dv}{dx} = \frac{3\alpha_g \omega}{R}$, and in the dimensionless form it became

$$\frac{d\bar{V}}{d\bar{x}} = \frac{3\alpha_g \bar{\omega}}{\bar{R}} \qquad (2.2.13)$$

Hence from (2 . 2 . 11) and (2 . 2 . 13)

$$\frac{d\bar{V}}{d\bar{x}} = \frac{\bar{V}}{\alpha_L} \quad \frac{d\alpha_g}{d\bar{x}} = \frac{3\alpha_g\bar{\omega}}{\bar{R}} \qquad (2.2.14a)$$

From integrals (2 . 2 . 3) , we obtain

$$\frac{\alpha_g}{\alpha_{g_o}} = \frac{\rho^o_{g_o} V_o}{\rho^o_g V} = \frac{V_o}{V} \left(\frac{R}{R_o}\right)^3$$

and in the dimensionless from it became

$$\frac{\alpha_g}{\alpha_{g_o}} = \frac{\bar{V}_o}{\bar{V}} \bar{R}^3 \qquad (2.2.14b)$$

From $\alpha_L V = \alpha_{L_o} V_o$ and $\alpha_L + \alpha_g = 1$, we get

$V = \alpha_{L_o} V_o + \alpha_g V$, and in the dimensionless form it can be written as

$$\bar{V} = \alpha_{L_o} \bar{V}_o + \alpha_g \bar{V}$$

by using (2 . 2 . 14 b) , we have

$$\bar{V} = \bar{V}_o (\alpha_{L_o} + \alpha_{g_o} \bar{R}^3) \qquad (2.2.14c)$$

From integrals (2 . 2 . 3) , we have

$$\rho^o_L \alpha_{L_o} V_o V + P_L = \rho^o_L \alpha_{L_o} V_o^2 + P_o$$

divide both sides by P_o and after some simplifications, we get

$$\bar{P}_L = 1 + \bar{V}_o^2 (1 - \bar{R}^3) \qquad (2.2.14d)$$

Upon some elementary transformations, and using (2 . 2 . 14), the system of

equations (2 . 2 . 1) may be represented in a form which expresses the parameter gradients in terms of $\bar{\omega}$ and $\bar{P}_{gL} = \bar{P}_g - \bar{P}_L$ (Nigmatulin 1990)

From (2 . 2 . 1) , we have

$$R^V \frac{d\omega}{dx} = \frac{P_g - P_L}{\rho_L^{\rho}} - \frac{4\,\mu_{eff}\omega}{R\rho_L^{\rho}} - \frac{3\,\omega^2}{2}$$

It can be written in the dimensionless form as follows

$$\bar{R}^{\bar{V}} \frac{d\bar{\omega}}{d\bar{x}} = \bar{P}_g - \bar{P}_L - \frac{4\mu_{eff}\,\bar{\omega}}{R\sqrt{P_o\,\rho_L^o}} - \frac{3\,\bar{\omega}^2}{2}$$

Hence $$\frac{d\bar{\omega}}{d\bar{x}} = \frac{1}{\bar{R}\bar{V}} (\bar{P}_g - \bar{P}_L) - \frac{4\,\mu^*\bar{\omega}}{\bar{R}^2\bar{V}} - \frac{3\,\bar{\omega}^2}{2\,\bar{R}\bar{V}}$$ (2.2.15a)

Where $$\mu^* = \frac{\mu_{eff}}{\rho_L^o R_o\, C_*}$$

we have $$\frac{dp_g}{dt} = \frac{-3\chi P_g}{R}\,\frac{dR}{dt}$$

Since $$\frac{dP_g}{dt} = \frac{dP_g}{dx} \cdot \frac{dx}{dt} = V\,\frac{dP_g}{dx}$$

Substitute for $\frac{dP_g}{dt}$ in the above equation, we get

$$V\,\frac{dP_g}{dx} = \frac{-3\chi P_g}{R}\,\frac{dR}{dt}$$

this equation canbe written in the dimensionless form as follows

$$\bar{V}\,\frac{d\bar{P}_g}{d\bar{x}} = \frac{-3\chi\bar{P}_g\bar{\omega}}{\bar{R}}$$ (2.2.15b)

From Euler equation in (2 . 2 . 1) , we have

$$\frac{dP}{dx} = - \rho V \frac{dv}{dx}$$

After some simplifications and using (2 . 2 . 13) Euler equation in dimension-less form can be written as

$$\frac{d\bar{P}}{d\bar{x}} = \frac{-3\ \bar{\nabla}_o^2 \bar{\omega} \bar{R}^2}{\bar{\nabla}} \qquad (2.2.15c)$$

from $\left(\frac{P_g}{P_{g_o}}\right) = \left(\frac{R_o}{R}\right)^{3\chi}$, and using $\bar{R} = \frac{R}{R_o}$, we obtain

$$\bar{R} = \bar{P}_g^{-1/3\chi} \qquad (2.2.15d)$$

From (2 . 2 . 14 c) substituting in (2 . 2 . 14 d) , we get

$$\bar{\nabla} = \bar{\nabla}_o - \frac{\alpha g_o\ (\bar{P} - 1)}{\bar{\nabla}_o} \approx \bar{\nabla}_o \qquad (2.2.15e)$$

2.3 Investigation of the solution behaviour before (up stream) the shock wave

Similar as the previous section we will consider nonsoluble gas bubbles. Points o and e correspond to the equilibrium states before ($x \to +\infty$) and behind ($x \to -\infty$) the wave, these are the critical points of the applicable system of differential equations in the space of physical parameters $\bar{\omega}$,$\bar{P}_L$ and $\bar{P}_g$. We shall investigate the behaviour of the solution in the neighborhood of the point o using the linearization approach. To describe the zone of the shock - wave front where deviations of variables $\bar{V}$, $\bar{R}$, $\bar{P}_L$, $\bar{P}_g$ and $\bar{\omega}$ from their values in the initial state o ($\bar{V} = \bar{V}_o$, $\bar{R} = \bar{P}_L = \bar{P}_g = 1$, $\bar{\omega} = 0$) are sufficiently small, the variable coefficients of $\bar{P}_g - \bar{P}_L$ and $\bar{\omega}$ may be assumed as constant quantities defined by the values of variables in state o.

Then, we obtain a system of linear equations

$$\frac{d\bar{\omega}}{d\bar{x}} = \frac{-4\mu^*\bar{\omega}}{\bar{V}_o} + \frac{1}{\bar{V}_o}\,\bar{P}_{g_L} \qquad (\bar{P}_{g_L} = \bar{P}_g - \bar{P}_L)$$

$$\frac{d\bar{P}_{g_L}}{d\bar{x}} = 3\left(\bar{V}_o - \frac{\chi}{\bar{V}_o}\right)\bar{\omega} \qquad (2.3.1)$$

$$\frac{d\bar{P}_L}{d\bar{x}} = -3\bar{V}_o\,\bar{\omega}$$

The fundamental solutions of this linear system are

$$\bar{\omega} = A^{(\omega)} \exp k_o\,(\bar{x} - \bar{x}_b)$$

$$\bar{P}_L = 1 + A_1^{(p)} \exp k_o\,(\bar{x} - \bar{x}_b) \qquad (2.3.2)$$

$$\bar{P}_g = 1 + A_2^{(p)} \exp k_o\,(\bar{x} - \bar{x}_b)$$

Where $\bar{x}_b$ is some point where we fixed thevalue of the parameter and k_o is in general a complex number. Only the real part of these expressions is understood as a solution of the physical problem. Since the parameter deviation from an equilibrium state before the wave at $x \rightarrow +\infty$ must approach zero : $\bar{P}_{gL}, \bar{\omega} \longrightarrow 0$ the real part k_o should be negative

$$\text{Re}\{k_o\} < 0 \qquad (2.3.3)$$

If the fundamental solutions are substituted in to (2. 3. 1) , we obtain the characteristic equation for k_o , which specifies the condition for existence of nontrivial solutions of the type (2 .3. 2) with nonzero constants $A^{(\omega)}$, $A_1^{(p)}$, and $A_2^{(p)}$

$$K_o^2 - 4\mu^* K_o - 3\chi(\bar{D}_o^2 C_o^{-2} - 1) = 0 \qquad (2.3.4)$$

Where $K_o = k_o \bar{D}_o = - k_o \bar{\nabla}_o$

In this case, the constants themselves are expressed by linear relations

$$A^{(\omega)} = - \frac{k_o \bar{\nabla}_o}{3\chi} A_2^{(p)} \quad , A_1^{(p)} = - \frac{\alpha_{Lo} \alpha_{go} \bar{\nabla}_o^2}{\chi} A_2^{(p)} \qquad (2.3.5)$$

The condition (2. 3. 3) leads to $D_o > C_o$. Thus in the medium under discussion, only stationary compression waves ($P_e > P_o$) are feasible. and the stationary rarefaction waves are impossible. Only one root of the characteristic equation

$$\bar{D}_o k_o = 2\mu^* - \sqrt{4\mu^{*2} + 3\chi (\bar{D}_o^2 C_o^{-2} - 1)} \qquad (D_o > C_o > 1) \qquad (2.3.6)$$

may be used as k_o , this expression is a real number which is in conformity with the monotonic profile, or the absence of oscillations before the wave.
(Nigmatulin 1990).

2.4 Finding a solution for shock - wave structure near critical point

We shall outline the procedure for finding a solution of a system of equations (2 . 2 . 1) or (2 . 2 . 15) with the boundary conditions (2 . 2 . 2). This solution passes through two critical points which relate to the initial and final states before and behind the shock wave. It describe the structure of this wave. The integral curves of the system of equations (2 . 2 . 1) or (2 . 2 . 15) allow a displacement a long axis x . Therefore, we shall fix a certain value of the gas dimensionless pressure with in bubbles $\overline{P}_g > 1$ or $A_2^{(p)} = \overline{P}_g(\overline{x}_b) - 1$ at $\overline{x} = \overline{x}_{b.}$, the quantity $\overline{P}_g(\overline{x}_b)$ must be sufficiently close to unity in this case for the linear solution (2 . 3 . 2) in the domain $\overline{x} > \overline{x}_b$ to be satisfied, the structure is determined for the entire shock wave in which the medium transits from the equilibrium state o in to an equilibrium state e . (Nigmatulin 1990)

2.5 Structure of stationary shock waves in a liquid with soluble gas bubbles

We shall now be concerned with stationary waves in a liquid with soluble gas bubbles similar to the analysis of such waves in a liquid with vapour bubbles (Nigmatulin 1990) .

Therefore, pressure, temperature, and density with in a bubble are said to be homogeneous. Thus, all parameters, except for C, depend only on the longitudinal coordinate x, and the concentration is C = C (x,r) , where r is the distance to the bubble center, As a result, the system of equations for a one dimensional stationary flow of liquid with soluble gas bubbles, similar to (2 . 2 . 1), assumes the form.

$$\frac{d}{dx}(\rho v) = 0 \quad , \quad \frac{d}{dx}(nv) = 0 \quad , \quad v\frac{d}{dx}(4/3\Pi R^3 \rho_g^o) = 4\Pi R^2 j$$

$$(\rho = \rho_L + \rho_g \quad , \quad \rho_i = \rho_i^o \alpha_i \quad (i = L, g) \quad , \quad \alpha_L + \alpha_g = 1 \quad , \quad \alpha_g = 4/3\Pi R^3 n)$$

$$\frac{d}{dx}(\rho v^2 + P_L) = 0 \qquad (2.5.1a)$$

$$v\frac{dR}{dx} = \omega + \frac{j}{\rho_L^o}$$

$$Rv\frac{d\omega}{dx} + \frac{3}{2}\omega^2 + \frac{4\mu_L \omega}{\rho_L^o R} = \frac{P_g - P_L - 2\sigma/R}{\rho_L^o}$$

Where μ_L is the coefficient of the viscosity of the liquid.

Transfer processes in a two - phase mixture are determined by the distributions

of microparameters near inhomogeneities.

For an analysis not to be too complicated. One has to make use of models that could considerably simplify the microprocess equations. A possible model is one that employs the concept of a cell with a test bubble in it at any point specified by a vector x. The cell dimensions are determined by the volume fractions of the phases and equal to $R\alpha_g^{-1/3}$, the cell center coinciding with the center of the test bubble. This cell moves with the macroscopic velocity v (t , x)of the gas phase at the point considered, The distributions of microparameters outside a cell are described by the equations for the corresponding microprocesses with the boundary conditions on the test bubble surface.

Consider a spherically symmetric test bubble with its center at a point x , the microparameters outside the bubble being dependent on time t, the position of the bubble center x, and the distance r of a microparticle from the center

$C = C (t , x , r)$

To determine the concentration distribution, we use the diffusion equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial r} + v \frac{\partial C}{\partial x} = \frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) , \quad r > R \qquad (2.5.1b)$$

The mass transfer rate may be found from the boundary condition on the bubble surface.

$$j = D \left. \frac{\partial C}{\partial r} \right|_R$$

Where u (r , t) is the radial velocity of the liquid at any point &

$u (r , t) = \frac{R^2}{r^2} \omega$, ω is the radial velocity of the liquid at the bubble surface.

$$\rho_L^o = \text{const.} \quad , \quad P_g = R_g \rho_g^o T_g \, , \, C \Big|_{r=R} = C_s = \Gamma P_g \, , \, u_g = C_{vg} T_g + u_{go}$$

We ignore both capillarity and viscosity effects, gas mass concentration, and gas density compared to liquid density, i.e

$$\frac{2\sigma}{RP_o} << 1\ ,\ \mu^* << 1\ ,\ \rho_g / \rho_L << 1\ ,\ \rho_g^o << \rho_L^o\ ,\text{where } \mu^* = \frac{\mu_L}{\rho_L^o R_o C_*}$$

We shall introduce dimensionless variables (2 . 2 . 5), and also the following variables

$$\eta = \frac{r}{R(x)}\ ,\ \bar{C} = \frac{C}{C_o}\ ,\ \bar{j} = \frac{j}{\rho_{go}^o C_*}\ ,\ \bar{n} = \frac{n}{n_o}\ ,\ \bar{\rho}_i^o = \frac{\rho_i^o}{\rho_{io}^o} \qquad (2.5.2)$$

The system of equation (2 . 5 . 1 a) has first integrals which, resulting from simplifications in dimensionless variables, may be written as

$$\frac{d}{dx}(\rho V) = 0$$

integrate we get

ρV = const.

since $\rho V = (\rho_g + \rho_L) V$ = const.

and $\rho_g << \rho_L$

$\therefore\ \rho_L V = \rho_{Lo} V_o$

And in the dimensionless form it is written as

$$\rho_L \bar{V} = \rho_{Lo} \bar{V}_o \qquad (2.5.3a)$$

After integrating $\frac{d}{dx}(nv) = 0$, we get

nv = const. and in the dimensionless form it became

$$\bar{n}\bar{v} = \bar{v}_o \qquad (2.5.3b)$$

integrate Euler equation in (2 . 5. 1a) we get

$$\rho v^2 + P = \text{const.}$$

i.e

$$\rho v^2 + P = \rho v_o^2 + P_o$$

Substitute for $\rho_L = \rho_L^o \alpha_L$ we get

$$\rho_L^o \alpha_L v^2 + P_L = \rho_L^o \alpha_{L_o} v_o^2 + P_o$$

divide both sides by P_o and after some simplification, we obtain

$$\frac{\bar{v}\bar{v}_o}{\alpha g_o} + \bar{P}_L = \frac{\bar{v}_o^2}{\alpha g_o} + 1 \qquad (2.5.3c)$$

Similar as chapter I we use $\eta = \frac{r}{R(t)}$ and $\left.\frac{\partial}{\partial t}\right|_r = \left.\frac{\partial}{\partial t}\right|_\eta + \frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial t}$

the diffusion equation became

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial x} + \frac{1}{R}\frac{dR}{dt}\left(-\eta + \frac{1}{\eta^2}\right)\frac{\partial C}{\partial \eta} = \frac{D}{R^2\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial C}{\partial \eta}\right) \quad (\eta \geq 1) \qquad (2.5.4)$$

with the boundary conditions at the bubble surface and remote from the bubble

$\eta = 1$: $C = C_s$

$\eta = \infty$: $C = C_o$

by using $\frac{\partial C}{\partial t} = 0$ and substituting in (2. 5. 4), and rewritten in dimensionless variables, as follows

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{x}} = \frac{\bar{\omega}}{\bar{R}}\frac{(\eta^3 - 1)}{\eta^2}\frac{\partial \bar{C}}{\partial \eta} + \frac{D^*}{\bar{R}^2\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2\frac{\partial \bar{C}}{\partial \eta}\right) \quad (\eta \geq 1) \qquad (2.5.4a)$$

Where $D^* = \frac{D}{R_o C_*}$

$$\eta = 1 : \bar{C} = \frac{C_s}{C_o} = \bar{C}_s$$

$$\eta = \infty : \bar{C} = \frac{C_o}{C_o} = 1$$

The differential equations (2 . 5 . 1 a) and

$$\frac{dP_g}{dt} = \frac{-3\chi P_g}{R}\frac{dR}{dt}$$

rewritten in dimensionless variables, as follows

$$\bar{v}\frac{d\bar{R}}{d\bar{x}} = \bar{\omega} + \bar{j}\frac{\rho_{go}}{\rho_L} \approx \bar{\omega}$$

$$\bar{R}\bar{v}\frac{d\bar{\omega}}{d\bar{x}} = \bar{P}_g - \bar{P}_L - \frac{3}{2}\bar{\omega}^2 \qquad (2.5.4b)$$

$$\bar{v}\frac{d\bar{P}_g}{d\bar{x}} = \frac{-3\chi\bar{P}_g}{\bar{R}}\bar{\omega}$$

The equation of state of gas

$$\bar{\rho}_g^0 = \frac{\bar{P}_g}{T(\bar{P}_g)}$$

$$\bar{v} = \bar{v}_o \quad (\alpha_{L_o} + \alpha g_o \bar{R}^3) \qquad (2.5.5)$$

$$\bar{P}_L = 1 + \bar{v}_o^2 (1 - \bar{R}^3)$$

and we have

$$\frac{dC_s}{dP} = \Gamma \qquad , \Gamma : \text{const.} \qquad (2.5.6)$$

The solution of this equation

$$C_s = \Gamma P \qquad (2.5.7)$$

The stationary - wave structure is defined by boundary conditions before and behind the wave. The initial equilibrium state before the wave is determined by parameters with subscript 0 .

$$\bar{v} = \bar{v}_o \quad , \quad \bar{P}_L = \bar{P}_g = 1 \quad , \quad \bar{C} = 1 \quad , \quad \bar{\omega} = 0 \quad , \quad \bar{R} = 1 \qquad (2.5.8)$$

and the final state e behind the wave in the form of liquid phase is defined by parameters with subscript e , parameters in this state being found from the finite relations (2 . 5 . 3 a) , (2 . 5 . 3 b) , (2 . 5 . 3 c) and (2 . 5 . 5)

$$\bar{v}_e = \alpha_{Lo} \bar{v}_o \, , \quad \bar{P}_e = 1 + \bar{v}_o^2 \, , \quad \bar{C} = 1 \quad , \quad \bar{R}_e = 0 \qquad (2.5.9)$$

In the case of the mixture of incompressible liquid with vapour bubbles, where $\bar{R}_e = 0$, this relationship has a form $\bar{P}_e = 1 + \bar{v}_o^2$, which is the same for soluble gas bubbles.

Note that, from the lasts formula (2 . 5 . 5)

$$\bar{P}_L = 1 + \bar{\nabla}_o^2 - \bar{\nabla}_o^2 \bar{R}^3$$

$$= \bar{P}_e - (\bar{P}_e - 1)\bar{R}^3 \leq \bar{P}_e \qquad (2.5.10)$$

The system (2 . 5 . 4b) simplifies and is reduced to the following equations

$$\bar{\nabla} \frac{d\bar{R}}{d\bar{x}} \approx \bar{\omega} \ , \ \bar{R}\bar{\nabla} \frac{d\bar{\omega}}{d\bar{x}} = 1 - \bar{P}_L - \frac{3}{2}\bar{\omega}^2 \qquad (2.5.11)$$

$$\bar{P}_L = 1 + (\bar{P}_e - 1)(1 - \bar{R}^3) \quad , \quad \bar{\nabla} = \bar{\nabla}_o (\alpha_{L_o} + \alpha g_o \bar{R}^3)$$

From equation (2 . 5 . 11) , we have

$$\bar{\nabla} \frac{d\bar{R}}{d\bar{x}} = \bar{\omega}$$

Since $\frac{d\bar{\omega}}{d\bar{x}} = \frac{d\bar{\omega}}{d\bar{R}} \cdot \frac{d\bar{R}}{d\bar{x}} = \frac{\bar{\omega}}{\bar{\nabla}} \frac{d\bar{\omega}}{d\bar{R}}$

$$\therefore \ \bar{\nabla} \frac{d\bar{\omega}}{d\bar{x}} = \bar{\omega} \frac{d\bar{\omega}}{d\bar{R}} = \frac{d}{d\bar{R}} \left(\frac{\bar{\omega}^2}{2} \right)$$

Hence $\bar{R} \frac{d}{d\bar{R}} \left(\frac{\bar{\omega}^2}{2} \right) + \frac{3}{2}\bar{\omega}^2 = - (\bar{P}_e - 1)(1 - \bar{R}^3)$

Let us find the solution of this equation, satisfying the condition $\bar{\omega} = 0$ at $\bar{R} = 1$ as follows

$$\frac{d}{d\bar{R}} \left(\frac{\bar{\omega}^2}{2} \right) + \frac{3}{\bar{R}} \frac{\bar{\omega}^2}{2} = \frac{- (\bar{P}_e - 1)(1 - \bar{R}^3)}{\bar{R}}$$

by using integrating factor

$$\int \frac{3}{\bar{R}}\, d\bar{R} \qquad 3\ln\bar{R}$$
$$e \quad = \quad e \quad = \bar{R}^3$$

$$\bar{R}^3 \frac{d}{d\bar{R}}\left(\frac{\bar{\omega}^2}{2}\right) + 3\bar{R}^2\left(\frac{\bar{\omega}^2}{2}\right) = -\,(\bar{P}_e - 1)(1 - \bar{R}^3)\,.\bar{R}^2$$

$$\left[\bar{R}^3\left(\frac{\bar{\omega}^2}{2}\right)\right] = -\,(\bar{P}_e - 1)(1 - \bar{R}^3)\,.\bar{R}^2$$

$$\bar{R}^3 \frac{\bar{\omega}^2}{2} = -\,(\bar{P}_e - 1)\left(\frac{\bar{R}^3}{3} - \frac{\bar{R}^6}{6}\right)$$

$$= -\,(\bar{P}_e - 1)\left(\frac{2\bar{R}^3 - \bar{R}^6}{6}\right) + B \qquad \text{, where B : const.}$$

$$\frac{\bar{\omega}^2}{2} = -\,(\bar{P}_e - 1)\left(\frac{2 - \bar{R}^3}{6}\right) + \frac{B}{\bar{R}^3}$$

$$0 = -\,(\bar{P}_e - 1)\left(\frac{1}{6}\right) + B$$

$$\therefore \; B = \frac{\bar{P}_e - 1}{6}$$

$$\therefore \; \frac{\bar{\omega}^2}{2} = -\,(\bar{P}_e - 1)\left(\frac{2 - \bar{R}^3}{6}\right) + \frac{\bar{P}_e - 1}{6\bar{R}^3}$$

$$= (\bar{P}_e - 1)\left[\frac{1}{6\bar{R}^3} - \frac{(2 - \bar{R}^3)}{6}\right]$$

$$\bar{\omega}^2 = (\bar{P}_e - 1)\left[\frac{1}{3\bar{R}^3} - \frac{(2 - \bar{R}^3)}{3}\right]$$

$$= \frac{(\bar{P}_e - 1)}{3}\left[\ \bar{R}^{-3} - 2 + \bar{R}^{3}\ \right]$$

$$= \frac{\bar{P}_e - 1}{3}\left(\bar{R}^{-3/2} - \bar{R}^{3/2}\right)^2 \qquad (2.5.12)$$

substitute (2 . 5 . 12) and the last equation (2 . 5 . 11) in the first equation (2 . 5 . 4b), we obtain

$$\frac{\alpha_{Lo} + \alpha_{go}\bar{R}^3}{\bar{R}^{-3/2} - \bar{R}^{3/2}}\, d\bar{R} = \frac{d\bar{x}}{\sqrt{3}}$$

integrate both sides, we get

$$\int_0^{\bar{R}} \frac{\alpha_{Lo} + \alpha_{go}\bar{R}^3}{\bar{R}^{-3/2} - \bar{R}^{3/2}}\, d\bar{R} = \frac{\bar{x}}{\sqrt{3}} \qquad (2.5.13)$$

Here, the constant of integration is chosen such that $\bar{R} = 0$ at $\bar{x} = 0$. The obtained solution defines $\bar{x}\ (\bar{R})$, which, together with the closed-form expressions for $\bar{P}_L\ (\bar{R})$, $\bar{v}\ (\bar{R})$, and $\bar{\omega}\ (\bar{R})$, determines the structure of the stationary shock - wave front in a parametric form with respect to $\bar{R}^4$. (Nigmatulin 1990).

Reverting to a more general case. described by the system of equations (2 . 5 . 4a) and (2 . 5 . 4b), we shall similar to the approach used in Sec.2.2-Sec.2.4 , investigate the asymptotic behaviour of a solution of this system of equations in the vicinity of an initial state before the wave. The solution of a linearized system is sought in a form of an exponential of type (2 . 3 . 2) decaying at $\bar{x} \longrightarrow \infty$, this solution for a function of two variables $C\ (\bar{x}, \eta)$ being sought in the form

$$\bar{C} = 1 + A_1^{(c)}(\eta) \exp k_o \bar{x} \quad , \quad \mathrm{Re}\{k_o\} < 0 \ , \ \bar{v}_o < 0 \qquad (2.5.14)$$

For constants $A^{(R)}$, $A^{(\omega)}$, $A_1^{(P)}$, and $A_2^{(P)}$, we obtain the following algebraic equations

Since $\bar{\omega} = A^{(\omega)} \exp k_o \bar{x}$

$$\bar{R} = 1 + A^{(R)} \exp k_o \bar{x}$$

$$\bar{v} = \bar{v}_o \quad , \bar{R} = 1$$

Substitute in the first equation (2 . 5 . 4b) , we get

$$\bar{v}_o k_o A^{(R)} \exp k_o \bar{x} = A^{(\omega)} \exp k_o \bar{x}$$

$$\therefore k_o \bar{v}_o A^{(R)} = A^{(\omega)} \qquad (2.5.15a)$$

Since $\bar{P}_g = 1 + A_2^{(P)} \exp k_o \bar{x}$

$$\bar{P}_L = 1 + A_1^{(P)} \exp k_o \bar{x}$$

From the second equation (2 . 5 . 4b) , we get

$$\bar{R}\bar{v} \frac{d\bar{\omega}}{d\bar{x}} = \bar{P}_g - \bar{P}_L - \frac{3}{2}\bar{\omega}^2$$

$$\bar{R}\bar{v} \frac{d\bar{\omega}}{d\bar{x}} = \bar{P}_g - \bar{P}_L$$

$$\bar{R}\bar{v}_o k_o A^{(\omega)} \exp k_o \bar{x} = (1 + A_2^{(P)} \exp k_o \bar{x}) - (1 + A_1^{(P)} \exp k_o \bar{x})$$

$$k_o \bar{v}_o A^{(\omega)} \exp k_o \bar{x} = (A_2^{(P)} - A_1^{(P)}) \exp k_o \bar{x}$$

$$k_o \bar{v}_o A^{(\omega)} = A_2^{(P)} - A_1^{(P)} \qquad (2.5.15b)$$

Substitute in the last equation (2 . 5 . 5), we get

$$1 + A_1^{(P)} \exp k_o \bar{x} = 1 + \bar{\nabla}_o^2 (1 - (1 + A^{(R)} \exp k_o \bar{x})^3)$$

$$A_1^{(P)} \exp k_o \bar{x} = \bar{\nabla}_o^2 (1 - (1 + 3A^{(R)} \exp k_o \bar{x}))$$

$$A_1^{(P)} \exp k_o \bar{x} = \bar{\nabla}_o^2 (- 3A^{(R)} \exp k_o \bar{x})$$

$$A_1^{(P)} = - 3 \bar{\nabla}_o^2 A^{(R)} \qquad (2.5.15c)$$

From (2 . 5 . 8) and substitute in the last equation (2 . 5 . 4b) , we get

$$\bar{\nabla}_o k_o \, A_2^{(P)} \exp k_o \bar{x} = - 3\chi A^{(\omega)} \exp k_o \bar{x}$$

$$\therefore k_o \bar{\nabla}_o A_2^{(P)} = - 3\chi A^{(\omega)} \qquad (2.5.15d)$$

and for the function $A_1^{(c)} (\eta)$, a differential equation with pertinent boundary conditions is obtained as
From (2 . 5 . 14) we get

$$\frac{\partial \bar{C}}{\partial \bar{x}} = k_o A_1^{(c)} \exp k_o \bar{x} \qquad (2.5.16)$$

$$\frac{\partial \bar{C}}{\partial \eta} = \exp k_o \bar{x} \; \frac{dA_1^{(c)}}{d\eta}$$

Substituting (2 . 5 .16) in (2 . 5 . 4a) and using (2 . 5 . 8) , we get

$$k_o \bar{\nabla}_o A_1^{(c)} = \frac{D^*}{\eta^2} \frac{d}{d\eta} (\eta^2 \frac{dA_1^{(c)}}{d\eta}) \qquad (2.5.17a)$$

when $\eta = 1 : C = C_s = \Gamma P_g$

substitute in (2 . 5 . 14) , we get

$$\bar{C} = \frac{C_s}{C_o} = 1 + A_1^{(c)} \exp k_o \bar{x}$$

$$\because \quad \frac{C_s}{C_o} = \frac{\Gamma P_g}{\Gamma P_o} = \frac{P_g}{P_o} = \bar{P}_g$$

And $\bar{P}_g = 1 + A_2^{(P)} \exp k_o \bar{x}$

Hence $1 + A_1^{(c)} \exp k_o \bar{x} = 1 + A_2^{(P)} \exp k_o \bar{x}$

$$\therefore \eta = 1 : A_1^{(c)} = A_2^{(P)} \qquad (2.5.17b)$$

When $\eta \longrightarrow \infty : C = C_o$, substitute in (2 . 5 . 14) , we get

$$\therefore \quad \frac{C_o}{C_o} = 1 = 1 + A_1^{(c)} \exp k_o \bar{x}$$

$$\therefore \quad \eta \longrightarrow \infty : A_1^{(c)} = 0 \qquad (2.5.17c)$$

Solution of this boundary - value problem i.e the equations (2 . 5 . 17a) (2 . 5 . 17 b) , (2 . 5 . 17c) is of the form

$$A_1^{(c)} = \frac{A_2^{(p)}}{\eta} \exp \left\{ \frac{K}{\sqrt{D^*}} (1 - \eta) \right\} \qquad \text{Where } K = \sqrt{k_o \nabla_o}$$

To satisfy the boundary condition at $\eta \longrightarrow \infty$, it is required that Re $K > 0$

The condition of existence of nonzero solutions of system of linear algebraic equations (2 . 5 . 15 a) , (2 . 5 . 15 b) , (2 . 5 . 15 c) , (2 . 5 . 15 d) with respect to $A^{(R)}$, $A^{(\omega)}$, $A_1^{(p)}$, $A_2^{(p)}$ Leads to an algebraic characteristic equation of the sixth order with respect to K,

where $K = \sqrt{k_o \bar{\nabla}_o}$ as follows:

Since $k_o \bar{\nabla}_o A^{(\omega)} = A_2^{(P)} - A_1^{(P)}$

$$k_o^2 \bar{\nabla}_o^2 A^{(R)} = \frac{-3\chi A^{(\omega)}}{k_o \bar{\nabla}_o} + 3\bar{\nabla}_o^2 A^{(R)}$$

$$K^4 A^{(R)} = \frac{-3\chi k_o \bar{\nabla}_o A^{(R)}}{K^2} + 3\bar{\nabla}_o^2 A^{(R)}$$

$$K^6 = -3\chi K^2 + 3\bar{\nabla}_o^2 K^2$$

$$= (3\bar{\nabla}_o^2 - 3\chi) K^2$$

Hence $K^6 + 3(\chi - \bar{\nabla}_o^2) K^2 = 0$ (2.5.18)

Two conditions for a characteristic root :

$$\text{Re}\{K\} > 0 \quad , \quad \text{Re}\{k_o \bar{\nabla}_o\} = \text{Re}\{K^2\} > 0 \qquad (2.5.19)$$

Let us analyze the characteristic root from equation (2 . 5 . 18)

$K^2 = 0$ or $K^4 + 3(\chi - \bar{\nabla}_o^2) = 0$

when $K^2 = 0$, so we have two roots are zero and doesn't satisfy the conditions (2 . 5 . 19)

when $K^4 + 3(\chi - \bar{\nabla}_o^2) = 0$

$$K^4 = 3(\bar{\nabla}_o^2 - \chi) \qquad (2.5.20)$$

$$K^2 = \pm\sqrt{3(\bar{\nabla}_o^2 - \chi)} \qquad (2.5.21)$$

Since $\bar{\nabla}_o$ and χ are real numbers so $(\bar{\nabla}_o^2 - \chi)$ is a real number

Hence (2 . 5 . 21) is a real number.

Note that, because of conditions (2 . 5 . 19) , only $\sqrt{3\,(\,\overline{\nabla}_o^2 - \chi\,)}$ is relevant

$\therefore \quad K^2 = \sqrt{3\,(\,\overline{\nabla}_o^2 - \chi\,)}$

if $\overline{\nabla}_o^2 > \chi$

$\therefore$ K^2 is real number and satisfy Re $K^2 > 0$, and

$K = \pm \sqrt[4]{3\,(\,\overline{\nabla}_o^2 - \chi\,)}$

Since Re $K > 0$

$\therefore \quad K = \sqrt[4]{3\,(\,\overline{\nabla}_o^2 - \chi\,)}$

$\therefore$ The solution exist and satisfy the conditions (2 . 5 . 19)

if $\overline{\nabla}_o^2 < \chi$

$\therefore \quad K^4 = -M$

i.e The R . H . S of expression (2 . 5 . 20) is not positive

$\therefore \quad K^2 = \pm \sqrt{M}\ i$

Which means that Re $K^2 = 0$, and it doesn't satisfy the conditions (2 . 5 . 19)

$\therefore$ The solution doesn't exist.

Hence the required root does exist and is unique, the unique root K is real, positive number.

The solution exist so the shock wave exist. Their integral curve connect the critical points.

In this case, the required root exist only at

$$\overline{v}_o^2 > \chi$$

$$\frac{v_o^2}{C_\alpha^2} > \chi$$

$$v_o^2 > \chi C_\alpha^2$$

$$v_o^2 > \frac{\chi P_o}{\rho_L^o \alpha_{L_o} \alpha_{g_o}} = C_o^2$$

Where C_o is the sound speed in the liquid with gas bubbles, v_o is the shock wave velocity.

If the wave has velocity greater than sound speed the steady shock wave exist.

So we proved the existence of the steady shock waves in liquids with soluble gas bubbles.

CONCLUSIONS

In chapter I we study the growth of spherical symmetric soluble gas bubble controlled by diffusion , and we find analytical self-similar solution by transformation of partial differential diffusion equation to ordinary differential equation.

It is possible only when $R\dot{R}$= constant.

Where R is the radius of the bubble and $\dot{R}$ is the time derivative of the radius , and conclude that the self - similar solution exists only for the bubble growth case when $h \geq 0$, $C_o > C_s$, where the parameter h controlling the radial motion, C_o is the concentration of the gas at remote from the bubble and C_s is the saturation concentration. And also we find that the radius is growing proportional to the square root of time , so the bubble growth law is parabolic.

In chapter II we study the structure of stationary shock waves in a liquid with soluble gas bubbles.

We proved that in the presence of diffusion there exists, unique solution for algebraic characteristic equation of sixth order with respect to $K = \sqrt{k_o \overline{v}_o}$ which was obtained from the condition of existence of nontrivial solution for the system of linear homogeneous equations in the amplitudes $A^{(\Phi)}$, where k_o is exponention coefficient and $\overline{v}_o$ is the dimensionless wave velocity. And the root K is real, positive number.

The solution exists so the shock wave exists. There is integral curve connecting the critical points.

The required root exists only if $v_o^2 > C_o^2$

where v_o is the wave velocity and C_o is the sound speed in the bubbly liquid .

And we conclude that if the wave has velocity greater than sound speed the steady shock wave exists

NOMENCLATURE

A = constant of integration, dimensionless

B = constant of integration, dimensionless

C = mass concentration of gas in the liquid $[M L^{-3}]$

$\mathbf{C_o}$ = sound speed in the bubbly liquid $[LT^{-1}]$

D = mass diffusivity $[L^2T^{-1}]$

$\mathbf{D_o}$ = wave velocity relative to the undisturbed medium $[LT^{-1}]$

$\mathbf{K_o}$ = complex number

n = number of bubbles per unit volume $[L^{-3}]$

$\mathbf{P_o}$ = ambient pressure $[ML^{-1}T^{-2}]$

$\mathbf{P_g}$ = pressure inside the bubble $[ML^{-1}T^{-2}]$

r = radial co-ordinate $[L]$

R = bubble radius $[L]$

$\dot{\mathbf{R}}$ = time derivative of the radius

$\mathbf{R_g}$ = gas constant, dimensionless

t = time co-ordinate $[T]$

u = radial velocity of the liquid at any point $[LT^{-1}]$

V = macroscopic velocity a long the x-axis $[LT^{-1}]$

ω = radial velocity of the liquid at the bubble surface $[LT^{-1}]$

χ = polytropic exponent of the gas

ρ = density $[M L^{-3}]$

ρ° = true density $[M L^{-3}]$

$\epsilon = 1 - \frac{\rho_g}{\rho_L}$, dimensionless

$\eta = \frac{r}{R(t)}$, dimensionless

α = volume content, dimensionless

μ_{eff} = effective viscosity of the liquid $[ML^{-1}T^{-1}]$

μ_L = coefficient of the viscosity of the liquid $[ML^{-1}T^{-1}]$

σ = surface tension $\left[MT^{-2}\right]$

ν = radial velocity of the gas $\left[LT^{-1}\right]$

Φ = any parameter ($v(x)$, $p(x)$, $R(x)$, $C(r,x)$)

Subscripts

g = gas.

L = liquid.

o = value at remote from the bubble.

S = saturation.

0 = values of the parameters in the initial state.

e = values of the parameters behind the wave.

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الملخص

الموجات الصادمة في سائل مع فقاعات غاز مذابة

تناقش هذه الأطروحة أثرانتشار الغاز على أنتفاخ فقاعة غازية كروية مذابة في سائل، بإستخدام المعادلات الأساسية التي تصف سلوك فقاعة الغاز المذابة في السائل. لقد تم إيجاد الحل المشابه الذاتي تحليلياً عن طريق تحويل المعادلات التفاضلية الجزئية إلى معادلات تفاضلية عادية. وهذا ممكن عندما $R\dot{R}$= ثابت، R هي نصف قطر الفقاعة ، $\dot{R}$ هي مشتقة نصف القطر بالنسبة للزمن. وإيضاً تم إيجاد صيغة لإنتفاخ الفقاعة والمعلم h الذي يتحكم في الحركة النصف قطرية (الكروية) ، بالإضافة إلى أشكال بيانيه توضح العلاقة بين المعلم h و $\frac{C_o - C_s}{\rho_g}$ لقيم مختلفة للمعلم اللابعدي ε الذي يساوي ρ_g / ρ_L ، C هي كتلة تركيز الغاز في السائل ، والرمز الدليلي o و s يعود إلى قيمة التركيز على مسافة بعيدة من الفقاعة والتشبع على التوالي. ρ هي الكثافة ، والرمز الدليلي g و L يعود إلى المعلمين الغاز والسائل على التوالي. وتم أيضاً مناقشة بنية الموجات الصادمة المستقرة في سائل يحتوي على فقاعات غاز مذابة وذلك بإستخدام المعادلات المستقرة للتدفق آحادي البعد. وتم إيجاد الحل لبنية الموجة الصادمة بالقرب من النقطة الحرجة وذلك بفحص سلوك الحل قبل الموجة الصادمة.

ولتقصي السلوك المقارب لحل منظومة المعادلات آحادية البعد لتدفق مستقر لسائل مع فقاعات الغاز المذابة بجوار حالة الإتزان الأولى ، فهذه المعادلات تتطلب أن تكون خطية بالنسبة لقيم المعالم عند النقطه o و حلها يكون على الصورة التالية

$$\Phi = \Phi_o \left(1+A^{(\Phi)} \exp k_o \bar{x} \right), \mathrm{Re}\, k_o < 0 \qquad (*)$$

وΦ هو أي معلم مثل السرعة V أو الضغط P أو نصف القطر R أو التركيز C وبعد إتمام العملية الخطية نحصل على منظومة من المعادلات الخطية المتجانسة من منظومة المعادلات آحادية البعد لتدفق مستقر لسائل مع فقاعات غاز مذابة. إن الشرط الأساسي لوجود حل غير مبتذل لهذه المنظومة يقودنا إلى معادلة جبرية من الدرجة السادسة بالنسبة لـ $K = \sqrt{k_o \bar{V}_o}$. و k_o هو المعامل الأسي و $\bar{V}_o$ هي سرعة الموجة اللابعدية. ولقد تم إثبات وجود حل وحيد أثناء الأنتشار للمعادلة (*) .

والجذر المطلوب موجود فقط عندما $V_o^2 > C_o^2$، حيث V_o هي سرعة الموجة و C_o هي سرعة الصوت في السائل الفقاعي. والحالتان أمام X عندما تؤول إلى $+\infty$ وخلف الموجة الصادمة عندما X تؤول إلى $-\infty$ هما حالتا الإتزان ولهذا يكون السلوك المقارب عندما X تؤول إلى $+\infty$ ضروري لتجاوز النقطة الحرجة ومنحنى التكامل يربط هاتين الحالتين ويحدد بنية الموجة الصادمة المستقرة.