FINITE DIFFERENCE SOLUTIONS FOR NAVIER-STOKES EQUATIONS

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To

My Family

&

My Friends

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ABSTRACT

The aim of this thesis is to present different finite difference methods to solve Navier-Stokes equations in one, two or three-dimensions.

In chapter one an introduction to finite difference methods using two different formula's connecting difference and differential operator's are given.

Chapter two presents a derivation of Navier-Stokes equations from first principle using a control volume.

The first section of chapter three discusses the linearization of Navier-Stokes equations using the perturbation method, while the other sections introduce the solutions of Navier-Stokes using explicit and implicit finite difference approximations for different space dimensions.

In chapter four several results are presented and various graphs for comparisons have been plotted.

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1.1 Introduction to finite difference

Approximation to derivatives

A numerical analysis is concerned with the solution of mathematical problems by arithmetic processes. Clearly, then, the need to approximate nonarithematic quantities-and to ascertain the errors associated with such approximations, lies at the heart of much of numerical analysis. In a given situation there will usually be several possible methods of obtaining the desired approximation [1]. Which of these to choose depends upon which of various possible criteria are used to judge how efficacious a given approximation is. In this work we will use finite difference approximations to solve partial differential equations. We will use here two formula's connecting difference and differential operator's [2].

First method:

If U and it's dervatives are single-valued, finite, and continuous functions of x, then by Taylor's theorem [3],

$$U(x+h)=U(x)+hU'(x)+\frac{1}{2}h^2U''(x)+\frac{1}{6}h^3U'''(x)+\ldots\ldots \qquad (1.1.1)$$

$$U(x-h)=U(x)-hU'(x)+\frac{1}{2}h^2U''(x)-\frac{1}{6}h^3U'''(x)+\ldots\ldots \qquad (1.1.2)$$

Adding (1.2.1) and (1.2.2) gives

$$U(x+h)+U(x-h)=2U(x)+h^2U''(x)+O(h^4), \qquad (1.1.3)$$

where $O(h^4)$ denotes terms containing fourth and higher powers of h. Assuming that these are negligible in comparison with lower powers of h it follows that [2],

$$U''(x)=\frac{U(x+h)-2U(x)+U(x-h)}{h^2}=\left(\frac{d^2U}{dx^2}\right)_{x=x}, \qquad (1.1.4)$$

with a leading error of order h^2 on the right hand side. Subracting equation (1.1.2) from equation (1.1.1) and neglecting terms of order h^3 leads to

$$U' = \left(\frac{dU}{dx}\right)_{x=x} = \frac{1}{2h}\left(U(x+h) - U(x-h)\right), \tag{1.1.5}$$

with an error of order h^2 as $h \to 0$.

Equation (1.1.5) clearly approximates the slope of the tangent at P in figure(1.1) by the slope of the chord AB ,and is called a central difference approximation. We can also approximate the slope of the tangent at the point P by either the slope of the chord PB, giving the forward-difference formula,

$$U'(x) = \frac{1}{h}\{U(x+h) - U(x)\}, \tag{1.1.6}$$

or the slope of the chord AP giving the backward-difference formula

$$U'(x) = \frac{1}{h}\{U(x) - U(x-h)\} \tag{1.1.7}$$

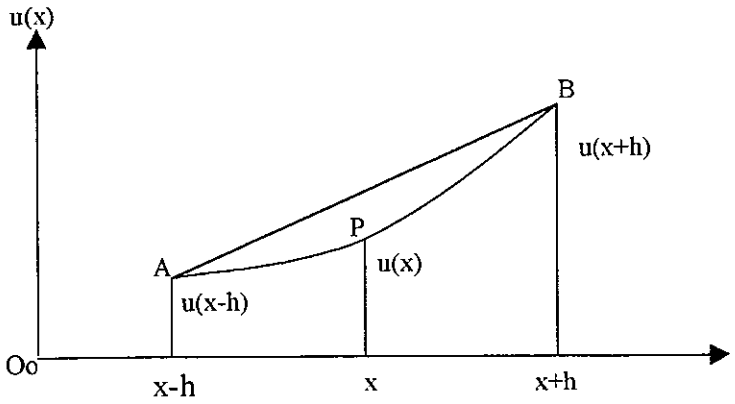


Figure 1.1

Both (1.1.6) and (1.1.7) can be concluded from equation (1.1.1) and (1.1.2). This shows that the leading errors in these forward and backward difference formulae are both O(h)

Notation for functions of several variables:

Let **U** be a function of the independent variables x and t. Subdivide the x-t plane into sets of equal rectangles of sides $\delta x=h$, $\delta t=k$, as shown in figure (1.2), by equally spaced grid lines parallel to Ot , defined by:

$\mathbf{x_i = ih}$ $\quad$ $\mathbf{,i = 0,1,2,3,....}$,

And equally space grid lines parallel to Ox , defined by:

$\mathbf{y_j = jk}$ $\quad$ $\mathbf{,j = 0,1,2,3,.....}$ as shown in the figure (1.2).

Denote the value of U at representative mesh piont P(ih, jk) by

$U_P = \mathbf{U(ih,jk) = U_{i,j}}$,

Then, by equation (1.1.4) we get

$$\left(\frac{\partial^2 U}{\partial x^2}\right)_P = \left(\frac{\partial^2 U}{\partial x^2}\right)_{i,j} = \frac{\mathbf{U((i+1)h,jk) - 2U(ih,jk) + U((i-1)h,jk)}}{\mathbf{h^2}}$$

i.e $$\frac{\partial^2 U}{\partial x^2} = \frac{\mathbf{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}}{\mathbf{h^2}}$$,

with a leading error of order h^2 .

Similarly,

$$\left(\frac{\partial^2 U}{\partial t^2}\right)_p = \frac{\mathbf{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}}{\mathbf{k^2}}$$,

with a leading error of order k^2.

With this notation the forward - difference approximation for $\frac{\partial U}{\partial t}$ at P is

$$\frac{\partial U}{\partial t} = \frac{\mathbf{U_{i,j+1} - U_{i,j}}}{\mathbf{k}}$$,

with a leading error of O(k).

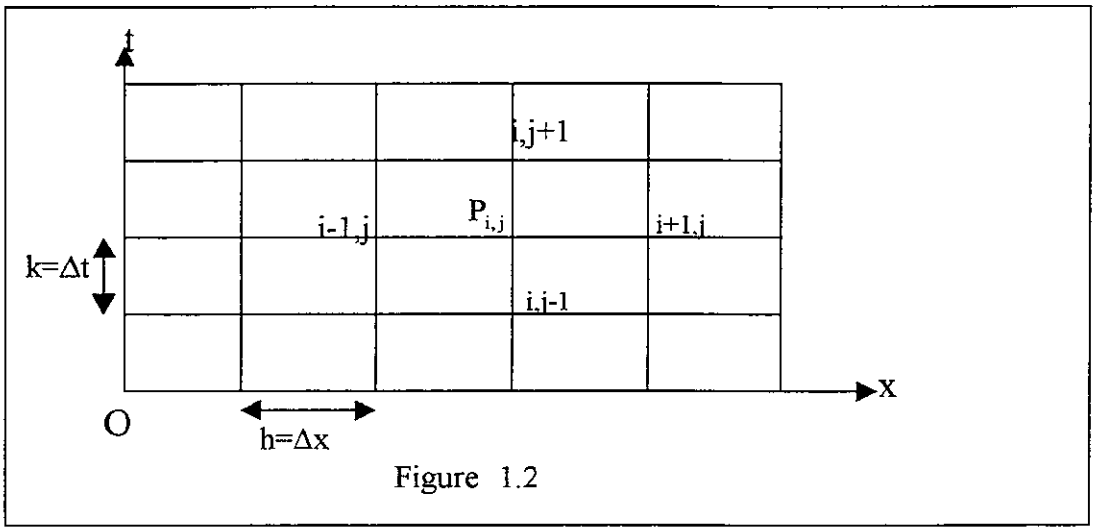


Figure 1.2

Second method:

If we return to equation (1.1.1) in the first way and using calculus, the total derivative will change to partial derivatives as in the following equation

$$\mathbf{u(x,t+k)} = (1 + \mathbf{k}\frac{\partial}{\partial t} + \frac{\mathbf{k}^2}{2}\frac{\partial^2}{\partial t^2} + ...)\,\mathbf{u(x,t)} \qquad (1.1.8)$$

$$= \mathbf{e}^{(\mathbf{k}\frac{\partial}{\partial t})}\,\mathbf{u(x,t)}$$

let $D = \dfrac{\partial}{\partial x}$

Now an exact formula connecting D and δ_x, the central difference opreator in x - direction is [2]

$$D = \frac{2}{h}\mathbf{sinh}^{-1}(\frac{\delta_x}{2}) = \frac{1}{h}(\delta_x - \frac{1^2}{2^2\cdot 3!}\delta_x^3 + \frac{1^2\cdot 3^2}{2^4\cdot 5!}\delta_x^5 .....) \qquad (1.1.9)$$

where $\delta_x \mathbf{U}_m^n = U_{m+\frac{1}{2}}^n - \mathbf{U}_{m-\frac{1}{2}}^n$ (1.1.10)

$$\delta_x^2 \mathbf{U}_m^n = \mathbf{U}_{m+1}^n - 2\mathbf{U}_m^n + \mathbf{U}_{m-1}^n \qquad (1.1.11)$$

CHAPTER TWO

Derivation of Navier-Stokes equation

2.1 Continuity equation:

To derive the continuity equation consider an infinitesimal fixed control volume (dx,dy,dz) as in figure(1.3) with a velocity field (u,v,w) and mass density ρ.

The conservation of mass states as described in [4]

$$\begin{bmatrix}\text{Net rate of mass flux through}\\ \text{the control surface}\end{bmatrix}+\begin{bmatrix}\text{Rate of change of mass inside}\\ \text{the control volume}\end{bmatrix}=0 \qquad (2.1.1)$$

So we get:

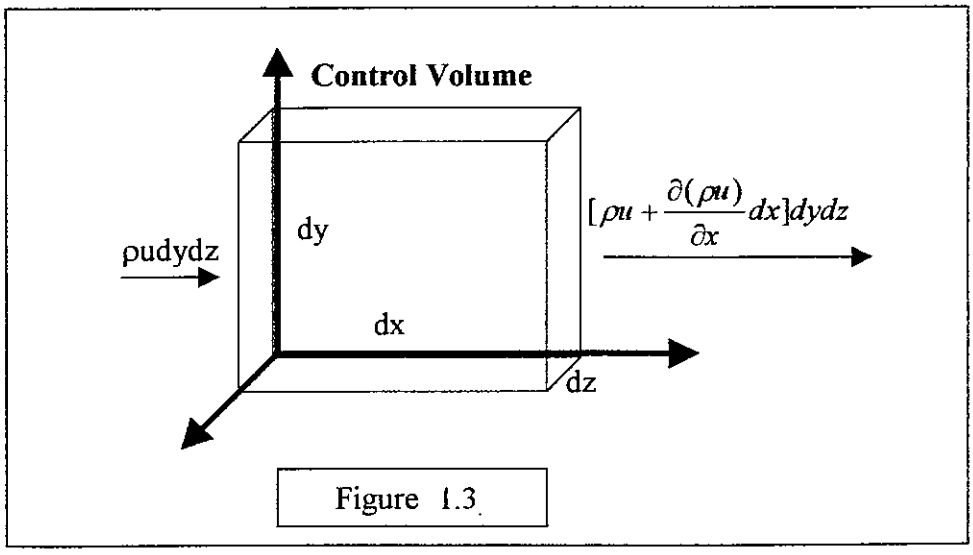


Figure 1.3

$$\int_{C\Omega}\frac{\partial \rho}{\partial t}d\Omega+\sum_{i}(\rho_i A_i V_i)_{out}-\sum_{i}(\rho_i A_i V_i)_{in}=0 \qquad (2.1.2)$$

where Ω is the volume and $C\Omega$ is the control volume

And assuming that the element is so small that the volume integral simply reduces to a differential term [5]

$$\int_{C\Omega} \frac{\partial \rho}{\partial t} d\Omega \approx \frac{\partial \rho}{\partial t} dxdydz \qquad (2.1.3)$$

Now, we have to calculate the rate of change in the inlet and outlet, using Taylor series expansion[3], e.g. the inlet in x-face is ρu multiply by area which is dydz, therefore we get:

$$(\rho u dydz)_{x+dx} = \left[\rho u + \left(\frac{\partial \rho u}{\partial x} \right) dx + \frac{\partial^2 (\rho u)}{\partial x^2} \frac{(dx)^2}{2!} + \cdots\cdots \right] dydz \qquad (2.1.4)$$

The calculation of the other faces is shown as below:[5]

Face	Inlet mass flow	Outlet mass flow
X	ρudydz	$\left[\rho u + \frac{\partial (\rho u)}{\partial x} dx \right] dydz$
Y	ρvdxdz	$\left[\rho v + \frac{\partial (\rho v)}{\partial y} dy \right] dxdz$
Z	ρwdxdy	$\left[\rho w + \frac{\partial (\rho w)}{\partial z} dz \right] dxdy$

Table 1

Neglecting higher order terms we conclude that the rate of change in the x-direction is

$$\left[\rho u + \frac{\partial(\rho u)}{\partial x}dx\right]dydz - \rho dydz = \frac{\partial(\rho u)}{\partial x}dxdydz \qquad (2.1.5)$$

Similarly, we may find the rate of changes for y- and z-faces from table 1. Substituting these values into equation (2.1.2) leads to the following [5]

$$\frac{\partial \rho}{\partial t}dxdydz + \frac{\partial(\rho u)}{\partial x}dxdydz + \frac{\partial(\rho v)}{\partial y}dxdydz + \frac{\partial(\rho w)}{\partial z}dxdydz = 0 \qquad (2.1.6)$$

Dividing equation (2.1.6) by dxdydz we get.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad (2.1.7)$$

The last equation is called the continuity equation or the equation of mass conservation.

And, we can write

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = \nabla \cdot (\rho V) \qquad (2.1.8)$$

where $$\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k \qquad (2.1.9)$$

So, we can write the continuity equation in the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \qquad (2.1.10)$$

There are two cases for the continuity, which is widely used in Fluid Mechanics. The first case when the fluid is incompressible i.e. ρ=constant, which simplify the continuity equation to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (2.1.11)$$

or $\nabla\cdot(V)=0$ (2.1.12)

The other case is the steady compressible flow, i.e. when the density is a function of space, but independent of t, therefore the continuity equation may be written as

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \qquad (2.1.13)$$

or $\nabla\cdot(\rho V)=0$ (2.1.14)

2.2 Derivation of Navier-Stokes equation(Conservation of momentum):

Newton's second law states that the force equals the mass multiply by the acceleration

$$\mathbf{F=ma} \tag{2.2.1}$$

If we choose a fluid element, there are two types of forces: body forces and surface forces, and they are related by the following equation [7].

$$\rho\frac{\mathbf{DV}}{\mathbf{Dt}} = \mathbf{f_{body}} + \mathbf{f_{surface}} \tag{2.2.2}$$

The body force applied to the entire mass of the fluid element. Such forces are usually due to external forces such as gravity force or an applied electromagnetic filed. Here we will consider only the gravity force and ignore the magnetohydrodynamic effects [6].

$$\mathbf{f_{body}} = \rho\mathbf{g} \tag{2.2.3}$$

Where **g** is the acceleration vector of gravity.

The surface forces include both normal forces and tangential (shear) forces. We shall consider the x-component of the forces acting on a differential element of mass dm and volume dV=dxdydz. Only these stresses that act in the x-direction will give rise to surface force in the x-direction. If the stresses at the center of the differential element to be σ_{xx}, τ_{yx} ,and τ_{zx} then the stesses acting in the x - direction on each face of the element may be obtained by Taylor series expansion .

The normal stresses on the left hand face is σ_{xx} as in [6]

Therefore, the rate of change on the right face is given by Taylor series as

$$\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx(dydz), \tag{2.2.4}$$

The product dydz being the area of the right hand face. The rate of change in the shear stresses are [6]

$$\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy(dzdx), \tag{2.2.5}$$

and

$$\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz(dydx), \tag{2.2.6}$$

And the net of change in x-direction is given by [6].

$$\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dxdydz = f_x, \tag{2.2.7}$$

Similarly

$$\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) dxdydz = f_y, \text{ and} \tag{2.2.8}$$

$$\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) dxdydz = f_z, \tag{2.2.9}$$

so the forces per unit volume as described in[6] is given by

$$F_x = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \tag{2.2.10)a}$$

$$F_y = \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{z} \right) \tag{2.2.10)b}$$

$$F_z = \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) \tag{2.2.10)c}$$

therefore, the Navier-Stokes equation can be written as

$$\rho\frac{Du}{Dt}=\rho g_x+F_x \qquad (2.2.11)$$

$$\rho\frac{Dv}{Dt}=\rho g_y+F_y \qquad (2.2.12)$$

$$\rho\frac{Dw}{Dt}=\rho g_z+F_z \qquad (2.2.13)$$

Note:

If the fluid at rest then the shear stress vanishes and the normal stresses equal to the hydrostatic pressure as in [6]

$$\sigma_{xx}=\sigma_{yy}=\sigma_{zz}=-P \qquad (2.2.14)$$

and $\tau_{ij}=0$ for all $i\neq j$

Stokes three postulates:

The Stokes three postulates are described in [8]

1- The fluid is continuous, and it's stress tensor is τ_{ij} at most a linear function of strain rates e_{ij}.

2- The fluid is isotropic; i.e. it's properties are independent of direction, and therefore the deformation law is independent of the coordinate axes in which it is expressed.

3- When the strain rates are zero, the deformation law must reduce to the hydrostatic pressure condition

$\tau_{ij}=-p\delta_{ij}$,where δ_{ij} is the kronecker delta function

($\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ if $i\neq j$ [1]

the terms –p is added to satisfy the hydrostatic condition, [6]

The following results have been taken from elasticity theory, in infinitesimally small stress when the liquid is less than 1% as described in [9].

$$e_{xx} = \frac{\partial u}{\partial x} \tag{2.2.16}$$

$$e_{yy} = \frac{\partial v}{\partial y} \tag{2.2.17}$$

$$e_{zz} = \frac{\partial w}{\partial z} \tag{2.2.18}$$

$$e_{xy} = e_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{2.2.19}$$

$$e_{yz} = e_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \tag{2.2.20}$$

$$e_{zx} = e_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \tag{2.2.21}$$

and the equations which relates the stress with strains are the following

$$\sigma_{xx} = 2\mu e_{xx} + \lambda\bar{\theta} \tag{2.2.22}$$

$$\sigma_{yy} = 2\mu e_{yy} + \lambda\bar{\theta} \tag{2.2.23}$$

$$\sigma_{zz} = 2\mu e_{zz} + \lambda\bar{\theta} \tag{2.2.24}$$

$$\tau_{xy} = \mu e_{xy} \tag{2.2.25}$$

$$\tau_{yz} = \mu e_{yz} \tag{2.2.26}$$

$$\tau_{zx} = \mu e_{zx} \tag{2.2.27}$$

$$\text{where } \bar{\theta} = e_{xx} + e_{yy} + e_{zz}\ , \quad \lambda = \frac{3\upsilon k}{1+\upsilon}, \text{ and} \tag{2.2.28}$$

$$\mu = \frac{E}{2(1+\upsilon)} \tag{2.2.29}$$

υ: is the Poisson's rates at it's dimensionless.

E: is Young's modulus and has the dimension of stress.

Using this in (2.2.11),(2.2.12) and (2.2.13) we get:

$$\rho\frac{\mathbf{Du}}{\mathbf{Dt}}=\rho g_x+\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}+\lambda\bar{\theta}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial z}\right)\right]-\frac{\partial P}{\partial x} \tag{2.2.29}$$

$$\rho\frac{\mathbf{Dv}}{\mathbf{Dt}}=\rho g_y-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial t}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[2\mu\frac{\partial v}{\partial y}+\lambda\bar{\theta}\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] \tag{2.2.30}$$

$$\rho\frac{\mathbf{Dw}}{\mathbf{Dt}}=\rho g_z-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[2\mu\frac{\partial w}{\partial z}+\lambda\bar{\theta}\right] \tag{2.2.31}$$

Using Stokes hypothesis as in [8]: $\lambda+(2/3)\mu=0$ and considering μ as a constant this implies that

$$\lambda=-\frac{2}{3}\mu$$

for incompressible fow : $\mathrm{div}(V)=0$ implies that $\bar{\theta}=0$

Equations (2.2.29), (2.2.30) and (2.3.31) can be written as [6]

$$\rho\frac{\mathbf{Du}}{\mathbf{Dt}}=\rho g_x+2\mu\frac{\partial^2 u}{\partial x^2}+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\mu\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right]-\frac{\partial p}{\partial x} \tag{2.2.32}$$

$$\rho\frac{\mathbf{Du}}{\mathbf{Dt}}=\rho g_x-\frac{\partial p}{\partial x}+2\mu\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+\mu\frac{\partial^2 u}{\partial x\partial y}+\mu\frac{\partial^2 u}{\partial z^2}+\mu\frac{\partial^2 w}{\partial x\partial z} \tag{2.2.33}$$

this implies that

$$\rho\frac{\mathbf{Du}}{\mathbf{Dt}}=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\right] \tag{2.2.34}$$

Using the continuity equation for incompressible flow we get :

$$\rho\frac{\mathbf{Du}}{\mathbf{Dt}}=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] \qquad (2.2.35)$$

Similar equations for y and z directions may be derived.

In vector notations, we can write the general equations as

$$\rho\frac{\mathbf{DV}}{\mathbf{Dt}}=\rho g-\Delta p+\mu\nabla^2 V \qquad (2.2.36)$$

Using the last equation and define the stream function ψ(x,y)as described in [5]

$$u=\frac{\partial\psi}{\partial y}\quad ,v=-\frac{\partial\psi}{\partial x} \qquad (2.2.37)$$

The stream function ψ(x,y) satisfies the continuity equation for incompressible flow, i.e.:

$$\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial\psi}{\partial x}\right)=0 \qquad (2.2.38)$$

therefore the volecity vector can be written as[5]

$$\mathbf{V}=u\bar{i}+v\bar{j}=\frac{\partial\psi}{\partial y}\bar{i}-\frac{\partial\psi}{\partial x}\bar{j}, \qquad (2.2.39)$$

and we can see that the ψ function replaces the two unknown u and v by one unknown.

If the flow is irrotational in x-y plane i.e. curl(V) equal to zero [5].

$$\frac{\partial \mathbf{v}}{\partial \mathbf{x}} - \frac{\partial \mathbf{u}}{\partial \mathbf{y}} = \frac{\partial}{\partial \mathbf{x}}\left(-\frac{\partial \psi}{\partial \mathbf{x}}\right) - \frac{\partial}{\partial \mathbf{y}}\left(\frac{\partial \psi}{\partial \mathbf{y}}\right) = 0 \qquad (2.2.40)$$

$$\frac{\partial^2 \psi}{\partial \mathbf{x}^2} + \frac{\partial^2 \psi}{\partial \mathbf{y}^2} = 0 \qquad (2.2.41)$$

That's mean that ψ satisfies Laplace equation.

2.3 Nondimensionalization of Navier-Stokes equation:

Many textbooks use pi-theorem method to find the dimensionless parameters of the Navier-Stokes equation. A powerful technique to find the dimensionless parameter for the incompressible flow continuity and momentum equations with constant viscosity, and assuming that the gravity in z-direction : [5]

continuity : $\nabla \cdot \mathbf{V} = 0$ (2.3.1)

Momentum : $\rho \frac{\mathbf{dV}}{\mathbf{dt}} = \rho \mathbf{g} - \nabla \mathbf{P} + \mu \nabla^2 \mathbf{V}$ (2.3.2)

Typical boundary condition for these two equations are:

Fixed solid surface $\mathbf{V}=0$

Inlet or outlet known $\mathbf{V}, \rho$

Free surface $z = \eta$, $w = \frac{d\eta}{dt}$, $P = P_a - Y(R_x^{-1} + R_y^{-1})$

The basic dimensions are M,L, and T represent mass, length and time respectively. The variables p,V (velocity), x,y,z and t can be nondimensionalized by using density and two reference constants which might be characteristic of particular fluid flow:[5]

Reference velocity =U

Reference length =L

For example U might be maximum velocity and L be the box length in x-direction.

Now define the dimensionless variables, denoting them by asterisk as in [5]:

$$\left.\begin{aligned} &V = UV^* \\ &x = Lx^* \qquad y = Ly^* \qquad z = Lz^* \\ &t = \frac{Lt^*}{U} \\ &p^* = \frac{p - \rho g z}{\rho U^2} \end{aligned}\right\} \tag{2.3.3}$$

All the dimensionless are obvious except the pressure which suggested by Bernoulli's equation.

Since ρ, μ, U and L are constants, then the derivatives can be found easily.

For example

$$\frac{\partial u}{\partial x} = \frac{\partial\left(Uu^*\right)}{\partial\left(Lx^*\right)} = \frac{U}{L}\frac{\partial u^*}{\partial x^*}$$

$$u\frac{\partial u}{\partial x} = Uu^*\frac{\partial\left(Uu^*\right)}{\partial\left(Lx^*\right)} = \frac{U^2}{L}u^*\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial\left(Ly^*\right)}\frac{\partial\left(Uu^*\right)}{\partial\left(Ly^*\right)} = \frac{U}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Substitute (2.3.3) in equations (2.3.1) and (2.3.2) and divide by the dimensional cofficient gives the following : [5]

continuity : $$\nabla\cdot(V^*) = 0 \tag{2.3.4}$$

momentum : $$\frac{dV^*}{dt^*} = -\nabla(P^*) + \frac{\mu}{\rho UL}\nabla^2(V^*) \tag{2.3.5}$$

The dimensionless boundary conditions are :

$$\left.\begin{aligned} &\text{fixed solid surface :} && V^* = 0 \\ &\text{inlet or outlet :} && \text{known } V^*, P^* \\ &\text{free surface, } z^* = \eta^*, && w^* = \frac{d\eta^*}{dt^*}, \\ & && p^* = \frac{p_a}{\rho U^2} + \frac{gL}{U^2}z^* + \frac{Y}{\rho U^2 L}\left(R_x^{*-1} + R_y^{*-1}\right) \end{aligned}\right\} \tag{2.3.6}$$

the constant $\frac{\rho UL}{\mu}$ is called Reynolds number and for simplicity we write

$$Re = \frac{\rho UL}{\mu}$$

It is named after the British engineer Osborne Reynolds (1842-1912) [5].

Substitute the Reynolds number in equation (2.3.5) we get:

$$\frac{\mathbf{dV}^*}{\mathbf{dt}^*} = -\nabla \mathbf{P}^* + \frac{1}{\mathbf{Re}} \nabla^2 (\mathbf{V}^*) \qquad (2.3.7)$$

CHAPTER THREE
3.1 Difference equations for Navier-Stokes Equations

If we rewrite the Navier-Stokes equation in three dimension in x-direction

$$\frac{\rho Du}{Dt} = \rho g_x - \frac{\partial P}{\partial x} + \mu \nabla^2 u \qquad (3.1.1)$$

and using calculus calculations to expand the differentail opreator D and ∇^2 we get

$$\left.\begin{aligned} \frac{Df}{Dt} &= \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} \\ u &= \frac{dx}{dt} \quad , v = \frac{dy}{dt} \quad , w = \frac{dz}{dt} \end{aligned}\right\} \qquad (3.1.2)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \qquad (3.1.3)$$

if we substitute these two formulas in equation(3.1.1) we get

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \qquad (3.1.4)$$

The same thing happens in the y and z directions that is:

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = \rho g_x - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \qquad (3.1.5)$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = \rho g_z - \frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] \qquad (3.1.6)$$

These three equations and the continuity equation are difficult to solve directly because the Navier-Stokes equations are non-linear.

The first idea is to linearize the Navier-Stokes equations, which may be done using perturbation method, that is.

$$u = U + \overline{U} \quad , v = V + \overline{V} \quad , w = W + \overline{W} \qquad (3.1.7)$$

where $U << \overline{U}$, $V << \overline{V}$, $W << \overline{W}$ **and** $\overline{U}, \overline{V}, \overline{W}$ are known and constant

That is, we want to investigate the velocity vector after a very short time period.

And if we substitute these values in the Navier-Stokes equations, we get

$$\rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right] = \rho\left[\frac{\partial U}{\partial t} + \overline{U}\frac{\partial U}{\partial x} + \overline{V}\frac{\partial U}{\partial y} + \overline{W}\frac{\partial U}{\partial z}\right]$$

$$\rho g_y - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right] = \rho\left[\frac{\partial V}{\partial t} + \overline{U}\frac{\partial V}{\partial x} + \overline{V}\frac{\partial V}{\partial y} + \overline{W}\frac{\partial V}{\partial z}\right]$$

$$\rho g_z - \frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2}\right] = \rho\left[\frac{\partial W}{\partial t} + \overline{U}\frac{\partial W}{\partial x} + \overline{V}\frac{\partial W}{\partial y} + \overline{W}\frac{\partial W}{\partial z}\right] \quad (3.1.8)$$

and the continuity equation

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 \quad (3.1.9)$$

We will solve these equations using an explicit finite difference method and the Crank-Nicolson method.

3.2 Explicit finite difference method for one-dimension flow:

If we take equation(3.1.8) for one dimension flow, the Navier-Stokes equation will be:

$$\rho g_x - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 U}{\partial x^2}\right) = \rho\left[\frac{\partial U}{\partial t} + \overline{U}\frac{\partial U}{\partial x}\right] \quad (3.2.1)$$

and the contiuity equation is:

$$\frac{\partial U}{\partial x} = 0 \quad (3.2.2)$$

An explicit finite difference method for the one dimension form is

$$\rho g_x - \frac{P_{i+1}^n - P_{i,j}^n}{\nabla x} + \mu\left[\frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{(\nabla x)^2}\right] = \rho\left[\frac{U_i^{n+1} - U_i^n}{(\nabla t)} + \overline{U}\frac{U_{i+1}^n - U_i^n}{(\nabla x)}\right] \quad (3.2.3)$$

As usual we will put $\Delta x = h, \Delta t = k$ and $\mathbf{A = -2\mu r + \rho\bar{U}hr + \rho, B = \mu r - \rho\bar{U}hr,}$

where $r = \frac{k}{h^2}$

Taking the (n+1) time level to the left hand side and the other terms in the right hand side. We get:

$$\rho U_i^{n+1} = k\rho g_x - hr(P_{i+1}^n - P_i^n) + \mu r U_{i-1}^n + AU_i^n + BU_{i+1}^n \qquad (3.2.4)$$

Drawing the mesh point of the last scheme, we can see from figure (3.1) that we have one unknown point at t=(n+1)k time level, and three unknown points at t=nk time level.

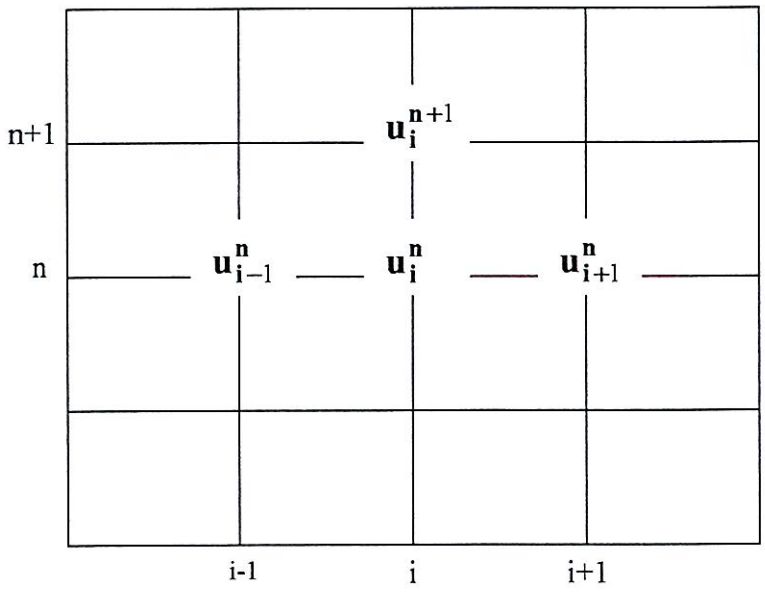


Figure 3.1
Explicit one dimension mesh point

Since we are interested in the velocity vector rather than the pressure gradient, we will therefore assume that the pressure gradient is known, either a constant or as a known function in space and time.

Equation (3.2.4) can be rewritten in matrix form:

$$\rho\begin{bmatrix}U_1\\U_2\\U_3\\U_4\\U_5\\U_6\\U_7\\U_8\\U_9\end{bmatrix}^{n+1}_{N-1}=\begin{bmatrix}A&B&0&0&0&0&0&0&0\\ \mu r&A&B&0&0&0&0&0&0\\0&\mu r&A&B&0&0&0&0&0\\0&0&\mu r&A&B&0&0&0&0\\0&0&0&\mu r&A&B&0&0&0\\0&0&0&0&\mu r&A&B&0&0\\0&0&0&0&0&\mu r&A&B&0\\0&0&0&0&0&0&\mu r&A&B\\0&0&0&0&0&0&0&\mu r&A\end{bmatrix}_{(N-1)\times(N-1)}\begin{bmatrix}U_1\\U_2\\U_3\\U_4\\U_5\\U_6\\U_7\\U_8\\U_9\end{bmatrix}^{n}_{N-1}+\begin{bmatrix}\mu r U_0^n+k\rho g_x-hr(p_2^n-p_1^n)\\k\rho g_x-hr(p_3^n-p_2^n)\\k\rho g_x-hr(p_4^n-p_3^n)\\k\rho g_x-hr(p_5^n-p_4^n)\\k\rho g_x-hr(p_6^n-p_5^n)\\k\rho g_x-hr(p_7^n-p_6^n)\\k\rho g_x-hr(p_8^n-p_7^n)\\k\rho g_x-hr(p_9^n-p_8^n)\\BU_{10}^n+k\rho g_x-hr(p_{10}^n-p_9^n)\end{bmatrix}_{N-1}$$

So, we can write the equation in more simple and compact form:

$$\rho U^{n+1}=CU^n+\text{Boundary} \qquad (3.2.5)$$

where the **U** is a cloumn vector and C is tridiagonal matrix. This is the one dimension explicit method for the Navier-Stokes equation.

3.3 Explicit finite difference method for two dimensional flow:

Consider Navier-Stokes equation in two-dimensions, we get from equation (3.1.8):

$$\left.\begin{aligned}\rho g_x-\frac{\partial P}{\partial x}+\mu\left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial^2 y}\right]&=\rho\left[\frac{\partial U}{\partial t}+\overline{U}\frac{\partial U}{\partial x}+\overline{V}\frac{\partial U}{\partial y}\right] \qquad (3.3.1a)\\ \rho g_y-\frac{\partial P}{\partial y}+\mu\left[\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial^2 y}\right]&=\rho\left[\frac{\partial V}{\partial t}+\overline{U}\frac{\partial V}{\partial x}+\overline{V}\frac{\partial V}{\partial y}\right] \qquad (3.3.1b)\end{aligned}\right\}(3.3.1)$$

and the continuity equation :

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \qquad (3.3.2)$$

The forward difference scheme for two dimensions using the following notations :

$\Delta t=k$ and $\Delta x=\Delta y=h$ will give

$$\rho U_{i,j}^{n+1} = k\rho g_x - hr(P_{i+1,j}^n - P_{i,j}^n) + (\rho - 4\mu r + \rho hr\overline{U} + \rho hr\overline{V})U_{i,j}^n +$$

$$(\mu r - \rho hr\overline{U})U_{i+1,j}^n + (\mu r - \rho hr\overline{V})U_{i,j+1}^n + \mu r U_{i-1,j}^n + \mu r U_{i,j-1}^n \qquad (3.3.3)$$

$i,j = 1,2,3,.....,N\text{-}1 \quad ,n = 0,1,2,3,.....$

and for simplicity we used:

$$\left.\begin{aligned} c_1 &= \rho - 4\mu r + \rho hr\overline{U} + \rho hr\overline{V} \\ c_2 &= \rho - \rho hr\overline{U} \quad ,c_3 = \mu r - \rho hr\overline{V} \end{aligned}\right\} \qquad (3.3.4)$$

therefore equation (3.3.3) becomes:

$$\rho U_{i,j}^{n+1} = k\rho g_x - hr(P_{i+1,j}^n - P_{i,j}^n) + c_1 U_{i,j}^n + c_2 U_{i+1,j}^n + c_3 U_{i,j+1}^n + \mu r U_{i-1,j}^n$$

$$+ \mu r U_{i,j-1}^n \qquad (3.3.5)$$

$i, j = 1,2,3,.....,N-1 \quad ,n = 0,1,2,3,.....$

Drawing the mesh point as in figure (3.2) we can see that we have one unknown point, located at t = (n + 1)k and five known points at t = nk. This method as usual is easier than the implicit method, because the second one needs the inverse of a large matrix, but it is less stable.

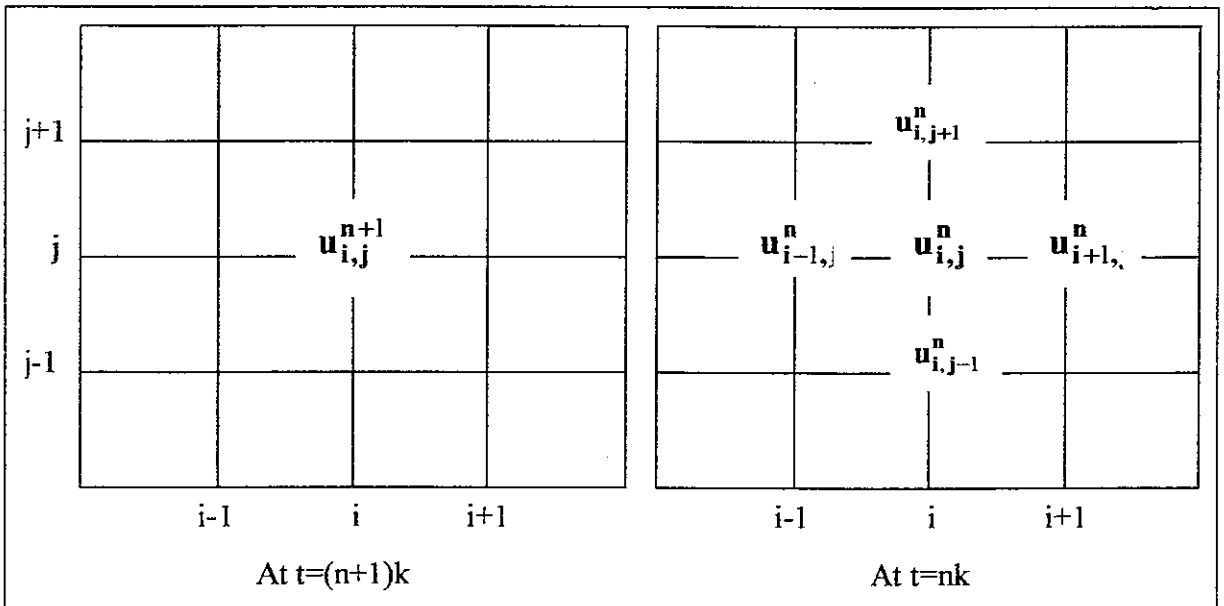


Figure 3.2

$$
\begin{bmatrix} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ U_{1,5} \\ \hline U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ U_{2,5} \\ \hline U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ U_{3,5} \\ \hline U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4} \\ U_{4,5} \\ \hline U_{5,1} \\ U_{5,2} \\ U_{5,3} \\ U_{5,4} \\ U_{5,5} \end{bmatrix}^{n+1}
=
\left[\begin{array}{ccccc:ccccc:ccccc:ccccc:ccccc}
c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu r & c_1 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
\mu r & 0 & 0 & 0 & 0 & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & 0 & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & 0 & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & c_2 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & 0 & c_1 & c_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1 & c_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 & \mu r & c_1
\end{array}\right]
\begin{bmatrix} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ U_{1,5} \\ \hline U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ U_{2,5} \\ \hline U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ U_{3,5} \\ \hline U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4} \\ U_{4,5} \\ \hline U_{5,1} \\ U_{5,2} \\ U_{5,3} \\ U_{5,4} \\ U_{5,5} \end{bmatrix}^{n}
+
\begin{bmatrix}
k\rho g_x - hr(p^n_{2,1} - p^n_{2,2}) + \mu r U^n_{0,1} + \mu r U^n_{1,0} \\
k\rho g_x - hr(p^n_{2,2} - p^n_{1,2}) + \mu r U^n_{0,2} \\
k\rho g_x - hr(p^n_{2,3} - p^n_{1,3}) + \mu r U^n_{0,3} \\
k\rho g_x - hr(p^n_{2,4} - p^n_{1,4}) + \mu r U^n_{0,4} \\
k\rho g_x - hr(p^n_{2,5} - p^n_{1,5}) + \mu r U^n_{0,5} + c_3 U^n_{1,6} \\
\hdashline
k\rho g_x - hr(p^n_{3,1} - p^n_{2,1}) + \mu r U^n_{2,0} \\
k\rho g_x - hr(p^n_{3,2} - p^n_{2,2}) \\
k\rho g_x - hr(p^n_{3,3} - p^n_{2,3}) \\
k\rho g_x - hr(p^n_{3,4} - p^n_{2,4}) \\
k\rho g_x - hr(p^n_{3,5} - p^n_{2,5}) + c_3 U^n_{2,6} \\
\hdashline
k\rho g_x - hr(p^n_{4,1} - p^n_{3,1}) + \mu r U^n_{3,0} \\
k\rho g_x - hr(p^n_{4,2} - p^n_{3,2}) \\
k\rho g_x - hr(p^n_{4,3} - p^n_{3,3}) \\
k\rho g_x - hr(p^n_{4,4} - p^n_{3,4}) \\
k\rho g_x - hr(p^n_{4,5} - p^n_{3,5}) + c_3 U^n_{3,6} \\
\hdashline
k\rho g_x - hr(p^n_{5,1} - p^n_{4,1}) + \mu r U^n_{4,0} \\
k\rho g_x - hr(p^n_{5,2} - p^n_{4,2}) \\
k\rho g_x - hr(p^n_{5,3} - p^n_{4,3}) \\
k\rho g_x - hr(p^n_{5,4} - p^n_{4,4}) \\
k\rho g_x - hr(p^n_{5,5} - p^n_{4,5}) + c_3 U^n_{4,6} \\
\hdashline
k\rho g_x - hr(p^n_{6,1} - p^n_{5,1}) + \mu r U^n_{5,0} \\
k\rho g_x - hr(p^n_{6,2} - p^n_{5,2}) \\
k\rho g_x - hr(p^n_{6,3} - p^n_{5,3}) \\
k\rho g_x - hr(p^n_{6,4} - p^n_{5,4}) \\
k\rho g_x - hr(p^n_{6,5} - p^n_{5,5}) + c_2 U^n_{6,5} + c_3 U^n_{5,6}
\end{bmatrix}
$$

The last matrix form corresponds to the explicit method for equation (3.3.5). Again, the last matrix can be written in the following simple form:

$$\rho U^{n+1} = AU^{n} + \text{Boundary} \qquad (3.3.6)$$

where boundary contains the boundary condition, the pressure gradient, and the gravity.

Where the matrix A is :

$$A = \begin{bmatrix} B_1 & B_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B_3 & B_1 & B_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B_3 & B_1 & B_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_3 & B_1 & B_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_3 & B_1 & B_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B_3 & B_1 & B_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & B_3 & B_1 & B_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & B_3 & B_1 & B_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_3 & B_1 & B_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_3 & B_1 \end{bmatrix}_{(N-1)\times(N-1)}$$

and

$$B_1 = \begin{bmatrix} c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu r & c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu r & c_1 & c_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & c_1 & c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r & c_1 \end{bmatrix}_{(N-1)\times(N-1)}$$

$B_2 = c_2 I_{(N-1)(N-1)}$ and $B_3 = \mu r I_{(N-1)(N-1)}$

where I is the identity matrix and O is the zero matrix . We have to calculate $(N-1)(N-1)$ unknown points at each time step.

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3.4 Implicit method for the two dimensional Navier-Stokes equation:

Again, consider equation (3.1.8) for the two dimensional Navier-Stokes equation:

$$\left.\begin{aligned} \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] &= \rho\left[\frac{\partial U}{\partial t} + \overline{U}\frac{\partial U}{\partial x} + \overline{V}\frac{\partial U}{\partial y}\right] \quad (3.4.1a) \\ \rho g_y - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right] &= \rho\left[\frac{\partial U}{\partial t} + \overline{U}\frac{\partial V}{\partial x} + \overline{V}\frac{\partial V}{\partial y}\right] \quad (3.4.1b) \end{aligned}\right\} (3.4.1)$$

and the continuity equation

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \qquad (3.4.2)$$

The implicit method for the two dimensional Navier - Stokes equation is :

$$(\rho - 4\mu r + \rho\overline{U}hr + \rho\overline{V}hr)U^n_{i,j} - (\mu r + \rho\overline{U}hr)U^n_{i-1,j} - (\mu r + \rho\overline{V}hr)U^n_{i,j-1}$$
$$- \mu r U^n_{i+1,j} - \mu r U^n_{i,j+1} = \rho U^{n-1}_{i,j} + k\rho g_x - hr(P^n_{i,j} - P^n_{i-1,j}) \qquad (3.4.3)$$

Also, we can use explicit method or central difference method for the pressure gradient .In the last scheme we have five unknown points and one known as it can be seen in figure (3.3a) at t=(n-1)k and figure (3.3b) at t=nk .

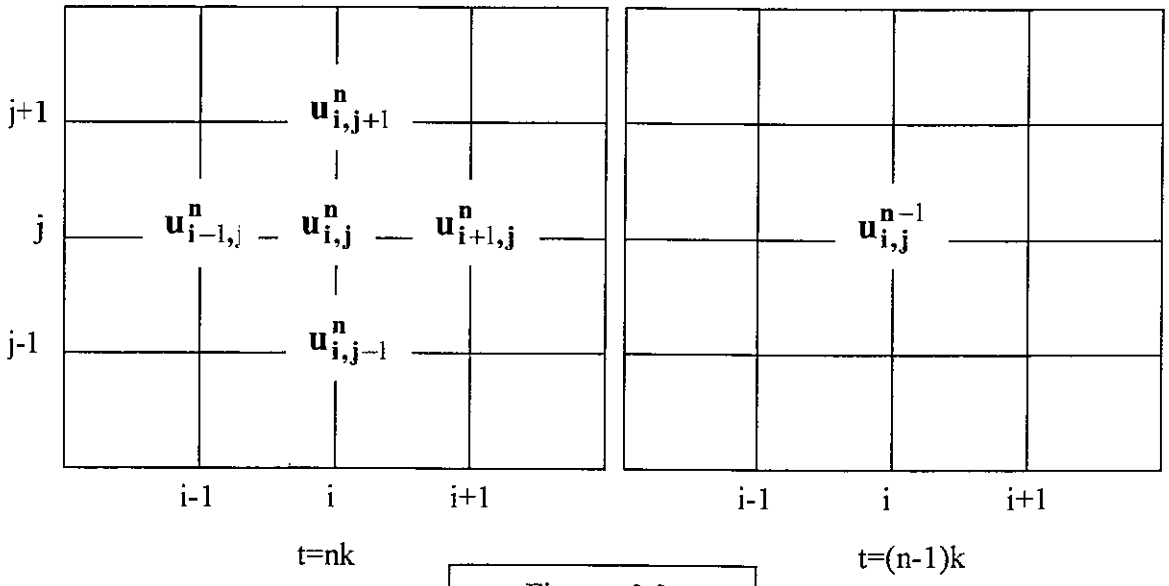


Figure 3.3

Return to equation (3.4.3), and let

$$c_1 = \rho + 4\mu r + \rho\bar{U}hr + \rho\bar{V}hr \qquad , c_2 = \mu r + \rho\bar{U}hr \qquad , c_3 = \mu r + \rho\bar{V}hr$$

so that $c_1 = c_2 + c_3 - 6\mu r + \rho$ and hence equation (3.4.4) gives :

$$c_1 U^n_{i,j} - c_2 U^n_{i-1,j} - c_3 U^n_{i,j-1} - \mu r U^n_{i+1,j} - \mu r U^n_{i,j+1} = \rho U^n_{i,j} + k\rho g_x$$

$$- hr(P^n_{i,j} - P^n_{i-1,j}), \text{ where } i, j = 1,2,....,N-1 \text{ and } n = 0,1,2,.... \qquad (3.4.4)$$

The last equation can be written in the following matrix form:

$$
\left[\begin{array}{cccc|cccc|cccc|cccc}
c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-c_3 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -c_3 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -c_3 & c_1 & 0 & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-c_2 & 0 & 0 & 0 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -c_2 & 0 & 0 & -c_3 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & 0 & 0 & 0 & -\mu r & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & -\mu r & 0 & 0 & -\mu r & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & 0 & 0 & 0 & -\mu r \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & c_1 & -\mu r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & -\mu r & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1 & -\mu r \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & -c_3 & c_1
\end{array}\right]
\left[\begin{array}{c} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ \hline U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ \hline U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ \hline U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4} \end{array}\right]^{n}
= \rho
\left[\begin{array}{c} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ \hline U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ \hline U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ \hline U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4} \end{array}\right]^{n-1}
+
\left[\begin{array}{c}
c_2 U_{0,1}^n + c_3 U_{1,0}^n + k\rho g_x - hr(p_{1,1}^n - p_{0,1}^n) \\
c_2 U_{0,2}^n + k\rho g_x - hr(p_{1,2}^n - p_{0,2}^n) \\
c_2 U_{0,3}^n + k\rho g_x - hr(p_{1,3}^n - p_{0,3}^n) \\
c_2 U_{0,4}^n + \mu r U_{1,5}^n + k\rho g_x - hr(p_{1,4}^n - p_{0,4}^n) \\
\hline
c_3 U_{2,0}^n + k\rho g_x - hr(p_{2,1}^n - p_{1,1}^n) \\
k\rho g_x - hr(p_{2,2}^n - p_{1,2}^n) \\
k\rho g_x - hr(p_{2,3}^n - p_{1,3}^n) \\
\mu r U_{2,5}^n + k\rho g_x - hr(p_{2,4}^n - p_{1,4}^n) \\
\hline
c_3 U_{3,0}^n + k\rho g_x - hr(p_{3,1}^n - p_{2,1}^n) \\
k\rho g_x - hr(p_{3,2}^n - p_{2,2}^n) \\
k\rho g_x - hr(p_{3,3}^n - p_{2,3}^n) \\
\mu r U_{3,5}^n + k\rho g_x - hr(p_{3,4}^n - p_{2,4}^n) \\
\hline
\mu r U_{5,1}^n + k\rho g_x - hr(p_{4,1}^n - p_{3,1}^n) \\
\mu r U_{5,2}^n + k\rho g_x - hr(p_{4,2}^n - p_{3,2}^n) \\
\mu r U_{5,3}^n + k\rho g_x - hr(p_{4,3}^n - p_{3,3}^n) \\
\mu r U_{5,4}^n + \mu r U_{4,5}^n + k\rho g_x - hr(p_{4,4}^n - p_{3,4}^n)
\end{array}\right]
$$

Again the last matrix form of the implicit method for Navier-Stokes equation can be written in a more simple way:

$$\mathbf{AU^n = \rho U^{n-1} + Boundary} \qquad (3.4.5)$$

Where as before we put the pressure difference, density, and gravity with the boundary as boundary. A is the block tri - diagonal matrix :

$$A=\begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & 0 & 0 & 0 \\ \mathbf{e}_3 & \mathbf{e}_1 & \mathbf{e}_2 & 0 & 0 \\ 0 & \mathbf{e}_3 & \mathbf{e}_1 & \mathbf{e}_2 & 0 \\ 0 & 0 & \mathbf{e}_3 & \mathbf{e}_1 & \mathbf{e}_2 \\ 0 & 0 & 0 & \mathbf{e}_3 & \mathbf{e}_1 \end{bmatrix}_{5\times5}$$

where

$$e_1=\begin{bmatrix} -\mathbf{c}_1 & -\mu\mathbf{r} & 0 & 0 & 0 \\ -\mathbf{c}_3 & -\mathbf{c}_1 & -\mu\mathbf{r} & 0 & 0 \\ 0 & -\mathbf{c}_3 & -\mathbf{c}_1 & -\mu\mathbf{r} & 0 \\ 0 & 0 & -\mathbf{c}_3 & -\mathbf{c}_1 & -\mu\mathbf{r} \\ 0 & 0 & 0 & -\mathbf{c}_3 & -\mathbf{c}_1 \end{bmatrix}_{5\times5}$$

$$\mathbf{e}_2=\begin{bmatrix} -\mu\mathbf{r} & 0 & 0 & 0 & 0 \\ 0 & -\mu\mathbf{r} & 0 & 0 & 0 \\ 0 & 0 & -\mu\mathbf{r} & 0 & 0 \\ 0 & 0 & 0 & -\mu\mathbf{r} & 0 \\ 0 & 0 & 0 & 0 & -\mu\mathbf{r} \end{bmatrix}_{5\times5}$$

i.e $e_2 = -\mu\mathbf{r}\mathbf{I}_{(N-1)(N-1)}$ where N here equals 6 .Similarly we can write e_3 in the same way that's mean $e_3 = -\mathbf{c}_2\mathbf{I}_{(N-1)(N-1)}$. $\mathbf{U^n}$ is the column vector of the volecity in x - direction and 0 is the matrix whose elements are all equal zero.

Although, the implicit method is more stable than the explicit method, but the inverse of the block matrix make it very expensive specially when h is very small, which means that we have a large system of equations.

3.5Crank-Nicolson method for two-dimensional Navier-Stokes equation:

Return back to equation (3.1.8) for two dimensions and incompressible continuity equation:

$$\rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] = \rho\left[\frac{\partial U}{\partial t} + \bar{U}\frac{\partial U}{\partial x} + \bar{V}\frac{\partial U}{\partial y}\right],$$

$$\rho g_y - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right] = \rho\left[\frac{\partial V}{\partial t} + \bar{U}\frac{\partial V}{\partial x} + \bar{V}\frac{\partial V}{\partial y}\right],$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0,$$

The Crank - Niclson method use $(n+\frac{1}{2})t$ for time , for second derviatives,and for first space derivative, as following

$$\frac{\partial^2 U}{\partial x^2} = \frac{1}{2}\left(\frac{U_{i+1,j}^{n+1} - 2U_{i,j}^{n+1} + U_{i-1,j}^{n+1}}{h^2} + \frac{U_{i+1,j}^{n} - 2U_{i,j}^{n} + U_{i-1,j}^{n}}{h^2}\right) \qquad (3.5.1)$$

$$\frac{\partial U}{\partial x} = \frac{1}{2}\left(\frac{U_{i+1,j}^{n+1} - U_{i,j}^{n+1}}{h} + \frac{U_{i+1,j}^{n} - U_{i,j}^{n}}{h}\right) \qquad (3.5.2)$$

$$\frac{\partial P}{\partial x} = \frac{P_{i+1,j}^{n} - P_{i,j}^{n}}{h} \qquad (3.5.3)$$

the same things for the others in space derviatives

Therefore, the scheme is given by :

$$k\rho\rho_x - hr(P_{i+1,j}^{n} - P_{i,j}^{n}) + \left(\frac{\mu r}{2} - \frac{\rho hr\bar{U}}{2}\right)U_{i+1,j}^{n+1} - \left(2\mu r - \frac{\rho hr\bar{U}}{2} - \frac{\rho hr\bar{V}}{2} + \rho\right)U_{i,j}^{n+1}$$

$$+\frac{\mu r}{2}U_{i-1,j}^{n+1} + \frac{\mu r}{2}U_{i,j-1}^{n+1} + \left(\frac{\mu r}{2} - \frac{\rho hr\bar{V}}{2}\right)U_{i,j+1}^{n+1} = \left(2\mu r - \frac{\rho hr\bar{U}}{2} - \frac{\rho hr\bar{V}}{2} - \rho\right)U_{i,j}^{n} -$$

$$\left(\frac{\mu r}{2} - \frac{\rho hr\bar{U}}{2}\right)U_{i+1,j}^{n} - \left(\frac{\mu r}{2} - \frac{\rho hr\bar{V}}{2}\right)U_{i,j+1}^{n} - \frac{\mu r}{2}U_{i-1,j}^{n} - \frac{\mu r}{2}U_{i,j-1}^{n} \qquad (3.5.4)$$

for simplicity let :

$$c_1 = 2\mu r - \frac{\rho hr\bar{U}}{2} - \frac{\rho hr\bar{V}}{2} - \rho \quad , c_2 = \frac{\mu r}{2} - \frac{\rho hr\bar{U}}{2},$$

$$c_3 = \frac{\mu r}{2} - \frac{\rho hr\bar{V}}{2} \quad , c_4 = 2\mu r - \rho - \frac{\rho hr\bar{U}}{2} - \frac{\rho hr\bar{V}}{2}$$

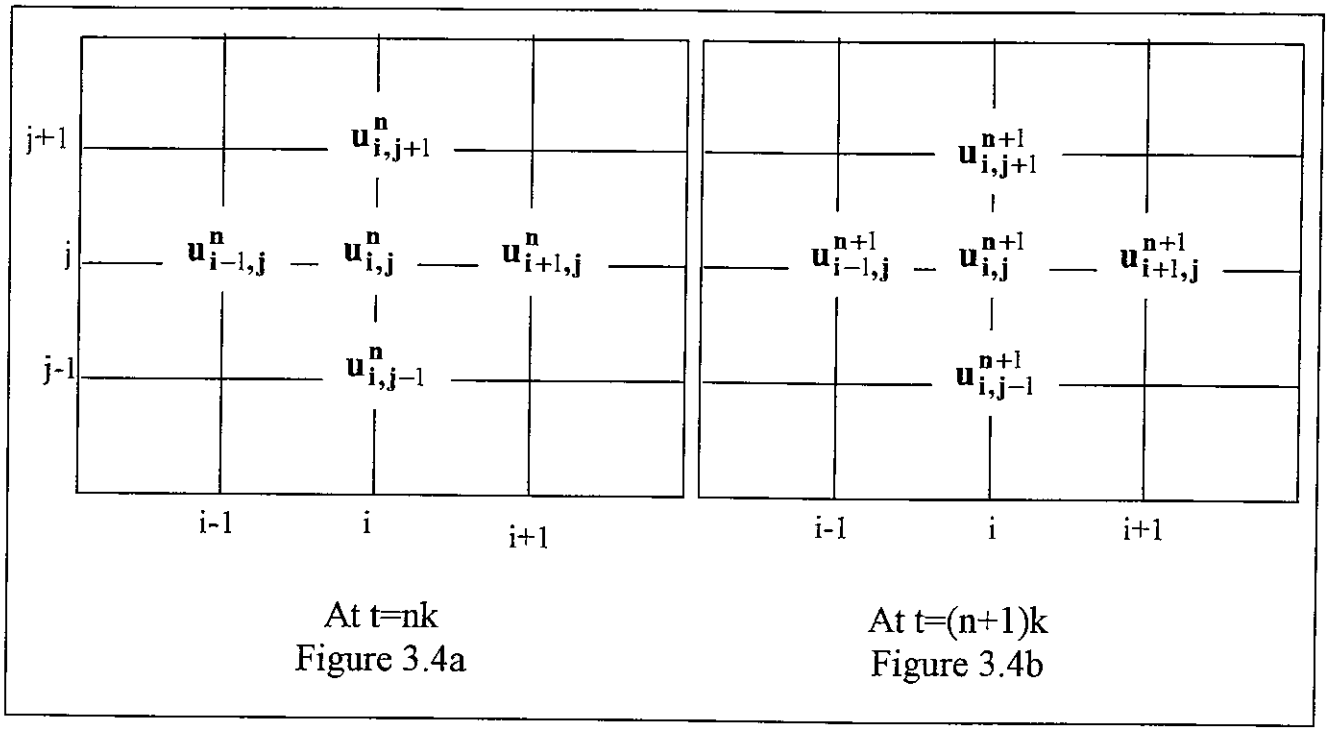
This simplifies equation(3.5.4) to :

$$k\rho g_x - hr(P_{i+1,j}^{n} - P_{i,j}^{n}) + c_2 U_{i+1,j}^{n+1} - c_1 U_{i,j}^{n} + \frac{\mu r}{2}U_{i-1,j}^{n+1} + c_3 U_{i,j+1}^{n+1} + \frac{\mu r}{2}U_{i,j-1}^{n+1}$$

$$= c_4 U_{i,j}^{n} - c_2 U_{i+1,j}^{n} - c_3 U_{i,j+1}^{n} - \frac{\mu r}{2}U_{i-1,j}^{n} - \frac{\mu r}{2}U_{i,j-1}^{n}, \qquad (3.5.5)$$

where $i, j = 1,2,3,....,N-1$ and $n = 0,1,2,3.....$

And looking at the mesh points in figure (3.4a) and figure (3.4b)



At t=nk
Figure 3.4a

At t=(n+1)k
Figure 3.4b

We conclude that we have five known points at t=nk and five unknown points at t=(n+1) k.

Forming the matrix method for Crank-Nicolson method of viscous fluid flow

$$
\left[\begin{array}{cccc|cccc|cccc|cccc}
-c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\mu r}{2} & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu r}{2} & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\mu r}{2} & -c_1 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\frac{\mu r}{2} & 0 & 0 & 0 & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & c_3 & 0 & 0 & c_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & c_3 & 0 & 0 & c_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & 0 & 0 & 0 & c_2 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & -c_1 & c_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & c_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1 & c_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & \frac{\mu r}{2} & -c_1
\end{array}\right]
\left[\begin{array}{c}
U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ \hline U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ \hline U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ \hline U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4}
\end{array}\right]^{n+1}
=
$$

$$
\left[\begin{array}{cccc|cccc|cccc|cccc}
c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\mu r}{2} & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\mu r}{2} & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\mu r}{2} & c_4 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-\frac{\mu r}{2} & 0 & 0 & 0 & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & -c_3 & 0 & 0 & -c_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & -c_3 & 0 & 0 & -c_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & 0 & 0 & 0 & -c_2 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & c_4 & -c_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & -c_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4 & -c_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & -\frac{\mu r}{2} & c_4
\end{array}\right]^{r}
\left[\begin{array}{c}
U_{1,1}^{n} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ \hline U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ \hline U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ \hline U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4}
\end{array}\right] +
$$

$$
\left[\begin{array}{c}
-k\rho g_x + hr\left(p^n_{2,1} - p^n_{1,1}\right) - \frac{\mu r}{2}\left(U^{n+1}_{1,0} + U^{n+1}_{0,1} + U^n_{0,1} + U^n_{1,0}\right) \\
-k\rho g_x + hr\left(p^n_{2,2} - p^n_{1,2}\right) - \frac{\mu r}{2}\left(U^{n+1}_{0,2} + U^n_{0,2}\right) \\
-k\rho g_x + hr\left(p^n_{2,3} - p^n_{1,3}\right) - \frac{\mu r}{2}\left(U^n_{0,3} + U^n_{0,3}\right) \\
-k\rho g_x + hr\left(p^n_{2,4} - p^n_{1,4}\right) - \frac{\mu r}{2}\left(U^{n+1}_{0,4} + U^n_{0,4}\right) - c3\left(U^{n+1}_{1,5} + U^n_{1,5}\right) \\
\hline
-k\rho g_x + hr\left(p^n_{3,1} - p^n_{2,1}\right) - \frac{\mu r}{2}\left(U^{n+1}_{2,0} + U^n_{2,0}\right) \\
-k\rho g_x + hr\left(p^n_{3,2} - p^n_{2,2}\right) \\
-k\rho g_x + hr\left(p^n_{3,3} - p^n_{2,3}\right) \\
-k\rho g_x + hr\left(p^n_{3,4} - p^n_{2,4}\right) - c3\left(U^{n+1}_{2,5} + U^n_{2,5}\right) \\
\hline
-k\rho g_x + hr\left(p^n_{4,1} - p^n_{3,1}\right) - \frac{\mu r}{2}\left(U^{n+1}_{3,0} + U^n_{3,0}\right) \\
-k\rho g_x + hr\left(p^n_{4,2} - p^n_{3,2}\right) \\
-k\rho g_x + hr\left(p^n_{4,3} - p^n_{3,3}\right) \\
-k\rho g_x + hr\left(p^n_{4,4} - p^n_{3,4}\right) - c3\left(U^{n+1}_{3,5} + U^n_{3,5}\right) \\
\hline
-k\rho g_x + hr\left(p^n_{5,1} - p^n_{4,1}\right) - \frac{\mu r}{2}\left(U^{n+1}_{4,0} + U^n_{4,0}\right) - c2\left(U^{n+1}_{5,1} + U^n_{5,1}\right) \\
-k\rho g_x + hr\left(p^n_{5,2} - p^n_{4,2}\right) - c2\left(U^{n+1}_{5,2} + U^n_{5,2}\right) \\
-k\rho g_x + hr\left(p^n_{5,3} - p^n_{4,3}\right) - c2\left(U^{n+1}_{5,3} + U^n_{5,3}\right) \\
-k\rho g_x + hr\left(p^n_{5,4} - p^n_{4,4}\right) - c2\left(U^{n+1}_{5,4} + U^n_{5,4}\right) - c3\left(U^{n+1}_{4,5} + U^n_{4,5}\right)
\end{array}\right]
$$

The last matrix method of Crank-Nicolson scheme can be written in a more simple form as:

$$\mathbf{AU}^{n+1} = \mathbf{BU}^{n} + \mathbf{Boundary} \qquad (3.5.6)$$

Where as usual we include the pressure gradient, the gravity and the boundary condition as Boundary. Also:

$$A = \begin{bmatrix} E & S & 0 & 0 \\ T & E & S & 0 \\ 0 & T & E & S \\ 0 & 0 & T & E \end{bmatrix}_{4\times4}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{V} & -\mathbf{S} & 0 & 0 \\ -\mathbf{T} & \mathbf{V} & -\mathbf{S} & 0 \\ 0 & -\mathbf{T} & \mathbf{V} & -\mathbf{S} \\ 0 & 0 & -\mathbf{T} & \mathbf{V} \end{bmatrix}_{4\times4}$$

where

$$E = \begin{bmatrix} \mathbf{c}_4 & \mathbf{c}_3 & 0 & 0 \\ \frac{\mu r}{2} & \mathbf{c}_4 & \mathbf{c}_3 & 0 \\ 0 & \frac{\mu r}{2} & \mathbf{c}_4 & \mathbf{c}_3 \\ 0 & 0 & \frac{\mu r}{2} & \mathbf{c}_4 \end{bmatrix}_{4\times4}$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{c}_1 & -\mathbf{c}_3 & 0 & 0 \\ -\frac{\mu r}{2} & \mathbf{c}_1 & -\mathbf{c}_3 & 0 \\ 0 & -\frac{\mu r}{2} & \mathbf{c}_1 & -\mathbf{c}_3 \\ 0 & 0 & -\frac{\mu r}{2} & \mathbf{c}_1 \end{bmatrix}_{4\times4}$$

$S = c_2\mathbf{I}_{(N-1)(N-1)}$, $T = \frac{\mu r}{2}\mathbf{I}_{(N-1)(N-1)}$, $N = 5$ where I is the identity matrix and 0 is the the matrix whose elements are all zeros. From the matrix method we can see that we have (N - 1)(N - 1) unkowns and we need to calculate them at each time step.

Also, we can do another approach to the viscous fluid flow using half time step for time level and the central difference method for the first space derivative, therefore the scheme will be:

$$\frac{\partial U}{\partial x}=\frac{1}{2}\left(\frac{U_{i+1,j}^{n+1}-U_{i-1,j}^{n+1}}{2h}+\frac{U_{i+1,j}^{n}-U_{i-1,j}^{n}}{2h}\right) \qquad (3.5.7)$$

$$k\rho g_x-hr(P_{i+1,j}^{n}-P_{i,j}^{n})+\left(\frac{\mu r}{2}-\frac{\rho hr\overline{U}}{4}\right)U_{i+1,j}^{n+1}-(2\mu r+\rho)U_{i,j}^{n+1}+$$

$$\left(\frac{\mu r}{2}+\frac{\rho hr\overline{U}}{4}\right)U_{i-1,j}^{n+1}+\left(\frac{\mu r}{2}-\frac{\rho hr\overline{V}}{4}\right)U_{i,j+1}^{n+1}+\left(\frac{\mu r}{2}+\frac{\rho hr\overline{V}}{4}\right)U_{i,j-1}^{n+1}=$$

$$(-\rho+2\mu r)U_{i,j}^{n}+\left(\frac{\rho hr\overline{U}}{4}-\frac{\mu r}{2}\right)U_{i+1,j}^{n}+\left(\frac{\rho hr\overline{V}}{4}-\frac{\mu r}{2}\right)U_{i,j+1}^{n}-\left(\frac{\rho hr\overline{U}}{4}+\frac{\mu r}{2}\right)U_{i-1,j}^{n}$$

$$-\left(\frac{\rho hr\overline{V}}{4}-\frac{\mu r}{2}\right)U_{i,j-1}^{n} \qquad (3.5.8)$$

Let $c_1=\frac{\mu r}{2}-\frac{\rho hr\overline{U}}{4}$, $c_2=2\mu r+\rho$, $c_3=\frac{\mu r}{2}+\frac{\rho hr\overline{U}}{4}$, $c_4=\frac{\mu r}{2}-\frac{\rho hr\overline{V}}{4}$

$c_5=\frac{\mu r}{2}+\frac{\rho hr\overline{V}}{4}$, $c_6=-\rho+2\mu r$

Which may lead to

$$k\rho g_x-hr(P_{i+1,j}^{n}-P_{i,j}^{n})+c_1U_{i+1,j}^{n+1}-c_2U_{i,j}^{n+1}+c_3U_{i-1,j}^{n+1}+c_4U_{i,j+1}^{n+1}+c_5U_{i,j-1}^{n+1}=$$

$$c_6U_{i,j}^{n}-c_1U_{i+1,j}^{n}-c_3U_{i-1,j}^{n}-c_4U_{i,j+1}^{n}-c_5U_{i,j-1}^{n} \qquad (3.5.9)$$

Since we use Crank - Nicolson method the matrix form of this scheme is:

$AU^{n+1}=BU^{n}+Boundary$ where $n=0,1,2,.....$

and

$$A=\begin{bmatrix} Q & Y & 0 & 0 & 0 & 0 & 0 & 0\\ R & Q & Y & 0 & 0 & 0 & 0 & 0\\ 0 & R & Q & Y & 0 & 0 & 0 & 0\\ 0 & 0 & R & Q & Y & 0 & 0 & 0\\ 0 & 0 & 0 & R & Q & Y & 0 & 0\\ 0 & 0 & 0 & 0 & R & Q & Y & 0\\ 0 & 0 & 0 & 0 & 0 & R & Q & Y\\ 0 & 0 & 0 & 0 & 0 & 0 & R & Q \end{bmatrix}_{8\times 8}$$

$$\mathbf{B}=\begin{bmatrix} \mathbf{a} & \mathbf{b} & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{c} & \mathbf{a} & \mathbf{b} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{c} & \mathbf{a} & \mathbf{b} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{c} & \mathbf{a} & \mathbf{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{c} & \mathbf{a} & \mathbf{b} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{c} & \mathbf{a} & \mathbf{b} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{c} & \mathbf{a} & \mathbf{b} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{c} & \mathbf{a} \end{bmatrix}_{8\times 8}$$

In the next page the scheme is written in full matrix form

$$\mathbf{Q}=\begin{bmatrix} -\mathbf{c}_2 & \mathbf{c}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{c}_5 & -\mathbf{c}_2 & \mathbf{c}_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{c}_5 & -\mathbf{c}_2 & \mathbf{c}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{c}_5 & -\mathbf{c}_2 & \mathbf{c}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{c}_5 & -\mathbf{c}_2 & \mathbf{c}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{c}_5 & -\mathbf{c}_2 & \mathbf{c}_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{c}_5 & -\mathbf{c}_2 & \mathbf{c}_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{c}_5 & -\mathbf{c}_2 \end{bmatrix}_{8\times 8}$$

$$\mathbf{a}=\begin{bmatrix} \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 \end{bmatrix}_{8\times 8}$$

$Y = c_1 \mathbf{I}_{(N-1)(N-1)}$ similarly $R = c_3 \mathbf{I}_{(N-1)(N-1)}$, $b = -c_1 \mathbf{I}_{(N-1)(N-1)}$ and $c = -R$ and $N = 9$

$$
\left[\begin{array}{cccc|cccc|cccc|cccc}
-c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_5 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & c_5 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & c_5 & -c_2 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
c_3 & 0 & 0 & 0 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & c_3 & 0 & 0 & c_5 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & c_3 & 0 & 0 & 0 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & c_4 & 0 & 0 & c_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & c_4 & 0 & 0 & c_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & 0 & 0 & 0 & c_1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & 0 & -c_2 & c_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & c_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2 & c_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & c_5 & -c_2
\end{array}\right]
\left[\begin{array}{c}
U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \\ \hline
U_{2,1} \\ U_{2,2} \\ U_{2,3} \\ U_{2,4} \\ \hline
U_{3,1} \\ U_{3,2} \\ U_{3,3} \\ U_{3,4} \\ \hline
U_{4,1} \\ U_{4,2} \\ U_{4,3} \\ U_{4,4}
\end{array}\right]^{n+1}
$$

$$
\begin{bmatrix}
\mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & 0 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\mathbf{c}_3 & 0 & 0 & 0 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & 0 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & 0 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 & -\mathbf{c}_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & 0 & 0 & 0 & -\mathbf{c}_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & 0 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6 & -\mathbf{c}_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{c}_3 & 0 & 0 & -\mathbf{c}_5 & \mathbf{c}_6
\end{bmatrix}
\begin{bmatrix}
\mathbf{U}_{1,1} \\ \mathbf{U}_{1,2} \\ \mathbf{U}_{1,3} \\ \mathbf{U}_{1,4} \\
\mathbf{U}_{2,1} \\ \mathbf{U}_{2,2} \\ \mathbf{U}_{2,3} \\ \mathbf{U}_{2,4} \\
\mathbf{U}_{3,1} \\ \mathbf{U}_{3,2} \\ \mathbf{U}_{3,3} \\ \mathbf{U}_{3,4} \\
\mathbf{U}_{4,1} \\ \mathbf{U}_{4,2} \\ \mathbf{U}_{4,3} \\ \mathbf{U}_{4,4}
\end{bmatrix}^{\mathbf{n}} +
$$

$$
\left[\begin{array}{c}
-k\rho g_x + hr(p^n_{2,1} - p^n_{1,1}) - c_3(U^{n+1}_{0,1} + U^n_{0,1}) - c_5(U^{n+1}_{1,0} + U^n_{1,0}) \\
-k\rho g_x + hr(p^n_{2,2} - p^n_{1,2}) - c_3(U^{n+1}_{0,2} + U^n_{0,2}) \\
-k\rho g_x + hr(p^n_{2,3} - p^n_{1,3}) - c_3(U^{n+1}_{0,3} + U^n_{0,3}) \\
-k\rho g_x + hr(p^n_{2,4} - p^n_{1,4}) - c_3(U^{n+1}_{0,4} + U^n_{0,4}) - c_4(U^{n+1}_{1,5} + U^n_{1,5}) \\
\hline
-k\rho g_x + hr(p^n_{3,1} - p^n_{2,1}) - c_5(U^{n+1}_{2,0} + U^n_{2,0}) \\
-k\rho g_x + hr(p^n_{3,2} - p^n_{2,2}) \\
-k\rho g_x + hr(p^n_{3,3} - p^n_{2,3}) \\
-k\rho g_x + hr(p^n_{3,4} - p^n_{2,3}) - c_4(U^{n+1}_{2,5} + U^n_{2,5}) \\
\hline
-k\rho g_x + hr(p^n_{4,1} - p^n_{3,1}) - c_5(U^{n+1}_{3,0} + U^n_{3,0}) \\
-k\rho g_x + hr(p^n_{4,2} - p^n_{3,2}) \\
-k\rho g_x + hr(p^n_{4,3} - p^n_{3,2}) \\
-k\rho g_x + hr(p^n_{4,4} - p^n_{3,4}) - c_4(U^{n+1}_{3,5} + U^n_{3,5}) \\
\hline
-k\rho g_x + hr(p^n_{5,1} - p^n_{4,1}) - c_1(U^{n+1}_{5,1} + U^n_{5,1}) - c_5(U^{n+1}_{4,0} + U^n_{4,0}) \\
-k\rho g_x + hr(p^n_{5,2} - p^n_{4,2}) - c_1(U^{n+1}_{5,2} + U^n_{5,2}) \\
-k\rho g_x + hr(p^n_{5,3} - p^n_{4,3}) - c_1(U^{n+1}_{5,3} + U^n_{5,3}) \\
-k\rho g_x + hr(p^n_{5,4} - p^n_{4,4}) - c_1(U^{n+1}_{5,4} + U^n_{5,4}) - c_4(U^{n+1}_{4,5} + U^n_{4,5})
\end{array}\right]
$$

3.6 Another explicit finite difference method for Navier-Stokes equation:

The nondimensional Navier-Stokes equation with primitive variables u,v, P for unsteady incompressible flow is [10]

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= \frac{1}{Re}\nabla^2 u - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\partial P}{\partial x} && (3.6.1a)\\ \frac{\partial v}{\partial t} &= \frac{1}{Re}\nabla^2 v - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{\partial P}{\partial y} && (3.6.1b) \end{aligned}\right\} (3.6.1)$$

and the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3.6.2)$$

Multiply equation(33.6.2) at first by (-u) and using this result in (3.6.1a) and again multiply the continuity equation by (-v) and use it in (3.6.1b) we get :

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= \frac{1}{Re}\nabla^2 u - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\partial P}{\partial x} - u\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial y} && (3.6.3a)\\ \frac{\partial v}{\partial t} &= \frac{1}{Re}\nabla^2 v - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{\partial P}{\partial y} - v\frac{\partial u}{\partial x} - v\frac{\partial v}{\partial y} && (3.6.3b) \end{aligned}\right\} (3.6.3)$$

The last equations can be written in the following compact form :

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= \frac{1}{Re}\nabla^2 u - \frac{\partial (u^2)}{\partial x} - \frac{\partial (uv)}{\partial y} - \frac{\partial P}{\partial x} && (3.6.4a)\\ \frac{\partial v}{\partial t} &= \frac{1}{Re}\nabla^2 v - \frac{\partial (uv)}{\partial x} - \frac{\partial (v^2)}{\partial y} - \frac{\partial P}{\partial y} && (3.6.4b) \end{aligned}\right\} (3.6.4)$$

Using the definition of the stream function (as described in chapter1)

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial y} && (3.6.5a)\\ v &= -\frac{\partial \psi}{\partial x} && (3.6.5b) \end{aligned}\right\} (3.6.5)$$

and define the vorticity ω as $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ (3.6.6)

Differenti ate (3.6.4a) with respect to y and (3.6.4b) with respect to x we get :

$$\frac{\partial^2 u}{\partial t \partial y} = \frac{1}{Re}\frac{\partial}{\partial y}(\nabla^2 u) - \frac{\partial^2 (u^{2})}{\partial y \partial x} - \frac{\partial^2 (uv)}{\partial y^2} - \frac{\partial^2 P}{\partial x \partial y} \qquad (3.6.7)$$

$$\frac{\partial^2 v}{\partial t \partial x} = \frac{1}{Re}\frac{\partial}{\partial x}(\nabla^2 v) - \frac{\partial^2}{\partial x^2}(uv) - \frac{\partial^2 (v^2)}{\partial x \partial y} - \frac{\partial^2 P}{\partial x \partial y} \qquad (3.6.7b)$$

Subtraction of (3.6.7a) from (3.6.7b) using the vorticity definition we get:

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{Re}\nabla^2\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{\partial^2 (uv)}{\partial x^2} + \frac{\partial (u^2)}{\partial y \partial x} - \frac{\partial (v^2)}{\partial y \partial x} + \frac{\partial^2 (uv)}{\partial y^2} \qquad (3.6.8)$$

Using the contiuity equation:

$$\frac{\partial \omega}{\partial t} = \frac{1}{Re}\nabla^2 \omega - u\frac{\partial \omega}{\partial x} - v\frac{\partial \omega}{\partial y} \qquad (3.6.9)$$

This can be written in compact form as it is described in [10]

$$\frac{\partial \omega}{\partial t} = \frac{1}{Re}\nabla^2 \omega - \nabla \cdot (\omega q) \qquad \text{where} \quad q = (u, v) \qquad (3.6.10)$$

Using the second method for finite difference method as described earlier, i.e.

$$\omega_{i,j}^{n+1} = \exp(kL)\omega_{i,j}^{n}$$

where the opreator $L = \frac{1}{Re}\nabla^2 \omega - u\frac{\partial \omega}{\partial x} - v\frac{\partial \omega}{\partial y}$ (3.6.11)

and expanding the exponential to a series with two tearms this leads to:

$$\omega_{i,j}^{n+1} = (1 + kL)\omega_{i,j}^{n} \qquad (3.6.12)$$

To approximate the $\nabla^2 \omega$ (Laplace equation) as usual we use the δ_x^2, δ_y^2 for x - direction and y - direction respectively, this leads to scheme as described in [10]

$$\omega_{i,j}^{n+1} = \left[1 + \frac{1}{Re}(\delta_x^2 + \delta_y^2) - \frac{k}{2h}(uH_x + vH_y)\right]\omega_{i,j}^{n} \qquad (3.6.13)$$

where $\omega_{i,j}^{n}$ is the vorticity at the grid point (ih, jh, nk) and $H_x = \Delta_x + \nabla_x$ $H_y = \Delta_y + \nabla_y$ and Δ and ∇ the forward and backward difference opreators respectively [10]. The calculations starts at $t = 0$ where $\psi(u, v)$ and ω is known. From (3.6.13) with $n = 0$ we obtain the vorticity ω at $t = k$ and henc u and v. This process continues with boundary values (if present) being included as the calculation proceeds in time[10]. Using the stream function for u and v and the explicit finite difference that is:

$$u = \frac{\partial \psi}{\partial y} \quad , v = -\frac{\partial \psi}{\partial x} \qquad (3.6.14a)$$

$$v = -\left(\frac{\psi^{n}_{i+1,j} - \psi^{n}_{i,j}}{h}\right) = -\Delta_x \psi^{n}_{i,j} \qquad (3.6.14b)$$

and substituting these values in eqaution (3.6.13) for u and v we get :

$$\omega^{n+1}_{i,j} = \left[1 + \frac{r}{Re}\left(\delta_x^2 + \delta_y^2\right) - \frac{k}{2h}\left(\Delta_y \psi^{n}_{i,j} H_x - \Delta_x \psi^{n}_{i,j} H_y\right)\right]\omega^{n}_{i,j} \qquad (3.6.15)$$

Return back to the definition of the stream function, where

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$

we may write

$$-\omega = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \qquad (3.6.16)$$

This equation can be approximated by the finite difference scheme as :

$$-h^2 \omega^{n}_{i,j} = \delta_x^2 \psi^{n}_{i,j} + \delta_y^2 \psi^{n}_{i,j} \qquad (3.6.17)$$

Equations (3.6.15) and (3.6.17) are solved together, and we may write each of them in a matrix form. First of all, equation (3.6.17) can be written in the following form :

$$-h^2 \omega^{n}_{i,j} = \psi^{n}_{i+1,j} + \psi^{n}_{i-1,j} + \psi^{n}_{i,j+1} + \psi^{n}_{i,j-1} - 4\psi^{n}_{i,j} \qquad (3.6.18)$$

and equation(3.6.15) becomes

$$\omega^{n+1}_{i,j} = \left(1 - \frac{4}{Re}\right)\omega^{n}_{i,j} + \left[\frac{r}{Re} - \frac{k}{2h}\left(\frac{\psi^{n}_{i,j+1} - \psi^{n}_{i,j}}{h}\right)\right]\omega^{n}_{i+1,j} +$$

$$\left[\frac{r}{Re} + \frac{k}{2h}\left(\frac{\psi^{n}_{i,j+1} - \psi^{n}_{i,j}}{h}\right)\right]\omega^{n}_{i-1,j} + \left[\frac{r}{Re} + \frac{k}{2h}\left(\frac{\psi^{n}_{i+1,j} - \psi^{n}_{i,j}}{h}\right)\right]\omega^{n}_{i,j+1}$$

$$+ \left[\frac{r}{Re} - \frac{k}{2h}\left(\frac{\psi^{n}_{i+1,j} - \psi^{n}_{i,j}}{h}\right)\right]\omega^{n}_{i,j-1} \qquad (3.6.19)$$

and we can see these points on figure (3.5).

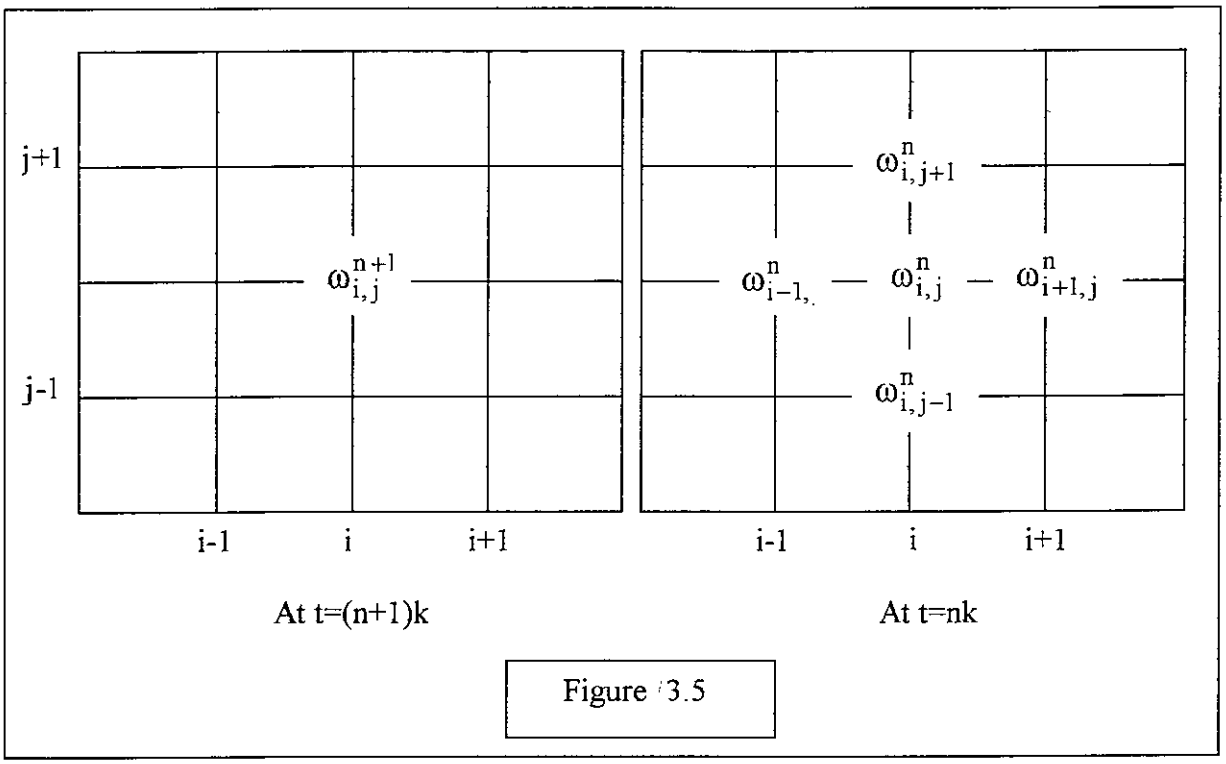


Figure 3.5

Now we have five known points (at t=nk), and one unknown point (at t=(n+1)k). Also we can see from figure (3.6) that the mesh points of equation (3.6.18) all lie on the same sheet, and we want to find the vortex function from this equation and substitute these values in equation (3.6.15).

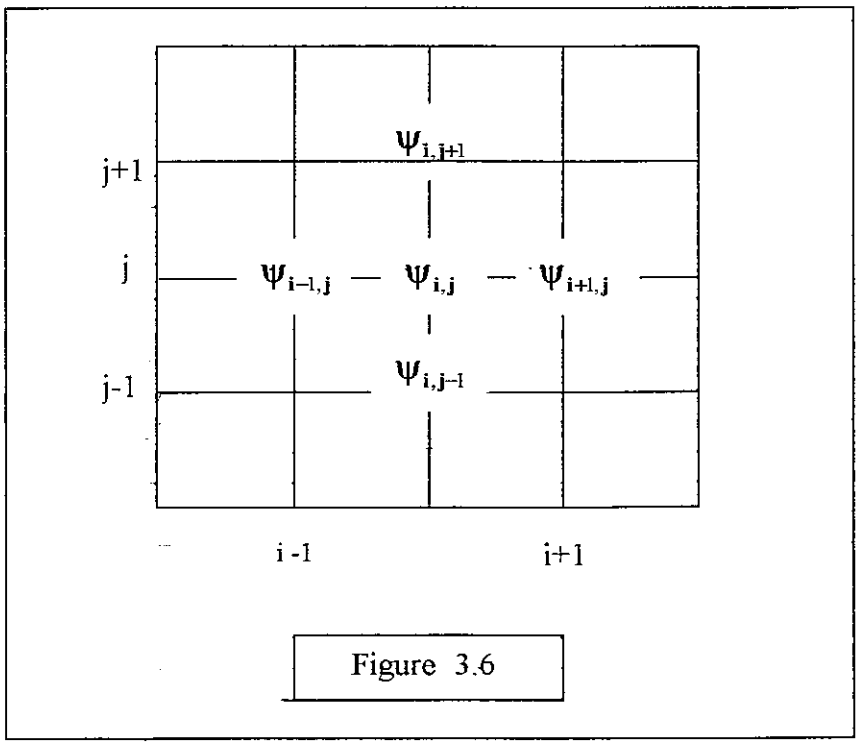


Figure 3.6

Equation (3.6.19) can be written in matrix form as follows :

$\omega^{n+1} = \mathbf{A}^n \omega^n + \mathbf{Boundary}$, where $n = 0,1,2,\cdots$,

and

$$\mathbf{A}^n = \begin{bmatrix} \mathbf{e}_1^n & \mathbf{b}_1^n & 0 & 0 & 0 & 0 \\ \mathbf{c}_1^n & \mathbf{e}_2^n & \mathbf{b}_2^n & 0 & 0 & 0 \\ 0 & \mathbf{c}_2^n & \mathbf{e}_3^n & \mathbf{b}_3^n & 0 & 0 \\ 0 & 0 & \mathbf{c}_3^n & \mathbf{e}_4^n & \mathbf{b}_4^n & 0 \\ 0 & 0 & 0 & \mathbf{c}_4^n & \mathbf{e}_5^n & \mathbf{b}_5^n \\ 0 & 0 & 0 & 0 & \mathbf{c}_5^n & \mathbf{e}_6^n \end{bmatrix},$$

$$\mathbf{e}_i^n = \begin{bmatrix} \left(1-\frac{4}{Re}\right) & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,1}^n-\psi_{i,1}^n}{h}\right)\right] & 0 & 0 & 0 & 0 \\ \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i+1,2}^n-\psi_{i,2}^n}{h}\right)\right] & \left(1-\frac{4}{Re}\right) & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,2}^n-\psi_{i,2}^n}{h}\right)\right] & 0 & 0 & 0 \\ 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i+1,3}^n-\psi_{i,3}^n}{h}\right)\right] & \left(1-\frac{4}{Re}\right) & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,3}^n-\psi_{i,3}^n}{h}\right)\right] & 0 & 0 \\ 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i+1,4}^n-\psi_{i,4}^n}{h}\right)\right] & \left(1-\frac{4}{Re}\right) & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,4}^n-\psi_{i,4}^n}{h}\right)\right] & 0 \\ 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i+1,5}^n-\psi_{i,5}^n}{h}\right)\right] & \left(1-\frac{4}{Re}\right) & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,5}^n-\psi_{i,5}^n}{h}\right)\right] \\ 0 & 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i+1,6}^n-\psi_{i,6}^n}{h}\right)\right] & \left(1-\frac{4}{Re}\right) \end{bmatrix},$$

$$\mathbf{b}_i^n = \begin{bmatrix} \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i,2}^n-\psi_{i,1}^n}{h}\right)\right] & 0 & 0 & 0 & 0 & 0 \\ 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i,3}^n-\psi_{i,2}^n}{h}\right)\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i,4}^n-\psi_{i,3}^n}{h}\right)\right] & 0 & 0 & 0 \\ 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i,5}^n-\psi_{i,4}^n}{h}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i,6}^n-\psi_{i,5}^n}{h}\right)\right] & 0 \\ 0 & 0 & 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{2h}\left(\frac{\psi_{i,7}^n-\psi_{i,6}^n}{h}\right)\right] \end{bmatrix}_{6\times 6},$$

$$c_1^n = \begin{bmatrix} \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,2}^n-\psi_{i+1,1}^n}{h}\right)\right] & 0 & 0 & 0 & 0 & 0 \\ 0 & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,3}^n-\psi_{i+1,1}^n}{h}\right)\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,4}^n-\psi_{i+1,3}^n}{h}\right)\right] & 0 & 0 & 0 \\ 0 & 0 & 0 & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,5}^n-\psi_{i+1,4}^n}{h}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & 0 & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,6}^n-\psi_{i+1,5}^n}{h}\right)\right] & 0 \\ 0 & 0 & 0 & 0 & 0 & \left[\frac{r}{Re}+\frac{k}{2h}\left(\frac{\psi_{i+1,7}^n-\psi_{i+1,6}^n}{h}\right)\right] \end{bmatrix}_{6\times6}$$

Also equation (3.6.18) can be written in matrix form as:

$$-h^2\omega^{n+1} = M\psi^{n+1} + \mathbf{Boundary} \qquad (3.6.20)$$

where ω^{n+1} and ψ^{n+1} is the column vector of the vortex and the stream function respectively and M is the block matrix,

$$M = \begin{bmatrix} N & I & 0 & 0 & 0 & 0 \\ I & N & I & 0 & 0 & 0 \\ 0 & I & N & I & 0 & 0 \\ 0 & 0 & I & N & I & 0 \\ 0 & 0 & 0 & I & N & I \\ 0 & 0 & 0 & 0 & I & N \end{bmatrix}_{6\times6}$$

where

$$N = \begin{bmatrix} -4 & 1 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}_{6\times6}$$

And I is the identity matrix having (N-1)(N-1) elements (N=7) and 0 is the matrix whose elements are all zero and have the same dimension of the matrix I.

We have to find the inverse of M to get ψ, and before that, we have to calculate initially the vortex from equation (3.6.15), then use these values in the last matrix form to find the stream function, and hence the velocity vector in two dimensions.

Return back to equation (3.6.9) and using the backward difference method for the last two terms (convection terms), we will get another scheme and assuming u and v positive we will get [10]

$$\omega_{i,j}^{n+1}=\left[1+\frac{r}{Re}(\delta_x^2+\delta_y^2)-\frac{k}{h}(u\nabla_x+v\nabla_y)\right]\omega_{i,j}^{n} \qquad (3.6.21)$$

Also this scheme will be solved together with equation (3.6.20). Equation (3.6.21) can write as:

$$\omega_{i,j}^{n+1}=\left(1-\frac{4r}{Re}-\frac{ku}{h}-\frac{kv}{h}\right)\omega_{i,j}^{n}+\frac{r}{Re}\omega_{i+1,j}^{n}+\frac{r}{Re}\omega_{i,j+1}^{n}+\left(\frac{r}{Re}+\frac{ku}{h}\right)\omega_{i-1,j}^{n}$$

$$+\left(\frac{r}{Re}+\frac{kv}{h}\right)\omega_{i,j-1}^{n} \qquad (3.6.22)$$

Using equation (3.6.14) for u and v leads to :

$$\omega_{i,j}^{n+1}=\left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,j+1}^{n}-\psi_{i+1,j}^{n}}{h}\right)\right]\omega_{i,j}^{n}+\frac{r}{Re}\omega_{i+1,j}^{n}+\frac{r}{Re}\omega_{i,j+1}^{n}+$$

$$\left[\frac{r}{Re}+\frac{k}{h}\left(\frac{\psi_{i,j+1}^{n}-\psi_{i,j}^{n}}{h}\right)\right]\omega_{i-1,j}^{n}+\left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,j}^{n}-\psi_{i,j}^{n}}{h}\right)\right]\omega_{i,j-1}^{n} \qquad (3.6.23)$$

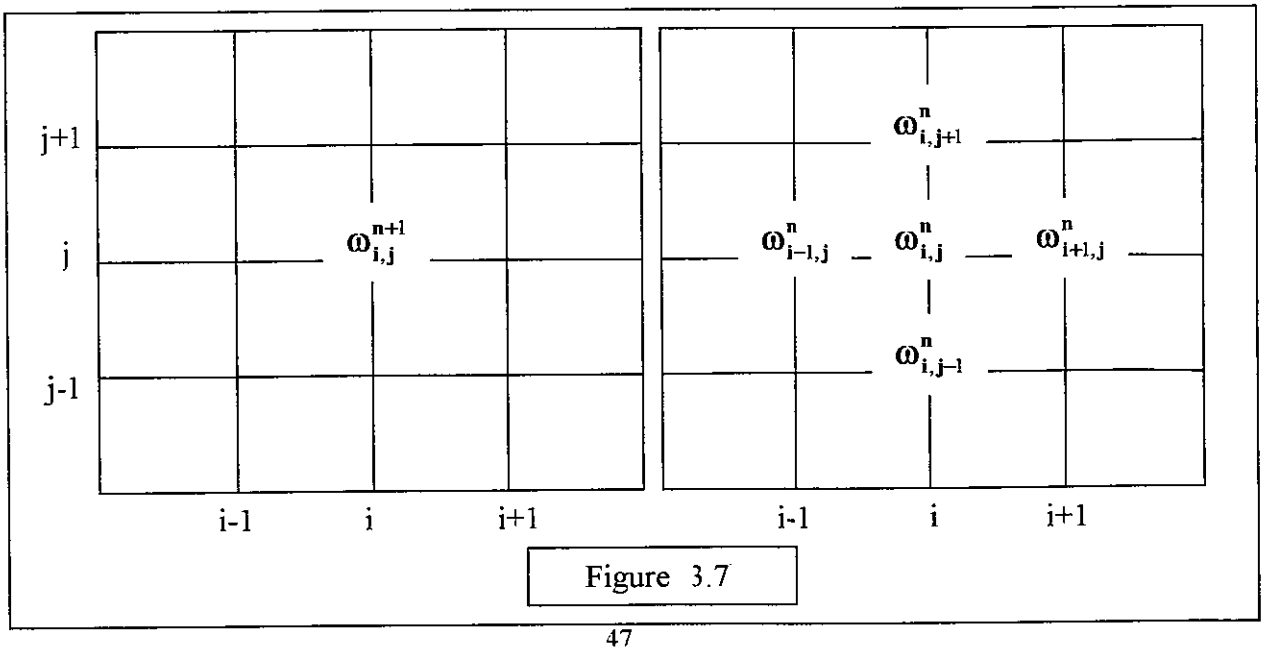


Figure 3.7

We can see that, we have five known points at t=nk, and one unknown point at t=(n+1)k, which we need to find.

We have from matrix from (N-1)(N-1) unknown at each time level matrix of size (N-1)(N-1)(N-1)(N-1) .Now the matrix method of (3.6.22) or (3.6.23)

$$\boldsymbol{\omega}^{n+1} = \mathbf{B}_i^n \boldsymbol{\omega}^n + \mathbf{Boundary} \qquad (3.6.24)$$

Where:

$$\mathbf{B}_i^n = \begin{bmatrix} \mathbf{K}_1^n & \mathbf{L}_1^n & 0 & 0 & 0 & 0 \\ \mathbf{M}_1^n & \mathbf{K}_2^n & \mathbf{L}_2^n & 0 & 0 & 0 \\ 0 & \mathbf{M}_2^n & \mathbf{K}_3^n & \mathbf{L}_3^n & 0 & 0 \\ 0 & 0 & \mathbf{M}_3^n & \mathbf{K}_4^n & \mathbf{L}_4^n & 0 \\ 0 & 0 & 0 & \mathbf{M}_4^n & \mathbf{K}_5^n & \mathbf{L}_5^n \\ 0 & 0 & 0 & 0 & \mathbf{M}_5^n & \mathbf{K}_6^n \end{bmatrix}_{6\times 6},$$

$$k_i^n = \begin{bmatrix} \left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,2}^n-\psi_{i+1,1}^n}{h}\right)\right] & \frac{r}{Re} & 0 & 0 & 0 & 0 \\ \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,2}^n-\psi_{i,2}^n}{h}\right)\right] & \left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,3}^n-\psi_{i+1,2}^n}{h}\right)\right] & \frac{r}{Re} & 0 & 0 & 0 \\ 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,3}^n-\psi_{i,3}^n}{h}\right)\right] & \left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,4}^n-\psi_{i+1,3}^n}{h}\right)\right] & \frac{r}{Re} & 0 & 0 \\ 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,4}^n-\psi_{i,4}^n}{h}\right)\right] & \left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,5}^n-\psi_{i+1,4}^n}{h}\right)\right] & \frac{r}{Re} & 0 \\ 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,5}^n-\psi_{i,5}^n}{h}\right)\right] & \left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,6}^n-\psi_{i+1,5}^n}{h}\right)\right] & \frac{r}{Re} \\ 0 & 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,6}^n-\psi_{i,6}^n}{h}\right)\right] & \left[1-\frac{4r}{Re}-\frac{k}{h}\left(\frac{\psi_{i,7}^n-\psi_{i+1,6}^n}{h}\right)\right] \end{bmatrix}_{6\times 6}$$

$$m_i^n = \begin{bmatrix} \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,2}^n-\psi_{i+1,1}^n}{h}\right)\right] & 0 & 0 & 0 & 0 & 0 \\ 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,3}^n-\psi_{i+1,2}^n}{h}\right)\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,4}^n-\psi_{i+1,3}^n}{h}\right)\right] & 0 & 0 & 0 \\ 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,5}^n-\psi_{i+1,4}^n}{h}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,6}^n-\psi_{i+1,5}^n}{h}\right)\right] & 0 \\ 0 & 0 & 0 & 0 & 0 & \left[\frac{r}{Re}-\frac{k}{h}\left(\frac{\psi_{i+1,7}^n-\psi_{i+1,6}^n}{h}\right)\right] \end{bmatrix}_{6\times 6}$$

l_i^n can be written as $l_i^n = \frac{r}{Re} I_{(N-1)(N-1)}$

3.7 Explicit Finite difference method for three dimensional Navier-Stokes equation:

$$\rho g_x - \frac{P^n_{i+1,j,l} - P^n_{i,j,l}}{h} + \mu\left[\frac{U^n_{i+1,j,l} - 2U^n_{i,j,l} + U^n_{i-1,j,l}}{h^2} + \frac{U^n_{i,j+1,l} - 2U^n_{i,j,l} + U^n_{i,j-1,l}}{h^2} + \right.$$

$$\left. \frac{U^n_{i,j,l+1} - 2U^n_{i,j,l} + U^n_{i,j,l-1}}{h^2}\right] = \rho\frac{U^{n+1}_{i,j,l} - U^n_{i,j,l}}{k} + \overline{U}\frac{U^n_{i+1,j,l} - U^n_{i,j,l}}{h} +$$

$$\left.\overline{V}\frac{U^n_{i,j+1,l} - U^n_{i,j,l}}{h} + \overline{W}\frac{U^n_{i,j,l+1} - U^n_{i,j,l}}{h}\right] \qquad (3.7.1)$$

If we let $r = \frac{k}{h^2}$, and let

$c_1 = \mu r - \rho hr\overline{U}$, $c_2 = -6\mu r + \rho + \rho hr\overline{U} + \rho hr\overline{V} + \rho hr\overline{W}$,

$c_3 = \mu r - \rho hr\overline{V}$, $c_4 = \mu r - \rho hr\overline{W}$, so that

$c_2 = -(c_1 + c_3 + c_4) - 3\mu r$,

and substitute these values in equation (3.7.1) we get:

$$k\rho g_x - hr\left(P^n_{i+1,j,l} - P^n_{i,j,l}\right) + c_1 U^n_{i+1,j,l} + c_2 U^n_{i,j,l} + \mu r U^n_{i-1,j,l} + c_3 U^n_{i,j+1,l} +$$

$$\mu r U^n_{i,j-1,l} + c_4 U^n_{i,j,l+1} + \mu r U^n_{i,j,l-1} = \rho U^{n+1}_{i,j,l} \qquad (3.7.2)$$

Similarly in the y and z directions we get:

$$k\rho g_y - hr\left(P^n_{i,j+1,l} - P^n_{i,j,l}\right) + c_1 V^n_{i+1,j,l} + c_2 V^n_{i,j,l} + \mu r V^n_{i-1,j,l} + c_3 V^n_{i,j+1,l} +$$

$$\mu r V^n_{i,j-1,l} + c_4 V^n_{i,j,l+1} + \mu r V^n_{i,j,l-1} = \rho V^{n+1}_{i,j,l} \qquad (3.7.3)$$

and

$$k\rho g_z - hr\left(P^n_{i,j,l+1} - P^n_{i,j,l}\right) + c_1 W^n_{i+1,j,l} + c_2 W^n_{i,j,l} + \mu r W^n_{i-1,j,l} + c_3 W^n_{i,j+1,l} +$$

$$\mu r W^n_{i,j-1,l} + c_4 W^n_{i,j,l+1} + \mu r W^n_{i,j,l-1} = \rho W^n_{i,j,l} \qquad (3.7.4)$$

Solving equation (3.1.7) by explicit finite difference scheme we will get:

$$\rho U^{n+1} = \Omega U^n + \mathbf{Boundary}, \qquad (3.7.5)$$

where

$$\Omega = \begin{bmatrix} \lambda & G & 0 & 0 & 0 & 0 & 0 & 0 \\ \theta & \lambda & G & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta & \lambda & G & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta & \lambda & G & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta & \lambda & G & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta & \lambda & G & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta & \lambda & G \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta & \lambda \end{bmatrix}_{(N-1)(N-1)},$$

$$\lambda = \begin{bmatrix} A & B & 0 & 0 & 0 & 0 & 0 & 0 \\ M & A & B & 0 & 0 & 0 & 0 & 0 \\ 0 & M & A & B & 0 & 0 & 0 & 0 \\ 0 & 0 & M & A & B & 0 & 0 & 0 \\ 0 & 0 & 0 & M & A & B & 0 & 0 \\ 0 & 0 & 0 & 0 & M & A & B & 0 \\ 0 & 0 & 0 & 0 & 0 & M & A & B \\ 0 & 0 & 0 & 0 & 0 & 0 & M & A \end{bmatrix}_{(N-1)(N-1)},$$

$$G = \begin{bmatrix} C & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & C & 0 \\ 0 & 0 & 0 & 0 & C \end{bmatrix}_{(N-1)(N-1)}, \theta = \begin{bmatrix} M & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 \\ 0 & 0 & 0 & M & 0 \\ 0 & 0 & 0 & 0 & M \end{bmatrix}_{(N-1)(N-1)},$$

and

$$A = \begin{bmatrix} c_2 & c_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu r & c_2 & c_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu r & c_2 & c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu r & c_2 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu r & c_2 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu r & c_2 & c_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu r & c_2 & c_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu r & c_2 \end{bmatrix}_{(N-1)(N-1)}$$

$B = c_3 I_{(N-1)(N-1)}$, $M = \mu r I_{(N-1)(N-1)}$, $C = c_1 I_{(N-1)(N-1)}$, where I is the identity matrix and 0 the matrix where elements are all zeros.

$$
\text{Boundary} = \left[\begin{array}{c}
k\rho g_x - hr(p^n_{2,1,1} - p^n_{1,1,1}) + \mu r U^n_{0,1,1} + \mu r U^n_{1,0,1} + \mu r U^n_{1,1,0} \\
k\rho g_x - hr(p^n_{2,1,2} - p^n_{1,1,2}) + \mu r U^n_{0,1,2} + \mu r U^n_{1,0,2} \\
k\rho g_x - hr(p^n_{2,1,3} - p^n_{1,1,3}) + \mu r U^n_{0,1,3} + \mu r U^n_{1,0,3} + c_4 U^n_{1,1,4} \\
\hline
k\rho g_x - hr(p^n_{2,2,1} - p^n_{1,2,1}) + \mu r U^n_{0,2,1} + \mu r U^n_{1,2,0} \\
k\rho g_x - hr(p^n_{2,2,2} - p^n_{1,2,2}) + \mu r U^n_{0,2,2} \\
k\rho g_x - hr(p^n_{2,2,3} - p^n_{1,2,3}) + \mu r U^n_{0,2,3} + c_4 U^n_{1,2,4} \\
\hline
k\rho g_x - hr(p^n_{2,3,1} - p^n_{1,3,1}) + c_3 U^n_{1,4,1} + \mu r U^n_{0,3,1} + \mu r U^n_{1,3,0} \\
k\rho g_x - hr(p^n_{2,3,2} - p^n_{1,3,2}) + \mu r U^n_{0,3,2} + c_3 U^n_{1,4,2} \\
k\rho g_x - hr(p^n_{2,3,3} - p^n_{1,3,3}) + \mu r U^n_{0,3,3} + c_3 U^n_{1,4,3} + c_4 U^n_{1,3,4} \\
\hline
k\rho g_x - hr(p^n_{3,1,1} - p^n_{2,1,1}) + \mu r U^n_{2,0,1} + \mu r U^n_{2,1,0} \\
k\rho g_x - hr(p^n_{3,1,2} - p^n_{2,1,2}) + \mu r U^n_{2,0,2} \\
k\rho g_x - hr(p^n_{3,1,3} - p^n_{2,1,3}) + \mu r U^n_{2,0,3} + c_4 U^n_{2,1,4} \\
\hline
k\rho g_x - hr(p^n_{3,2,1} - p^n_{2,2,1}) + \mu r U^n_{2,2,0} \\
k\rho g_x - hr(p^n_{3,2,2} - p^n_{2,2,2}) \\
k\rho g_x - hr(p^n_{3,2,3} - p^n_{2,2,3}) + c_4 U^n_{2,2,4} \\
\hline
k\rho g_x - hr(p^n_{3,3,1} - p^n_{2,3,1}) + \mu r U^n_{2,3,0} + c_3 U^n_{2,4,1} \\
k\rho g_x - hr(p^n_{3,3,2} - p^n_{2,3,2}) + c_3 U^n_{2,4,2} \\
k\rho g_x - hr(p^n_{3,3,3} - p^n_{2,3,3}) + c_3 U^n_{2,4,3} + c_4 U^n_{2,3,4} \\
\hline
k\rho g_x - hr(p^n_{4,1,1} - p^n_{3,1,1}) + c_1 U^n_{4,1,1} + \mu r U^n_{3.0.1} + \mu r U^n_{3,1,0} \\
k\rho g_x - hr(p^n_{4,1,2} - p^n_{3,1,2}) + c_1 U^n_{4,1,2} + \mu r U^n_{3,0,2} \\
k\rho g_x - hr(p^n_{4,1,3} - p^n_{3,1,3}) + c_1 U^n_{4,1,3} + \mu r U^n_{3,0,3} + c_4 U^n_{3,1,4} \\
\hline
k\rho g_x - hr(p^n_{4,2,1} - p^n_{3,2,1}) + c_1 U^n_{4,2,1} + \mu r U^n_{3,2,0} \\
k\rho g_x - hr(p^n_{4,2,2} - p^n_{3,2,2}) + c_1 U^n_{4,2,2} \\
k\rho g_x - hr(p^n_{4,2,3} - p^n_{3,2,3}) + c_1 U^n_{3,2,4} \\
\hline
k\rho g_x - hr(p^n_{4,3,1} - p^n_{3,3,1}) + c_1 U^n_{4,3,1} + c_3 U^n_{3,4,1} + \mu r U^n_{3,3,0} \\
k\rho g_x - hr(p^n_{4,3,2} - p^n_{3,3,2}) + c_1 U^n_{4,3,2} + c_3 U^n_{3,4,2} \\
k\rho g_x - hr(p^n_{4,3,3} - p^n_{3,3,3}) + c_1 U^n_{4,2,3} + c_3 U^n_{3,4,3} + c_4 U^n_{3,3,4}
\end{array}\right]
$$

Boundary for three dimension Navier-Stokes equation (explicit method)

3.8 Implicit Finite difference method for three dimensional Navier-Stokes equation:

Return to the viscous fluid flow equation (3.1.8) and descretize it using a fully implicit method, We get:

$$\mu r U^{n}_{i+1,j,l} + (\mu r + \rho \overline{U} hr) U^{n}_{i-1,j,l} + \mu r U^{n}_{i,j+1,l} + (\mu r + \rho hr \overline{V}) U^{n}_{i,j-1,l} + \mu r U^{n}_{i,j,l+1} + (\mu r + \rho hr \overline{W}) U^{n}_{i,j,l-1} - (6\mu r + \rho + \rho hr \overline{U} + \rho hr \overline{V} + \rho hr \overline{W}) U^{n}_{i,j,l} = -\rho U^{n-1}_{i,j,l} \quad (3.8.1)$$

To simplify these secheme we will let as usual :

$$c_1 = \mu r + \rho hr \overline{U}, \quad c_2 = \mu r + \rho hr \overline{V}, \quad c_3 = \mu r + \rho hr \overline{W}$$

$$c_4 = 6\mu r + \rho + \rho hr \overline{U} + \rho hr \overline{V} + \rho hr \overline{W} \quad (3.8.2)$$

We can see that there is a simple relation between these terms for example :

$c_4 = c_1 + c_2 + c_3 + 3\mu r + \rho$.

Substituting these constants in equation (3.8.1) we get :

$$\mu r U^{n}_{i+1,j,l} + c_1 U^{n}_{i-1,j,l} + \mu r U^{n}_{i,j+1,l} + c_2 U^{n}_{i,j-1,l} + \mu r U^{n}_{i,j,l+1} + c_3 U^{n}_{i,j.,l-1} - c_4 U^{n}_{i,j,l} = -\rho U^{n-1}_{i,j,l} \quad (3.8.3)$$

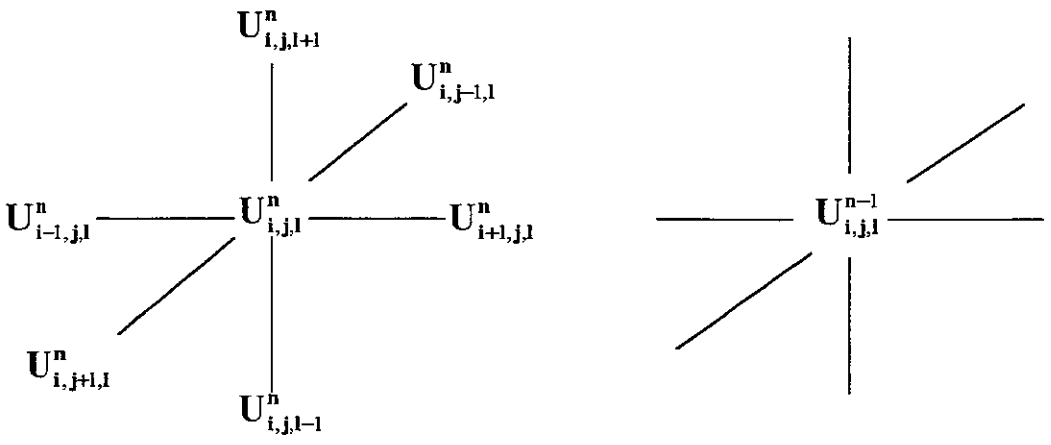


Figure 3.8

Form figure (3.8) we can see that we have seven unknown points located at t=nl, and one known point at t=(n-1)l. To solve this system of equations we need to find the inverse of the sparse matrix. Now we will form the matrix for the implicit method for viscous fluid flow:

$$\mathbf{AU^n = U^{n-1} + Boundary,} \qquad \mathbf{for\ n} = 1,2,3.....\ ,$$

where $\mathbf{U^n}$ and $\mathbf{U^{n-1}}$ are the column vectors of the velocity in x - direction and Boundary is the boundary condition which also include the pressure gradient, which is known (assumption) and A is the sparse matrix

$$A = \begin{bmatrix}
\alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma & \alpha & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\omega & 0 & 0 & 0 & 0 & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & 0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 & \beta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \alpha & 0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha & \beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & \gamma & \alpha
\end{bmatrix}_{(N-1)(N-1)(N-1)(N-1}$$

where

$$\alpha = \begin{bmatrix} -c_4 & \mu r & 0 & 0 & 0 & 0 & 0 & 0 \\ c_3 & -c_4 & \mu r & 0 & 0 & 0 & 0 & 0 \\ 0 & c_3 & -c_4 & \mu r & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & -c_4 & \mu r & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & -c_4 & \mu r & 0 & 0 \\ 0 & 0 & 0 & 0 & c_3 & -c_4 & \mu r & 0 \\ 0 & 0 & 0 & 0 & 0 & c_3 & -c_4 & \mu r \\ 0 & 0 & 0 & 0 & 0 & 0 & c_3 & -c_4 \end{bmatrix}_{(N-1)(N-1)},$$

$$\beta = \begin{bmatrix} \mu r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu r \end{bmatrix}_{(N-1)(N-1)},$$

$$\gamma = \begin{bmatrix} c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 \end{bmatrix}_{(N-1)(N-1)},$$

$$\omega = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 \end{bmatrix}_{(N-1)(N-1)},$$

$$\text{boundary} = \left[\begin{array}{c}
-k\rho g_x + hr(p^n_{1,1,1} - p^n_{0,1,1}) - c_1 U^n_{0,1,1} - c_2 U^n_{1,0,1} - c_3 U^n_{1,1,0} \\
-k\rho g_x + hr(p^n_{1,1,2} - p^n_{0,1,2}) - c_1 U^n_{0,1,2} - c_2 U^n_{1,0,2} \\
-k\rho g_x + hr(p^n_{1,1,3} - p^n_{0,1,3}) - c_1 U^n_{0,1,3} - c_2 U^n_{1,0,3} - \mu r U^n_{1,1,4} \\
\hline
-k\rho g_x + hr(p^n_{1,2,1} - p^n_{0,2,1}) - c_1 U^n_{0,2,1} - c_3 U^n_{1,2,0} \\
-k\rho g_x + hr(p^n_{1,2,2} - p^n_{0,2,2}) - c_1 U^n_{0,2,2} \\
-k\rho g_x + hr(p^n_{1,2,3} - p^n_{0,2,3}) - c_1 U^n_{0,2,3} - \mu r U^n_{1,2,4} \\
\hline
-k\rho g_x + hr(p^n_{1,3,1} - p^n_{0,3,1}) - c_1 U^n_{0,3,1} - \mu r U^n_{1,4,1} - c_3 U^n_{1,3,0} \\
-k\rho g_x + hr(p^n_{1,3,2} - p^n_{0,3,2}) - c_1 U^n_{0,3,2} - \mu r U^n_{1,4,2} \\
-k\rho g_x + hr(p^n_{1,3,3} - p^n_{0,3,3}) - c_1 U^n_{0,3,3} - \mu r U^n_{1,4,3} - \mu r U^n_{1,3,4} \\
\hline
-k\rho g_x + hr(p^n_{2,1,1} - p^n_{1,1,1}) - c_2 U^n_{2,0,1} - c_3 U^n_{2,1,0} \\
-k\rho g_x + hr(p^n_{2,1,2} - p^n_{1,1,2}) - c_2 U^n_{2,0,2} \\
-k\rho g_x + hr(p^n_{2,1,3} - p^n_{1,1,3}) - c_2 U^n_{2,0,3} - \mu r U^n_{2,1,4} \\
\hline
-k\rho g_x + hr(p^n_{2,2,1} - p^n_{1,2,1}) - c_3 U^n_{2,2,0} \\
-k\rho g_x + hr(p^n_{2,2,2} - p^n_{1,2,2}) \\
-k\rho g_x + hr(p^n_{2,2,3} - p^n_{1,2,3}) - \mu r U^n_{2,2,4} \\
\hline
-k\rho g_x + hr(p^n_{2,3,1} - p^n_{1,3,1}) - \mu r U^n_{2,4,1} - c_3 U^n_{2,3,0} \\
-k\rho g_x + hr(p^n_{2,3,2} - p^n_{1,3,2}) - \mu r U^n_{2,4,2} \\
-k\rho g_x + hr(p^n_{2,3,3} - p^n_{1,3,3}) - \mu r U^n_{2,4,3} - \mu r U^n_{2,3,4} \\
\hline
-k\rho g_x + hr(p^n_{3,1,1} - p^n_{2,1,1}) - \mu r U^n_{4,1,1} - c_2 U^n_{3,0,1} - c_3 U^n_{3,1,0} \\
-k\rho g_x + hr(p^n_{3,1,2} - p^n_{2,1,2}) - \mu r U^n_{4,1,2} - c_2 U^n_{3,0,2} \\
-k\rho g_x + hr(p^n_{3,1,3} - p^n_{2,1,3}) - \mu r U^n_{4,1,3} - c_2 U^n_{3,0,3} - \mu r U^n_{3,1,4} \\
\hline
-k\rho g_x + hr(p^n_{3,2,1} - p^n_{2,2,1}) - \mu r U^n_{4,2,1} - c_3 U^n_{3,2,0} \\
-k\rho g_x + hr(p^n_{3,2,2} - p^n_{2,2,2}) - \mu r U^n_{4,2,2} \\
-k\rho g_x + hr(p^n_{3,2,3} - p^n_{2,2,3}) - \mu r U^n_{4,2,3} - \mu r U^n_{3,2,4} \\
\hline
-k\rho g_x + hr(p^n_{3,3,1} - p^n_{2,3,1}) - \mu r U^n_{4,3,1} - \mu r U^n_{3,4,1} - c_3 U^n_{3,3,0} \\
-k\rho g_x + hr(p^n_{3,3,2} - p^n_{2,3,2}) - \mu r U^n_{4,3,2} - \mu r U^n_{3,4,2} \\
-k\rho g_x + hr(p^n_{3,3,3} - p^n_{2,3,3}) - \mu r U^n_{4,3,3} - \mu r U^n_{3,4,3} - \mu r U^n_{3,3,4}
\end{array}\right]$$

Crank-Nicolson method for three dimensional Navier-Stokes equation:

Returning to equation (3.1.8) and using half time step and replace the first and the second order partial differential equation by it's finite difference approximations at the n and (n+1) time steps:

$$\left(\frac{\partial U}{\partial x}\right)_{i,j}^{n+\frac{1}{2}} = \frac{1}{2}\left\{\frac{U_{i+1,j,k}^{n+1} - U_{i,j,k}^{n+1}}{h} + \frac{U_{i+1,j,k}^{n} - U_{i,j,k}^{n}}{h}\right\} \qquad (3.9.1a)$$

$$\left(\frac{\partial^2 U}{\partial x^2}\right)_{i,j,k}^{n+\frac{1}{2}} = \frac{1}{2}\left\{\frac{U_{i+1,j,k}^{n+1} - 2U_{i,j,k}^{n+1} + U_{i-1,j,k}^{n+1}}{h^2} + \frac{U_{i+1,j,k}^{n} - 2U_{i,j,k}^{n} + U_{i-1,j,k}^{n}}{h^2}\right\} \qquad (3.9.1b)$$

$$\left(\frac{\partial U}{\partial t}\right)_{i,j,k}^{n+\frac{1}{2}} = \frac{U_{i,j,k}^{n+1} - U_{i,j,k}^{n}}{(\Delta t)} \qquad (3.9.1c)$$

The scheme of equation (3.1.8) using the last methods will be:

$$k\rho g_x - hr(P_{i+1,j,k}^{n} - P_{i,j,k}^{n}) + \left(\frac{\mu r - \rho hr\overline{U}}{2}\right)U_{i+1,j,k}^{n+1} + \left(\frac{\mu r - \rho hr\overline{V}}{2}\right)U_{i,j+1,k}^{n+1}$$
$$+ \left(\frac{\mu r - \rho hr\overline{W}}{2}\right)U_{i,j,k+1}^{n} + \frac{\mu r}{2}U_{i-1,j,k}^{n+1} + \frac{\mu r}{2}U_{i,j-1,k}^{n} + \frac{\mu r}{2}U_{i,j,k-1}^{n} -$$
$$\left(3\mu r + \rho - \frac{\rho hr\overline{U} + \rho hr\overline{V} + \rho hr\overline{W}}{2}\right)U_{i,j,k}^{n} = \left(\frac{\rho hr\overline{U} - \mu r}{2}\right)U_{i+1,j,k}^{n} +$$
$$\left(\frac{\rho hr\overline{V} - \mu r}{2}\right)U_{i,j+1,k}^{n} + \left(\frac{\mu hr\overline{W} - \mu r}{2}\right)U_{i,j,k+1}^{n} - \frac{\mu r}{2}U_{i-1,j,k}^{n} - \frac{\mu r}{2}U_{i,j-1,k}^{n} -$$
$$\frac{\mu r}{2}U_{i,j,k-1}^{n} - \left(\rho + \frac{\rho hr\overline{U} + \rho hr\overline{V} + \rho hr\overline{W}}{2} - 3\mu r\right)U_{i,j,k}^{n} \qquad (3.9.2)$$

As we can see from figure(3.9) we have seven unknown mesh points at t=(n+1)k, and seven known at t=nk

.

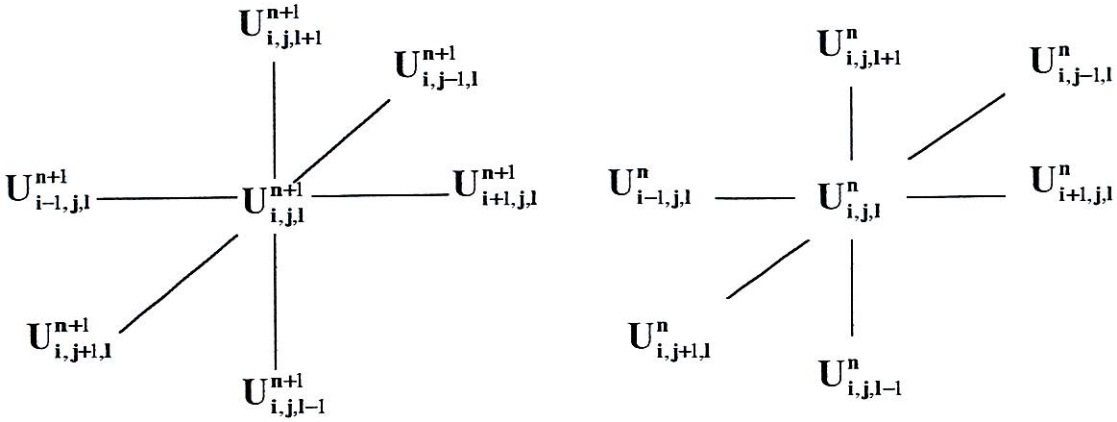


Figure 3.9

For simplicity we will let:

$$\left.\begin{aligned} c_1 &= \frac{\mu r - \rho hr\overline{U}}{2}, \qquad c_2 = \frac{\mu r - \rho hr\overline{V}}{2}, \qquad c_3 = \frac{\mu r - \rho hr\overline{W}}{2} \\ c_4 &= 3\mu r + \rho - \frac{\rho hr\overline{U} + \rho hr\overline{V} + \rho hr\overline{W}}{2} \\ c_5 &= \rho + \frac{\rho hr\overline{U} + \rho hr\overline{V} + \rho hr\overline{W}}{2} - 3\mu r \\ \Delta_x P^n_{i,j,k} &= P^n_{i+1,j,k} - P^n_{i,j,k} \end{aligned}\right\} \qquad (3.9.3)$$

Therefore equation (3.9.2) becomes

$$k\rho g_x - hr\Delta P^n_{i,j,k} + c_1 U^{n+1}_{i+1,j,k} + c_2 U^{n+1}_{i,j+1,k} + c_3 U^{n+1}_{i,j,k+1} + \frac{\mu r}{2} U^{n+1}_{i-1,j,k} +$$

$$\frac{\mu r}{2} U^{n+1}_{i,j-1,k} + \frac{\mu r}{2} U^{n+1}_{i,j,k-1} - c_4 U^{n+1}_{i,j,k} = -c_1 U^{n}_{i+1,j,k} - c_2 U^{n}_{i,j+1,k} -$$

$$c_3 U^{n}_{i,j,k+1} - \frac{\mu r}{2} U^{n}_{i-1,j,k} - \frac{\mu r}{2} U^{n}_{i,j,k-1} - c_5 U^{n}_{i,j,k} \qquad (3.9.4)$$

After these calculations we as usual will form the matrix form for equation (3.9.4)

$$\mathbf{AU^{n+1}} = \mathbf{BU^{n}} + \mathbf{Boundary} \quad , \qquad (3.9.5)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{L} & \mathbf{M} & 0 & 0 & 0 \\ \mathbf{N} & \mathbf{L} & \mathbf{M} & 0 & 0 \\ 0 & \mathbf{N} & \mathbf{L} & \mathbf{M} & 0 \\ 0 & 0 & \mathbf{N} & \mathbf{L} & \mathbf{M} \\ 0 & 0 & 0 & \mathbf{N} & \mathbf{L} \end{bmatrix}_{5\times5} ,$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{R} & -\mathbf{M} & 0 & 0 & 0 \\ -\mathbf{N} & \mathbf{R} & -\mathbf{M} & 0 & 0 \\ 0 & -\mathbf{N} & \mathbf{R} & -\mathbf{M} & 0 \\ 0 & 0 & -\mathbf{N} & \mathbf{R} & -\mathbf{M} \\ 0 & 0 & 0 & -\mathbf{N} & \mathbf{R} \end{bmatrix}_{5\times5}$$

The matrices A and B are block matrices, so the components of both of them are also matrices as we will show below

$$
L = \begin{bmatrix}
-c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 & 0 & c_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & 0 & 0 & 0 & 0 & c_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_4 & c_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4 & c_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_4
\end{bmatrix}_{r}
$$

$$
M = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}_{(N-1)(N-1)(N-1)(N-1)}
$$

$$N =$$

$$\begin{bmatrix}
\frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu r}{2}
\end{bmatrix}_{(N-1)(N-1)(N-1)(N-1)}$$

$$
S = \left[\begin{array}{ccccc|ccccc|ccccc|ccccc|ccccc}
-c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-\frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & -c_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 & -c_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & 0 & 0 & 0 & -c_2 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & 0 & -c_5 & -c_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5 & -c_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu r}{2} & 0 & 0 & 0 & \frac{\mu r}{2} & -c_5
\end{array}\right]
$$

$$
\text{Boundary} = \left[\begin{array}{c}
-k\rho g_x + hr(p^n_{2,1,1} - p^n_{1,1,1}) - \frac{\mu r}{2}(U^n_{0,1,1} + U^{n+1}_{0,1,1}) - \frac{\mu r}{2}(U^n_{1,0,1} + U^{n+1}_{1,0,1}) - \frac{\mu r}{2}(U^n_{1,1,0} + U^{n+1}_{1,1,0}) \\
-k\rho g_x + hr(p^n_{2,1,2} - p^n_{1,1,2}) - \frac{\mu r}{2}(U^n_{0,1,2} + U^{n+1}_{0,1,2}) - \frac{\mu r}{2}(U^n_{1,0,2} + U^{n+1}_{1,0,2}) \\
-k\rho g_x + hr(p^n_{2,1,3} - p^n_{1,1,3}) - c_3(U^n_{1,1,4} + U^{n+1}_{1,1,4}) - \frac{\mu r}{2}(U^n_{0,1,3} + U^{n+1}_{0,1,3}) - \frac{\mu r}{2}(U^n_{1,0,3} + U^{n+1}_{1,0,3}) \\
\hline
-k\rho g_x + hr(p^n_{2,2,1} - p^n_{1,2,1}) - \frac{\mu r}{2}(U^n_{0,2,1} + U^{n+1}_{0,2,1}) - \frac{\mu r}{2}(U^n_{1,2,0} + U^{n+1}_{1,2,0}) \\
-k\rho g_x + hr(P^n_{2,2,2} - p^n_{1,2,2}) - \frac{\mu r}{2}(U^n_{0,2,2} + U^{n+1}_{0,2,2}) \\
-k\rho g_x + hr(p^n_{2,2,3} - p^n_{1,2,3}) - c_3(U^n_{1,2,4} + U^{n+1}_{1,2,4}) - \frac{\mu r}{2}(U^n_{0,2,3} + U^{n+1}_{0,2,3}) \\
\hline
-k\rho g_x + hr(p^n_{2,3,1} - p^n_{1,3,1}) - \frac{\mu r}{2}(U^n_{0,3,1} + U^{n+1}_{0,3,1}) - c_2(U^n_{1,4,1} + U^{n+1}_{1,4,1}) - \frac{\mu r}{2}(U^n_{1,3,0} + U^{n+1}_{1,3,0}) \\
-k\rho g_x + hr(p^n_{2,3,2} - p^n_{1,3,2}) - \frac{\mu r}{2}(U^n_{0,3,2} + U^{n+1}_{0,3,2}) - c_2(U^n_{1,4,2} + U^{n+1}_{1,4,2}) \\
-k\rho g_x + hr(p^n_{2,3,3} - p^n_{1,3,3}) - \frac{\mu r}{2}(U^n_{0,3,3} + U^{n+1}_{0,3,3}) - c_2(U^n_{1,4,3} + U^{n+1}_{1,4,3}) - c_3(U^n_{1,3,4} + U^{n+1}_{1,3,4}) \\
\hline
-k\rho g_x + hr(p^n_{3,1,1} - p^n_{2,1,1}) - \frac{\mu r}{2}(U^n_{2,0,1} + U^{n+1}_{2,0,1}) - \frac{\mu r}{2}(U^n_{2,1,0} + U^{n+1}_{2,1,0}) \\
-k\rho g_x + hr(p^n_{3,1,2} - p^n_{2,1,2}) - \frac{\mu r}{2}(U^n_{2,0,2} + U^{n+1}_{2,0,2}) \\
-k\rho g_x + hr(p^n_{3,1,3} - p^n_{2,1,3}) - \frac{\mu r}{2}(U^n_{2,0,3} + U^{n+1}_{2,0,3}) - c_3(U^n_{2,1,4} + U^{n+1}_{2,1,4}) \\
\hline
-k\rho g_x + hr(p^n_{3,2,1} - p^n_{2,2,1}) - \frac{\mu r}{2}(U^n_{2,2,0} + U^{n+1}_{2,2,0}) \\
-k\rho g_x + hr(p^n_{3,2,2} - p^n_{2,2,2}) \\
-k\rho g_x + hr(p^n_{3,2,3} - p^n_{2,2,3}) - c_3(U^n_{2,2,4} + U^n_{2,2,4}) \\
\hline
-k\rho g_x + hr(p^n_{3,3,1} - p^n_{2,3,1}) - c_2(U^n_{2,4,1} + U^n_{2,4,1}) - \frac{\mu r}{2}(U^n_{2,3,0} + U^{n+1}_{2,3,0}) \\
-k\rho g_x + hr(p^n_{3,3,2} - p^n_{2,3,2}) - c_2(U^n_{2,4,2} + U^{n+1}_{2,4,2}) \\
-k\rho g_x + hr(p^n_{3,3,3} - p^n_{2,3,3}) - c_2(U^n_{2,4,3} + U^{n+1}_{2,4,3}) - c_3(U^n_{2,3,4} + U^{n+1}_{2,3,4}) \\
\hline
-k\rho g_x + hr(p^n_{4,1,1} - p^n_{3,1,1}) - c_1(U^n_{4,1,1} + U^n_{4,1,1}) - \frac{\mu r}{2}(U^n_{3,0,1} + U^{n+1}_{3,0,1}) - \frac{\mu r}{2}(U^n_{3,1,0} + U^{n+1}_{3,1,0}) \\
-k\rho g_x + hr(p^n_{4,1,2} - p^n_{3,1,2}) - c_1(U^n_{4,1,2} + U^n_{4,1,2}) - \frac{\mu r}{2}(U^n_{3,0,2} + U^n_{3,0,2}) \\
-k\rho g_x + hr(p^n_{4,1,3} - p^n_{3,1,3}) - c_1(U^n_{4,1,3} + U^{n+1}_{4,1,3}) - \frac{\mu r}{2}(U^n_{3,0,3} + U^{n+1}_{3,0,3}) - c_3(U^n_{3,1,4} + U^{n+1}_{3,1,4}) \\
\hline
-k\rho g_x + hr(p^n_{4,2,1} - p^n_{3,2,1}) - c_1(U^n_{4,2,1} + U^{n+1}_{4,2,1}) - \frac{\mu r}{2}(U^n_{3,2,0} + U^{n+1}_{3,2,0}) \\
-k\rho g_x + hr(p^n_{4,2,2} - p^n_{3,2,2}) - c_1(U^n_{4,2,2} + U^{n+1}_{4,2,2}) \\
-k\rho g_x + hr(p^n_{4,2,3} - p^n_{3,2,3}) - c_1(U^n_{4,2,3} + U^{n+1}_{4,2,3}) - c_3(U^n_{3,2,4} + U^{n+1}_{3,2,4}) \\
\hline
-k\rho g_x + hr(p^n_{4,3,1} - p^n_{3,3,1}) - c_1(U^n_{4,3,1} + U^n_{4,3,1}) - c_2(U^n_{3,4,1} + U^{n+1}_{3,4,1}) - \frac{\mu r}{2}(U^n_{3,3,0} + U^{n+1}_{3,3,0}) \\
-k\rho g_x + hr(p^n_{4,3,2} - p^n_{3,3,2}) - c_1(U^n_{4,3,2} + U^{n+1}_{4,3,2}) - c_2(U^n_{3,4,2} + U^{n+1}_{3,4,2}) \\
-k\rho g_x + hr(p^n_{4,3,3} - p^n_{3,3,3}) - c_1(U^n_{4,3,3} + U^{n+1}_{4,3,3}) - c_2(U^n_{3,4,3} + U^{n+1}_{3,4,3}) - c_3(U^n_{3,3,4} + U^{n+1}_{3,3,4})
\end{array}\right]
$$

CHAPTER 4

4.1 RESULTS AND DISCUSSIONS

Explicit method for one dimension flow using r=0.1

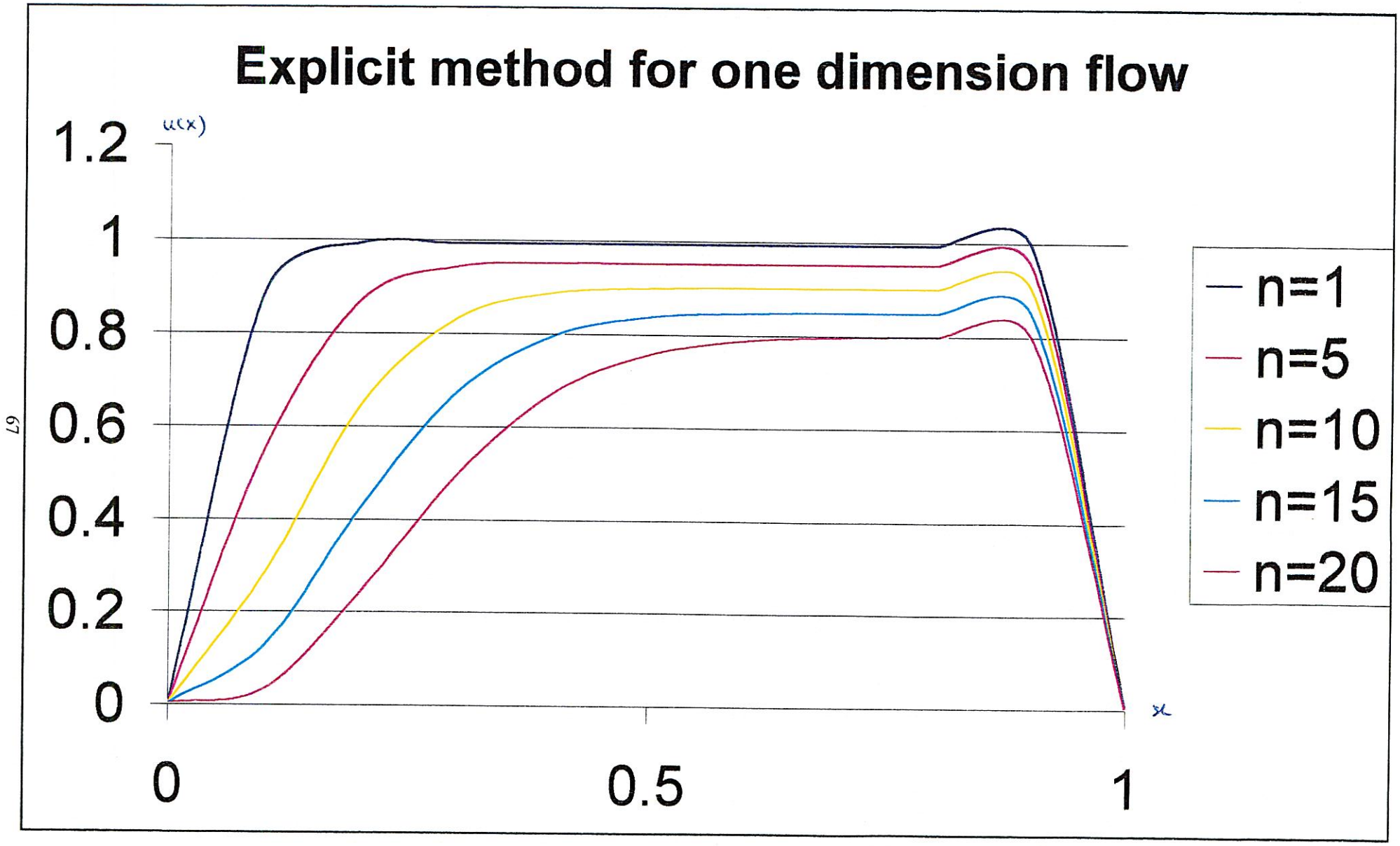
x	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15	n=16	n=17	n=18	n=19	n=20
0.1	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282	0.244	0.209	0.178	0.15	0.125	0.102	0.082	0.063	0.047	0.032
0.2	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643	0.596	0.551	0.507	0.464	0.422	0.382	0.344	0.308	0.273	0.24
0.3	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829	0.8	0.77	0.738	0.704	0.67	0.635	0.6	0.564	0.528	0.492
0.4	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885	0.87	0.852	0.834	0.814	0.793	0.771	0.747	0.722	0.696	0.669
0.5	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896	0.885	0.873	0.861	0.848	0.835	0.82	0.805	0.789	0.772	0.754
0.6	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898	0.888	0.877	0.866	0.856	0.845	0.834	0.822	0.81	0.798	0.785
0.7	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898	0.888	0.878	0.867	0.857	0.847	0.836	0.826	0.815	0.805	0.794
0.8	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898	0.888	0.878	0.867	0.857	0.847	0.837	0.826	0.816	0.806	0.796
0.9	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898	0.888	0.878	0.867	0.857	0.847	0.837	0.827	0.816	0.806	0.796

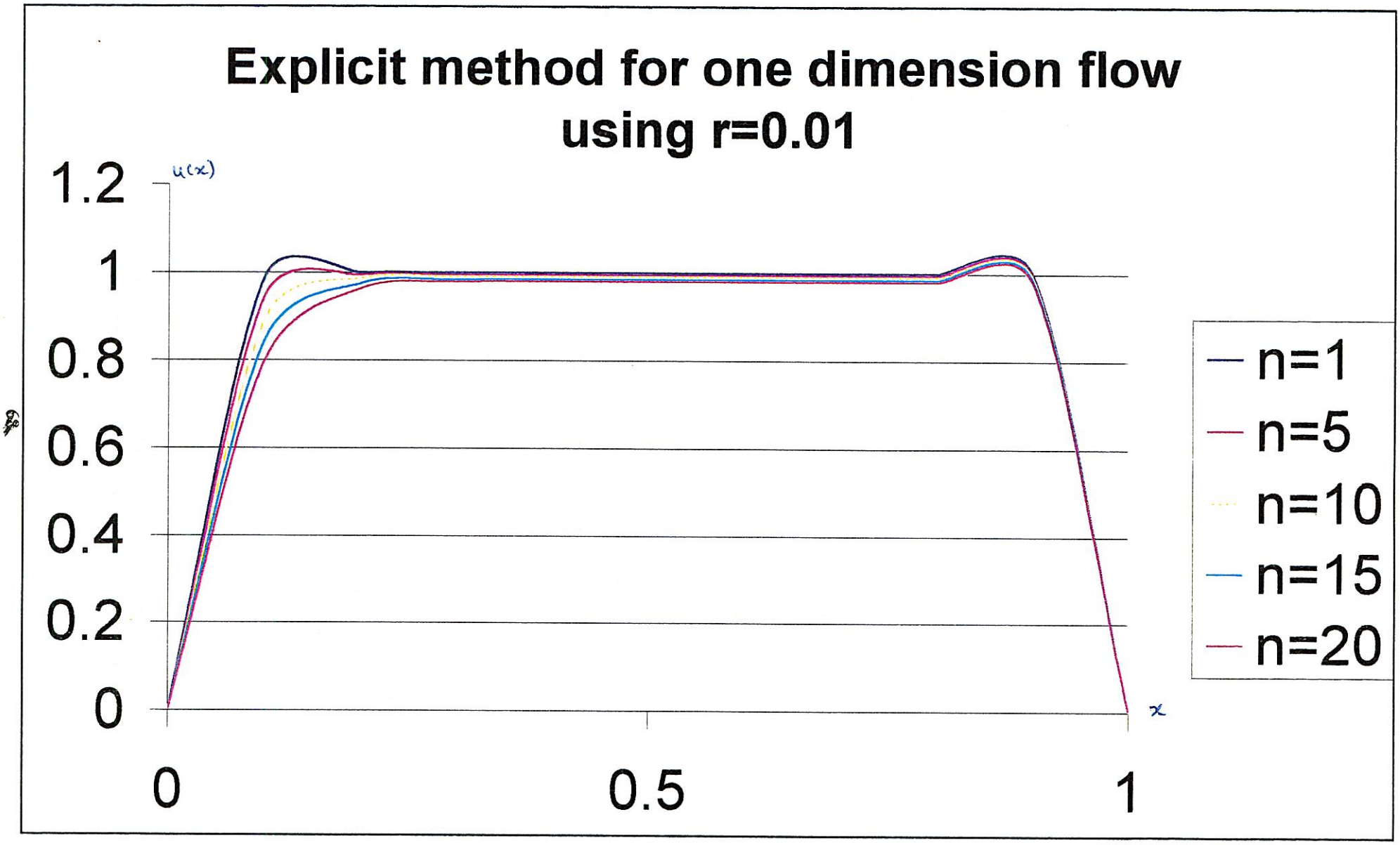
Explicit method for one dimension flow using r=0.01

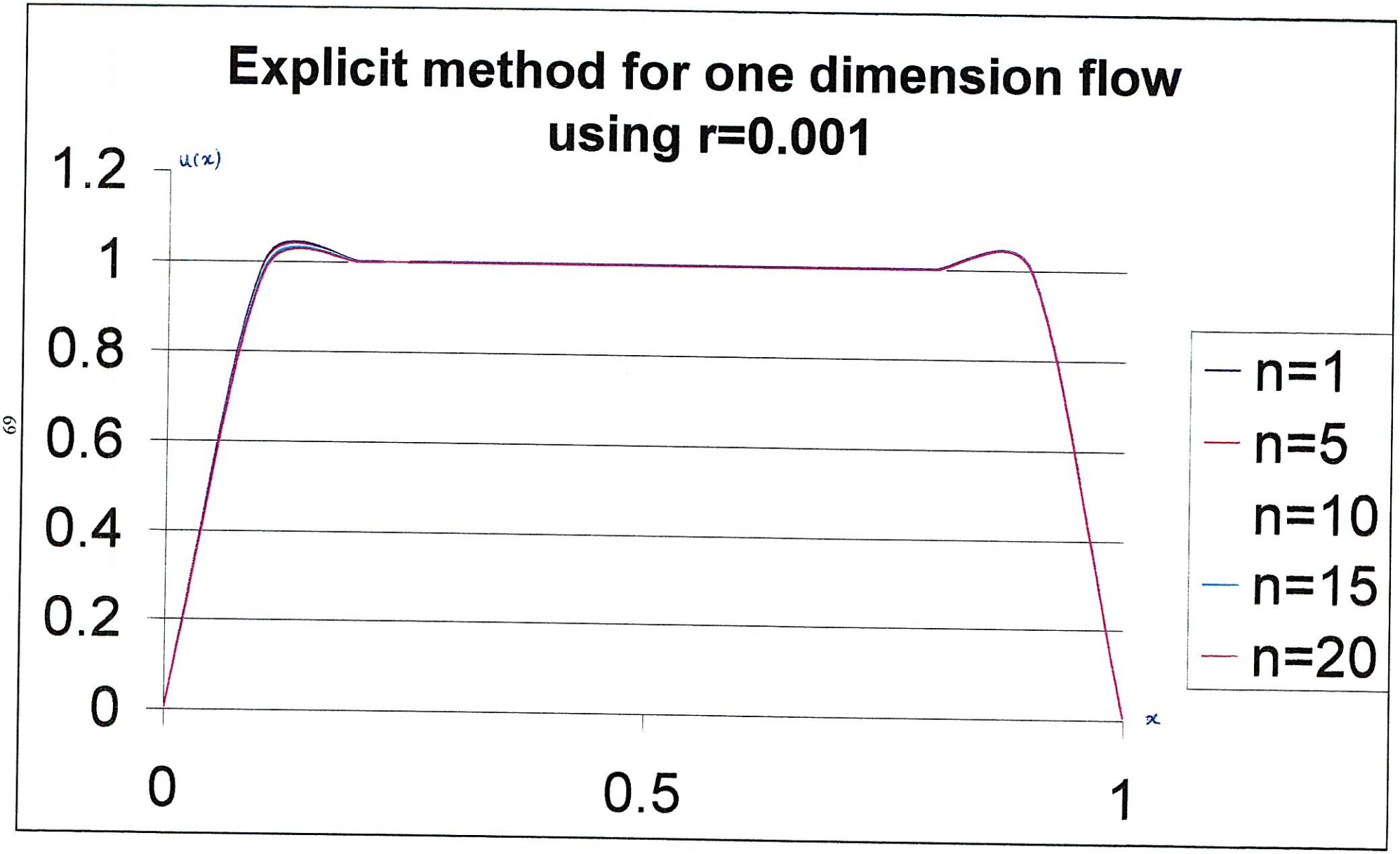
x	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15	n=16	n=17	n=18	n=19	n=20
0.1	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895	0.885	0.875	0.865	0.855	0.846	0.836	0.827	0.818	0.808	0.799
0.2	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986	0.984	0.982	0.98	0.977	0.975	0.973	0.97	0.968	0.965	0.963
0.3	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.986	0.985	0.984	0.983	0.982	0.981	0.98	0.979
0.4	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.983	0.982	0.981	0.98
0.5	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.983	0.982	0.981	0.98
0.6	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.983	0.982	0.981	0.98
0.7	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.983	0.982	0.981	0.98
0.8	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.983	0.982	0.981	0.98
0.9	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.983	0.982	0.981	0.98

Explicit method for one dimension flow using r=0.001

x	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15	n=16	n=17	n=18	n=19	n=20
0.1	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989	0.988	0.987	0.986	0.985	0.984	0.982	0.981	0.98	0.979	0.978
0.2	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998	0.998
0.3	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998
0.4	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998
0.5	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998
0.6	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998
0.7	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998
0.8	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998
0.9	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998	0.998


Explicit method for one dimension flow
u(x)
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
x
n=1
n=5
n=10
n=15
n=20


Explicit method for one dimension flow
using r=0.01
u(x)
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
x
n=1
n=5
n=10
n=15
n=20


Explicit method for one dimension flow
using r=0.001
u(x)
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
x
n=1
n=5
n=10
n=15
n=20

Result of 2dimension Explicit method using r=0.1

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.79	0.622	0.487	0.379	0.293	0.225	0.169	0.125	0.09	0.062
(0.1,0.2)	1	0.89	0.781	0.676	0.58	0.491	0.412	0.342	0.28	0.227	0.18
(0.1,0.3)	1	0.89	0.791	0.7	0.618	0.542	0.473	0.409	0.351	0.299	0.252
(0.1,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.423	0.369	0.32	0.276
(0.1,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.324	0.281
(0.1,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.1,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.1,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.1,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.2,0.1)	1	0.89	0.781	0.676	0.58	0.491	0.412	0.342	0.28	0.227	0.18
(0.2,0.2)	1	0.99	0.96	0.914	0.856	0.791	0.721	0.649	0.577	0.508	0.441
(0.2,0.3)	1	0.99	0.97	0.941	0.904	0.86	0.811	0.758	0.702	0.644	0.586
(0.2,0.4)	1	0.99	0.97	0.942	0.907	0.868	0.825	0.779	0.731	0.682	0.632
(0.2,0.5)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.641
(0.2,0.6)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.2,0.7)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.2,0.8)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.2,0.9)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.3,0.1)	1	0.89	0.791	0.7	0.618	0.542	0.473	0.409	0.351	0.299	0.252
(0.3,0.2)	1	0.99	0.97	0.941	0.904	0.86	0.811	0.758	0.702	0.644	0.586
(0.3,0.3)	1	0.99	0.98	0.967	0.952	0.932	0.907	0.878	0.844	0.805	0.763
(0.3,0.4)	1	0.99	0.98	0.968	0.955	0.94	0.922	0.9	0.876	0.848	0.817
(0.3,0.5)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.88	0.855	0.828
(0.3,0.6)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.3,0.7)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.3,0.8)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.3,0.9)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.4,0.1)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.423	0.369	0.32	0.276
(0.4,0.2)	1	0.99	0.97	0.942	0.907	0.868	0.825	0.779	0.731	0.682	0.632
(0.4,0.3)	1	0.99	0.98	0.968	0.955	0.94	0.922	0.9	0.876	0.848	0.817
(0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.936	0.923	0.908	0.892	0.873
(0.4,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.937	0.926	0.913	0.899	0.884
(0.4,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.4,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.4,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.4,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.5,0.1)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.324	0.281
(0.5,0.2)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.641
(0.5,0.3)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.88	0.855	0.828
(0.5,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.937	0.926	0.913	0.899	0.884
(0.5,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.906	0.895
(0.5,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.5,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.5,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.5,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.6,0.1)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.2)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.6,0.3)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829

(0.6,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.6,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.6,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.6,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.6,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.6,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.1)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.2)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.7,0.3)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.7,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.7,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.7,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.1)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.2)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.8,0.3)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.8,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.1)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.2)	1	0.99	0.97	0.942	0.907	0.868	0.826	0.782	0.736	0.689	0.643
(0.9,0.3)	1	0.99	0.98	0.968	0.956	0.94	0.923	0.903	0.881	0.856	0.829
(0.9,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.913	0.9	0.885
(0.9,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.896
(0.9,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898

Result of 2dimension Explicit method using r=0.01

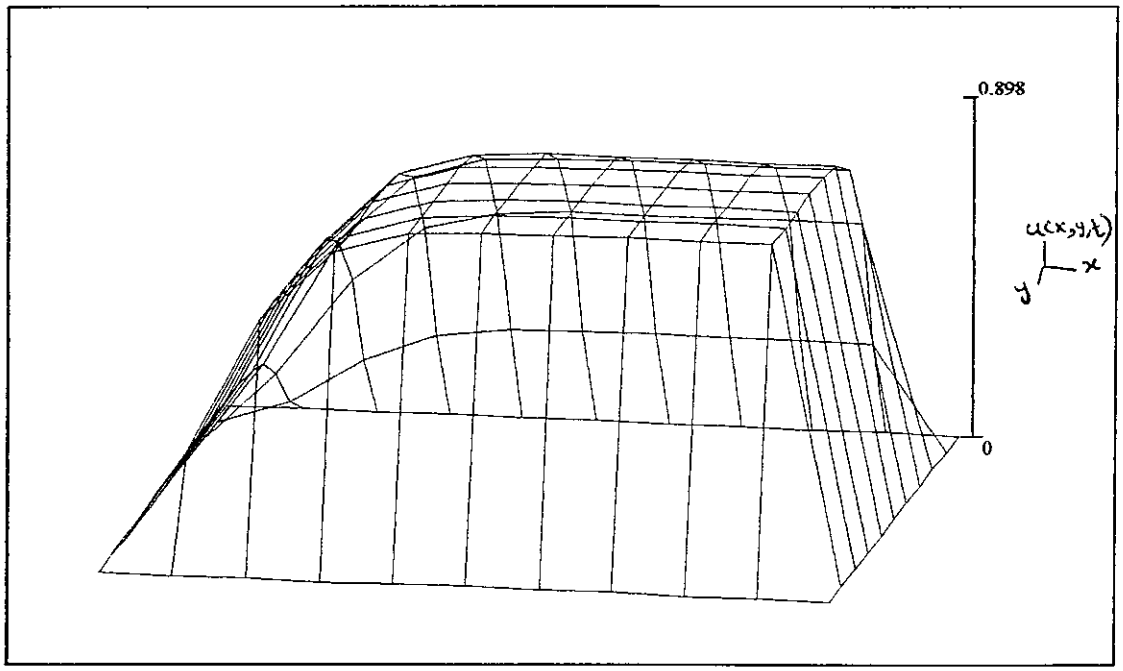
(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.979	0.958	0.938	0.918	0.899	0.88	0.861	0.843	0.825	0.808
(0.1,0.2)	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.902	0.891
(0.1,0.3)	1	0.989	0.978	0.967	0.957	0.946	0.935	0.925	0.915	0.905	0.895
(0.1,0.4)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.5)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.6)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.7)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.8)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.9)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.2,0.1)	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.902	0.891
(0.2,0.2)	1	0.999	0.998	0.996	0.995	0.993	0.991	0.989	0.986	0.984	0.981
(0.2,0.3)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.2,0.4)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.2,0.5)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.2,0.6)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.2,0.7)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.2,0.8)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.2,0.9)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.3,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.935	0.925	0.915	0.905	0.895
(0.3,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.3,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.4,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.4,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.5,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.5,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.6,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.6,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99

(0.6,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.7,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.7,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.8,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.8,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.9,0.2)	1	0.999	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.987	0.986
(0.9,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99

Result of 2dimension Explicit method using r=0.001

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.991	0.99	0.989
(0.1,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.2,0.1)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.991	0.99	0.989
(0.2,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.3,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.4,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.5,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.6,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999

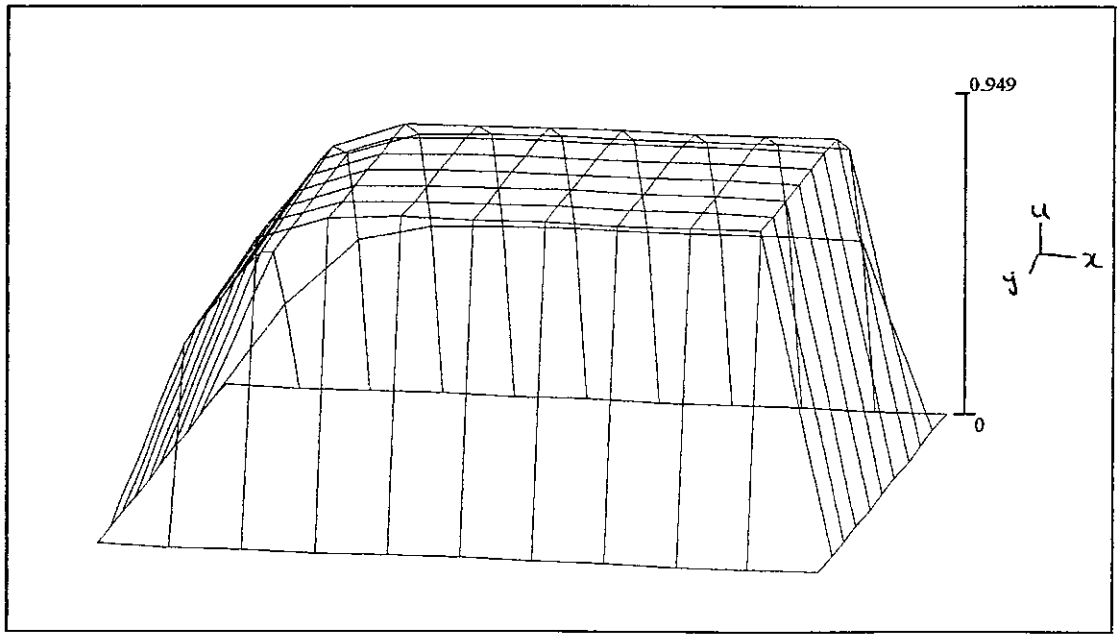
(0.6,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.7,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999



f

2dimension explicit method

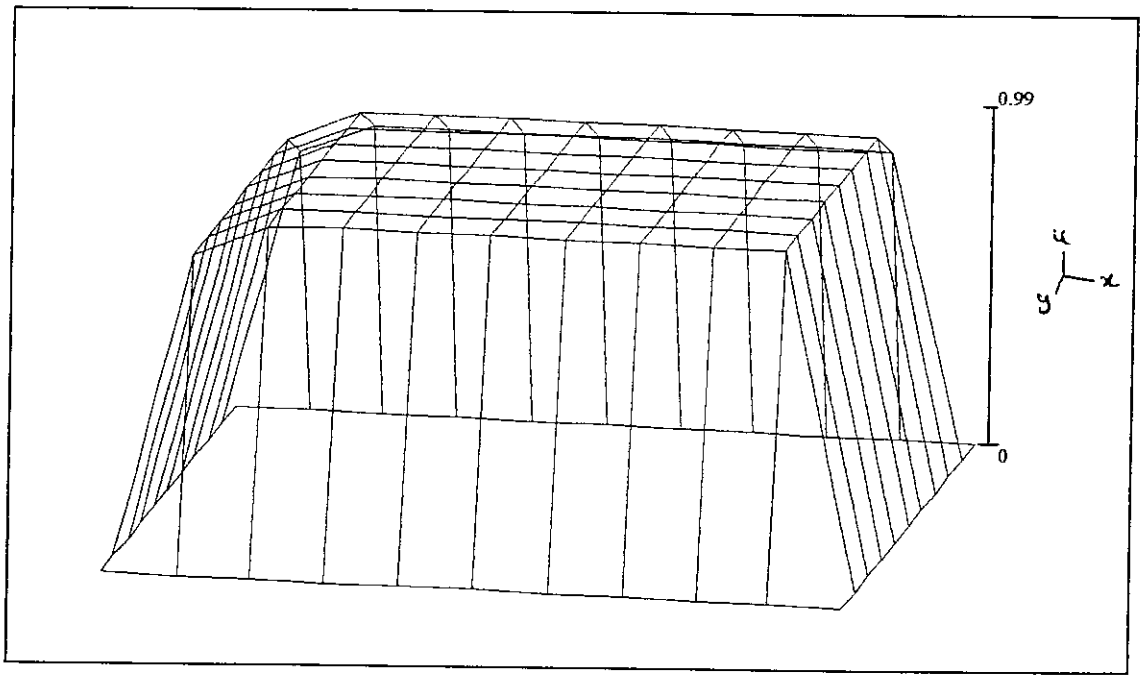
This graph for n=10 and r=0.1



f2

2dimension explicit method

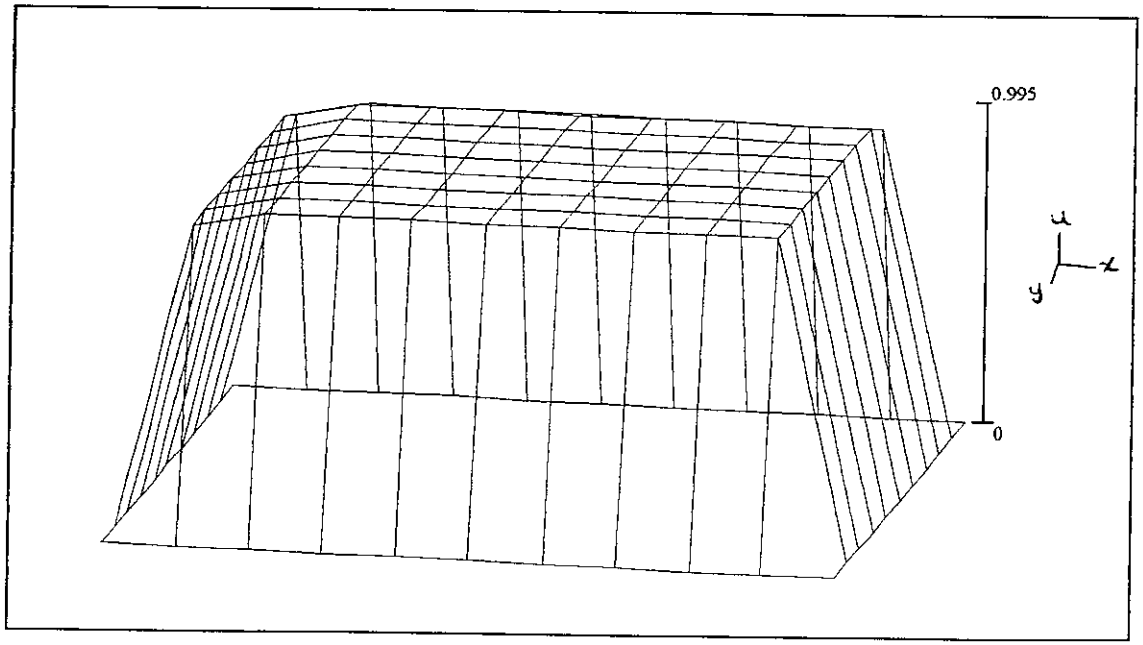
This graph for n=5 and r=0.1



f

2dimension explicit method (n=10)

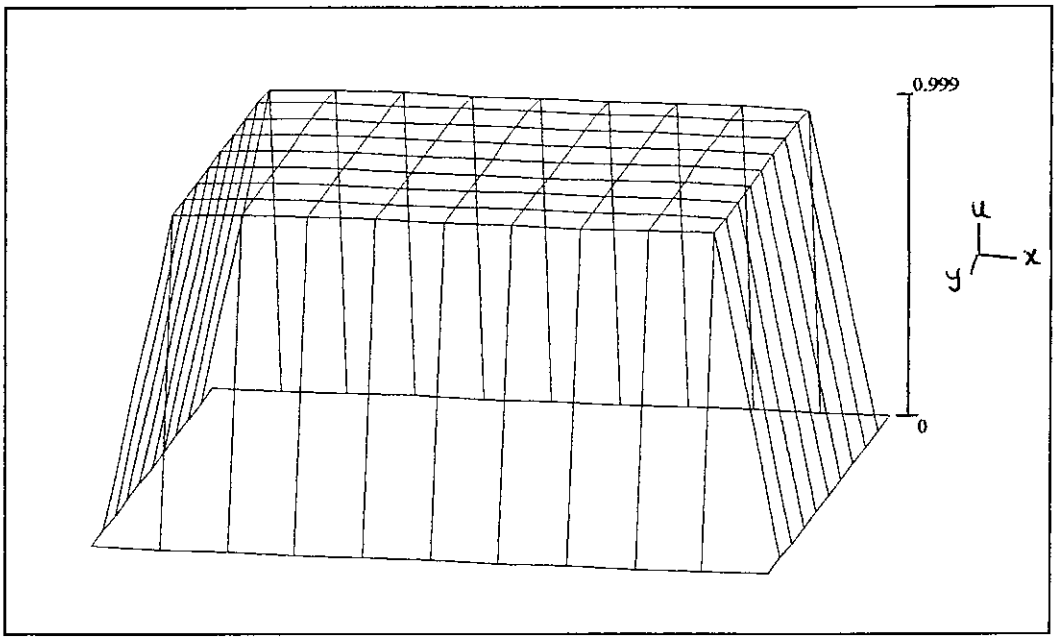
This graph for n=10 and r=0.01



f2

2dimension Explicit method(n=5)

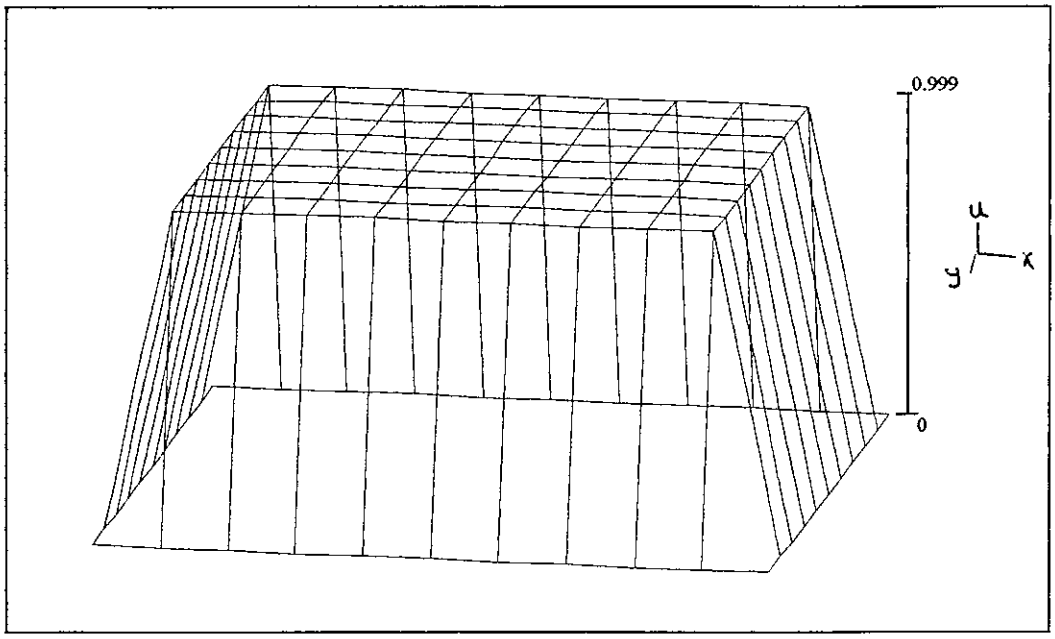
This graph for n=10 and r=0.01



f

2dimension explicit method

This graph for n=10 and r=0.001



f2

2 dimension explicit method

This graph for n=5 and r=0.001

Result of 2dimension Implicit method using r=0.1

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.721	0.528	0.392	0.294	0.223	0.17	0.13	0.1	0.076	0.058
(0.1,0.2)	1	0.82	0.67	0.547	0.446	0.364	0.298	0.243	0.198	0.162	0.131
(0.1,0.3)	1	0.833	0.699	0.589	0.498	0.422	0.358	0.303	0.257	0.218	0.184
(0.1,0.4)	1	0.835	0.704	0.599	0.512	0.44	0.38	0.328	0.284	0.246	0.213
(0.1,0.5)	1	0.836	0.705	0.601	0.516	0.445	0.387	0.337	0.295	0.258	0.226
(0.1,0.6)	1	0.836	0.705	0.601	0.516	0.446	0.388	0.339	0.297	0.261	0.23
(0.1,0.7)	1	0.835	0.705	0.6	0.514	0.444	0.385	0.336	0.294	0.258	0.227
(0.1,0.8)	1	0.832	0.697	0.588	0.499	0.426	0.367	0.317	0.274	0.239	0.208
(0.1,0.9)	1	0.778	0.617	0.497	0.406	0.335	0.28	0.236	0.2	0.17	0.146
(0.2,0.1)	1	0.82	0.67	0.547	0.446	0.364	0.298	0.243	0.198	0.162	0.131
(0.2,0.2)	1	0.944	0.864	0.776	0.687	0.602	0.523	0.452	0.389	0.333	0.284
(0.2,0.3)	1	0.963	0.908	0.843	0.773	0.702	0.633	0.566	0.504	0.447	0.394
(0.2,0.4)	1	0.965	0.916	0.859	0.798	0.736	0.674	0.615	0.558	0.505	0.455
(0.2,0.5)	1	0.966	0.918	0.862	0.804	0.745	0.688	0.632	0.579	0.529	0.482
(0.2,0.6)	1	0.966	0.918	0.863	0.805	0.747	0.691	0.637	0.585	0.537	0.491
(0.2,0.7)	1	0.965	0.917	0.861	0.802	0.743	0.685	0.631	0.579	0.53	0.485
(0.2,0.8)	1	0.96	0.905	0.841	0.776	0.712	0.651	0.593	0.54	0.49	0.445
(0.2,0.9)	1	0.893	0.794	0.705	0.626	0.557	0.495	0.442	0.394	0.352	0.315
(0.3,0.1)	1	0.833	0.699	0.589	0.498	0.422	0.358	0.303	0.257	0.218	0.184
(0.3,0.2)	1	0.963	0.908	0.843	0.773	0.702	0.633	0.566	0.504	0.447	0.394
(0.3,0.3)	1	0.982	0.955	0.918	0.874	0.823	0.769	0.713	0.657	0.601	0.546
(0.3,0.4)	1	0.986	0.965	0.938	0.904	0.865	0.822	0.777	0.729	0.68	0.632
(0.3,0.5)	1	0.986	0.967	0.942	0.912	0.878	0.84	0.8	0.758	0.714	0.671
(0.3,0.6)	1	0.986	0.967	0.942	0.913	0.88	0.844	0.805	0.766	0.725	0.683
(0.3,0.7)	1	0.986	0.966	0.94	0.909	0.875	0.837	0.798	0.757	0.716	0.675
(0.3,0.8)	1	0.98	0.952	0.918	0.879	0.837	0.794	0.749	0.705	0.662	0.62
(0.3,0.9)	1	0.91	0.833	0.766	0.706	0.652	0.602	0.557	0.515	0.476	0.44
(0.4,0.1)	1	0.835	0.704	0.599	0.512	0.44	0.38	0.328	0.284	0.246	0.213
(0.4,0.2)	1	0.965	0.916	0.859	0.798	0.736	0.674	0.615	0.558	0.505	0.455
(0.4,0.3)	1	0.986	0.965	0.938	0.904	0.865	0.822	0.777	0.729	0.68	0.632
(0.4,0.4)	1	0.989	0.975	0.958	0.936	0.911	0.881	0.848	0.811	0.772	0.731
(0.4,0.5)	1	0.989	0.977	0.962	0.945	0.924	0.9	0.874	0.844	0.812	0.778
(0.4,0.6)	1	0.989	0.977	0.963	0.946	0.927	0.905	0.88	0.853	0.824	0.793
(0.4,0.7)	1	0.989	0.976	0.96	0.942	0.921	0.897	0.872	0.844	0.814	0.783
(0.4,0.8)	1	0.983	0.962	0.937	0.91	0.881	0.85	0.818	0.786	0.752	0.719
(0.4,0.9)	1	0.912	0.841	0.781	0.73	0.685	0.644	0.607	0.573	0.541	0.511
(0.5,0.1)	1	0.836	0.705	0.601	0.516	0.445	0.387	0.337	0.295	0.258	0.226
(0.5,0.2)	1	0.966	0.918	0.862	0.804	0.745	0.688	0.632	0.579	0.529	0.482
(0.5,0.3)	1	0.986	0.967	0.942	0.912	0.878	0.84	0.8	0.758	0.714	0.671
(0.5,0.4)	1	0.989	0.977	0.962	0.945	0.924	0.9	0.874	0.844	0.812	0.778
(0.5,0.5)	1	0.99	0.979	0.967	0.953	0.938	0.921	0.901	0.879	0.854	0.827
(0.5,0.6)	1	0.99	0.979	0.967	0.955	0.941	0.925	0.908	0.889	0.868	0.844
(0.5,0.7)	1	0.989	0.978	0.965	0.951	0.935	0.918	0.899	0.879	0.857	0.834
(0.5,0.8)	1	0.984	0.964	0.942	0.918	0.894	0.869	0.844	0.818	0.792	0.765
(0.5,0.9)	1	0.913	0.842	0.784	0.736	0.694	0.658	0.626	0.596	0.569	0.543
(0.6,0.1)	1	0.836	0.705	0.601	0.516	0.446	0.388	0.339	0.297	0.261	0.23
(0.6,0.2)	1	0.966	0.918	0.863	0.805	0.747	0.691	0.637	0.585	0.537	0.491
(0.6,0.3)	1	0.986	0.967	0.942	0.913	0.88	0.844	0.805	0.766	0.725	0.683

(0.6,0.4)	1	0.989	0.977	0.963	0.946	0.927	0.905	0.88	0.853	0.824	0.793
(0.6,0.5)	1	0.99	0.979	0.967	0.955	0.941	0.925	0.908	0.889	0.868	0.844
(0.6,0.6)	1	0.99	0.979	0.968	0.956	0.944	0.93	0.915	0.899	0.881	0.862
(0.6,0.7)	1	0.989	0.978	0.965	0.952	0.938	0.923	0.906	0.889	0.87	0.851
(0.6,0.8)	1	0.984	0.964	0.942	0.92	0.897	0.873	0.85	0.827	0.804	0.781
(0.6,0.9)	1	0.913	0.842	0.785	0.737	0.696	0.661	0.63	0.603	0.578	0.554
(0.7,0.1)	1	0.835	0.705	0.6	0.514	0.444	0.385	0.336	0.294	0.258	0.227
(0.7,0.2)	1	0.965	0.917	0.861	0.802	0.743	0.685	0.631	0.579	0.53	0.485
(0.7,0.3)	1	0.986	0.966	0.94	0.909	0.875	0.837	0.798	0.757	0.716	0.675
(0.7,0.4)	1	0.989	0.976	0.96	0.942	0.921	0.897	0.872	0.844	0.814	0.783
(0.7,0.5)	1	0.989	0.978	0.965	0.951	0.935	0.918	0.899	0.879	0.857	0.834
(0.7,0.6)	1	0.989	0.978	0.965	0.952	0.938	0.923	0.906	0.889	0.87	0.851
(0.7,0.7)	1	0.989	0.976	0.963	0.948	0.932	0.915	0.897	0.879	0.86	0.84
(0.7,0.8)	1	0.983	0.963	0.94	0.916	0.891	0.866	0.842	0.818	0.794	0.771
(0.7,0.9)	1	0.912	0.841	0.783	0.734	0.692	0.656	0.624	0.596	0.571	0.547
(0.8,0.1)	1	0.832	0.697	0.588	0.499	0.426	0.367	0.317	0.274	0.239	0.208
(0.8,0.2)	1	0.96	0.905	0.841	0.776	0.712	0.651	0.593	0.54	0.49	0.445
(0.8,0.3)	1	0.98	0.952	0.918	0.879	0.837	0.794	0.749	0.705	0.662	0.62
(0.8,0.4)	1	0.983	0.962	0.937	0.91	0.881	0.85	0.818	0.786	0.752	0.719
(0.8,0.5)	1	0.984	0.964	0.942	0.918	0.894	0.869	0.844	0.818	0.792	0.765
(0.8,0.6)	1	0.984	0.964	0.942	0.92	0.897	0.873	0.85	0.827	0.804	0.781
(0.8,0.7)	1	0.983	0.963	0.94	0.916	0.891	0.866	0.842	0.818	0.794	0.771
(0.8,0.8)	1	0.978	0.949	0.918	0.885	0.852	0.821	0.79	0.761	0.734	0.708
(0.8,0.9)	1	0.908	0.83	0.765	0.71	0.663	0.622	0.586	0.555	0.527	0.502
(0.9,0.1)	1	0.778	0.617	0.497	0.406	0.335	0.28	0.236	0.2	0.17	0.146
(0.9,0.2)	1	0.893	0.794	0.705	0.626	0.557	0.495	0.442	0.394	0.352	0.315
(0.9,0.3)	1	0.91	0.833	0.766	0.706	0.652	0.602	0.557	0.515	0.476	0.44
(0.9,0.4)	1	0.912	0.841	0.781	0.73	0.685	0.644	0.607	0.573	0.541	0.511
(0.9,0.5)	1	0.913	0.842	0.784	0.736	0.694	0.658	0.626	0.596	0.569	0.543
(0.9,0.6)	1	0.913	0.842	0.785	0.737	0.696	0.661	0.63	0.603	0.578	0.554
(0.9,0.7)	1	0.912	0.841	0.783	0.734	0.692	0.656	0.624	0.596	0.571	0.547
(0.9,0.8)	1	0.908	0.83	0.765	0.71	0.663	0.622	0.586	0.555	0.527	0.502
(0.9,0.9)	1	0.846	0.73	0.641	0.572	0.517	0.472	0.435	0.404	0.378	0.356

Result of 2dimension Implicit method using r=0.01

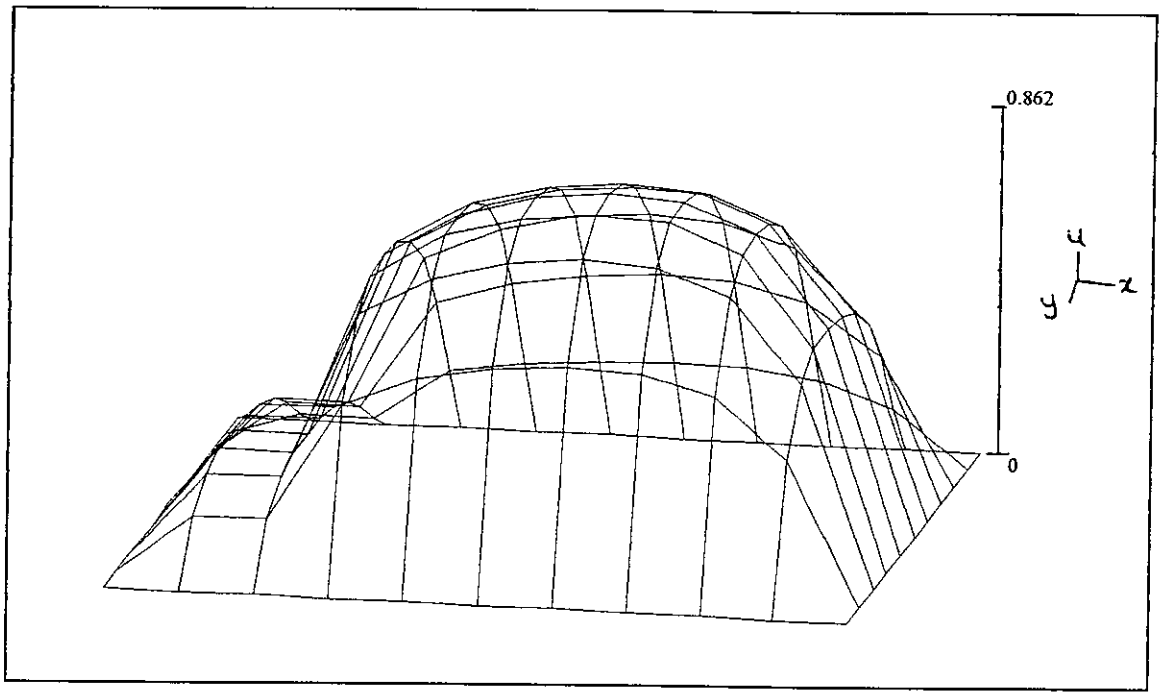
(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.2)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.806
(0.1,0.3)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.4)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.5)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82
(0.1,0.6)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82
(0.1,0.7)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.8)	1	0.979	0.959	0.94	0.921	0.902	0.884	0.867	0.849	0.832	0.816
(0.1,0.9)	1	0.97	0.942	0.914	0.888	0.863	0.838	0.815	0.793	0.771	0.75
(0.2,0.1)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.806
(0.2,0.2)	1	0.998	0.996	0.993	0.989	0.984	0.98	0.974	0.968	0.962	0.955
(0.2,0.3)	1	0.999	0.997	0.995	0.992	0.989	0.986	0.983	0.979	0.975	0.971
(0.2,0.4)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.2,0.5)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.2,0.6)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.2,0.7)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.2,0.8)	1	0.999	0.997	0.994	0.991	0.988	0.985	0.981	0.977	0.973	0.968
(0.2,0.9)	1	0.989	0.978	0.967	0.955	0.944	0.933	0.922	0.911	0.9	0.889
(0.3,0.1)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.3,0.2)	1	0.999	0.997	0.995	0.992	0.989	0.986	0.983	0.979	0.975	0.971
(0.3,0.3)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.99	0.989	0.987
(0.3,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.988
(0.3,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.992	0.991	0.99	0.988
(0.3,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.992	0.991	0.99	0.988
(0.3,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.988
(0.3,0.8)	1	0.999	0.998	0.996	0.995	0.993	0.992	0.99	0.988	0.986	0.984
(0.3,0.9)	1	0.989	0.979	0.969	0.959	0.949	0.939	0.93	0.921	0.912	0.903
(0.4,0.1)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.4,0.2)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.4,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.988
(0.4,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.8)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.99	0.989	0.987	0.985
(0.4,0.9)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.5,0.1)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82
(0.5,0.2)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.5,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.992	0.991	0.99	0.988
(0.5,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.8)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.5,0.9)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.6,0.1)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82
(0.6,0.2)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.6,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.992	0.991	0.99	0.988

(0.6,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.8)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.6,0.9)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.7,0.1)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.7,0.2)	1	0.999	0.997	0.995	0.992	0.99	0.987	0.983	0.98	0.976	0.972
(0.7,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.988
(0.7,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.7,0.8)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.99	0.989	0.987	0.985
(0.7,0.9)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.8,0.1)	1	0.979	0.959	0.94	0.921	0.902	0.884	0.867	0.849	0.832	0.816
(0.8,0.2)	1	0.999	0.997	0.994	0.991	0.988	0.985	0.981	0.977	0.973	0.968
(0.8,0.3)	1	0.999	0.998	0.996	0.995	0.993	0.992	0.99	0.988	0.986	0.984
(0.8,0.4)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.99	0.989	0.987	0.985
(0.8,0.5)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.8,0.6)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.8,0.7)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.99	0.989	0.987	0.985
(0.8,0.8)	1	0.999	0.997	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981
(0.8,0.9)	1	0.989	0.979	0.968	0.958	0.948	0.938	0.929	0.919	0.91	0.901
(0.9,0.1)	1	0.97	0.942	0.914	0.888	0.863	0.838	0.815	0.793	0.771	0.75
(0.9,0.2)	1	0.989	0.978	0.967	0.955	0.944	0.933	0.922	0.911	0.9	0.889
(0.9,0.3)	1	0.989	0.979	0.969	0.959	0.949	0.939	0.93	0.921	0.912	0.903
(0.9,0.4)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.9,0.5)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.9,0.6)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.9,0.7)	1	0.989	0.979	0.969	0.959	0.949	0.94	0.931	0.922	0.913	0.905
(0.9,0.8)	1	0.989	0.979	0.968	0.958	0.948	0.938	0.929	0.919	0.91	0.901
(0.9,0.9)	1	0.98	0.96	0.941	0.923	0.906	0.889	0.873	0.857	0.842	0.827

Result of 2dimension Implicit method using r=0.001

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.9)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.2,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.2,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.9)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.991	0.99	0.989
(0.3,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.3,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.4,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.4,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.5,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.5,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.6,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.6,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999

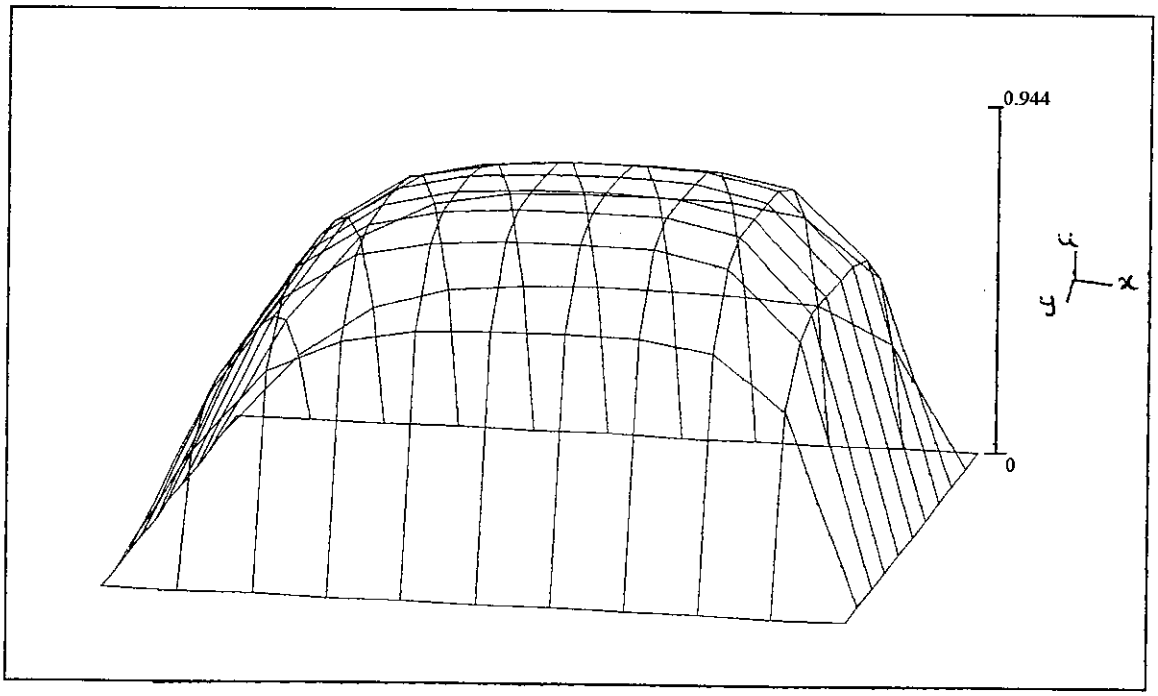
(0.6,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.7,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.7,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.8,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.1)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.9,0.2)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.991	0.99	0.989
(0.9,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.9)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.983	0.981	0.979



f2

2dimension implicit method

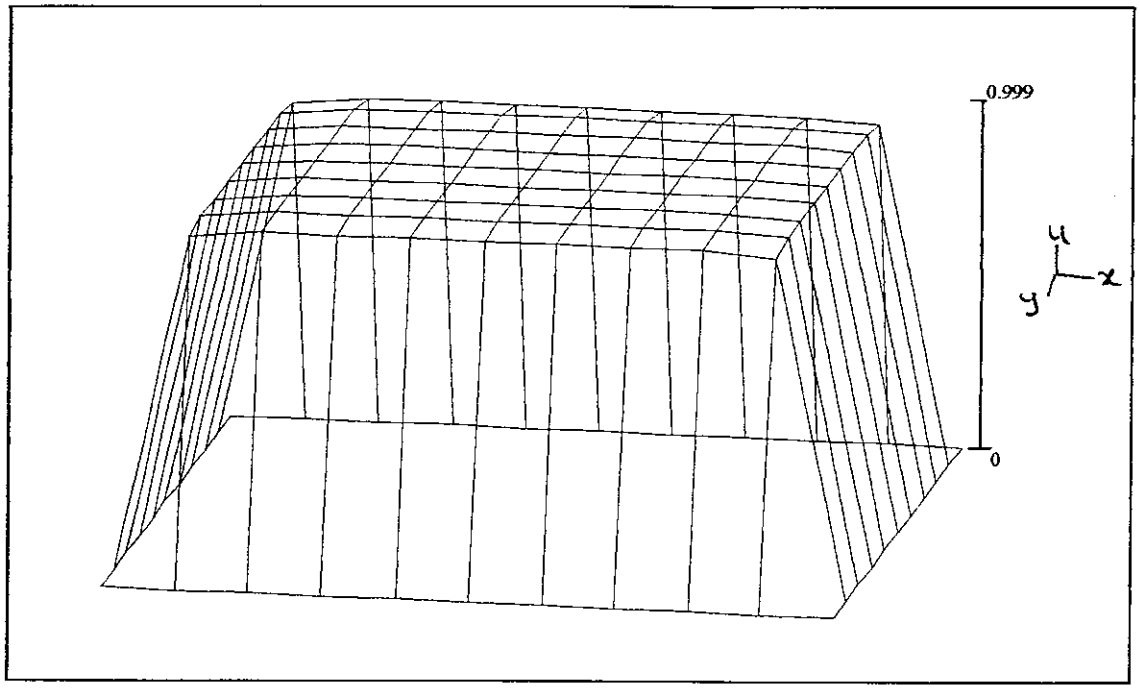
this graph for n=10 and r=0.1



f

2dimension Implicit method

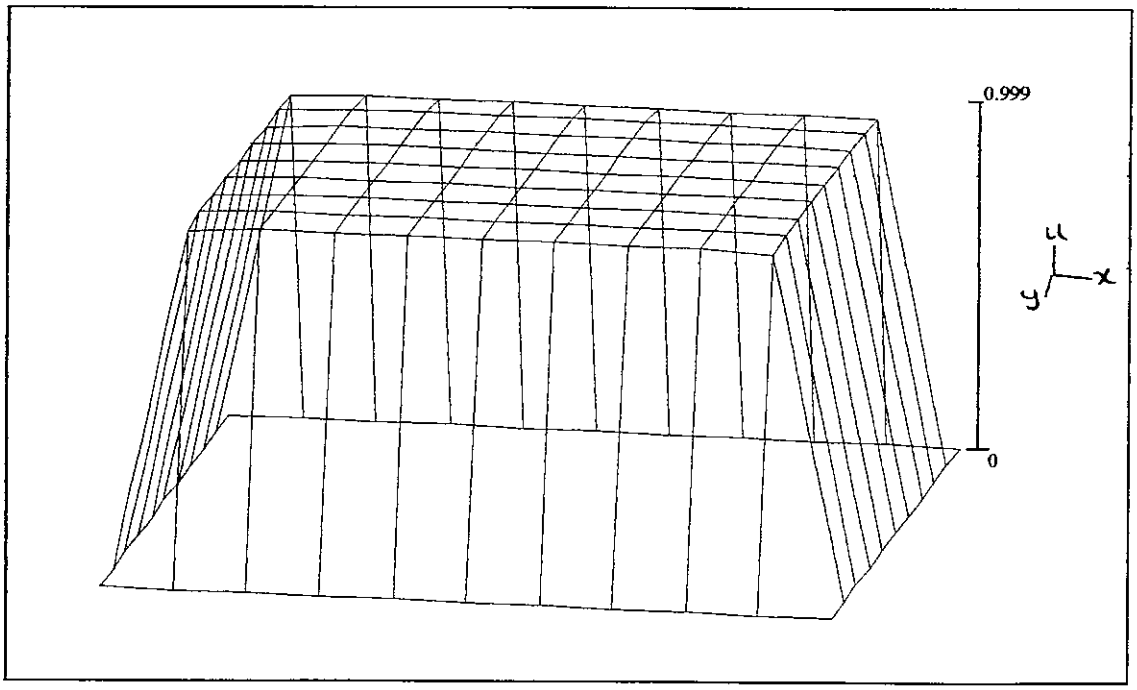
this graph for n=5 and r=0.1



f

2dimension Implicit method

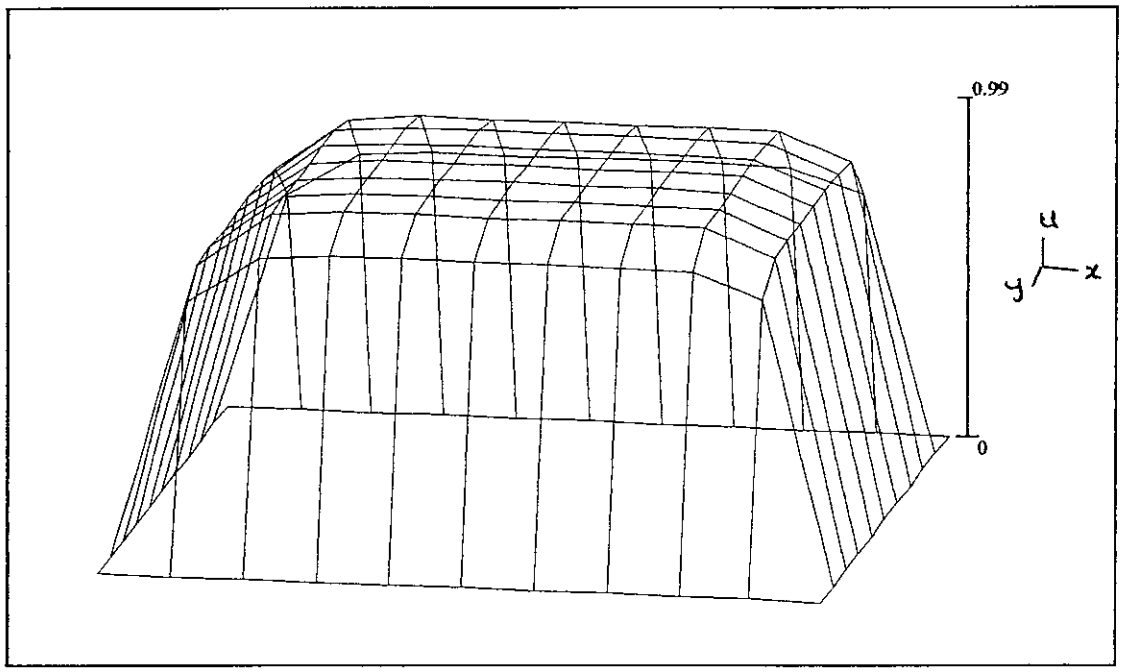
This graph for n=10 and r=0.001



f2

2dimension Implicit method

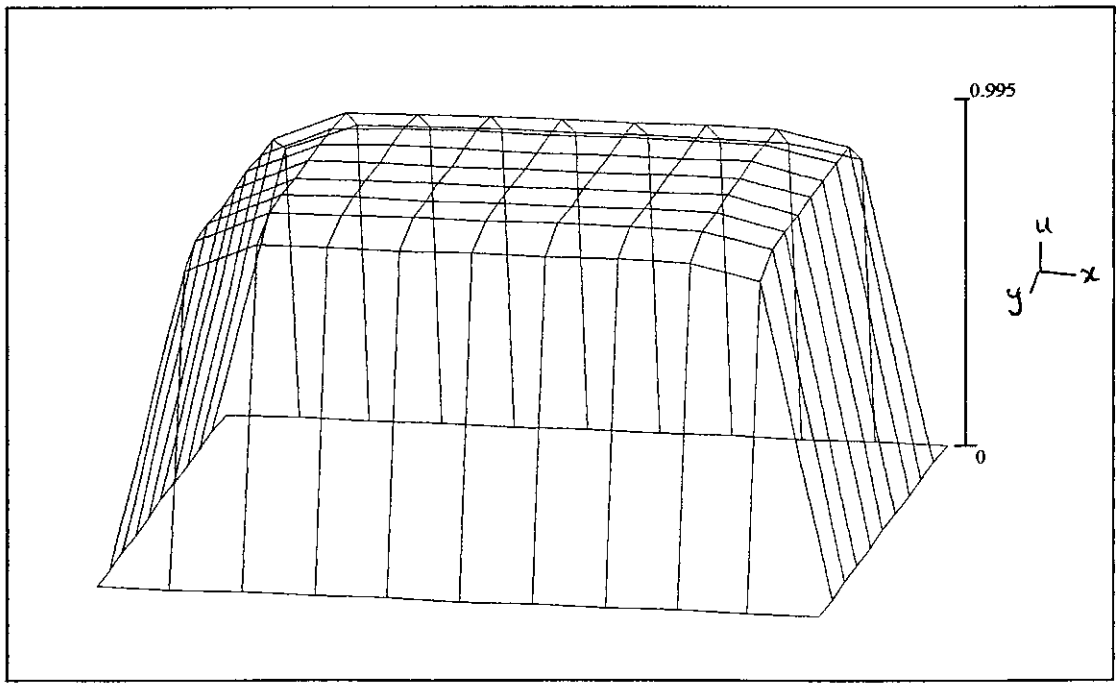
This graph for n=5 and r=0.001



f

2dimension Implicit method(n=10)

This graph for n=10 and r=0.01



f2

2dimension Implicit method(n=5)

This graph for n=10 and r=0.01

Result of 2dimension Crank-Nicolson method using r=0.1[equation(3.5.5) page 30]

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.809	0.653	0.525	0.42	0.334	0.264	0.207	0.16	0.122	0.09
(0.1,0.2)	1	0.891	0.786	0.688	0.596	0.513	0.438	0.37	0.31	0.257	0.211
(0.1,0.3)	1	0.895	0.799	0.712	0.631	0.558	0.49	0.428	0.372	0.321	0.275
(0.1,0.4)	1	0.895	0.8	0.714	0.636	0.565	0.501	0.442	0.389	0.34	0.296
(0.1,0.5)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.392	0.345	0.302
(0.1,0.6)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.1,0.7)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.1,0.8)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.1,0.9)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.2,0.1)	1	0.891	0.786	0.688	0.596	0.513	0.438	0.37	0.31	0.257	0.211
(0.2,0.2)	1	0.981	0.946	0.898	0.843	0.781	0.716	0.65	0.584	0.52	0.459
(0.2,0.3)	1	0.985	0.961	0.93	0.892	0.848	0.8	0.749	0.696	0.642	0.588
(0.2,0.4)	1	0.985	0.962	0.933	0.898	0.859	0.817	0.773	0.726	0.679	0.631
(0.2,0.5)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.732	0.687	0.642
(0.2,0.6)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.644
(0.2,0.7)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.645
(0.2,0.8)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.645
(0.2,0.9)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.645
(0.3,0.1)	1	0.895	0.799	0.712	0.631	0.558	0.49	0.428	0.372	0.321	0.275
(0.3,0.2)	1	0.985	0.961	0.93	0.892	0.848	0.8	0.749	0.696	0.642	0.588
(0.3,0.3)	1	0.989	0.977	0.962	0.943	0.921	0.894	0.863	0.828	0.79	0.749
(0.3,0.4)	1	0.99	0.978	0.965	0.95	0.933	0.913	0.89	0.864	0.835	0.803
(0.3,0.5)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.894	0.871	0.845	0.817
(0.3,0.6)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.3,0.7)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.3,0.8)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.3,0.9)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.4,0.1)	1	0.895	0.8	0.714	0.636	0.565	0.501	0.442	0.389	0.34	0.296
(0.4,0.2)	1	0.985	0.962	0.933	0.898	0.859	0.817	0.773	0.726	0.679	0.631
(0.4,0.3)	1	0.99	0.978	0.965	0.95	0.933	0.913	0.89	0.864	0.835	0.803
(0.4,0.4)	1	0.99	0.979	0.969	0.958	0.945	0.932	0.917	0.901	0.882	0.861
(0.4,0.5)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.922	0.908	0.893	0.876
(0.4,0.6)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.4,0.7)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.4,0.8)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.4,0.9)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.5,0.1)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.392	0.345	0.302
(0.5,0.2)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.732	0.687	0.642
(0.5,0.3)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.894	0.871	0.845	0.817
(0.5,0.4)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.922	0.908	0.893	0.876
(0.5,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.916	0.903	0.891
(0.5,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.905	0.894
(0.5,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.906	0.894
(0.5,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.906	0.894
(0.5,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.906	0.894
(0.6,0.1)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.6,0.2)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.644
(0.6,0.3)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82

(0.6,0.4)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.6,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.905	0.894
(0.6,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.907	0.897
(0.6,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.908	0.897
(0.6,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.908	0.897
(0.6,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.908	0.897
(0.7,0.1)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.7,0.2)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.645
(0.7,0.3)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.7,0.4)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.7,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.906	0.894
(0.7,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.908	0.897
(0.7,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.1)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.8,0.2)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.645
(0.8,0.3)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.8,0.4)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.906	0.894
(0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.908	0.897
(0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.1)	1	0.895	0.8	0.714	0.636	0.566	0.502	0.445	0.393	0.346	0.303
(0.9,0.2)	1	0.985	0.962	0.933	0.899	0.861	0.82	0.777	0.733	0.689	0.645
(0.9,0.3)	1	0.99	0.978	0.966	0.951	0.935	0.916	0.895	0.872	0.847	0.82
(0.9,0.4)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.923	0.909	0.895	0.879
(0.9,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.928	0.917	0.906	0.894
(0.9,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.928	0.918	0.908	0.897
(0.9,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898

Result of 2dimension Crank-Nicolson method using r=0.01[equation(3.5.5) page 30]

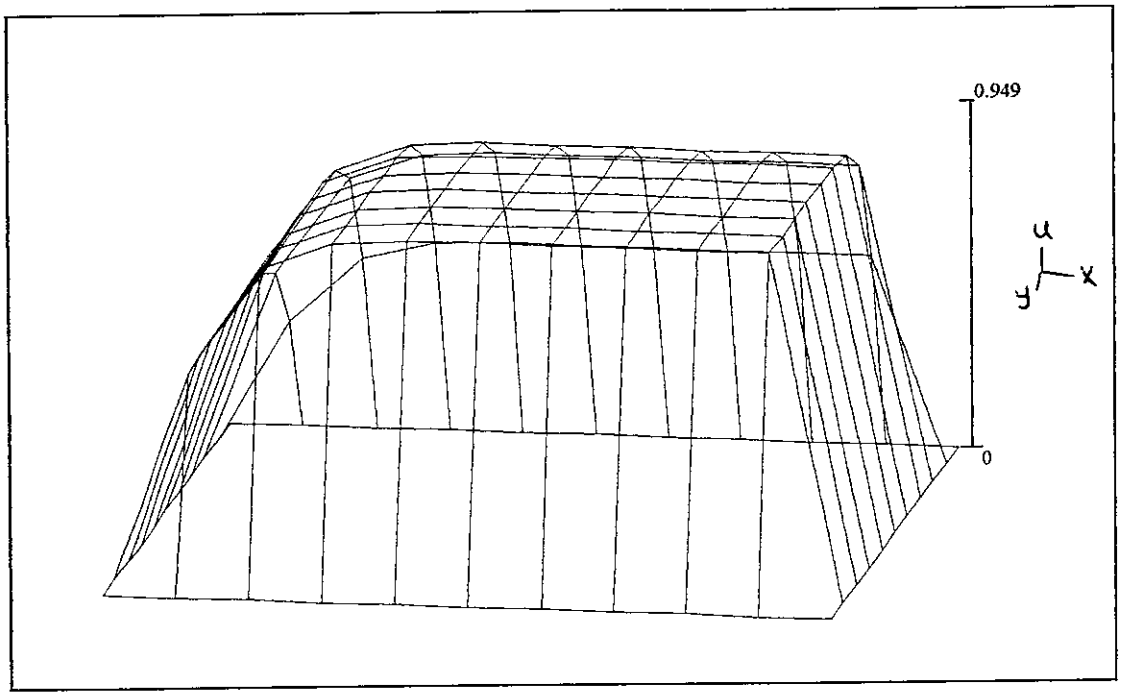
(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.979	0.959	0.939	0.919	0.9	0.881	0.863	0.845	0.827	0.809
(0.1,0.2)	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.902	0.891
(0.1,0.3)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.4)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.5)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.6)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.7)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.8)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.1,0.9)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.2,0.1)	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.902	0.891
(0.2,0.2)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.983	0.98
(0.2,0.3)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.99	0.989	0.987	0.985
(0.2,0.4)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.2,0.5)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.2,0.6)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.2,0.7)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.2,0.8)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.2,0.9)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.3,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.3,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.99	0.989	0.987	0.985
(0.3,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.3,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.3,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.4,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.4,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.5,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.5,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.6,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.6,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99

(0.6,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.7,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.7,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.8,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.8,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.8,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.1)	1	0.989	0.978	0.967	0.957	0.946	0.936	0.925	0.915	0.905	0.895
(0.9,0.2)	1	0.999	0.998	0.996	0.995	0.994	0.992	0.991	0.989	0.987	0.985
(0.9,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.9,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99

Result of 2dimension Crank-Nicolson method using r=0.001[equation(3.5.5) page 30]

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.991	0.99	0.989
(0.1,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.1,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.2,0.1)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.991	0.99	0.989
(0.2,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.3,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.4,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.5,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.6,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999

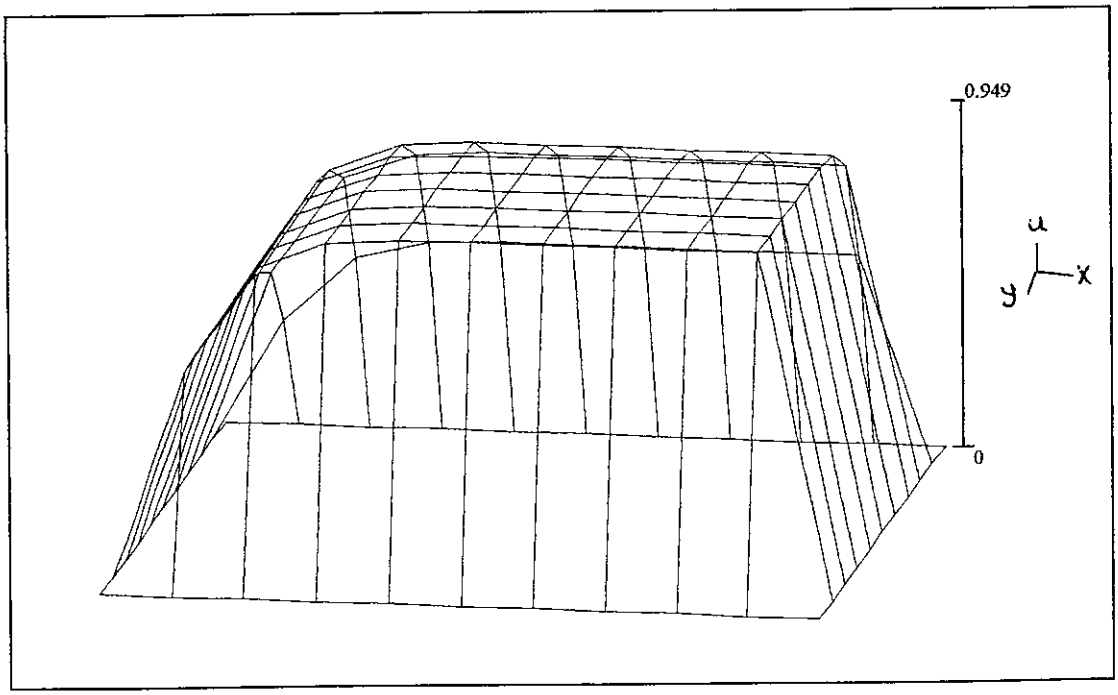
(0.6,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.7,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.1)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.9,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.9,0.9)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999



f2

2dimension Crank-Niclson mehod

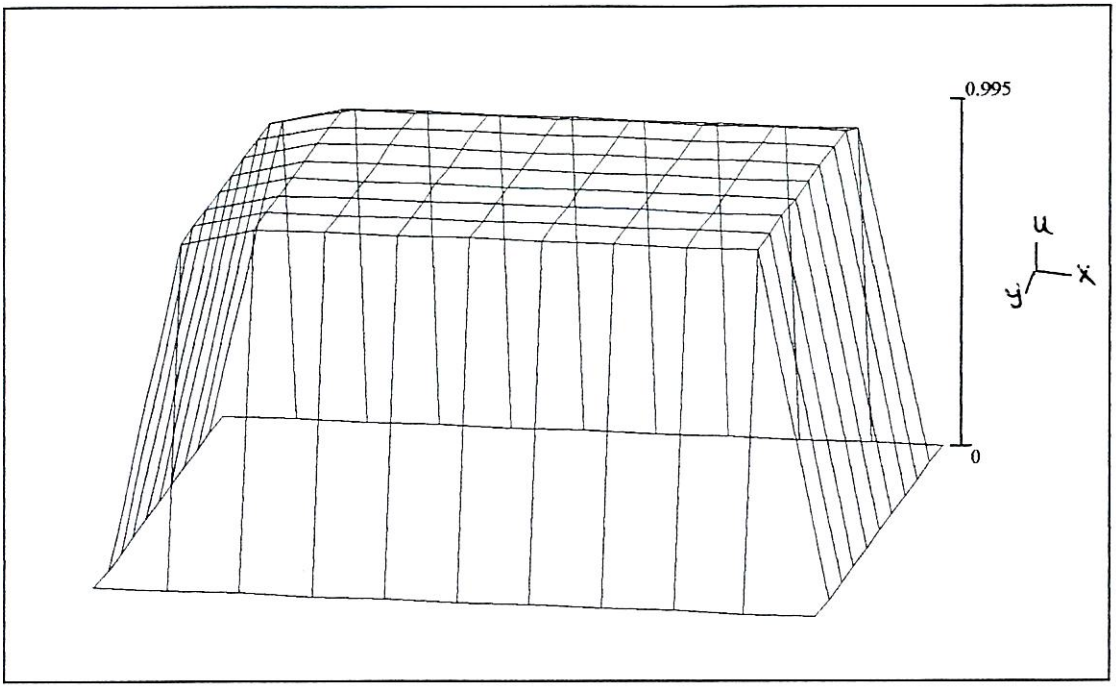
This graph for n=5 and r=0.1



f2

2dimension Crank-Niclson mehod

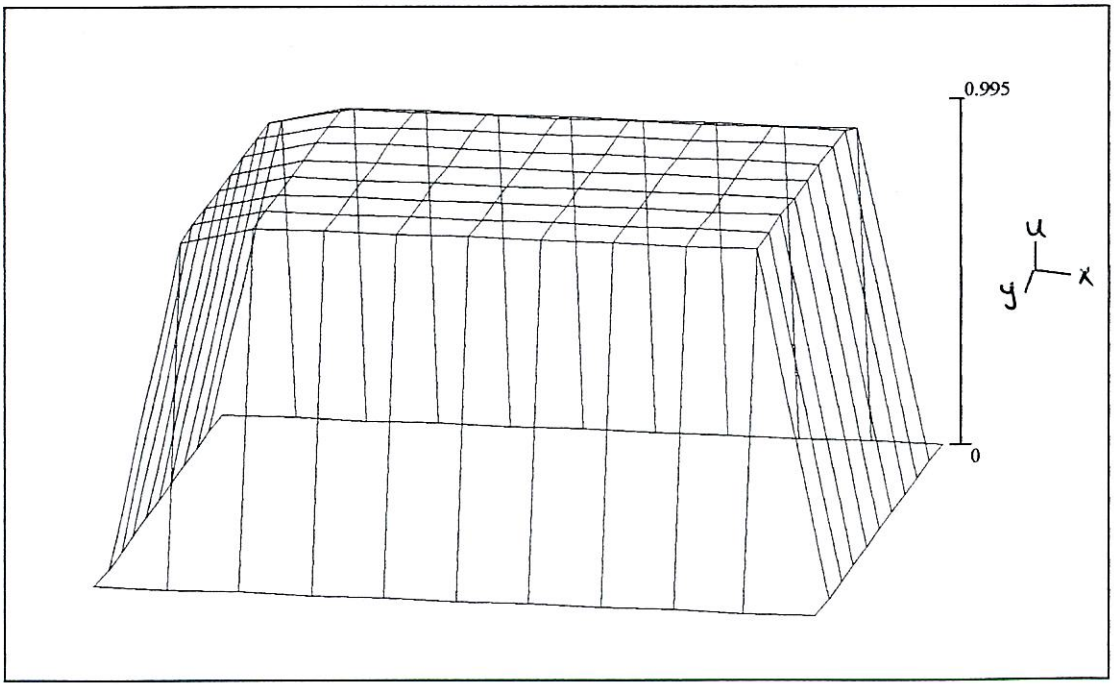
This graph for n=5 and r=0.1



f2

2dimension Crank-Niclson mehod

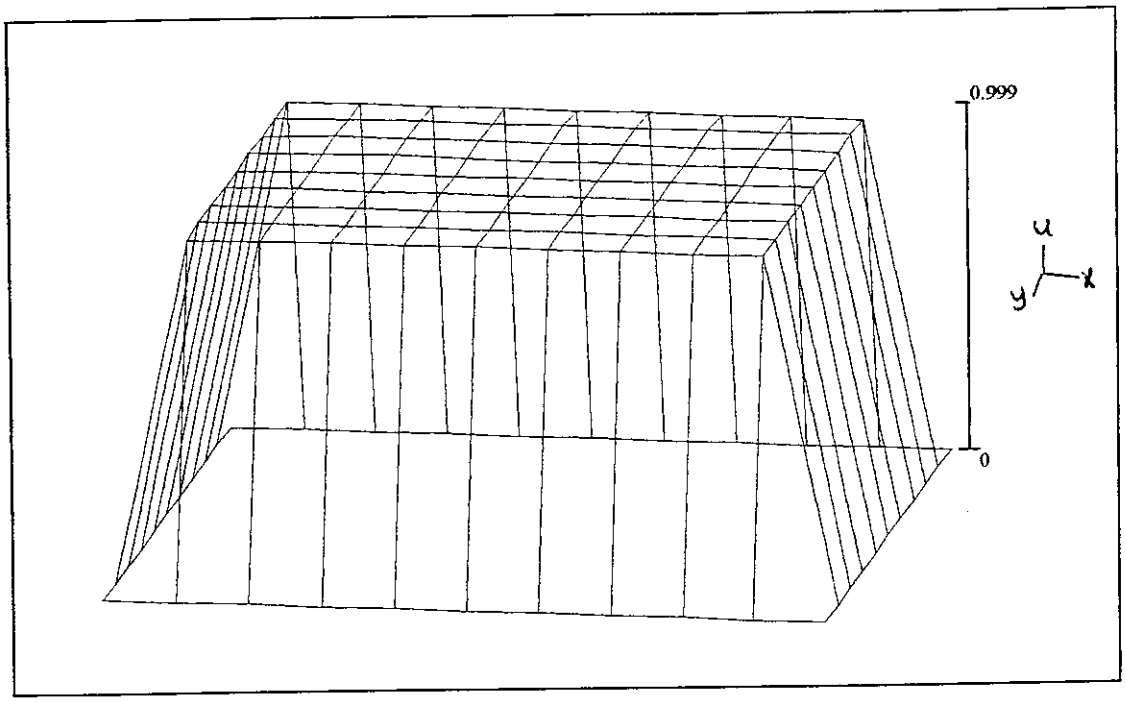
This graph for n=5 and r=0.01



f2

2dimension Crank-Niclson mehod

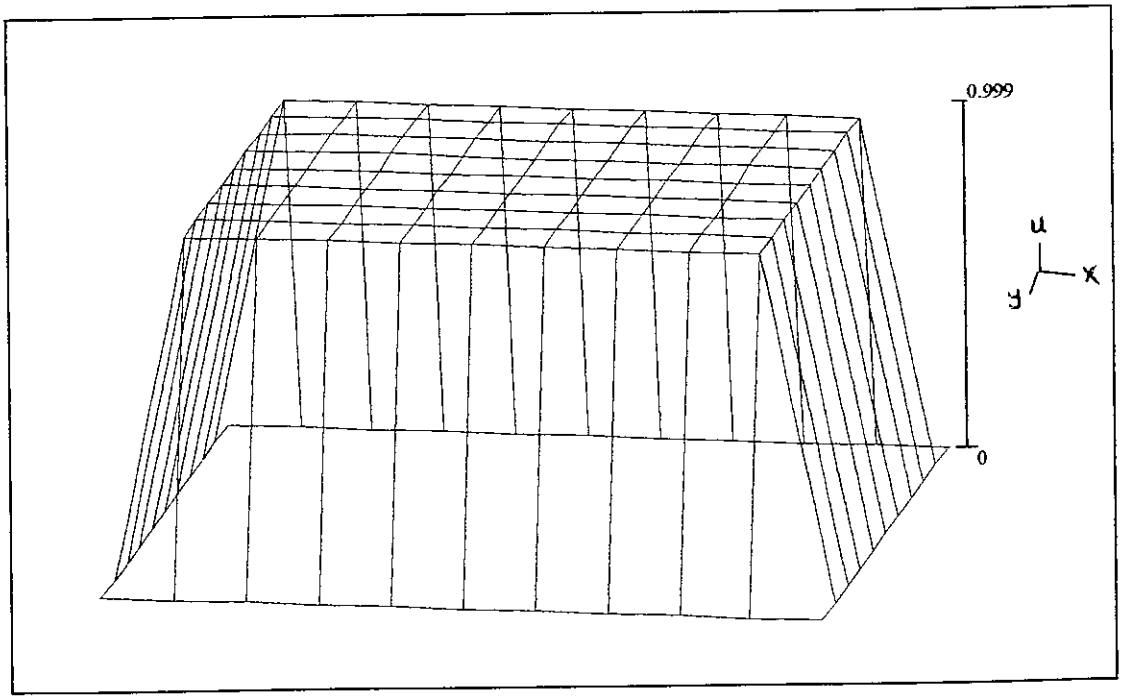
This graph for n=5 and r=0.01



f2

2dimension Crank-Niclson mehod

This graph for n=5 and r=0.001



f2

2dimension Crank-Niclson mehod

This graph for n=5 and r=0.001

Result of 2dimension Crank-Nicolson method using r=0.1[equation(3.5.7) page 36]

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.735	0.546	0.41	0.309	0.235	0.178	0.135	0.102	0.076	0.055
(0.1,0.2)	1	0.846	0.708	0.588	0.486	0.4	0.328	0.268	0.218	0.176	0.141
(0.1,0.3)	1	0.853	0.731	0.626	0.537	0.459	0.392	0.334	0.284	0.24	0.202
(0.1,0.4)	1	0.854	0.733	0.632	0.546	0.474	0.411	0.357	0.309	0.268	0.231
(0.1,0.5)	1	0.854	0.733	0.633	0.548	0.476	0.415	0.362	0.317	0.277	0.242
(0.1,0.6)	1	0.854	0.733	0.633	0.548	0.477	0.416	0.364	0.318	0.279	0.244
(0.1,0.7)	1	0.854	0.733	0.632	0.548	0.476	0.415	0.363	0.317	0.278	0.244
(0.1,0.8)	1	0.853	0.73	0.628	0.541	0.468	0.406	0.353	0.308	0.268	0.234
(0.1,0.9)	1	0.814	0.671	0.558	0.469	0.396	0.337	0.288	0.247	0.212	0.182
(0.2,0.1)	1	0.846	0.708	0.588	0.486	0.4	0.328	0.268	0.218	0.176	0.141
(0.2,0.2)	1	0.971	0.914	0.839	0.757	0.674	0.593	0.518	0.448	0.385	0.328
(0.2,0.3)	1	0.98	0.943	0.893	0.835	0.771	0.706	0.641	0.577	0.515	0.457
(0.2,0.4)	1	0.98	0.946	0.901	0.85	0.795	0.738	0.682	0.626	0.571	0.518
(0.2,0.5)	1	0.981	0.946	0.902	0.852	0.799	0.746	0.692	0.64	0.589	0.54
(0.2,0.6)	1	0.981	0.946	0.902	0.852	0.8	0.747	0.694	0.643	0.593	0.546
(0.2,0.7)	1	0.98	0.946	0.901	0.851	0.799	0.746	0.693	0.641	0.592	0.545
(0.2,0.8)	1	0.98	0.942	0.895	0.842	0.786	0.73	0.675	0.622	0.572	0.524
(0.2,0.9)	1	0.936	0.867	0.797	0.73	0.667	0.607	0.553	0.502	0.455	0.413
(0.3,0.1)	1	0.853	0.731	0.626	0.537	0.459	0.392	0.334	0.284	0.24	0.202
(0.3,0.2)	1	0.98	0.943	0.893	0.835	0.771	0.706	0.641	0.577	0.515	0.457
(0.3,0.3)	1	0.989	0.973	0.95	0.92	0.882	0.839	0.791	0.739	0.686	0.632
(0.3,0.4)	1	0.989	0.976	0.958	0.936	0.909	0.877	0.841	0.801	0.758	0.713
(0.3,0.5)	1	0.989	0.976	0.959	0.939	0.914	0.885	0.854	0.819	0.782	0.743
(0.3,0.6)	1	0.989	0.976	0.96	0.939	0.915	0.887	0.856	0.823	0.787	0.751
(0.3,0.7)	1	0.989	0.976	0.959	0.938	0.914	0.885	0.854	0.821	0.785	0.749
(0.3,0.8)	1	0.988	0.972	0.952	0.928	0.899	0.867	0.833	0.797	0.759	0.721
(0.3,0.9)	1	0.944	0.894	0.849	0.805	0.763	0.722	0.683	0.644	0.606	0.57
(0.4,0.1)	1	0.854	0.733	0.632	0.546	0.474	0.411	0.357	0.309	0.268	0.231
(0.4,0.2)	1	0.98	0.946	0.901	0.85	0.795	0.738	0.682	0.626	0.571	0.518
(0.4,0.3)	1	0.989	0.976	0.958	0.936	0.909	0.877	0.841	0.801	0.758	0.713
(0.4,0.4)	1	0.99	0.979	0.967	0.953	0.937	0.917	0.894	0.867	0.838	0.805
(0.4,0.5)	1	0.99	0.979	0.968	0.956	0.942	0.926	0.908	0.887	0.863	0.837
(0.4,0.6)	1	0.99	0.979	0.968	0.956	0.943	0.927	0.91	0.891	0.87	0.847
(0.4,0.7)	1	0.99	0.979	0.968	0.955	0.942	0.926	0.908	0.889	0.868	0.844
(0.4,0.8)	1	0.989	0.976	0.961	0.944	0.926	0.907	0.886	0.863	0.839	0.813
(0.4,0.9)	1	0.944	0.897	0.856	0.82	0.786	0.756	0.726	0.698	0.671	0.644
(0.5,0.1)	1	0.854	0.733	0.633	0.548	0.476	0.415	0.362	0.317	0.277	0.242
(0.5,0.2)	1	0.981	0.946	0.902	0.852	0.799	0.746	0.692	0.64	0.589	0.54
(0.5,0.3)	1	0.989	0.976	0.959	0.939	0.914	0.885	0.854	0.819	0.782	0.743
(0.5,0.4)	1	0.99	0.979	0.968	0.956	0.942	0.926	0.908	0.887	0.863	0.837
(0.5,0.5)	1	0.99	0.98	0.969	0.958	0.947	0.935	0.921	0.907	0.89	0.872
(0.5,0.6)	1	0.99	0.98	0.969	0.959	0.948	0.936	0.924	0.911	0.897	0.881
(0.5,0.7)	1	0.99	0.979	0.969	0.958	0.947	0.935	0.922	0.909	0.894	0.879
(0.5,0.8)	1	0.989	0.976	0.962	0.947	0.932	0.916	0.899	0.882	0.865	0.846
(0.5,0.9)	1	0.945	0.897	0.857	0.822	0.791	0.763	0.737	0.714	0.692	0.67
(0.6,0.1)	1	0.854	0.733	0.633	0.548	0.477	0.416	0.364	0.318	0.279	0.244
(0.6,0.2)	1	0.981	0.946	0.902	0.852	0.8	0.747	0.694	0.643	0.593	0.546
(0.6,0.3)	1	0.989	0.976	0.96	0.939	0.915	0.887	0.856	0.823	0.787	0.751

(0.6,0.4)	1	0.99	0.979	0.968	0.956	0.943	0.927	0.91	0.891	0.87	0.847
(0.6,0.5)	1	0.99	0.98	0.969	0.959	0.948	0.936	0.924	0.911	0.897	0.881
(0.6,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.903	0.89
(0.6,0.7)	1	0.99	0.979	0.969	0.958	0.948	0.936	0.925	0.913	0.901	0.888
(0.6,0.8)	1	0.989	0.976	0.962	0.947	0.932	0.917	0.902	0.886	0.871	0.855
(0.6,0.9)	1	0.945	0.897	0.857	0.822	0.791	0.764	0.74	0.717	0.697	0.678
(0.7,0.1)	1	0.854	0.733	0.632	0.548	0.476	0.415	0.363	0.317	0.278	0.244
(0.7,0.2)	1	0.98	0.946	0.901	0.851	0.799	0.746	0.693	0.641	0.592	0.545
(0.7,0.3)	1	0.989	0.976	0.959	0.938	0.914	0.885	0.854	0.821	0.785	0.749
(0.7,0.4)	1	0.99	0.979	0.968	0.955	0.942	0.926	0.908	0.889	0.868	0.844
(0.7,0.5)	1	0.99	0.979	0.969	0.958	0.947	0.935	0.922	0.909	0.894	0.879
(0.7,0.6)	1	0.99	0.979	0.969	0.958	0.948	0.936	0.925	0.913	0.901	0.888
(0.7,0.7)	1	0.99	0.979	0.969	0.958	0.946	0.935	0.923	0.911	0.899	0.886
(0.7,0.8)	1	0.989	0.976	0.962	0.947	0.931	0.916	0.9	0.884	0.869	0.853
(0.7,0.9)	1	0.944	0.897	0.857	0.822	0.791	0.763	0.738	0.716	0.695	0.676
(0.8,0.1)	1	0.853	0.73	0.628	0.541	0.468	0.406	0.353	0.308	0.268	0.234
(0.8,0.2)	1	0.98	0.942	0.895	0.842	0.786	0.73	0.675	0.622	0.572	0.524
(0.8,0.3)	1	0.988	0.972	0.952	0.928	0.899	0.867	0.833	0.797	0.759	0.721
(0.8,0.4)	1	0.989	0.976	0.961	0.944	0.926	0.907	0.886	0.863	0.839	0.813
(0.8,0.5)	1	0.989	0.976	0.962	0.947	0.932	0.916	0.899	0.882	0.865	0.846
(0.8,0.6)	1	0.989	0.976	0.962	0.947	0.932	0.917	0.902	0.886	0.871	0.855
(0.8,0.7)	1	0.989	0.976	0.962	0.947	0.931	0.916	0.9	0.884	0.869	0.853
(0.8,0.8)	1	0.988	0.972	0.954	0.936	0.916	0.897	0.877	0.858	0.84	0.822
(0.8,0.9)	1	0.944	0.894	0.85	0.812	0.778	0.747	0.719	0.694	0.672	0.651
(0.9,0.1)	1	0.814	0.671	0.558	0.469	0.396	0.337	0.288	0.247	0.212	0.182
(0.9,0.2)	1	0.936	0.867	0.797	0.73	0.667	0.607	0.553	0.502	0.455	0.413
(0.9,0.3)	1	0.944	0.894	0.849	0.805	0.763	0.722	0.683	0.644	0.606	0.57
(0.9,0.4)	1	0.944	0.897	0.856	0.82	0.786	0.756	0.726	0.698	0.671	0.644
(0.9,0.5)	1	0.945	0.897	0.857	0.822	0.791	0.763	0.737	0.714	0.692	0.67
(0.9,0.6)	1	0.945	0.897	0.857	0.822	0.791	0.764	0.74	0.717	0.697	0.678
(0.9,0.7)	1	0.944	0.897	0.857	0.822	0.791	0.763	0.738	0.716	0.695	0.676
(0.9,0.8)	1	0.944	0.894	0.85	0.812	0.778	0.747	0.719	0.694	0.672	0.651
(0.9,0.9)	1	0.901	0.822	0.758	0.704	0.66	0.622	0.589	0.561	0.536	0.514

Result of 2dimension Crank-Nicolson method using r=0.01[equation(3.5.7) page 36]

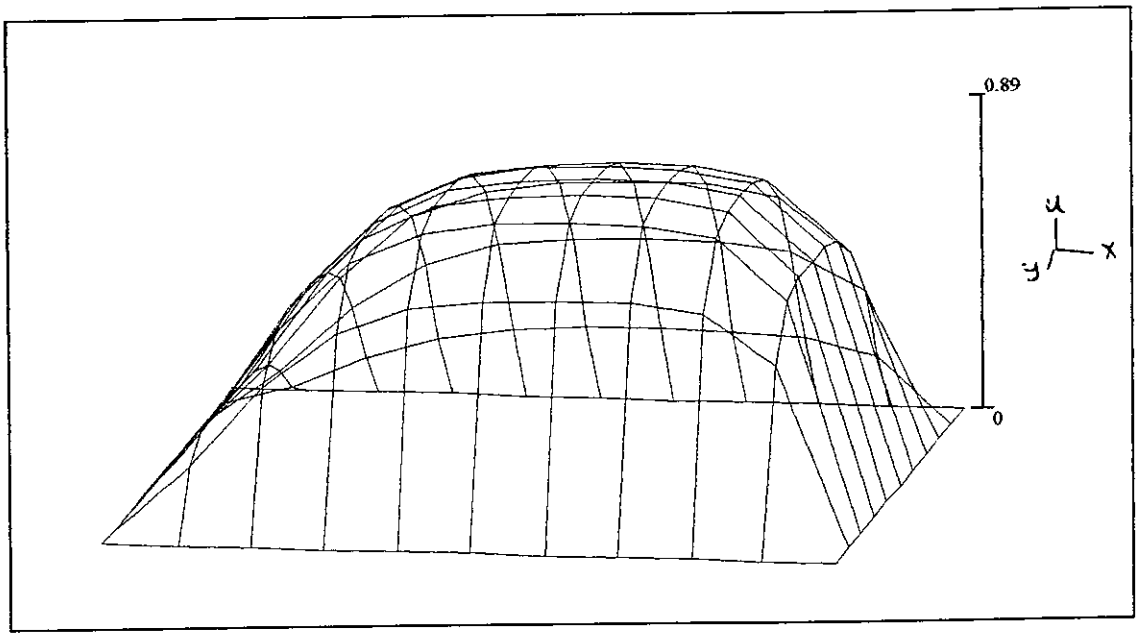
(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.97	0.94	0.912	0.884	0.858	0.832	0.807	0.783	0.76	0.737
(0.1,0.2)	1	0.984	0.968	0.952	0.937	0.921	0.906	0.891	0.876	0.861	0.846
(0.1,0.3)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.867	0.854
(0.1,0.4)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.1,0.5)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.1,0.6)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.1,0.7)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.1,0.8)	1	0.984	0.969	0.953	0.938	0.923	0.909	0.895	0.881	0.867	0.853
(0.1,0.9)	1	0.979	0.959	0.939	0.92	0.902	0.883	0.866	0.849	0.832	0.815
(0.2,0.1)	1	0.984	0.968	0.952	0.937	0.921	0.906	0.891	0.876	0.861	0.846
(0.2,0.2)	1	0.999	0.997	0.995	0.993	0.99	0.986	0.983	0.979	0.975	0.97
(0.2,0.3)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.982	0.979
(0.2,0.4)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.2,0.5)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.2,0.6)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.2,0.7)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.2,0.8)	1	0.999	0.997	0.996	0.994	0.992	0.99	0.987	0.985	0.982	0.979
(0.2,0.9)	1	0.994	0.988	0.981	0.975	0.969	0.962	0.955	0.949	0.942	0.935
(0.3,0.1)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.867	0.854
(0.3,0.2)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.982	0.979
(0.3,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.991	0.99	0.989
(0.3,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.3,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.3,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.3,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.3,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.988
(0.3,0.9)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.949	0.944
(0.4,0.1)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.4,0.2)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.4,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.4,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.4,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.4,0.9)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.5,0.1)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.5,0.2)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.5,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.5,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.5,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.5,0.9)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.6,0.1)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.6,0.2)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.6,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989

(0.6,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.6,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.6,0.9)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.7,0.1)	1	0.984	0.969	0.953	0.938	0.924	0.909	0.895	0.881	0.868	0.854
(0.7,0.2)	1	0.999	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.98
(0.7,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.99	0.989
(0.7,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.7,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.7,0.9)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.8,0.1)	1	0.984	0.969	0.953	0.938	0.923	0.909	0.895	0.881	0.867	0.853
(0.8,0.2)	1	0.999	0.997	0.996	0.994	0.992	0.99	0.987	0.985	0.982	0.979
(0.8,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.988
(0.8,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989
(0.8,0.8)	1	0.999	0.998	0.997	0.996	0.994	0.993	0.992	0.99	0.989	0.988
(0.8,0.9)	1	0.994	0.988	0.982	0.977	0.971	0.965	0.96	0.954	0.949	0.944
(0.9,0.1)	1	0.979	0.959	0.939	0.92	0.902	0.883	0.866	0.849	0.832	0.815
(0.9,0.2)	1	0.994	0.988	0.981	0.975	0.969	0.962	0.955	0.949	0.942	0.935
(0.9,0.3)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.949	0.944
(0.9,0.4)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.9,0.5)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.9,0.6)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.9,0.7)	1	0.994	0.988	0.982	0.977	0.971	0.966	0.96	0.955	0.95	0.945
(0.9,0.8)	1	0.994	0.988	0.982	0.977	0.971	0.965	0.96	0.954	0.949	0.944
(0.9,0.9)	1	0.989	0.978	0.968	0.958	0.948	0.938	0.929	0.92	0.91	0.902

Result of 2dimension Crank-Nicolson method using r=0.001[equation(3.5.7) page 36]

(x,y) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.2)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.3)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.4)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.5)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.6)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.7)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.8)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.1,0.9)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.2,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.2,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.2,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.3,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.3,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.3,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.4,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.4,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.4,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.5,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.5,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.5,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.6,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.6,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999

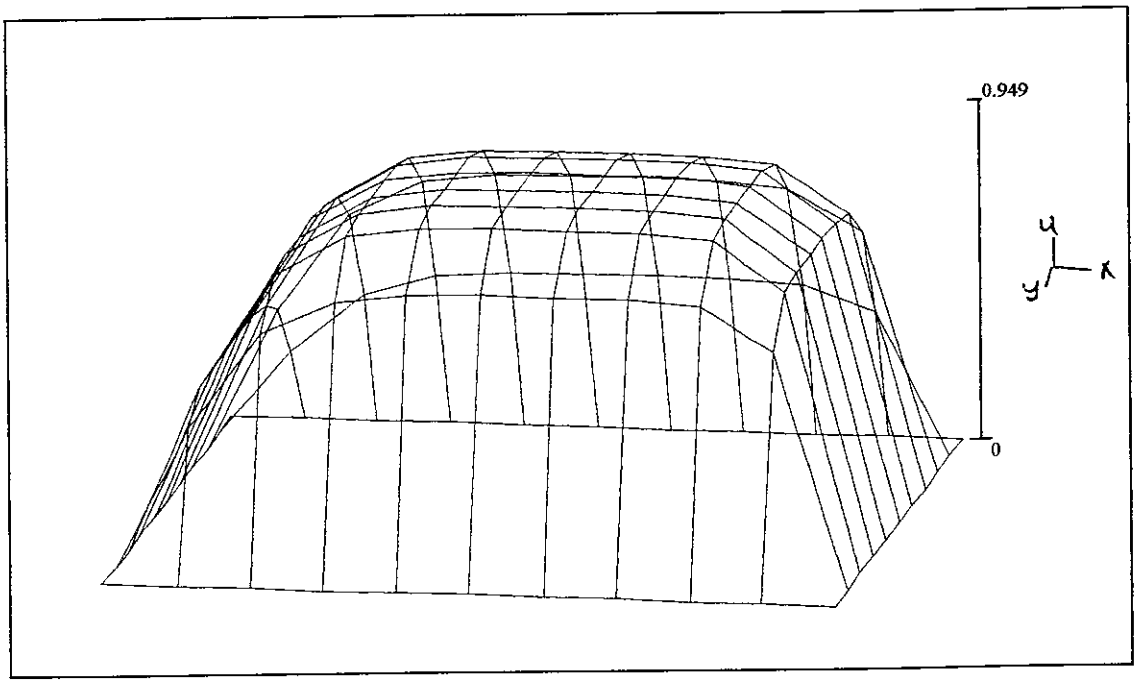
(0.6,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.6,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.7,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.7,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.7,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.8,0.1)	1	0.998	0.997	0.995	0.994	0.992	0.99	0.989	0.987	0.986	0.984
(0.8,0.2)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.3)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.4)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.5)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.6)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.7)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.8)	1	1	1	1	1	0.999	0.999	0.999	0.999	0.999	0.999
(0.8,0.9)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.9,0.2)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.3)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.4)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.5)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.6)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.7)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.8)	1	0.999	0.999	0.998	0.998	0.997	0.996	0.996	0.995	0.995	0.994
(0.9,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.993	0.992	0.991	0.99	0.989



f

2dimension Crank-Niclson method

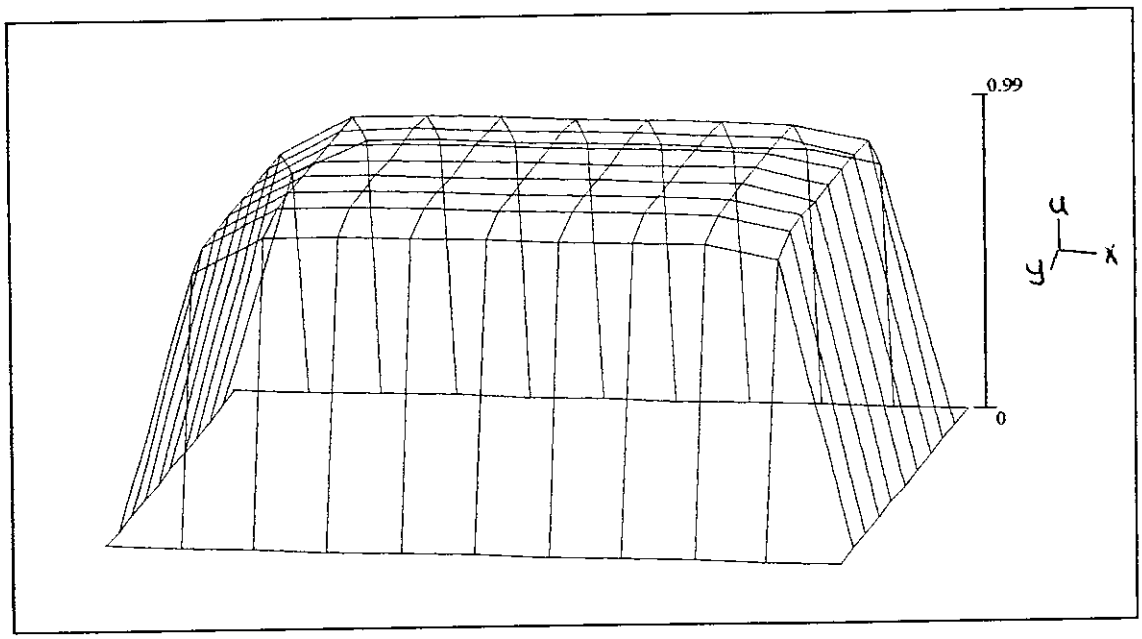
This graph for r=0.1 and n=10



f2

2dimension Crank-Niclson method

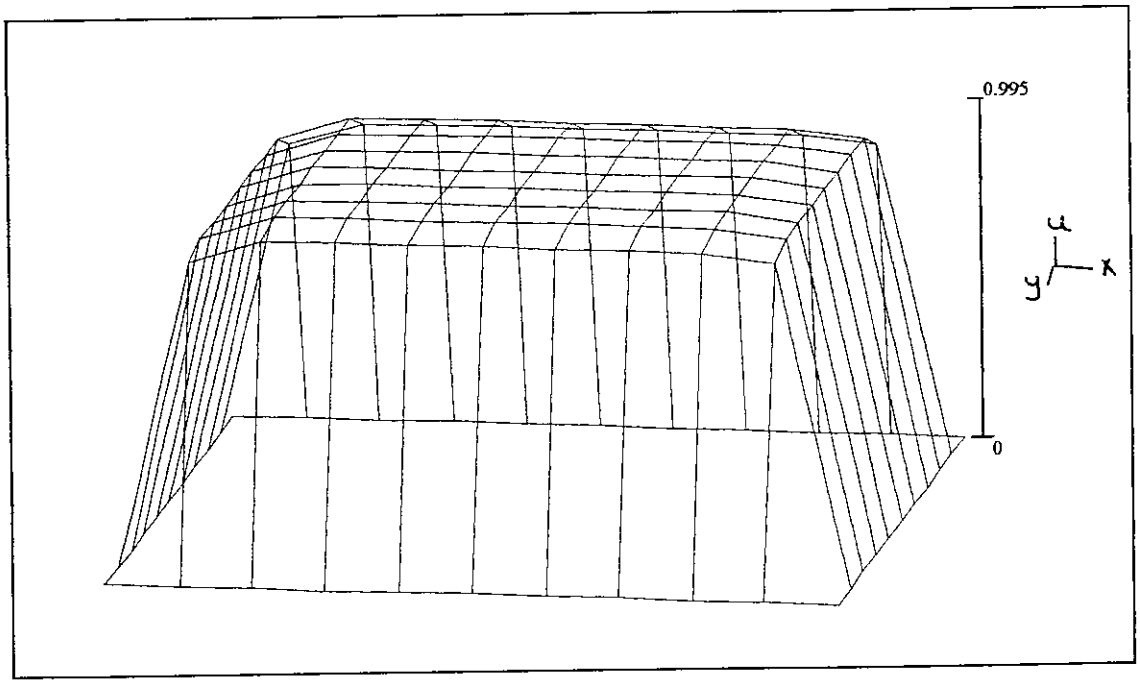
This graph for r=0.1 and n=5



f

2dimension Crank-Niclson method

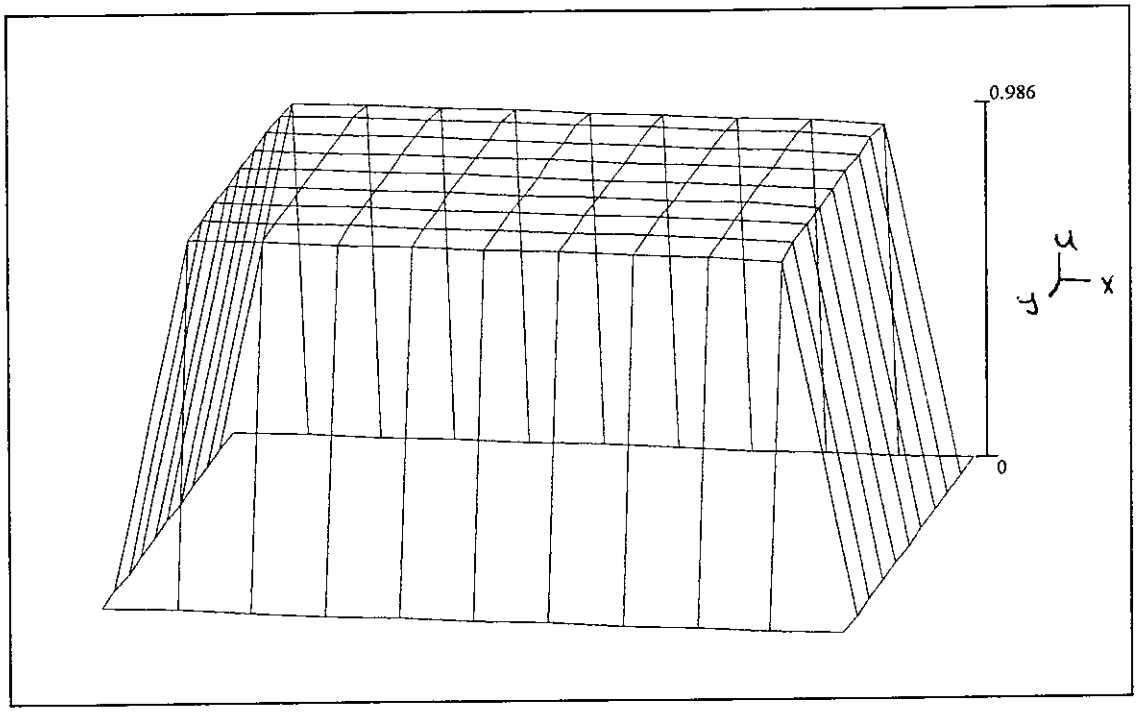
This graph for r=0.01 and n=10



f2

2dimension Crank-Niclson method

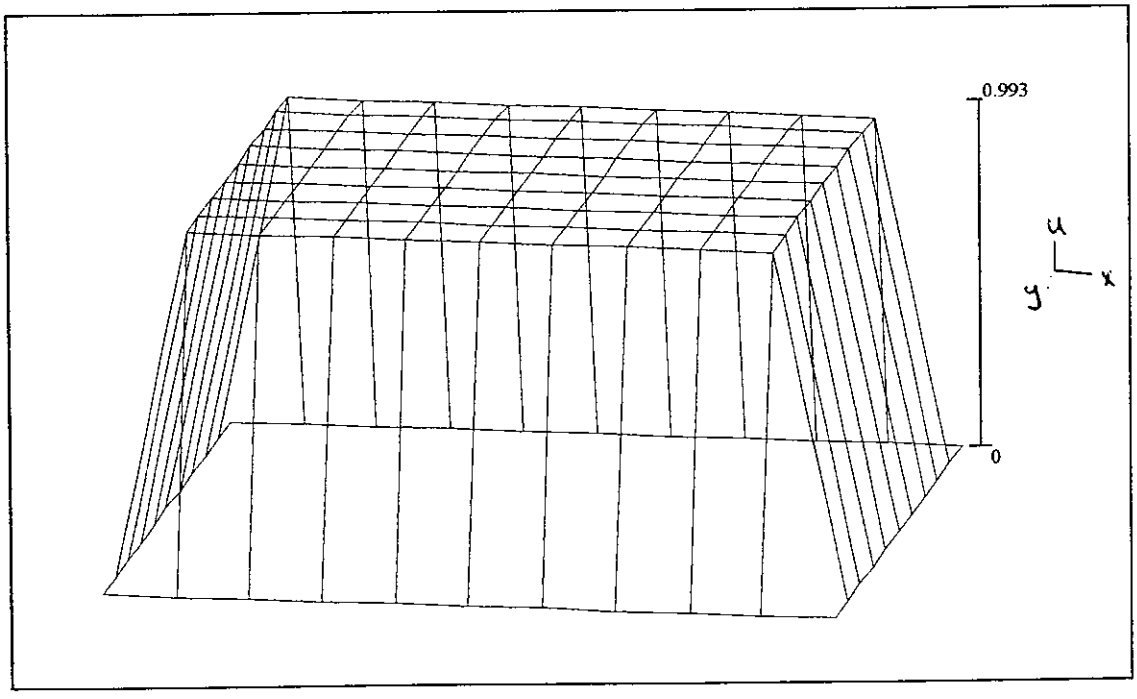
This graph for r=0.01 and n=5



f

2dimension Crank-Niclson method

This graph for r=0.001 and n=10



f2

2dimension Crank-Niclson method

this graph for r=0.001 and n=5

Result of 2dimension vortex method using r=0.1 & n=10 [equation(3.6.19) on page 43]

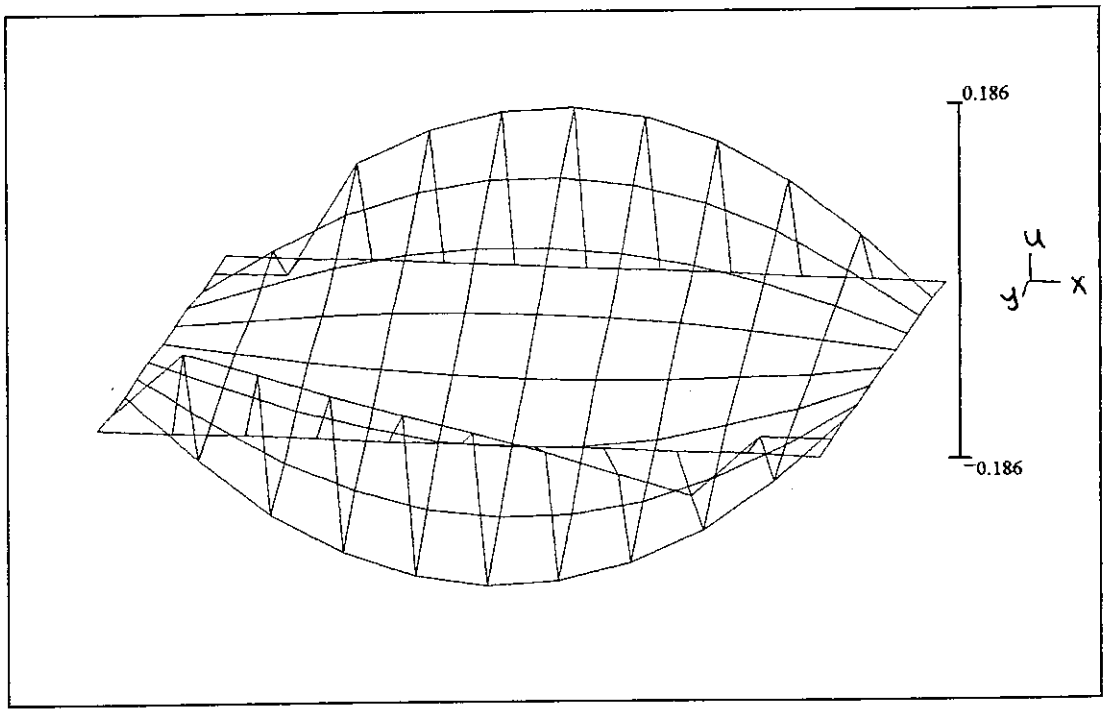
(x,y)	stream function	u	v	votex function
(0.1,0.1)	0.007876	0	-0.0648	0.257027
(0.1,0.2)	0.014355	0.04494	-0.11864	0.419189
(0.1,0.3)	0.018849	0.0256	-0.15657	0.486866
(0.1,0.4)	0.021409	0.00827	-0.17849	0.506499
(0.1,0.5)	0.022236	-0.00822	-0.1856	0.510246
(0.1,0.6)	0.021414	-0.02559	-0.17849	0.507185
(0.1,0.7)	0.018855	-0.04497	-0.15657	0.487853
(0.1,0.8)	0.014358	-0.06483	-0.11864	0.419735
(0.1,0.9)	0.007875	0.06481	-0.06479	0.256543
(0.2,0.1)	0.014356	0.11863	-0.04495	0.419521
(0.2,0.2)	0.026219	0.08287	-0.08288	0.682272
(0.2,0.3)	0.034506	0.04752	-0.1102	0.79042
(0.2,0.4)	0.039258	0.01538	-0.12632	0.821074
(0.2,0.5)	0.040796	-0.01533	-0.13162	0.826623
(0.2,0.6)	0.039263	-0.04751	-0.1263	0.821692
(0.2,0.7)	0.034512	-0.0829	-0.11018	0.791219
(0.2,0.8)	0.026222	-0.11868	-0.08286	0.682475
(0.2,0.9)	0.014354	0.04497	-0.04494	0.418698
(0.3,0.1)	0.018851	0.15656	-0.02558	0.487404
(0.3,0.2)	0.034507	0.11019	-0.04749	0.790791
(0.3,0.3)	0.045526	0.06364	-0.06362	0.914283
(0.3,0.4)	0.05189	0.02068	-0.0733	0.948653
(0.3,0.5)	0.053958	-0.02065	-0.0765	0.95459
(0.3,0.6)	0.051893	-0.06363	-0.0733	0.948894
(0.3,0.7)	0.04553	-0.11022	-0.06362	0.914484
(0.3,0.8)	0.034508	-0.1566	-0.04751	0.790515
(0.3,0.9)	0.018848	0.02561	-0.02562	0.486616
(0.4,0.1)	0.021409	0.17847	-0.00822	0.50672
(0.4,0.2)	0.039256	0.12632	-0.01533	0.821249
(0.4,0.3)	0.051888	0.07332	-0.02064	0.948688
(0.4,0.4)	0.05922	0.02388	-0.02385	0.983898
(0.4,0.5)	0.061608	-0.02385	-0.02493	0.989854
(0.4,0.6)	0.059223	-0.07331	-0.02385	0.983955
(0.4,0.7)	0.051892	-0.12633	-0.02066	0.948708
(0.4,0.8)	0.039259	-0.17849	-0.01537	0.821101
(0.4,0.9)	0.02141	0.00821	-0.00826	0.506422
(0.5,0.1)	0.022231	0.18558	0.00827	0.509792
(0.5,0.2)	0.040789	0.13163	0.01538	0.826191
(0.5,0.3)	0.053952	0.07653	0.02067	0.954389
(0.5,0.4)	0.061605	0.02496	0.02388	0.989799
(0.5,0.5)	0.064101	-0.02493	0.02496	0.995765
(0.5,0.6)	0.061608	-0.0765	0.02388	0.989853
(0.5,0.7)	0.053958	-0.13162	0.02067	0.95459
(0.5,0.8)	0.040796	-0.1856	0.01534	0.826623
(0.5,0.9)	0.022236	-0.00832	0.00822	0.510243
(0.6,0.1)	0.021404	0.17847	0.0256	0.506034
(0.6,0.2)	0.039251	0.12634	0.04751	0.820631
(0.6,0.3)	0.051885	0.07332	0.06363	0.948446
(0.6,0.4)	0.059217	0.02388	0.07331	0.983842

(0.6,0.5)	0.061605	-0.02385	0.07653	0.989799
(0.6,0.6)	0.05922	-0.07329	0.07333	0.983898
(0.6,0.7)	0.051891	-0.12629	0.06365	0.948837
(0.6,0.8)	0.039262	-0.17848	0.04752	0.821665
(0.6,0.9)	0.021414	-0.0257	0.02559	0.507259
(0.7,0.1)	0.018844	0.15656	0.04491	0.486416
(0.7,0.2)	0.0345	0.11022	0.08283	0.78999
(0.7,0.3)	0.045522	0.06364	0.11017	0.914081
(0.7,0.4)	0.051886	0.02066	0.12631	0.948631
(0.7,0.5)	0.053952	-0.02065	0.13163	0.954389
(0.7,0.6)	0.051887	-0.06361	0.12635	0.948502
(0.7,0.7)	0.045526	-0.11016	0.11024	0.914281
(0.7,0.8)	0.03451	-0.15655	0.08291	0.791122
(0.7,0.9)	0.018855	-0.04502	0.04496	0.488101
(0.8,0.1)	0.014353	0.11864	0.06476	0.418972
(0.8,0.2)	0.026217	0.08288	0.1186	0.682065
(0.8,0.3)	0.034505	0.0475	0.15654	0.790694
(0.8,0.4)	0.039255	0.01534	0.17846	0.821221
(0.8,0.5)	0.040789	-0.01537	0.18558	0.826191
(0.8,0.6)	0.039252	-0.0475	0.17848	0.820657
(0.8,0.7)	0.034502	-0.08283	0.15658	0.790084
(0.8,0.8)	0.026219	-0.1186	0.11867	0.682268
(0.8,0.9)	0.014359	-0.06482	0.06483	0.420227
(0.9,0.1)	0.007877	0.0648	0	0.257511
(0.9,0.2)	0.014357	0.04494	0	0.420011
(0.9,0.3)	0.018851	0.02558	0	0.487652
(0.9,0.4)	0.021409	0.00822	0	0.506793
(0.9,0.5)	0.022231	-0.00827	0	0.50979
(0.9,0.6)	0.021404	-0.0256	0	0.505957
(0.9,0.7)	0.018844	-0.04492	0	0.486167
(0.9,0.8)	0.014352	-0.06476	0	0.418483
(0.9,0.9)	0.007876	0	0	0.257028

Result of 2dimension vortex method using r=0.1 & n=100 [eqaution(3.6.19) on page 43]

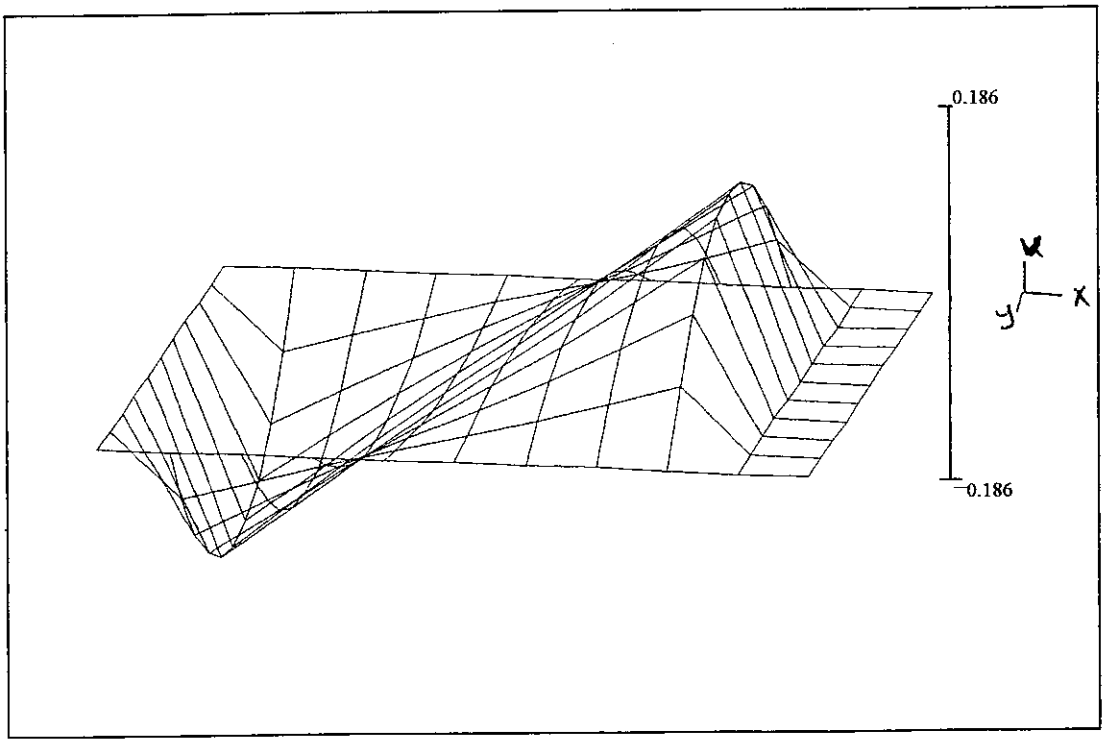
(x,y)	stream function	u	v	votex function
(0.1,0.1)	0.001099	0	-0.0099	0.021099
(0.1,0.2)	0.00209	0.00786	-0.01884	0.040132
(0.1,0.3)	0.002876	0.00506	-0.02595	0.055238
(0.1,0.4)	0.003382	0.00174	-0.03049	0.064942
(0.1,0.5)	0.003556	-0.00174	-0.03207	0.068296
(0.1,0.6)	0.003382	-0.00505	-0.03051	0.064968
(0.1,0.7)	0.002877	-0.00786	-0.02596	0.055279
(0.1,0.8)	0.002091	-0.00992	-0.01885	0.040168
(0.1,0.9)	0.001099	0.0099	-0.00992	0.021112
(0.2,0.1)	0.002089	0.01885	-0.00787	0.04012
(0.2,0.2)	0.003974	0.01497	-0.01496	0.076314
(0.2,0.3)	0.005471	0.0096	-0.02058	0.10504
(0.2,0.4)	0.006431	0.00332	-0.0242	0.123494
(0.2,0.5)	0.006763	-0.0033	-0.02544	0.129873
(0.2,0.6)	0.006433	-0.0096	-0.0242	0.123548
(0.2,0.7)	0.005473	-0.01497	-0.02059	0.105125
(0.2,0.8)	0.003976	-0.01885	-0.01496	0.076391
(0.2,0.9)	0.002091	0.00785	-0.00786	0.040153
(0.3,0.1)	0.002876	0.02594	-0.00504	0.0552
(0.3,0.2)	0.00547	0.02059	-0.00959	0.104997
(0.3,0.3)	0.007529	0.01322	-0.0132	0.144521
(0.3,0.4)	0.008851	0.00456	-0.01553	0.169911
(0.3,0.5)	0.009307	-0.00454	-0.01633	0.17869
(0.3,0.6)	0.008853	-0.01321	-0.01553	0.169992
(0.3,0.7)	0.007532	-0.0206	-0.01321	0.144648
(0.3,0.8)	0.005472	-0.02595	-0.00961	0.105117
(0.3,0.9)	0.002877	0.00503	-0.00505	0.05526
(0.4,0.1)	0.00338	0.03049	-0.00174	0.064868
(0.4,0.2)	0.006429	0.0242	-0.0033	0.123385
(0.4,0.3)	0.008849	0.01555	-0.00455	0.169829
(0.4,0.4)	0.010404	0.00536	-0.00534	0.199666
(0.4,0.5)	0.01094	-0.00534	-0.00562	0.209984
(0.4,0.6)	0.010406	-0.01553	-0.00534	0.199766
(0.4,0.7)	0.008853	-0.0242	-0.00454	0.16999
(0.4,0.8)	0.006433	-0.03051	-0.0033	0.123541
(0.4,0.9)	0.003382	0.00172	-0.00174	0.064955
(0.5,0.1)	0.003554	0.03205	0.00175	0.068187
(0.5,0.2)	0.006759	0.02545	0.00331	0.129697
(0.5,0.3)	0.009304	0.01634	0.00456	0.178515
(0.5,0.4)	0.010938	0.00564	0.00536	0.209876
(0.5,0.5)	0.011502	-0.00562	0.00564	0.220721
(0.5,0.6)	0.01094	-0.01633	0.00536	0.209984
(0.5,0.7)	0.009307	-0.02544	0.00456	0.178689
(0.5,0.8)	0.006763	-0.03207	0.00331	0.129872
(0.5,0.9)	0.003556	-0.00177	0.00174	0.068295
(0.6,0.1)	0.003379	0.03049	0.00504	0.064842
(0.6,0.2)	0.006428	0.0242	0.0096	0.123331
(0.6,0.3)	0.008848	0.01554	0.01322	0.169748
(0.6,0.4)	0.010402	0.00536	0.01554	0.199565

(0.6,0.5)	0.010938	-0.00534	0.01634	0.209875
(0.6,0.6)	0.010404	-0.01553	0.01555	0.199665
(0.6,0.7)	0.008851	-0.02419	0.01322	0.169912
(0.6,0.8)	0.006432	-0.0305	0.00961	0.123499
(0.6,0.9)	0.003382	-0.00507	0.00506	0.064955
(0.7,0.1)	0.002875	0.02593	0.00786	0.055159
(0.7,0.2)	0.005468	0.02058	0.01495	0.104912
(0.7,0.3)	0.007526	0.01322	0.02058	0.144393
(0.7,0.4)	0.008848	0.00456	0.0242	0.16975
(0.7,0.5)	0.009304	-0.00455	0.02545	0.178514
(0.7,0.6)	0.008849	-0.0132	0.0242	0.169826
(0.7,0.7)	0.007529	-0.02058	0.02059	0.14452
(0.7,0.8)	0.005471	-0.02595	0.01497	0.105046
(0.7,0.9)	0.002876	-0.00787	0.00786	0.055257
(0.8,0.1)	0.002089	0.01884	0.00991	0.040084
(0.8,0.2)	0.003973	0.01495	0.01884	0.076236
(0.8,0.3)	0.005468	0.0096	0.02593	0.104919
(0.8,0.4)	0.006428	0.00331	0.03049	0.123337
(0.8,0.5)	0.006759	-0.0033	0.03205	0.129697
(0.8,0.6)	0.006429	-0.00959	0.03049	0.123378
(0.8,0.7)	0.00547	-0.01496	0.02595	0.104989
(0.8,0.8)	0.003974	-0.01884	0.01885	0.076313
(0.8,0.9)	0.00209	-0.00992	0.00991	0.040147
(0.9,0.1)	0.001098	0.00991	0	0.021085
(0.9,0.2)	0.002089	0.00786	0	0.040099
(0.9,0.3)	0.002875	0.00504	0	0.055179
(0.9,0.4)	0.003379	0.00175	0	0.064854
(0.9,0.5)	0.003554	-0.00174	0	0.068187
(0.9,0.6)	0.00338	-0.00505	0	0.064854
(0.9,0.7)	0.002875	-0.00786	0	0.05518
(0.9,0.8)	0.002089	-0.0099	0	0.040105
(0.9,0.9)	0.001099	0	0	0.021098



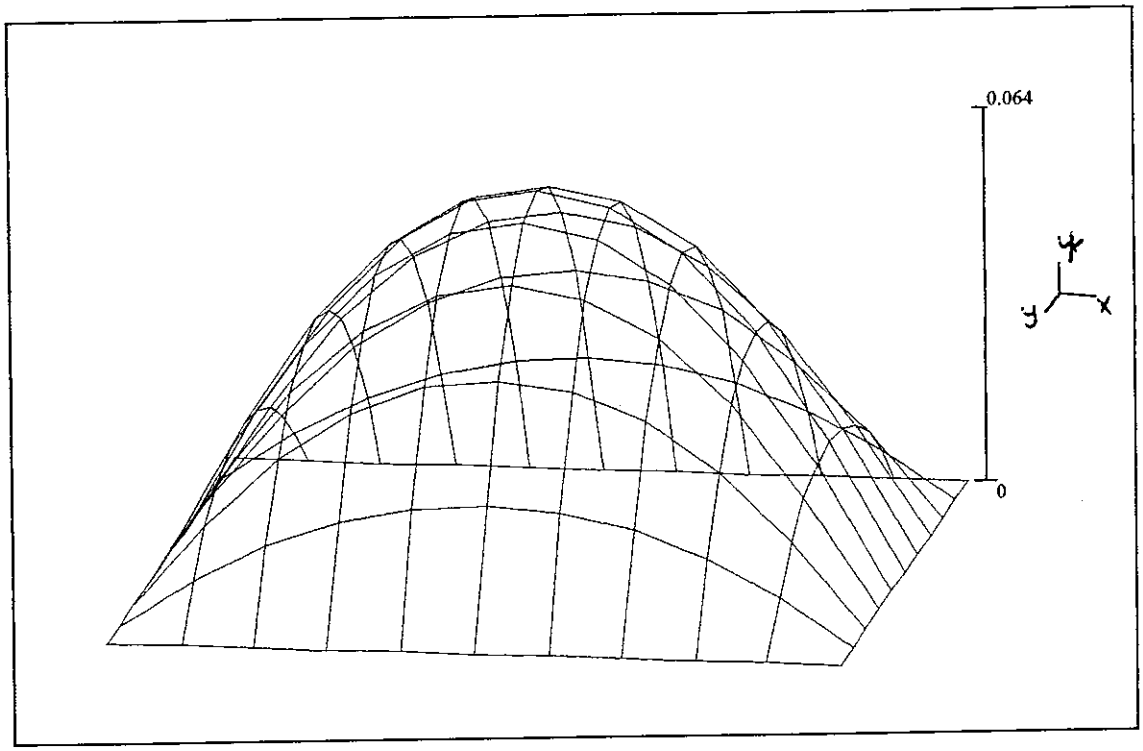
f

u at n=10



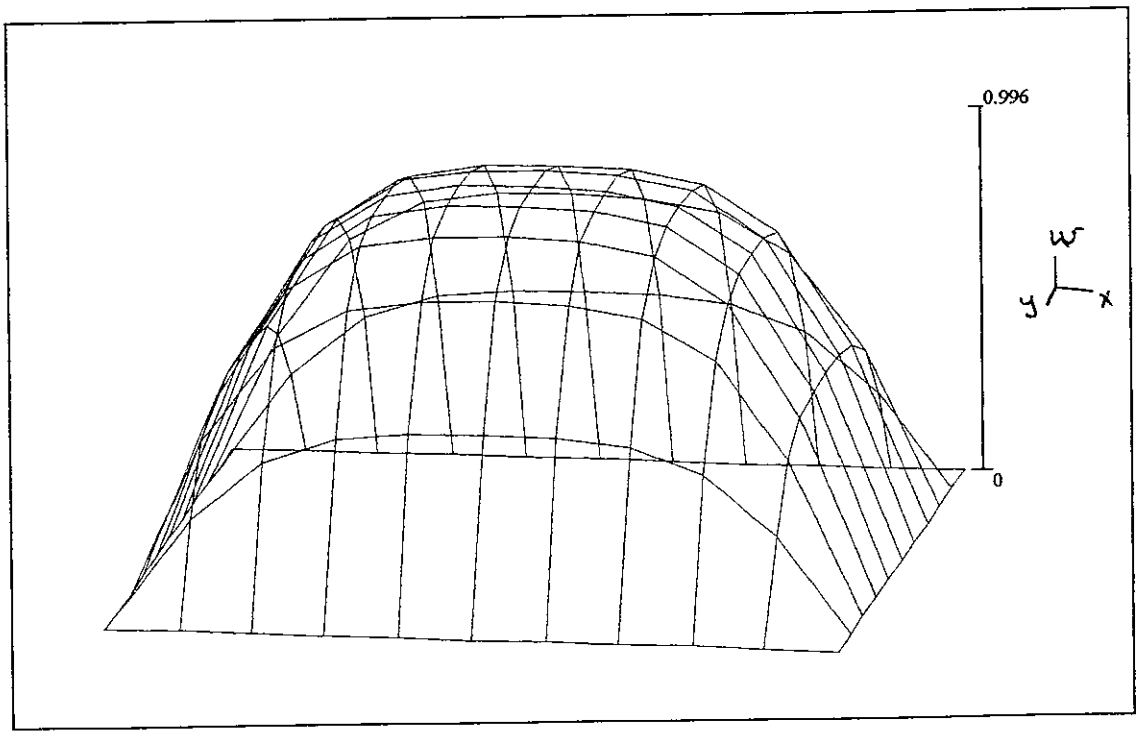
f2

v at n=10



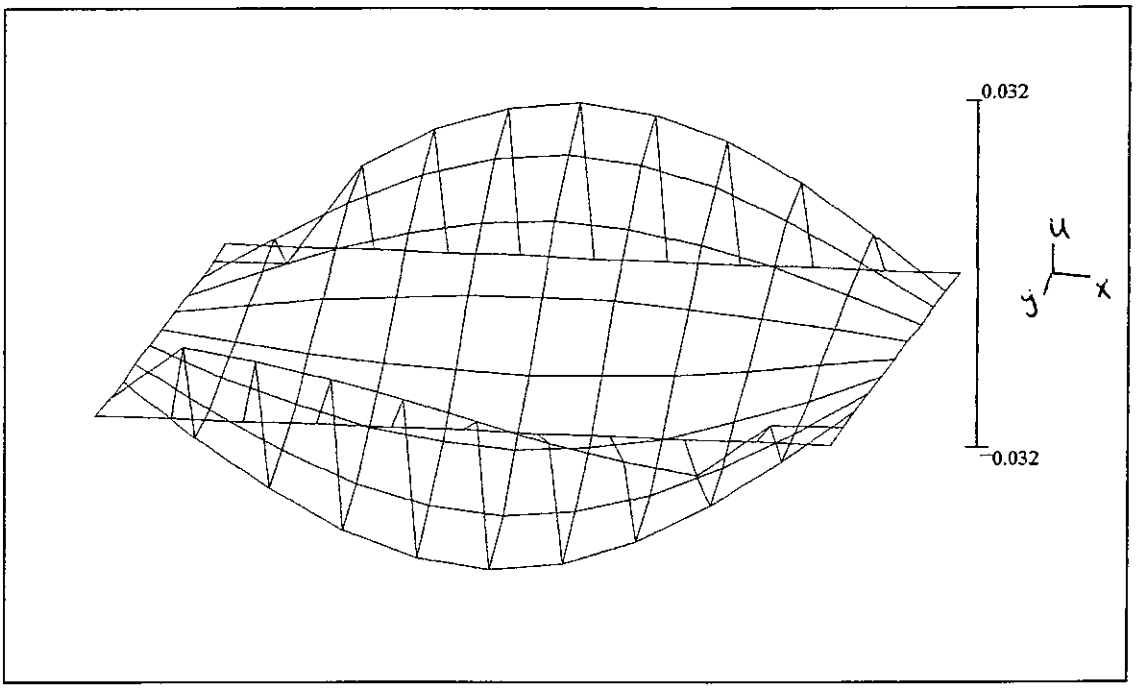
f2c

stream function at n=10



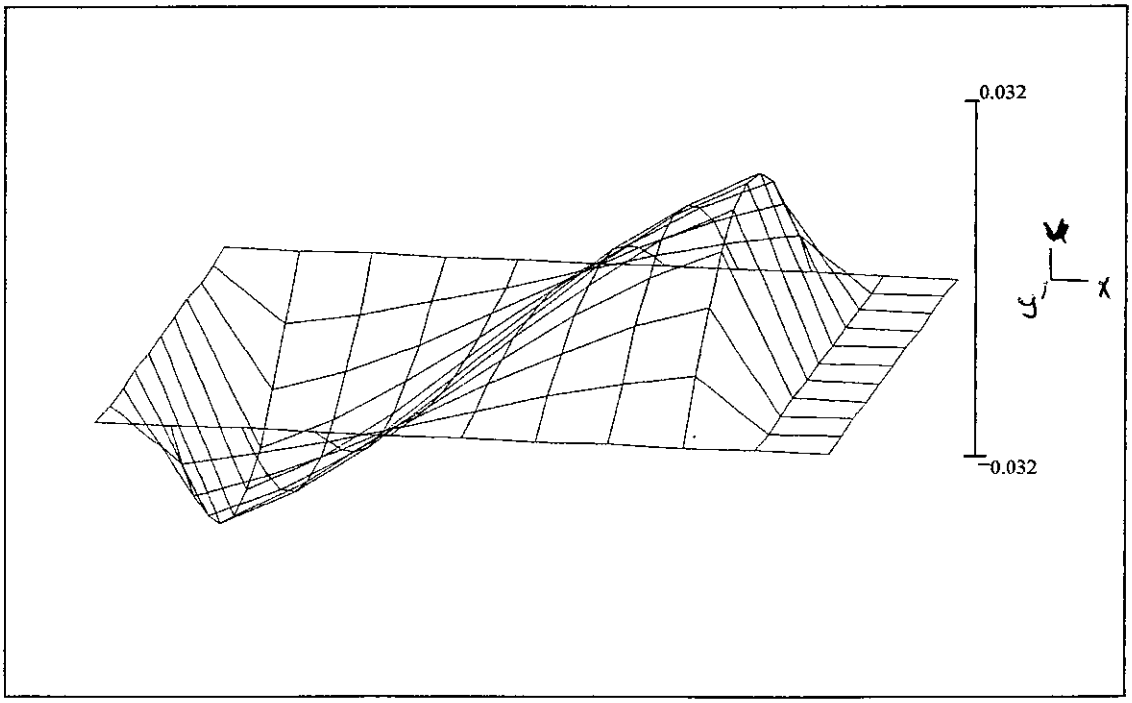
f2a

vortex function at n=10



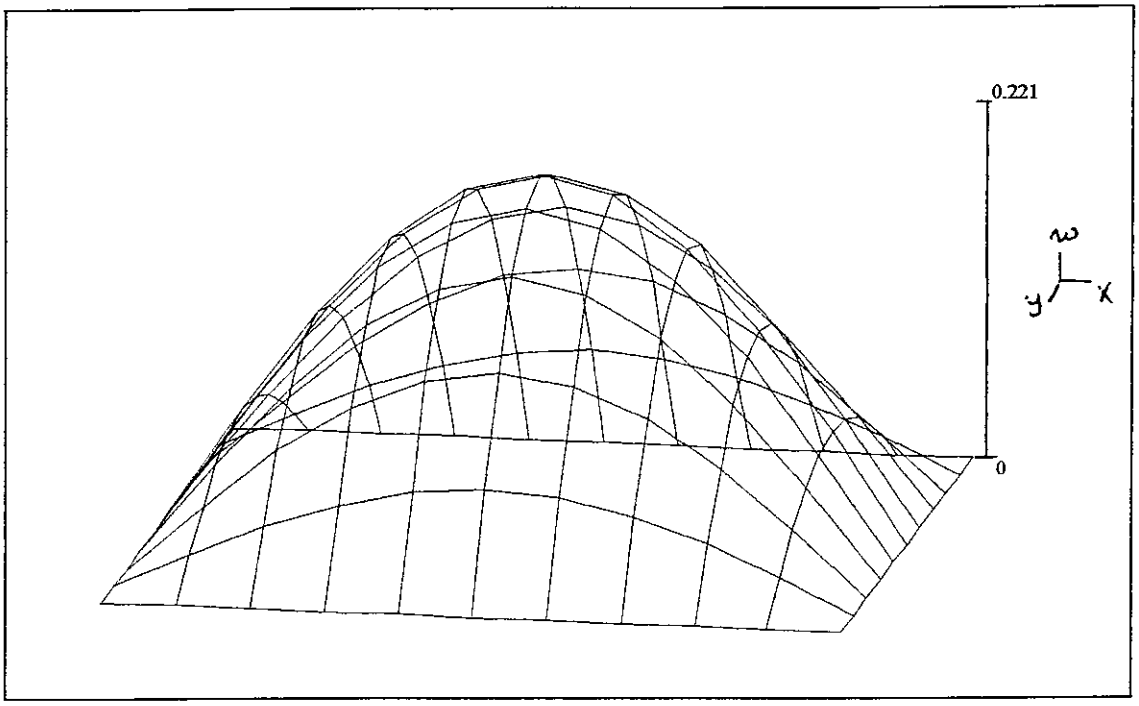
f

u at n=100



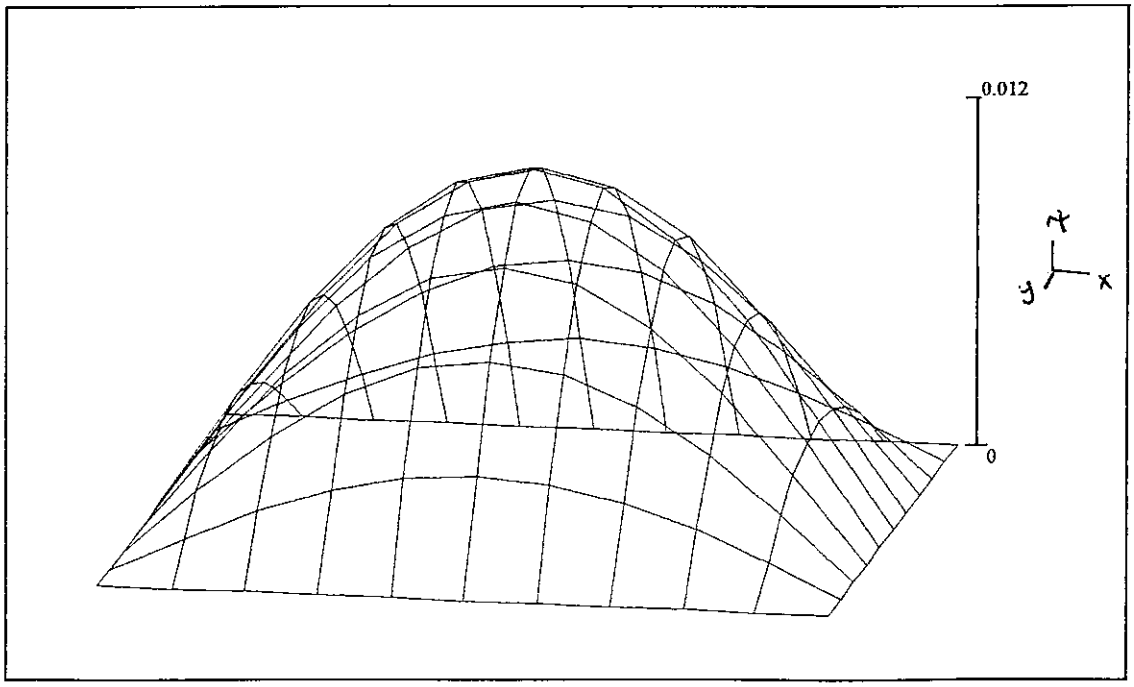
f2

v at n=100



fb

vortex function at n=100



fc

stream function at n=100

Result of 3dimension Explicit method using r=0.1

(x,y,z) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.84	0.578	0.394	0.266	0.176	0.113	0.069	0.038	0.016	0.001
(0.1,0.1,0.2)	1	0.89	0.697	0.535	0.404	0.299	0.217	0.153	0.104	0.066	0.038
(0.1,0.1,0.3)	1	0.89	0.702	0.551	0.429	0.33	0.251	0.187	0.136	0.095	0.063
(0.1,0.1,0.4)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.195	0.145	0.105	0.073
(0.1,0.1,0.5)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.1,0.6)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.1,0.7)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.1,0.8)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.1,0.9)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.2,0.1)	1	0.89	0.697	0.535	0.404	0.299	0.217	0.153	0.104	0.066	0.038
(0.1,0.2,0.2)	1	0.94	0.826	0.707	0.592	0.485	0.389	0.305	0.234	0.174	0.125
(0.1,0.2,0.3)	1	0.94	0.831	0.724	0.622	0.527	0.441	0.362	0.293	0.232	0.179
(0.1,0.2,0.4)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.373	0.307	0.248	0.197
(0.1,0.2,0.5)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.251	0.201
(0.1,0.2,0.6)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.2,0.7)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.2,0.8)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.2,0.9)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.3,0.1)	1	0.89	0.702	0.551	0.429	0.33	0.251	0.187	0.136	0.095	0.063
(0.1,0.3,0.2)	1	0.94	0.831	0.724	0.622	0.527	0.441	0.362	0.293	0.232	0.179
(0.1,0.3,0.3)	1	0.94	0.836	0.741	0.653	0.572	0.495	0.425	0.359	0.3	0.246
(0.1,0.3,0.4)	1	0.94	0.836	0.741	0.655	0.576	0.504	0.437	0.375	0.319	0.268
(0.1,0.3,0.5)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.3,0.6)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.3,0.7)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.3,0.8)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.3,0.9)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.4,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.195	0.145	0.105	0.073
(0.1,0.4,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.373	0.307	0.248	0.197
(0.1,0.4,0.3)	1	0.94	0.836	0.741	0.655	0.576	0.504	0.437	0.375	0.319	0.268
(0.1,0.4,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.449	0.391	0.339	0.291
(0.1,0.4,0.5)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.342	0.295
(0.1,0.4,0.6)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.4,0.7)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.4,0.8)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.4,0.9)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.5,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.5,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.251	0.201
(0.1,0.5,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.5,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.342	0.295
(0.1,0.5,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.451	0.396	0.346	0.3
(0.1,0.5,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.5,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.5,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.5,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.6,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.6,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.6,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.6,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296

(0.1,0.6,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.6,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.6,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.6,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.6,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.7,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.7,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.7,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.7,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.7,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.7,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.7,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.7,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.7,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.8,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.8,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.8,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.8,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.8,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.8,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.8,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.8,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.8,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.9,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.1,0.9,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.1,0.9,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.1,0.9,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.1,0.9,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.1,0.9,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.9,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.9,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.1,0.9,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.2,0.1,0.1)	1	0.89	0.697	0.535	0.404	0.299	0.217	0.153	0.104	0.066	0.038
(0.2,0.1,0.2)	1	0.94	0.826	0.707	0.592	0.485	0.389	0.305	0.234	0.174	0.125
(0.2,0.1,0.3)	1	0.94	0.831	0.724	0.622	0.527	0.441	0.362	0.293	0.232	0.179
(0.2,0.1,0.4)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.373	0.307	0.248	0.197
(0.2,0.1,0.5)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.251	0.201
(0.2,0.1,0.6)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.1,0.7)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.1,0.8)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.1,0.9)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.2,0.1)	1	0.94	0.826	0.707	0.592	0.485	0.389	0.305	0.234	0.174	0.125
(0.2,0.2,0.2)	1	0.99	0.965	0.913	0.841	0.756	0.664	0.572	0.482	0.397	0.32
(0.2,0.2,0.3)	1	0.99	0.97	0.931	0.878	0.813	0.74	0.662	0.583	0.505	0.429
(0.2,0.2,0.4)	1	0.99	0.97	0.932	0.88	0.818	0.75	0.679	0.606	0.534	0.463
(0.2,0.2,0.5)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.538	0.47
(0.2,0.2,0.6)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.2,0.2,0.7)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.2,0.2,0.8)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.2,0.2,0.9)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.2,0.3,0.1)	1	0.94	0.831	0.724	0.622	0.527	0.441	0.362	0.293	0.232	0.179
(0.2,0.3,0.2)	1	0.99	0.97	0.931	0.878	0.813	0.74	0.662	0.583	0.505	0.429
(0.2,0.3,0.3)	1	0.99	0.975	0.95	0.915	0.871	0.819	0.761	0.697	0.63	0.562
(0.2,0.3,0.4)	1	0.99	0.975	0.95	0.917	0.877	0.83	0.778	0.722	0.663	0.602
(0.2,0.3,0.5)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.725	0.668	0.61

(0.2,0.3,0.6)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.2,0.3,0.7)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.2,0.3,0.8)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.2,0.3,0.9)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.2,0.4,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.373	0.307	0.248	0.197
(0.2,0.4,0.2)	1	0.99	0.97	0.932	0.88	0.818	0.75	0.679	0.606	0.534	0.463
(0.2,0.4,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.83	0.778	0.722	0.663	0.602
(0.2,0.4,0.4)	1	0.99	0.975	0.951	0.919	0.882	0.841	0.796	0.747	0.696	0.644
(0.2,0.4,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.797	0.751	0.702	0.652
(0.2,0.4,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.2,0.4,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.2,0.4,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.2,0.4,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.2,0.5,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.251	0.201
(0.2,0.5,0.2)	1	1	0.977	0.937	0.883	0.821	0.753	0.682	0.61	0.539	0.471
(0.2,0.5,0.3)	1	1	0.983	0.956	0.922	0.881	0.834	0.782	0.727	0.67	0.611
(0.2,0.5,0.4)	1	1	0.983	0.957	0.925	0.887	0.845	0.8	0.753	0.703	0.653
(0.2,0.5,0.5)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.709	0.661
(0.2,0.5,0.6)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.2,0.5,0.7)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.2,0.5,0.8)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.2,0.5,0.9)	1	0.99	0.976	0.952	0.921	0.885	0.844	0.801	0.756	0.709	0.662
(0.2,0.6,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.6,0.2)	1	0.99	0.971	0.933	0.881	0.82	0.752	0.682	0.61	0.54	0.472
(0.2,0.6,0.3)	1	0.99	0.976	0.952	0.919	0.879	0.833	0.782	0.727	0.67	0.612
(0.2,0.6,0.4)	1	0.99	0.976	0.952	0.921	0.885	0.844	0.8	0.753	0.704	0.654
(0.2,0.6,0.5)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.801	0.756	0.71	0.662
(0.2,0.6,0.6)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.802	0.756	0.71	0.663
(0.2,0.6,0.7)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.802	0.756	0.71	0.664
(0.2,0.6,0.8)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.802	0.756	0.71	0.664
(0.2,0.6,0.9)	1	0.99	0.975	0.951	0.92	0.884	0.844	0.801	0.756	0.71	0.663
(0.2,0.7,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.7,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.752	0.681	0.61	0.539	0.472
(0.2,0.7,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.781	0.726	0.67	0.612
(0.2,0.7,0.4)	1	0.99	0.975	0.951	0.92	0.883	0.842	0.798	0.752	0.703	0.654
(0.2,0.7,0.5)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.662
(0.2,0.7,0.6)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.663
(0.2,0.7,0.7)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.663
(0.2,0.7,0.8)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.663
(0.2,0.7,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.663
(0.2,0.8,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.8,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.472
(0.2,0.8,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.2,0.8,0.4)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.703	0.653
(0.2,0.8,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.754	0.708	0.661
(0.2,0.8,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.2,0.8,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.2,0.8,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.2,0.8,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.2,0.9,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.2,0.9,0.2)	1	1	0.977	0.937	0.883	0.821	0.753	0.682	0.61	0.54	0.472
(0.2,0.9,0.3)	1	1	0.983	0.956	0.922	0.881	0.834	0.782	0.727	0.67	0.612
(0.2,0.9,0.4)	1	1	0.983	0.957	0.925	0.887	0.845	0.8	0.753	0.704	0.654
(0.2,0.9,0.5)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.2,0.9,0.6)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.757	0.71	0.663

(0.2,0.9,0.7)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.757	0.71	0.664
(0.2,0.9,0.8)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.757	0.71	0.664
(0.2,0.9,0.9)	1	0.99	0.976	0.952	0.921	0.885	0.844	0.801	0.756	0.71	0.663
(0.3,0.1,0.1)	1	0.89	0.702	0.551	0.429	0.33	0.251	0.187	0.136	0.095	0.063
(0.3,0.1,0.2)	1	0.94	0.831	0.724	0.622	0.527	0.441	0.362	0.293	0.232	0.179
(0.3,0.1,0.3)	1	0.94	0.836	0.741	0.653	0.572	0.495	0.425	0.359	0.3	0.246
(0.3,0.1,0.4)	1	0.94	0.836	0.741	0.655	0.576	0.504	0.437	0.375	0.319	0.268
(0.3,0.1,0.5)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.1,0.6)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.1,0.7)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.1,0.8)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.1,0.9)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.2,0.1)	1	0.94	0.831	0.724	0.622	0.527	0.441	0.362	0.293	0.232	0.179
(0.3,0.2,0.2)	1	0.99	0.97	0.931	0.878	0.813	0.74	0.662	0.583	0.505	0.429
(0.3,0.2,0.3)	1	0.99	0.975	0.95	0.915	0.871	0.819	0.761	0.697	0.63	0.562
(0.3,0.2,0.4)	1	0.99	0.975	0.95	0.917	0.877	0.83	0.778	0.722	0.663	0.602
(0.3,0.2,0.5)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.725	0.668	0.61
(0.3,0.2,0.6)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.3,0.2,0.7)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.3,0.2,0.8)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.3,0.2,0.9)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.3,0.3,0.1)	1	0.94	0.836	0.741	0.653	0.572	0.495	0.425	0.359	0.3	0.246
(0.3,0.3,0.2)	1	0.99	0.975	0.95	0.915	0.871	0.819	0.761	0.697	0.63	0.562
(0.3,0.3,0.3)	1	0.99	0.98	0.968	0.952	0.931	0.903	0.867	0.825	0.777	0.723
(0.3,0.3,0.4)	1	0.99	0.98	0.968	0.954	0.937	0.914	0.886	0.852	0.813	0.769
(0.3,0.3,0.5)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.819	0.778
(0.3,0.3,0.6)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.3,0.3,0.7)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.3,0.3,0.8)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.3,0.3,0.9)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.3,0.4,0.1)	1	0.94	0.836	0.741	0.655	0.576	0.504	0.437	0.375	0.319	0.268
(0.3,0.4,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.83	0.778	0.722	0.663	0.602
(0.3,0.4,0.3)	1	0.99	0.98	0.968	0.954	0.937	0.914	0.886	0.852	0.813	0.769
(0.3,0.4,0.4)	1	0.99	0.98	0.969	0.957	0.942	0.925	0.904	0.879	0.851	0.818
(0.3,0.4,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.856	0.826
(0.3,0.4,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.3,0.4,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.3,0.4,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.3,0.4,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.3,0.5,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.5,0.2)	1	1	0.983	0.956	0.922	0.881	0.834	0.782	0.727	0.67	0.611
(0.3,0.5,0.3)	1	1	0.989	0.976	0.961	0.943	0.92	0.892	0.859	0.822	0.78
(0.3,0.5,0.4)	1	1	0.989	0.977	0.964	0.949	0.931	0.911	0.887	0.86	0.829
(0.3,0.5,0.5)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.89	0.866	0.838
(0.3,0.5,0.6)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.839
(0.3,0.5,0.7)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.84
(0.3,0.5,0.8)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.84
(0.3,0.5,0.9)	1	0.99	0.981	0.971	0.959	0.945	0.929	0.91	0.889	0.865	0.839
(0.3,0.6,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.6,0.2)	1	0.99	0.976	0.952	0.919	0.879	0.833	0.782	0.727	0.67	0.612
(0.3,0.6,0.3)	1	0.99	0.981	0.97	0.957	0.94	0.918	0.891	0.859	0.822	0.782
(0.3,0.6,0.4)	1	0.99	0.981	0.971	0.959	0.946	0.929	0.909	0.886	0.86	0.83
(0.3,0.6,0.5)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.866	0.84
(0.3,0.6,0.6)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.867	0.841
(0.3,0.6,0.7)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.867	0.841

(0.3,0.6,0.8)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.867	0.841
(0.3,0.6,0.9)	1	0.99	0.98	0.969	0.957	0.944	0.928	0.91	0.889	0.865	0.84
(0.3,0.7,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.7,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.781	0.726	0.67	0.612
(0.3,0.7,0.3)	1	0.99	0.98	0.969	0.955	0.937	0.915	0.889	0.857	0.821	0.781
(0.3,0.7,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.907	0.884	0.858	0.829
(0.3,0.7,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.864	0.838
(0.3,0.7,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.865	0.839
(0.3,0.7,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.865	0.839
(0.3,0.7,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.865	0.839
(0.3,0.7,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.888	0.864	0.839
(0.3,0.8,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.8,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.3,0.8,0.3)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.78
(0.3,0.8,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.3,0.8,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.3,0.8,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.3,0.8,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.3,0.8,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.3,0.8,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.3,0.9,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.3,0.9,0.2)	1	1	0.983	0.956	0.922	0.881	0.834	0.782	0.727	0.67	0.612
(0.3,0.9,0.3)	1	1	0.989	0.976	0.961	0.943	0.92	0.892	0.859	0.822	0.782
(0.3,0.9,0.4)	1	1	0.989	0.977	0.964	0.949	0.931	0.911	0.887	0.86	0.83
(0.3,0.9,0.5)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.84
(0.3,0.9,0.6)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.867	0.841
(0.3,0.9,0.7)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.867	0.841
(0.3,0.9,0.8)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.867	0.841
(0.3,0.9,0.9)	1	0.99	0.981	0.971	0.959	0.945	0.929	0.911	0.889	0.866	0.84
(0.4,0.1,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.195	0.145	0.105	0.073
(0.4,0.1,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.373	0.307	0.248	0.197
(0.4,0.1,0.3)	1	0.94	0.836	0.741	0.655	0.576	0.504	0.437	0.375	0.319	0.268
(0.4,0.1,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.449	0.391	0.339	0.291
(0.4,0.1,0.5)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.342	0.295
(0.4,0.1,0.6)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.1,0.7)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.1,0.8)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.1,0.9)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.2,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.373	0.307	0.248	0.197
(0.4,0.2,0.2)	1	0.99	0.97	0.932	0.88	0.818	0.75	0.679	0.606	0.534	0.463
(0.4,0.2,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.83	0.778	0.722	0.663	0.602
(0.4,0.2,0.4)	1	0.99	0.975	0.951	0.919	0.882	0.841	0.796	0.747	0.696	0.644
(0.4,0.2,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.797	0.751	0.702	0.652
(0.4,0.2,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.4,0.2,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.4,0.2,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.4,0.2,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.4,0.3,0.1)	1	0.94	0.836	0.741	0.655	0.576	0.504	0.437	0.375	0.319	0.268
(0.4,0.3,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.83	0.778	0.722	0.663	0.602
(0.4,0.3,0.3)	1	0.99	0.98	0.968	0.954	0.937	0.914	0.886	0.852	0.813	0.769
(0.4,0.3,0.4)	1	0.99	0.98	0.969	0.957	0.942	0.925	0.904	0.879	0.851	0.818
(0.4,0.3,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.856	0.826
(0.4,0.3,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.4,0.3,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.4,0.3,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828

(0.4,0.3,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.4,0.4,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.449	0.391	0.339	0.291
(0.4,0.4,0.2)	1	0.99	0.975	0.951	0.919	0.882	0.841	0.796	0.747	0.696	0.644
(0.4,0.4,0.3)	1	0.99	0.98	0.969	0.957	0.942	0.925	0.904	0.879	0.851	0.818
(0.4,0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.936	0.923	0.907	0.888	0.867
(0.4,0.4,0.5)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.91	0.894	0.876
(0.4,0.4,0.6)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.4,0.4,0.7)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.4,0.4,0.8)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.4,0.4,0.9)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.4,0.5,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.342	0.295
(0.4,0.5,0.2)	1	1	0.983	0.957	0.925	0.887	0.845	0.8	0.753	0.703	0.653
(0.4,0.5,0.3)	1	1	0.989	0.977	0.964	0.949	0.931	0.911	0.887	0.86	0.829
(0.4,0.5,0.4)	1	1	0.989	0.978	0.967	0.955	0.943	0.93	0.915	0.898	0.879
(0.4,0.5,0.5)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.904	0.889
(0.4,0.5,0.6)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.4,0.5,0.7)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.4,0.5,0.8)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.4,0.5,0.9)	1	0.99	0.981	0.971	0.961	0.951	0.941	0.929	0.917	0.904	0.889
(0.4,0.6,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.6,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.844	0.8	0.753	0.704	0.654
(0.4,0.6,0.3)	1	0.99	0.981	0.971	0.959	0.946	0.929	0.909	0.886	0.86	0.83
(0.4,0.6,0.4)	1	0.99	0.981	0.971	0.962	0.951	0.94	0.928	0.914	0.899	0.881
(0.4,0.6,0.5)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.89
(0.4,0.6,0.6)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.891
(0.4,0.6,0.7)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.891
(0.4,0.6,0.8)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.891
(0.4,0.6,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.939	0.928	0.916	0.904	0.89
(0.4,0.7,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.7,0.2)	1	0.99	0.975	0.951	0.92	0.883	0.842	0.798	0.752	0.703	0.654
(0.4,0.7,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.907	0.884	0.858	0.829
(0.4,0.7,0.4)	1	0.99	0.98	0.97	0.959	0.949	0.938	0.926	0.912	0.896	0.879
(0.4,0.7,0.5)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.902	0.888
(0.4,0.7,0.6)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.903	0.889
(0.4,0.7,0.7)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.903	0.889
(0.4,0.7,0.8)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.903	0.889
(0.4,0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.889
(0.4,0.8,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.8,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.703	0.653
(0.4,0.8,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.4,0.8,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.878
(0.4,0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.901	0.887
(0.4,0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.4,0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.4,0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.4,0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.4,0.9,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.4,0.9,0.2)	1	1	0.983	0.957	0.925	0.887	0.845	0.8	0.753	0.704	0.654
(0.4,0.9,0.3)	1	1	0.989	0.977	0.964	0.949	0.931	0.911	0.887	0.86	0.83
(0.4,0.9,0.4)	1	1	0.989	0.978	0.967	0.955	0.943	0.93	0.915	0.899	0.881
(0.4,0.9,0.5)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.4,0.9,0.6)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.906	0.891
(0.4,0.9,0.7)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.906	0.891
(0.4,0.9,0.8)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.906	0.891
(0.4,0.9,0.9)	1	0.99	0.981	0.971	0.961	0.951	0.941	0.929	0.917	0.904	0.89

(0.5,0.1,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.5,0.1,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.251	0.201
(0.5,0.1,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.5,0.1,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.342	0.295
(0.5,0.1,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.451	0.396	0.346	0.3
(0.5,0.1,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.1,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.1,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.1,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.2,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.251	0.201
(0.5,0.2,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.538	0.47
(0.5,0.2,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.725	0.668	0.61
(0.5,0.2,0.4)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.797	0.751	0.702	0.652
(0.5,0.2,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.707	0.66
(0.5,0.2,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.5,0.2,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.5,0.2,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.5,0.2,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.5,0.3,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.5,0.3,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.725	0.668	0.61
(0.5,0.3,0.3)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.819	0.778
(0.5,0.3,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.856	0.826
(0.5,0.3,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.886	0.862	0.835
(0.5,0.3,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.5,0.3,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.5,0.3,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.5,0.3,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.5,0.4,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.342	0.295
(0.5,0.4,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.797	0.751	0.702	0.652
(0.5,0.4,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.856	0.826
(0.5,0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.91	0.894	0.876
(0.5,0.4,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.926	0.914	0.9	0.885
(0.5,0.4,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.5,0.4,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.5,0.4,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.5,0.4,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.5,0.5,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.451	0.396	0.346	0.3
(0.5,0.5,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.709	0.661
(0.5,0.5,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.89	0.866	0.838
(0.5,0.5,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.904	0.889
(0.5,0.5,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.922	0.911	0.898
(0.5,0.5,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.5,0.5,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.5,0.5,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.5,0.5,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.898
(0.5,0.6,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.6,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.801	0.756	0.71	0.662
(0.5,0.6,0.3)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.866	0.84
(0.5,0.6,0.4)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.89
(0.5,0.6,0.5)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.899
(0.5,0.6,0.6)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.9
(0.5,0.6,0.7)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.901
(0.5,0.6,0.8)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.901
(0.5,0.6,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.899
(0.5,0.7,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301

(0.5,0.7,0.2)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.662
(0.5,0.7,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.864	0.838
(0.5,0.7,0.4)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.902	0.888
(0.5,0.7,0.5)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.929	0.919	0.908	0.897
(0.5,0.7,0.6)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.929	0.919	0.909	0.898
(0.5,0.7,0.7)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.929	0.919	0.909	0.898
(0.5,0.7,0.8)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.929	0.919	0.909	0.898
(0.5,0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.5,0.8,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.8,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.754	0.708	0.661
(0.5,0.8,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.5,0.8,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.901	0.887
(0.5,0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.907	0.896
(0.5,0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.5,0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.5,0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.5,0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.5,0.9,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.5,0.9,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.5,0.9,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.84
(0.5,0.9,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.5,0.9,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.5,0.9,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.5,0.9,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.5,0.9,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.5,0.9,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.6,0.1,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.6,0.1,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.6,0.1,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.6,0.1,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.6,0.1,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.6,0.1,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.6,0.1,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.6,0.1,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.6,0.1,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.6,0.2,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.6,0.2,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.6,0.2,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.6,0.2,0.4)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.6,0.2,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.6,0.2,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.6,0.2,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.6,0.2,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.6,0.2,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.6,0.3,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.6,0.3,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.6,0.3,0.3)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.6,0.3,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.6,0.3,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.6,0.3,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.6,0.3,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.6,0.3,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.6,0.3,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.6,0.4,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.6,0.4,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653

(0.6,0.4,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.6,0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.6,0.4,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.6,0.4,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.887
(0.6,0.4,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.6,0.4,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.6,0.4,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.6,0.5,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.6,0.5,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.6,0.5,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.839
(0.6,0.5,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.6,0.5,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.6,0.5,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.9
(0.6,0.5,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.6,0.5,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.6,0.5,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.6,0.6,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.6,0.6,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.802	0.756	0.71	0.663
(0.6,0.6,0.3)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.867	0.841
(0.6,0.6,0.4)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.891
(0.6,0.6,0.5)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.9
(0.6,0.6,0.6)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.6,0.6,0.7)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.6,0.6,0.8)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.6,0.6,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
(0.6,0.7,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.6,0.7,0.2)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.663
(0.6,0.7,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.865	0.839
(0.6,0.7,0.4)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.903	0.889
(0.6,0.7,0.5)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.929	0.919	0.909	0.898
(0.6,0.7,0.6)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.6,0.7,0.7)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.6,0.7,0.8)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.6,0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.6,0.8,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.6,0.8,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.6,0.8,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.6,0.8,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.6,0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.6,0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.6,0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.6,0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.6,0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.6,0.9,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.6,0.9,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.757	0.71	0.663
(0.6,0.9,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.867	0.841
(0.6,0.9,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.906	0.891
(0.6,0.9,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.6,0.9,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.6,0.9,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.6,0.9,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.6,0.9,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.911	0.901
(0.7,0.1,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.7,0.1,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.7,0.1,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273

(0.7,0.1,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.7,0.1,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.7,0.1,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.1,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.1,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.1,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.2,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.7,0.2,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.7,0.2,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.7,0.2,0.4)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.7,0.2,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.7,0.2,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.7,0.2,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.7,0.2,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.7,0.2,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.7,0.3,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.7,0.3,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.7,0.3,0.3)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.7,0.3,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.7,0.3,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.7,0.3,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.7,0.3,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.7,0.3,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.7,0.3,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.7,0.4,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.7,0.4,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.7,0.4,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.7,0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.7,0.4,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.7,0.4,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.7,0.4,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.7,0.4,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.7,0.4,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.7,0.5,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.7,0.5,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.7,0.5,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.84
(0.7,0.5,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.7,0.5,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.7,0.5,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.7,0.5,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.7,0.5,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.7,0.5,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.7,0.6,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.6,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.802	0.756	0.71	0.664
(0.7,0.6,0.3)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.867	0.841
(0.7,0.6,0.4)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.891
(0.7,0.6,0.5)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.901
(0.7,0.6,0.6)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.7,0.6,0.7)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.7,0.6,0.8)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.7,0.6,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
(0.7,0.7,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.7,0.2)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.663
(0.7,0.7,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.865	0.839
(0.7,0.7,0.4)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.903	0.889

(0.7,0.7,0.5)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.929	0.919	0.909	0.898
(0.7,0.7,0.6)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.7,0.7,0.7)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.7,0.7,0.8)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.7,0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.7,0.8,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.8,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.7,0.8,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.7,0.8,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.7,0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.7,0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.7,0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.898
(0.7,0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.898
(0.7,0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.7,0.9,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.7,0.9,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.757	0.71	0.664
(0.7,0.9,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.867	0.841
(0.7,0.9,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.906	0.891
(0.7,0.9,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.7,0.9,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.7,0.9,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.7,0.9,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.7,0.9,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.911	0.901
(0.8,0.1,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.8,0.1,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.8,0.1,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.8,0.1,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.8,0.1,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.8,0.1,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.1,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.1,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.1,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.2,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.8,0.2,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.8,0.2,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.8,0.2,0.4)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.8,0.2,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.8,0.2,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.8,0.2,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.8,0.2,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.8,0.2,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.8,0.3,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.8,0.3,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.8,0.3,0.3)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.8,0.3,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.8,0.3,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.8,0.3,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.8,0.3,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.8,0.3,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.8,0.3,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.8,0.4,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.8,0.4,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.8,0.4,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.8,0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.8,0.4,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886

(0.8,0.4,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.8,0.4,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.8,0.4,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.8,0.4,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.8,0.5,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.8,0.5,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.756	0.71	0.662
(0.8,0.5,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.866	0.84
(0.8,0.5,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.905	0.89
(0.8,0.5,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.911	0.899
(0.8,0.5,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.8,0.5,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.8,0.5,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.8,0.5,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.8,0.6,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.6,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.845	0.802	0.756	0.71	0.664
(0.8,0.6,0.3)	1	0.99	0.981	0.971	0.959	0.946	0.93	0.911	0.89	0.867	0.841
(0.8,0.6,0.4)	1	0.99	0.981	0.971	0.962	0.952	0.941	0.93	0.918	0.905	0.891
(0.8,0.6,0.5)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.911	0.901
(0.8,0.6,0.6)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.8,0.6,0.7)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.8,0.6,0.8)	1	0.99	0.981	0.971	0.962	0.952	0.942	0.932	0.922	0.912	0.902
(0.8,0.6,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
(0.8,0.7,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.7,0.2)	1	0.99	0.975	0.951	0.92	0.883	0.843	0.8	0.755	0.709	0.663
(0.8,0.7,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.909	0.888	0.865	0.839
(0.8,0.7,0.4)	1	0.99	0.98	0.97	0.959	0.949	0.939	0.928	0.916	0.903	0.889
(0.8,0.7,0.5)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.929	0.919	0.909	0.898
(0.8,0.7,0.6)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.8,0.7,0.7)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.8,0.7,0.8)	1	0.99	0.98	0.97	0.959	0.949	0.94	0.93	0.92	0.91	0.9
(0.8,0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.8,0.8,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.8,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.8,0.8,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.8,0.8,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.8,0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.8,0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.8,0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.898
(0.8,0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.898
(0.8,0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.8,0.9,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.8,0.9,0.2)	1	1	0.983	0.957	0.925	0.887	0.846	0.802	0.757	0.71	0.664
(0.8,0.9,0.3)	1	1	0.989	0.977	0.964	0.949	0.932	0.913	0.891	0.867	0.841
(0.8,0.9,0.4)	1	1	0.989	0.978	0.967	0.955	0.944	0.932	0.919	0.906	0.891
(0.8,0.9,0.5)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.912	0.901
(0.8,0.9,0.6)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.8,0.9,0.7)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.8,0.9,0.8)	1	1	0.989	0.978	0.967	0.956	0.945	0.934	0.923	0.913	0.902
(0.8,0.9,0.9)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.911	0.901
(0.9,0.1,0.1)	1	0.89	0.702	0.551	0.431	0.334	0.257	0.196	0.146	0.107	0.075
(0.9,0.1,0.2)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.9,0.1,0.3)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.9,0.1,0.4)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.9,0.1,0.5)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.9,0.1,0.6)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302

(0.9,0.1,0.7)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.1,0.8)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.1,0.9)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.2,0.1)	1	0.94	0.831	0.724	0.624	0.532	0.449	0.375	0.309	0.252	0.202
(0.9,0.2,0.2)	1	0.99	0.97	0.932	0.88	0.819	0.751	0.681	0.609	0.539	0.471
(0.9,0.2,0.3)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.9,0.2,0.4)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.9,0.2,0.5)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.799	0.754	0.708	0.661
(0.9,0.2,0.6)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.708	0.662
(0.9,0.2,0.7)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.9,0.2,0.8)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.9,0.2,0.9)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.9,0.3,0.1)	1	0.94	0.836	0.741	0.655	0.577	0.504	0.438	0.378	0.323	0.273
(0.9,0.3,0.2)	1	0.99	0.975	0.95	0.917	0.877	0.831	0.78	0.726	0.669	0.611
(0.9,0.3,0.3)	1	0.99	0.98	0.968	0.955	0.937	0.915	0.888	0.856	0.82	0.779
(0.9,0.3,0.4)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.9,0.3,0.5)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.863	0.837
(0.9,0.3,0.6)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.9,0.3,0.7)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.9,0.3,0.8)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.9,0.3,0.9)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.9,0.4,0.1)	1	0.94	0.836	0.742	0.657	0.581	0.512	0.45	0.394	0.343	0.296
(0.9,0.4,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.842	0.798	0.751	0.702	0.653
(0.9,0.4,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.926	0.906	0.883	0.857	0.828
(0.9,0.4,0.4)	1	0.99	0.98	0.969	0.959	0.948	0.937	0.925	0.911	0.895	0.877
(0.9,0.4,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.914	0.901	0.886
(0.9,0.4,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.9,0.4,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.9,0.4,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.9,0.4,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.9,0.5,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.301
(0.9,0.5,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.844	0.801	0.756	0.709	0.662
(0.9,0.5,0.3)	1	0.99	0.981	0.971	0.959	0.945	0.929	0.91	0.889	0.865	0.839
(0.9,0.5,0.4)	1	0.99	0.981	0.971	0.961	0.951	0.941	0.929	0.917	0.904	0.889
(0.9,0.5,0.5)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.898
(0.9,0.5,0.6)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.9,0.5,0.7)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.9,0.5,0.8)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.9,0.5,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.909	0.898
(0.9,0.6,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.6,0.2)	1	0.99	0.975	0.951	0.92	0.884	0.844	0.801	0.756	0.71	0.663
(0.9,0.6,0.3)	1	0.99	0.98	0.969	0.957	0.944	0.928	0.91	0.889	0.865	0.84
(0.9,0.6,0.4)	1	0.99	0.98	0.97	0.96	0.95	0.939	0.928	0.916	0.904	0.89
(0.9,0.6,0.5)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.899
(0.9,0.6,0.6)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
(0.9,0.6,0.7)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
(0.9,0.6,0.8)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9
(0.9,0.6,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.9,0.7,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.7,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.663
(0.9,0.7,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.888	0.864	0.839
(0.9,0.7,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.889
(0.9,0.7,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.908	0.898
(0.9,0.7,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.9,0.7,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899

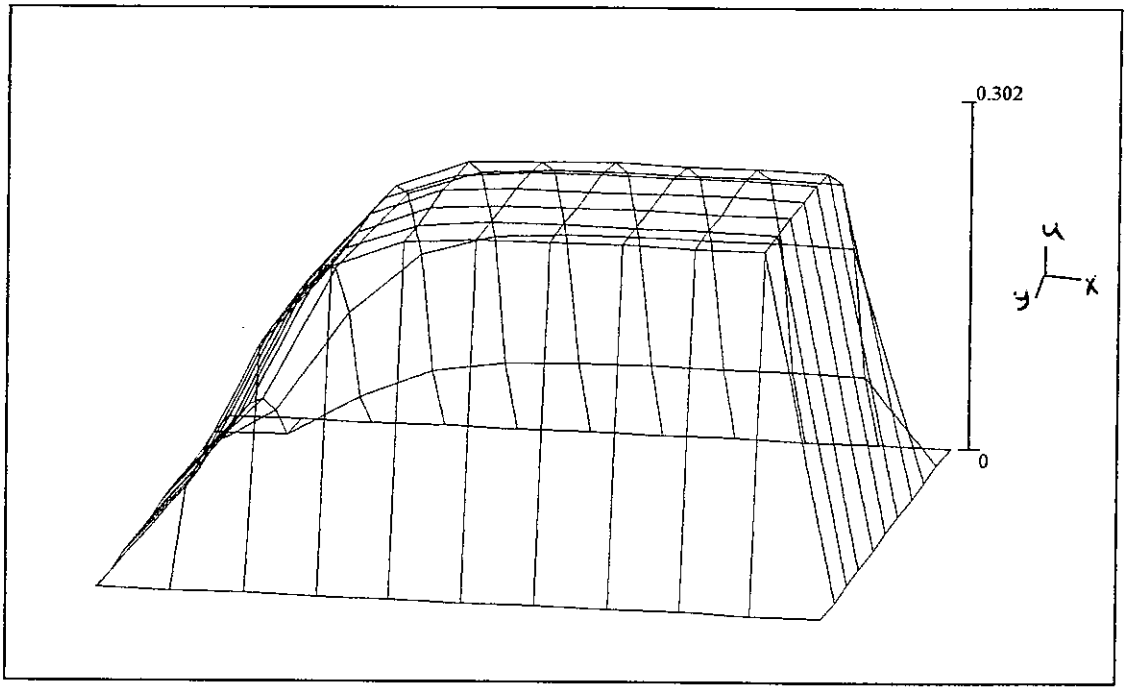
(0.9,0.7,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.9,0.7,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.919	0.909	0.899
(0.9,0.8,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.8,0.2)	1	0.99	0.975	0.951	0.919	0.883	0.843	0.8	0.755	0.709	0.662
(0.9,0.8,0.3)	1	0.99	0.98	0.969	0.957	0.943	0.927	0.908	0.887	0.864	0.838
(0.9,0.8,0.4)	1	0.99	0.98	0.969	0.959	0.949	0.938	0.927	0.915	0.902	0.888
(0.9,0.8,0.5)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.897
(0.9,0.8,0.6)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.8,0.7)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.8,0.8)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.8,0.9)	1	0.99	0.98	0.969	0.959	0.949	0.939	0.929	0.918	0.908	0.898
(0.9,0.9,0.1)	1	0.94	0.836	0.742	0.657	0.582	0.513	0.452	0.396	0.346	0.302
(0.9,0.9,0.2)	1	0.99	0.976	0.952	0.921	0.885	0.844	0.801	0.756	0.71	0.663
(0.9,0.9,0.3)	1	0.99	0.981	0.971	0.959	0.945	0.929	0.911	0.889	0.866	0.84
(0.9,0.9,0.4)	1	0.99	0.981	0.971	0.961	0.951	0.941	0.929	0.917	0.904	0.89
(0.9,0.9,0.5)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.91	0.899
(0.9,0.9,0.6)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.911	0.901
(0.9,0.9,0.7)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.911	0.901
(0.9,0.9,0.8)	1	0.99	0.981	0.971	0.961	0.952	0.941	0.931	0.921	0.911	0.901
(0.9,0.9,0.9)	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.9

Result of 3dimension Explicit method using r=0.01

(x,y,z) n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.984	0.953	0.924	0.895	0.867	0.84	0.814	0.789	0.764	0.74
(0.1,0.1,0.2)	1	0.989	0.968	0.948	0.927	0.908	0.888	0.869	0.85	0.831	0.813
(0.1,0.1,0.3)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.1,0.4)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.1,0.5)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.1,0.6)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.1,0.7)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.1,0.8)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.1,0.9)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.2,0.1)	1	0.989	0.968	0.948	0.927	0.908	0.888	0.869	0.85	0.831	0.813
(0.1,0.2,0.2)	1	0.994	0.983	0.972	0.961	0.949	0.938	0.927	0.915	0.904	0.892
(0.1,0.2,0.3)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.2,0.4)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.2,0.5)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.2,0.6)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.2,0.7)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.2,0.8)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.2,0.9)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.3,0.1)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.3,0.2)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.3,0.3)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.3,0.4)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.3,0.5)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.3,0.6)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.3,0.7)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.3,0.8)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.3,0.9)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.4,0.1)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.4,0.2)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.4,0.3)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.4,0.4)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.4,0.5)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.4,0.6)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.4,0.7)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.4,0.8)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.4,0.9)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.5,0.1)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.5,0.2)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.5,0.3)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.5,0.4)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.5,0.5)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.5,0.6)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.5,0.7)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.5,0.8)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.5,0.9)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899
(0.1,0.6,0.1)	1	0.989	0.968	0.948	0.928	0.908	0.889	0.87	0.852	0.834	0.816
(0.1,0.6,0.2)	1	0.994	0.983	0.972	0.961	0.95	0.939	0.928	0.917	0.906	0.896
(0.1,0.6,0.3)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.919	0.909	0.899
(0.1,0.6,0.4)	1	0.994	0.983	0.972	0.961	0.951	0.94	0.93	0.92	0.909	0.899

Result of 3dimension Explicit method using r=0.001

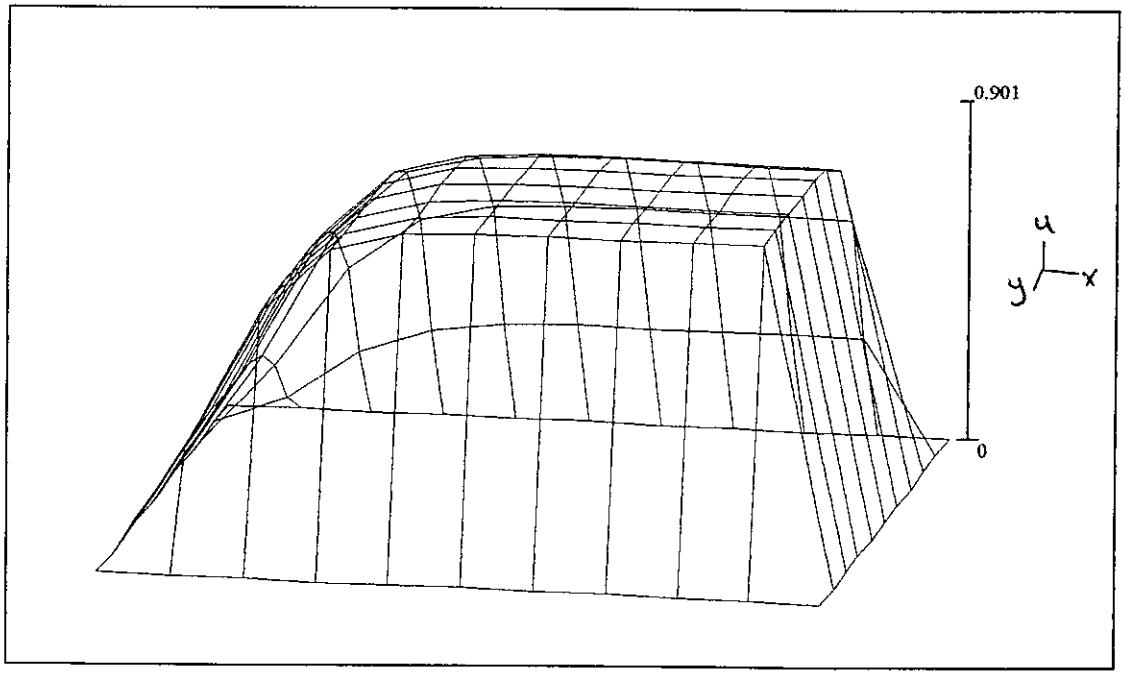
(x,y,z) n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.998	0.995	0.992	0.989	0.986	0.983	0.98	0.977	0.974	0.971
(0.1,0.1,0.2)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.3)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.4)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.5)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.6)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.7)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.8)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.1,0.9)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.2,0.1)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.2,0.2)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.2,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.3,0.1)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.3,0.2)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.3,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.3,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.3,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.3,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.3,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.3,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.3,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.1)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.4,0.2)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.4,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.4,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.1)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.5,0.2)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.5,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.5)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.6)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.7)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.8)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.5,0.9)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.6,0.1)	1	0.999	0.997	0.995	0.993	0.991	0.988	0.986	0.984	0.982	0.98
(0.1,0.6,0.2)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.989
(0.1,0.6,0.3)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99
(0.1,0.6,0.4)	1	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991	0.99



f

3dimension explicit method

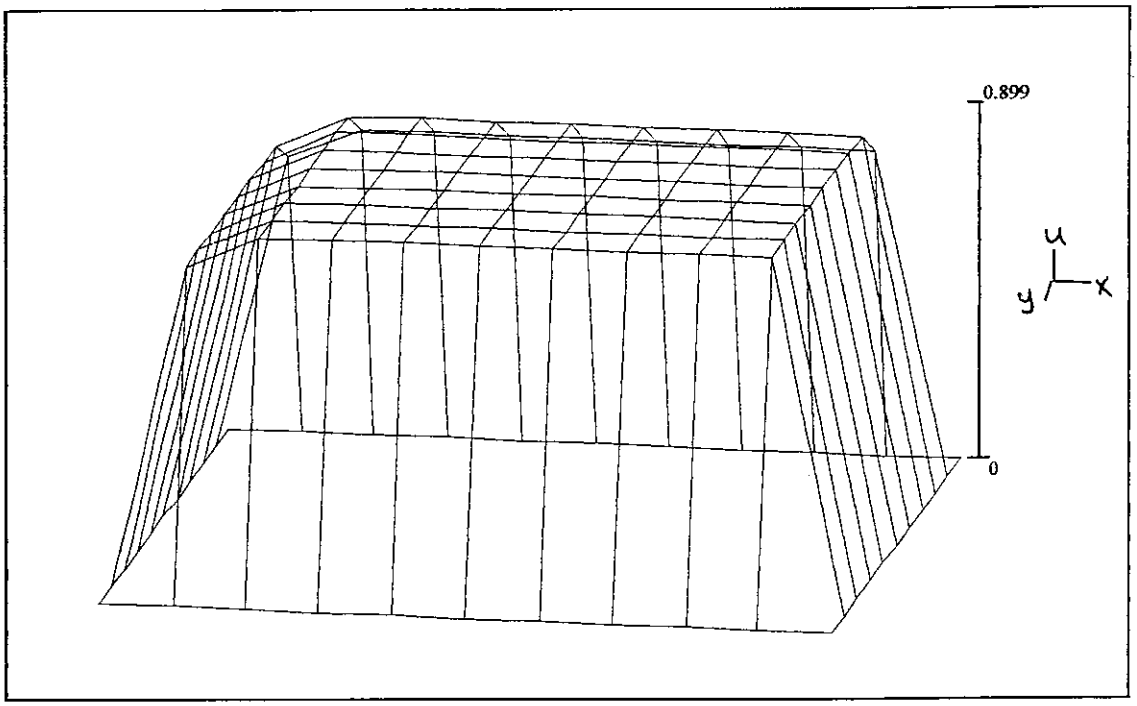
x=0.1 and y & z are varible and r=0.1 at n=10



f2

3dimension explicit method

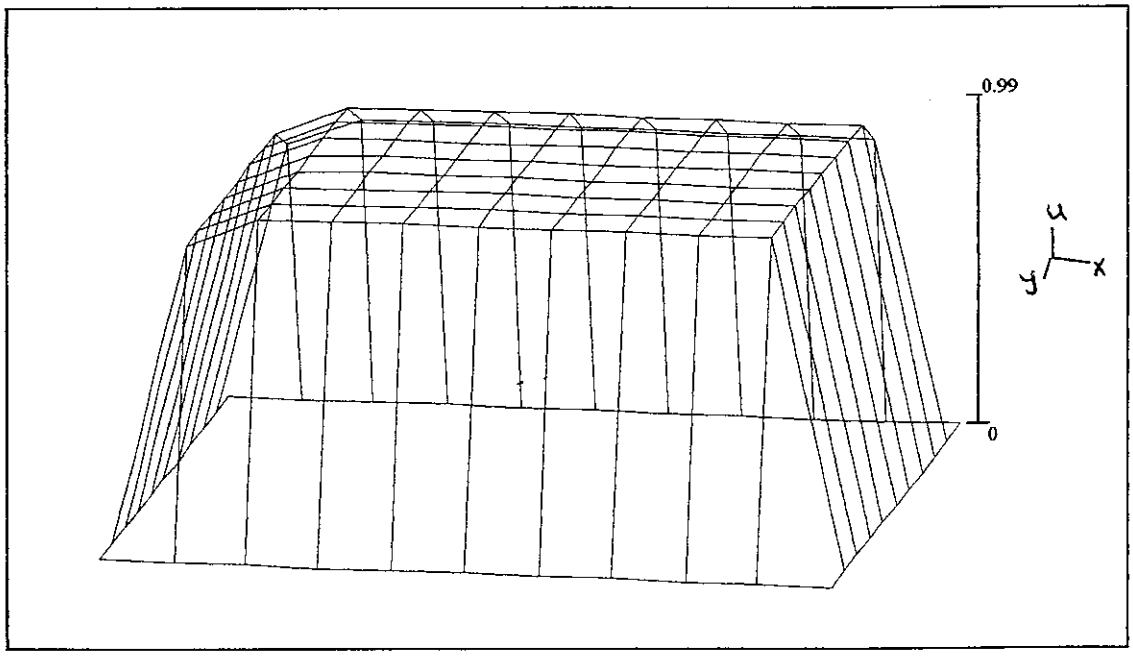
x=0.9 and y & z are varible and r=0.1 at n=10



f

3dimensoin explicit method

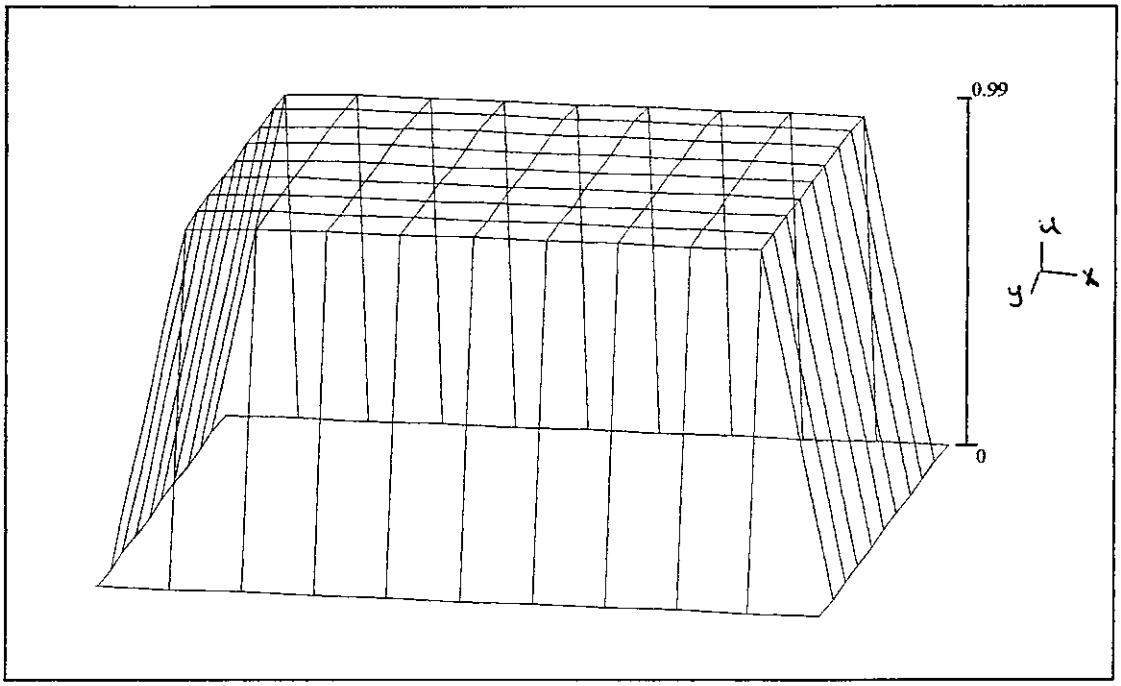
x=0.1 y & z are varible and r=0.01 at n=10



f2

3dimension explicit method

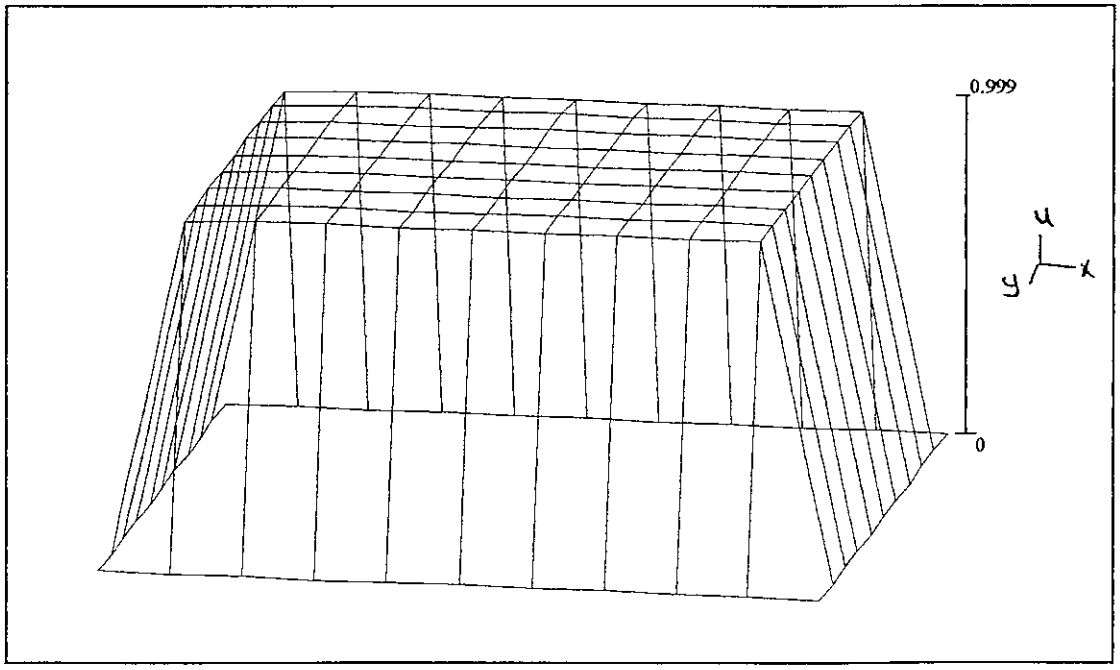
x=0.9 y & z are varible and r=0.01 at n=10



f

3dimension explicit method

x=0.1 and y & z are varible and r=0.001 at n=10



f2

3dimension explicit method

x=0.9 and y & z are varible and r=0.001 at n=10

Result of 3dimension Implicit method using r=0.1

(x,y,z) n	1	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.633	0.408	0.266	0.176	0.117	0.077	0.05	0.032	0.018	0.009
(0.1,0.1,0.2)	1	0.71	0.506	0.362	0.26	0.186	0.134	0.095	0.067	0.045	0.029
(0.1,0.1,0.3)	1	0.72	0.524	0.386	0.286	0.213	0.158	0.118	0.087	0.063	0.045
(0.1,0.1,0.4)	1	0.721	0.527	0.391	0.292	0.22	0.167	0.126	0.095	0.071	0.052
(0.1,0.1,0.5)	1	0.721	0.528	0.392	0.294	0.222	0.169	0.129	0.099	0.075	0.056
(0.1,0.1,0.6)	1	0.721	0.528	0.392	0.294	0.223	0.17	0.13	0.099	0.075	0.057
(0.1,0.1,0.7)	1	0.721	0.528	0.391	0.293	0.222	0.169	0.129	0.098	0.074	0.056
(0.1,0.1,0.8)	1	0.718	0.523	0.384	0.286	0.214	0.161	0.122	0.092	0.069	0.051
(0.1,0.1,0.9)	1	0.677	0.468	0.329	0.235	0.17	0.124	0.09	0.066	0.047	0.033
(0.1,0.2,0.1)	1	0.71	0.506	0.362	0.26	0.186	0.134	0.095	0.067	0.045	0.029
(0.1,0.2,0.2)	1	0.805	0.638	0.5	0.389	0.301	0.231	0.176	0.133	0.099	0.073
(0.1,0.2,0.3)	1	0.818	0.664	0.536	0.431	0.345	0.275	0.219	0.172	0.134	0.104
(0.1,0.2,0.4)	1	0.82	0.669	0.545	0.443	0.359	0.291	0.235	0.189	0.151	0.12
(0.1,0.2,0.5)	1	0.82	0.67	0.546	0.445	0.363	0.296	0.241	0.196	0.158	0.127
(0.1,0.2,0.6)	1	0.82	0.67	0.546	0.446	0.364	0.297	0.242	0.197	0.16	0.129
(0.1,0.2,0.7)	1	0.82	0.669	0.545	0.444	0.362	0.295	0.24	0.195	0.158	0.127
(0.1,0.2,0.8)	1	0.816	0.662	0.535	0.432	0.348	0.281	0.226	0.182	0.146	0.116
(0.1,0.2,0.9)	1	0.765	0.588	0.454	0.353	0.275	0.216	0.169	0.132	0.103	0.08
(0.1,0.3,0.1)	1	0.72	0.524	0.386	0.286	0.213	0.158	0.118	0.087	0.063	0.045
(0.1,0.3,0.2)	1	0.818	0.664	0.536	0.431	0.345	0.275	0.219	0.172	0.134	0.104
(0.1,0.3,0.3)	1	0.831	0.693	0.577	0.48	0.399	0.33	0.272	0.222	0.181	0.146
(0.1,0.3,0.4)	1	0.833	0.698	0.587	0.494	0.415	0.349	0.293	0.245	0.204	0.168
(0.1,0.3,0.5)	1	0.833	0.699	0.588	0.497	0.42	0.355	0.3	0.253	0.213	0.178
(0.1,0.3,0.6)	1	0.833	0.699	0.589	0.497	0.421	0.357	0.302	0.255	0.216	0.181
(0.1,0.3,0.7)	1	0.833	0.698	0.587	0.496	0.419	0.354	0.299	0.253	0.213	0.179
(0.1,0.3,0.8)	1	0.83	0.69	0.576	0.481	0.403	0.337	0.282	0.236	0.197	0.164
(0.1,0.3,0.9)	1	0.777	0.612	0.487	0.392	0.317	0.258	0.211	0.172	0.14	0.114
(0.1,0.4,0.1)	1	0.721	0.527	0.391	0.292	0.22	0.167	0.126	0.095	0.071	0.052
(0.1,0.4,0.2)	1	0.82	0.669	0.545	0.443	0.359	0.291	0.235	0.189	0.151	0.12
(0.1,0.4,0.3)	1	0.833	0.698	0.587	0.494	0.415	0.349	0.293	0.245	0.204	0.168
(0.1,0.4,0.4)	1	0.835	0.703	0.596	0.508	0.434	0.371	0.317	0.27	0.23	0.195
(0.1,0.4,0.5)	1	0.835	0.704	0.598	0.511	0.439	0.377	0.325	0.28	0.24	0.206
(0.1,0.4,0.6)	1	0.835	0.704	0.598	0.512	0.439	0.379	0.327	0.282	0.243	0.21
(0.1,0.4,0.7)	1	0.835	0.704	0.597	0.51	0.437	0.376	0.324	0.279	0.24	0.207
(0.1,0.4,0.8)	1	0.831	0.696	0.585	0.495	0.42	0.358	0.305	0.261	0.222	0.19
(0.1,0.4,0.9)	1	0.778	0.616	0.495	0.402	0.33	0.273	0.227	0.19	0.159	0.133
(0.1,0.5,0.1)	1	0.721	0.528	0.392	0.294	0.222	0.169	0.129	0.099	0.075	0.056
(0.1,0.5,0.2)	1	0.82	0.67	0.546	0.445	0.363	0.296	0.241	0.196	0.158	0.127
(0.1,0.5,0.3)	1	0.833	0.699	0.588	0.497	0.42	0.355	0.3	0.253	0.213	0.178
(0.1,0.5,0.4)	1	0.835	0.704	0.598	0.511	0.439	0.377	0.325	0.28	0.24	0.206
(0.1,0.5,0.5)	1	0.836	0.705	0.6	0.515	0.444	0.384	0.333	0.29	0.252	0.218
(0.1,0.5,0.6)	1	0.836	0.705	0.601	0.515	0.445	0.385	0.335	0.292	0.255	0.222
(0.1,0.5,0.7)	1	0.835	0.705	0.599	0.513	0.442	0.383	0.332	0.289	0.252	0.219
(0.1,0.5,0.8)	1	0.832	0.696	0.587	0.498	0.425	0.364	0.313	0.27	0.233	0.201
(0.1,0.5,0.9)	1	0.778	0.617	0.496	0.405	0.334	0.278	0.233	0.197	0.166	0.141
(0.1,0.6,0.1)	1	0.721	0.528	0.392	0.294	0.223	0.17	0.13	0.099	0.075	0.057
(0.1,0.6,0.2)	1	0.82	0.67	0.546	0.446	0.364	0.297	0.242	0.197	0.16	0.129
(0.1,0.6,0.3)	1	0.833	0.699	0.589	0.497	0.421	0.357	0.302	0.255	0.216	0.181
(0.1,0.6,0.4)	1	0.835	0.704	0.598	0.512	0.439	0.379	0.327	0.282	0.243	0.21
(0.1,0.6,0.5)	1	0.836	0.705	0.601	0.515	0.445	0.385	0.335	0.292	0.255	0.222

(0.1,0.6,0.6)	1	0.836	0.705	0.601	0.516	0.446	0.387	0.337	0.295	0.258	0.226
(0.1,0.6,0.7)	1	0.835	0.705	0.6	0.514	0.443	0.384	0.334	0.292	0.255	0.223
(0.1,0.6,0.8)	1	0.832	0.697	0.587	0.499	0.426	0.365	0.315	0.272	0.236	0.205
(0.1,0.6,0.9)	1	0.778	0.617	0.496	0.405	0.335	0.279	0.235	0.198	0.168	0.143
(0.1,0.7,0.1)	1	0.721	0.528	0.391	0.293	0.222	0.169	0.129	0.098	0.074	0.056
(0.1,0.7,0.2)	1	0.82	0.669	0.545	0.444	0.362	0.295	0.24	0.195	0.158	0.127
(0.1,0.7,0.3)	1	0.833	0.698	0.587	0.496	0.419	0.354	0.299	0.253	0.213	0.179
(0.1,0.7,0.4)	1	0.835	0.704	0.597	0.51	0.437	0.376	0.324	0.279	0.24	0.207
(0.1,0.7,0.5)	1	0.835	0.705	0.599	0.513	0.442	0.383	0.332	0.289	0.252	0.219
(0.1,0.7,0.6)	1	0.835	0.705	0.6	0.514	0.443	0.384	0.334	0.292	0.255	0.223
(0.1,0.7,0.7)	1	0.835	0.704	0.598	0.512	0.441	0.381	0.331	0.289	0.252	0.22
(0.1,0.7,0.8)	1	0.831	0.696	0.586	0.497	0.423	0.363	0.312	0.269	0.233	0.202
(0.1,0.7,0.9)	1	0.778	0.616	0.495	0.404	0.333	0.277	0.232	0.196	0.166	0.141
(0.1,0.8,0.1)	1	0.718	0.523	0.384	0.286	0.214	0.161	0.122	0.092	0.069	0.051
(0.1,0.8,0.2)	1	0.816	0.662	0.535	0.432	0.348	0.281	0.226	0.182	0.146	0.116
(0.1,0.8,0.3)	1	0.83	0.69	0.576	0.481	0.403	0.337	0.282	0.236	0.197	0.164
(0.1,0.8,0.4)	1	0.831	0.696	0.585	0.495	0.42	0.358	0.305	0.261	0.222	0.19
(0.1,0.8,0.5)	1	0.832	0.696	0.587	0.498	0.425	0.364	0.313	0.27	0.233	0.201
(0.1,0.8,0.6)	1	0.832	0.697	0.587	0.499	0.426	0.365	0.315	0.272	0.236	0.205
(0.1,0.8,0.7)	1	0.831	0.696	0.586	0.497	0.423	0.363	0.312	0.269	0.233	0.202
(0.1,0.8,0.8)	1	0.828	0.688	0.574	0.482	0.407	0.345	0.294	0.251	0.215	0.185
(0.1,0.8,0.9)	1	0.775	0.61	0.486	0.392	0.32	0.264	0.219	0.183	0.154	0.129
(0.1,0.9,0.1)	1	0.677	0.468	0.329	0.235	0.17	0.124	0.09	0.066	0.047	0.033
(0.1,0.9,0.2)	1	0.765	0.588	0.454	0.353	0.275	0.216	0.169	0.132	0.103	0.08
(0.1,0.9,0.3)	1	0.777	0.612	0.487	0.392	0.317	0.258	0.211	0.172	0.14	0.114
(0.1,0.9,0.4)	1	0.778	0.616	0.495	0.402	0.33	0.273	0.227	0.19	0.159	0.133
(0.1,0.9,0.5)	1	0.778	0.617	0.496	0.405	0.334	0.278	0.233	0.197	0.166	0.141
(0.1,0.9,0.6)	1	0.778	0.617	0.496	0.405	0.335	0.279	0.235	0.198	0.168	0.143
(0.1,0.9,0.7)	1	0.778	0.616	0.495	0.404	0.333	0.277	0.232	0.196	0.166	0.141
(0.1,0.9,0.8)	1	0.775	0.61	0.486	0.392	0.32	0.264	0.219	0.183	0.154	0.129
(0.1,0.9,0.9)	1	0.728	0.542	0.413	0.32	0.253	0.202	0.163	0.133	0.109	0.09
(0.2,0.1,0.1)	1	0.71	0.506	0.362	0.26	0.186	0.134	0.095	0.067	0.045	0.029
(0.2,0.1,0.2)	1	0.805	0.638	0.5	0.389	0.301	0.231	0.176	0.133	0.099	0.073
(0.2,0.1,0.3)	1	0.818	0.664	0.536	0.431	0.345	0.275	0.219	0.172	0.134	0.104
(0.2,0.1,0.4)	1	0.82	0.669	0.545	0.443	0.359	0.291	0.235	0.189	0.151	0.12
(0.2,0.1,0.5)	1	0.82	0.67	0.546	0.445	0.363	0.296	0.241	0.196	0.158	0.127
(0.2,0.1,0.6)	1	0.82	0.67	0.546	0.446	0.364	0.297	0.242	0.197	0.16	0.129
(0.2,0.1,0.7)	1	0.82	0.669	0.545	0.444	0.362	0.295	0.24	0.195	0.158	0.127
(0.2,0.1,0.8)	1	0.816	0.662	0.535	0.432	0.348	0.281	0.226	0.182	0.146	0.116
(0.2,0.1,0.9)	1	0.765	0.588	0.454	0.353	0.275	0.216	0.169	0.132	0.103	0.08
(0.2,0.2,0.1)	1	0.805	0.638	0.5	0.389	0.301	0.231	0.176	0.133	0.099	0.073
(0.2,0.2,0.2)	1	0.924	0.817	0.703	0.592	0.491	0.402	0.326	0.261	0.207	0.162
(0.2,0.2,0.3)	1	0.941	0.856	0.759	0.661	0.568	0.482	0.404	0.336	0.277	0.226
(0.2,0.2,0.4)	1	0.943	0.863	0.772	0.681	0.592	0.511	0.437	0.37	0.312	0.26
(0.2,0.2,0.5)	1	0.944	0.864	0.775	0.685	0.599	0.52	0.447	0.383	0.325	0.275
(0.2,0.2,0.6)	1	0.944	0.864	0.776	0.686	0.6	0.521	0.45	0.386	0.329	0.279
(0.2,0.2,0.7)	1	0.944	0.863	0.774	0.683	0.597	0.518	0.446	0.382	0.325	0.276
(0.2,0.2,0.8)	1	0.939	0.853	0.758	0.663	0.574	0.493	0.42	0.357	0.301	0.253
(0.2,0.2,0.9)	1	0.875	0.751	0.638	0.538	0.451	0.377	0.314	0.261	0.216	0.178
(0.2,0.3,0.1)	1	0.818	0.664	0.536	0.431	0.345	0.275	0.219	0.172	0.134	0.104
(0.2,0.3,0.2)	1	0.941	0.856	0.759	0.661	0.568	0.482	0.404	0.336	0.277	0.226
(0.2,0.3,0.3)	1	0.959	0.898	0.823	0.742	0.66	0.58	0.504	0.434	0.371	0.313
(0.2,0.3,0.4)	1	0.962	0.906	0.838	0.765	0.69	0.617	0.546	0.479	0.417	0.36
(0.2,0.3,0.5)	1	0.962	0.907	0.842	0.771	0.699	0.628	0.56	0.496	0.436	0.381
(0.2,0.3,0.6)	1	0.962	0.907	0.842	0.772	0.701	0.63	0.563	0.5	0.442	0.388

(0.2,0.3,0.7)	1	0.962	0.906	0.84	0.769	0.697	0.626	0.558	0.495	0.436	0.383
(0.2,0.3,0.8)	1	0.957	0.895	0.822	0.745	0.668	0.595	0.526	0.462	0.404	0.352
(0.2,0.3,0.9)	1	0.89	0.786	0.689	0.602	0.524	0.454	0.392	0.338	0.291	0.249
(0.2,0.4,0.1)	1	0.82	0.669	0.545	0.443	0.359	0.291	0.235	0.189	0.151	0.12
(0.2,0.4,0.2)	1	0.943	0.863	0.772	0.681	0.592	0.511	0.437	0.37	0.312	0.26
(0.2,0.4,0.3)	1	0.962	0.906	0.838	0.765	0.69	0.617	0.546	0.479	0.417	0.36
(0.2,0.4,0.4)	1	0.965	0.914	0.854	0.79	0.723	0.656	0.592	0.529	0.47	0.415
(0.2,0.4,0.5)	1	0.965	0.916	0.858	0.796	0.732	0.669	0.608	0.549	0.492	0.439
(0.2,0.4,0.6)	1	0.965	0.916	0.858	0.797	0.734	0.672	0.612	0.554	0.499	0.447
(0.2,0.4,0.7)	1	0.965	0.915	0.856	0.794	0.73	0.667	0.606	0.548	0.493	0.441
(0.2,0.4,0.8)	1	0.96	0.903	0.837	0.768	0.7	0.633	0.57	0.511	0.456	0.406
(0.2,0.4,0.9)	1	0.892	0.792	0.702	0.62	0.547	0.483	0.425	0.374	0.328	0.287
(0.2,0.5,0.1)	1	0.82	0.67	0.546	0.445	0.363	0.296	0.241	0.196	0.158	0.127
(0.2,0.5,0.2)	1	0.944	0.864	0.775	0.685	0.599	0.52	0.447	0.383	0.325	0.275
(0.2,0.5,0.3)	1	0.962	0.907	0.842	0.771	0.699	0.628	0.56	0.496	0.436	0.381
(0.2,0.5,0.4)	1	0.965	0.916	0.858	0.796	0.732	0.669	0.608	0.549	0.492	0.439
(0.2,0.5,0.5)	1	0.966	0.917	0.861	0.802	0.742	0.682	0.625	0.569	0.516	0.465
(0.2,0.5,0.6)	1	0.966	0.917	0.862	0.803	0.744	0.685	0.629	0.574	0.523	0.474
(0.2,0.5,0.7)	1	0.965	0.916	0.86	0.8	0.739	0.68	0.623	0.568	0.516	0.468
(0.2,0.5,0.8)	1	0.96	0.904	0.841	0.774	0.709	0.646	0.586	0.53	0.478	0.43
(0.2,0.5,0.9)	1	0.893	0.794	0.704	0.625	0.554	0.492	0.437	0.387	0.344	0.304
(0.2,0.6,0.1)	1	0.82	0.67	0.546	0.446	0.364	0.297	0.242	0.197	0.16	0.129
(0.2,0.6,0.2)	1	0.944	0.864	0.776	0.686	0.6	0.521	0.45	0.386	0.329	0.279
(0.2,0.6,0.3)	1	0.962	0.907	0.842	0.772	0.701	0.63	0.563	0.5	0.442	0.388
(0.2,0.6,0.4)	1	0.965	0.916	0.858	0.797	0.734	0.672	0.612	0.554	0.499	0.447
(0.2,0.6,0.5)	1	0.966	0.917	0.862	0.803	0.744	0.685	0.629	0.574	0.523	0.474
(0.2,0.6,0.6)	1	0.966	0.918	0.862	0.804	0.746	0.688	0.633	0.58	0.53	0.483
(0.2,0.6,0.7)	1	0.965	0.917	0.86	0.801	0.741	0.683	0.627	0.574	0.523	0.476
(0.2,0.6,0.8)	1	0.96	0.904	0.841	0.775	0.711	0.648	0.59	0.535	0.484	0.438
(0.2,0.6,0.9)	1	0.893	0.794	0.705	0.625	0.555	0.494	0.439	0.391	0.348	0.31
(0.2,0.7,0.1)	1	0.82	0.669	0.545	0.444	0.362	0.295	0.24	0.195	0.158	0.127
(0.2,0.7,0.2)	1	0.944	0.863	0.774	0.683	0.597	0.518	0.446	0.382	0.325	0.276
(0.2,0.7,0.3)	1	0.962	0.906	0.84	0.769	0.697	0.626	0.558	0.495	0.436	0.383
(0.2,0.7,0.4)	1	0.965	0.915	0.856	0.794	0.73	0.667	0.606	0.548	0.493	0.441
(0.2,0.7,0.5)	1	0.965	0.916	0.86	0.8	0.739	0.68	0.623	0.568	0.516	0.468
(0.2,0.7,0.6)	1	0.965	0.917	0.86	0.801	0.741	0.683	0.627	0.574	0.523	0.476
(0.2,0.7,0.7)	1	0.965	0.915	0.858	0.798	0.737	0.678	0.621	0.567	0.517	0.47
(0.2,0.7,0.8)	1	0.96	0.903	0.839	0.772	0.706	0.643	0.584	0.529	0.478	0.432
(0.2,0.7,0.9)	1	0.892	0.793	0.703	0.623	0.552	0.49	0.435	0.387	0.344	0.306
(0.2,0.8,0.1)	1	0.816	0.662	0.535	0.432	0.348	0.281	0.226	0.182	0.146	0.116
(0.2,0.8,0.2)	1	0.939	0.853	0.758	0.663	0.574	0.493	0.42	0.357	0.301	0.253
(0.2,0.8,0.3)	1	0.957	0.895	0.822	0.745	0.668	0.595	0.526	0.462	0.404	0.352
(0.2,0.8,0.4)	1	0.96	0.903	0.837	0.768	0.7	0.633	0.57	0.511	0.456	0.406
(0.2,0.8,0.5)	1	0.96	0.904	0.841	0.774	0.709	0.646	0.586	0.53	0.478	0.43
(0.2,0.8,0.6)	1	0.96	0.904	0.841	0.775	0.711	0.648	0.59	0.535	0.484	0.438
(0.2,0.8,0.7)	1	0.96	0.903	0.839	0.772	0.706	0.643	0.584	0.529	0.478	0.432
(0.2,0.8,0.8)	1	0.955	0.892	0.82	0.748	0.677	0.611	0.55	0.494	0.443	0.397
(0.2,0.8,0.9)	1	0.888	0.783	0.688	0.604	0.53	0.466	0.41	0.361	0.318	0.281
(0.2,0.9,0.1)	1	0.765	0.588	0.454	0.353	0.275	0.216	0.169	0.132	0.103	0.08
(0.2,0.9,0.2)	1	0.875	0.751	0.638	0.538	0.451	0.377	0.314	0.261	0.216	0.178
(0.2,0.9,0.3)	1	0.89	0.786	0.689	0.602	0.524	0.454	0.392	0.338	0.291	0.249
(0.2,0.9,0.4)	1	0.892	0.792	0.702	0.62	0.547	0.483	0.425	0.374	0.328	0.287
(0.2,0.9,0.5)	1	0.893	0.794	0.704	0.625	0.554	0.492	0.437	0.387	0.344	0.304
(0.2,0.9,0.6)	1	0.893	0.794	0.705	0.625	0.555	0.494	0.439	0.391	0.348	0.31
(0.2,0.9,0.7)	1	0.892	0.793	0.703	0.623	0.552	0.49	0.435	0.387	0.344	0.306

(0.2,0.9,0.8)	1	0.888	0.783	0.688	0.604	0.53	0.466	0.41	0.361	0.318	0.281
(0.2,0.9,0.9)	1	0.829	0.691	0.58	0.489	0.416	0.355	0.305	0.263	0.228	0.198
(0.3,0.1,0.1)	1	0.72	0.524	0.386	0.286	0.213	0.158	0.118	0.087	0.063	0.045
(0.3,0.1,0.2)	1	0.818	0.664	0.536	0.431	0.345	0.275	0.219	0.172	0.134	0.104
(0.3,0.1,0.3)	1	0.831	0.693	0.577	0.48	0.399	0.33	0.272	0.222	0.181	0.146
(0.3,0.1,0.4)	1	0.833	0.698	0.587	0.494	0.415	0.349	0.293	0.245	0.204	0.168
(0.3,0.1,0.5)	1	0.833	0.699	0.588	0.497	0.42	0.355	0.3	0.253	0.213	0.178
(0.3,0.1,0.6)	1	0.833	0.699	0.589	0.497	0.421	0.357	0.302	0.255	0.216	0.181
(0.3,0.1,0.7)	1	0.833	0.698	0.587	0.496	0.419	0.354	0.299	0.253	0.213	0.179
(0.3,0.1,0.8)	1	0.83	0.69	0.576	0.481	0.403	0.337	0.282	0.236	0.197	0.164
(0.3,0.1,0.9)	1	0.777	0.612	0.487	0.392	0.317	0.258	0.211	0.172	0.14	0.114
(0.3,0.2,0.1)	1	0.818	0.664	0.536	0.431	0.345	0.275	0.219	0.172	0.134	0.104
(0.3,0.2,0.2)	1	0.941	0.856	0.759	0.661	0.568	0.482	0.404	0.336	0.277	0.226
(0.3,0.2,0.3)	1	0.959	0.898	0.823	0.742	0.66	0.58	0.504	0.434	0.371	0.313
(0.3,0.2,0.4)	1	0.962	0.906	0.838	0.765	0.69	0.617	0.546	0.479	0.417	0.36
(0.3,0.2,0.5)	1	0.962	0.907	0.842	0.771	0.699	0.628	0.56	0.496	0.436	0.381
(0.3,0.2,0.6)	1	0.962	0.907	0.842	0.772	0.701	0.63	0.563	0.5	0.442	0.388
(0.3,0.2,0.7)	1	0.962	0.906	0.84	0.769	0.697	0.626	0.558	0.495	0.436	0.383
(0.3,0.2,0.8)	1	0.957	0.895	0.822	0.745	0.668	0.595	0.526	0.462	0.404	0.352
(0.3,0.2,0.9)	1	0.89	0.786	0.689	0.602	0.524	0.454	0.392	0.338	0.291	0.249
(0.3,0.3,0.1)	1	0.831	0.693	0.577	0.48	0.399	0.33	0.272	0.222	0.181	0.146
(0.3,0.3,0.2)	1	0.959	0.898	0.823	0.742	0.66	0.58	0.504	0.434	0.371	0.313
(0.3,0.3,0.3)	1	0.979	0.944	0.896	0.837	0.771	0.702	0.631	0.562	0.496	0.433
(0.3,0.3,0.4)	1	0.982	0.953	0.913	0.865	0.809	0.748	0.685	0.622	0.559	0.499
(0.3,0.3,0.5)	1	0.982	0.955	0.917	0.872	0.82	0.763	0.704	0.645	0.586	0.528
(0.3,0.3,0.6)	1	0.982	0.955	0.918	0.873	0.822	0.766	0.709	0.651	0.593	0.537
(0.3,0.3,0.7)	1	0.982	0.954	0.916	0.869	0.817	0.76	0.702	0.644	0.586	0.53
(0.3,0.3,0.8)	1	0.977	0.941	0.895	0.841	0.783	0.722	0.661	0.601	0.543	0.488
(0.3,0.3,0.9)	1	0.907	0.824	0.748	0.677	0.611	0.549	0.492	0.439	0.391	0.346
(0.3,0.4,0.1)	1	0.833	0.698	0.587	0.494	0.415	0.349	0.293	0.245	0.204	0.168
(0.3,0.4,0.2)	1	0.962	0.906	0.838	0.765	0.69	0.617	0.546	0.479	0.417	0.36
(0.3,0.4,0.3)	1	0.982	0.953	0.913	0.865	0.809	0.748	0.685	0.622	0.559	0.499
(0.3,0.4,0.4)	1	0.985	0.963	0.932	0.894	0.849	0.799	0.745	0.689	0.632	0.575
(0.3,0.4,0.5)	1	0.985	0.964	0.936	0.902	0.861	0.816	0.767	0.715	0.662	0.609
(0.3,0.4,0.6)	1	0.985	0.965	0.937	0.903	0.863	0.819	0.772	0.722	0.672	0.621
(0.3,0.4,0.7)	1	0.985	0.963	0.934	0.899	0.858	0.813	0.764	0.714	0.663	0.613
(0.3,0.4,0.8)	1	0.98	0.95	0.913	0.869	0.822	0.771	0.719	0.666	0.614	0.563
(0.3,0.4,0.9)	1	0.909	0.831	0.762	0.699	0.64	0.586	0.535	0.487	0.442	0.4
(0.3,0.5,0.1)	1	0.833	0.699	0.588	0.497	0.42	0.355	0.3	0.253	0.213	0.178
(0.3,0.5,0.2)	1	0.962	0.907	0.842	0.771	0.699	0.628	0.56	0.496	0.436	0.381
(0.3,0.5,0.3)	1	0.982	0.955	0.917	0.872	0.82	0.763	0.704	0.645	0.586	0.528
(0.3,0.5,0.4)	1	0.985	0.964	0.936	0.902	0.861	0.816	0.767	0.715	0.662	0.609
(0.3,0.5,0.5)	1	0.986	0.966	0.941	0.909	0.873	0.833	0.789	0.743	0.695	0.646
(0.3,0.5,0.6)	1	0.986	0.966	0.941	0.911	0.876	0.837	0.795	0.75	0.705	0.659
(0.3,0.5,0.7)	1	0.986	0.965	0.939	0.907	0.87	0.83	0.787	0.742	0.696	0.65
(0.3,0.5,0.8)	1	0.98	0.952	0.917	0.877	0.833	0.787	0.74	0.692	0.644	0.597
(0.3,0.5,0.9)	1	0.91	0.833	0.765	0.704	0.649	0.598	0.55	0.505	0.464	0.424
(0.3,0.6,0.1)	1	0.833	0.699	0.589	0.497	0.421	0.357	0.302	0.255	0.216	0.181
(0.3,0.6,0.2)	1	0.962	0.907	0.842	0.772	0.701	0.63	0.563	0.5	0.442	0.388
(0.3,0.6,0.3)	1	0.982	0.955	0.918	0.873	0.822	0.766	0.709	0.651	0.593	0.537
(0.3,0.6,0.4)	1	0.985	0.965	0.937	0.903	0.863	0.819	0.772	0.722	0.672	0.621
(0.3,0.6,0.5)	1	0.986	0.966	0.941	0.911	0.876	0.837	0.795	0.75	0.705	0.659
(0.3,0.6,0.6)	1	0.986	0.967	0.942	0.912	0.878	0.84	0.8	0.758	0.715	0.671
(0.3,0.6,0.7)	1	0.986	0.965	0.939	0.908	0.873	0.834	0.793	0.75	0.706	0.662
(0.3,0.6,0.8)	1	0.98	0.952	0.917	0.878	0.835	0.79	0.745	0.699	0.653	0.609

(0.3,0.6,0.9)	1	0.91	0.833	0.766	0.705	0.65	0.6	0.554	0.51	0.47	0.432
(0.3,0.7,0.1)	1	0.833	0.698	0.587	0.496	0.419	0.354	0.299	0.253	0.213	0.179
(0.3,0.7,0.2)	1	0.962	0.906	0.84	0.769	0.697	0.626	0.558	0.495	0.436	0.383
(0.3,0.7,0.3)	1	0.982	0.954	0.916	0.869	0.817	0.76	0.702	0.644	0.586	0.53
(0.3,0.7,0.4)	1	0.985	0.963	0.934	0.899	0.858	0.813	0.764	0.714	0.663	0.613
(0.3,0.7,0.5)	1	0.986	0.965	0.939	0.907	0.87	0.83	0.787	0.742	0.696	0.65
(0.3,0.7,0.6)	1	0.986	0.965	0.939	0.908	0.873	0.834	0.793	0.75	0.706	0.662
(0.3,0.7,0.7)	1	0.985	0.964	0.937	0.904	0.867	0.827	0.785	0.742	0.698	0.654
(0.3,0.7,0.8)	1	0.98	0.951	0.915	0.874	0.83	0.784	0.738	0.691	0.645	0.601
(0.3,0.7,0.9)	1	0.909	0.832	0.764	0.702	0.647	0.596	0.548	0.505	0.464	0.427
(0.3,0.8,0.1)	1	0.83	0.69	0.576	0.481	0.403	0.337	0.282	0.236	0.197	0.164
(0.3,0.8,0.2)	1	0.957	0.895	0.822	0.745	0.668	0.595	0.526	0.462	0.404	0.352
(0.3,0.8,0.3)	1	0.977	0.941	0.895	0.841	0.783	0.722	0.661	0.601	0.543	0.488
(0.3,0.8,0.4)	1	0.98	0.95	0.913	0.869	0.822	0.771	0.719	0.666	0.614	0.563
(0.3,0.8,0.5)	1	0.98	0.952	0.917	0.877	0.833	0.787	0.74	0.692	0.644	0.597
(0.3,0.8,0.6)	1	0.98	0.952	0.917	0.878	0.835	0.79	0.745	0.699	0.653	0.609
(0.3,0.8,0.7)	1	0.98	0.951	0.915	0.874	0.83	0.784	0.738	0.691	0.645	0.601
(0.3,0.8,0.8)	1	0.974	0.938	0.894	0.845	0.795	0.744	0.693	0.644	0.597	0.552
(0.3,0.8,0.9)	1	0.905	0.821	0.747	0.68	0.62	0.565	0.516	0.471	0.429	0.392
(0.3,0.9,0.1)	1	0.777	0.612	0.487	0.392	0.317	0.258	0.211	0.172	0.14	0.114
(0.3,0.9,0.2)	1	0.89	0.786	0.689	0.602	0.524	0.454	0.392	0.338	0.291	0.249
(0.3,0.9,0.3)	1	0.907	0.824	0.748	0.677	0.611	0.549	0.492	0.439	0.391	0.346
(0.3,0.9,0.4)	1	0.909	0.831	0.762	0.699	0.64	0.586	0.535	0.487	0.442	0.4
(0.3,0.9,0.5)	1	0.91	0.833	0.765	0.704	0.649	0.598	0.55	0.505	0.464	0.424
(0.3,0.9,0.6)	1	0.91	0.833	0.766	0.705	0.65	0.6	0.554	0.51	0.47	0.432
(0.3,0.9,0.7)	1	0.909	0.832	0.764	0.702	0.647	0.596	0.548	0.505	0.464	0.427
(0.3,0.9,0.8)	1	0.905	0.821	0.747	0.68	0.62	0.565	0.516	0.471	0.429	0.392
(0.3,0.9,0.9)	1	0.843	0.722	0.627	0.549	0.485	0.43	0.384	0.343	0.308	0.277
(0.4,0.1,0.1)	1	0.721	0.527	0.391	0.292	0.22	0.167	0.126	0.095	0.071	0.052
(0.4,0.1,0.2)	1	0.82	0.669	0.545	0.443	0.359	0.291	0.235	0.189	0.151	0.12
(0.4,0.1,0.3)	1	0.833	0.698	0.587	0.494	0.415	0.349	0.293	0.245	0.204	0.168
(0.4,0.1,0.4)	1	0.835	0.703	0.596	0.508	0.434	0.371	0.317	0.27	0.23	0.195
(0.4,0.1,0.5)	1	0.835	0.704	0.598	0.511	0.439	0.377	0.325	0.28	0.24	0.206
(0.4,0.1,0.6)	1	0.835	0.704	0.598	0.512	0.439	0.379	0.327	0.282	0.243	0.21
(0.4,0.1,0.7)	1	0.835	0.704	0.597	0.51	0.437	0.376	0.324	0.279	0.24	0.207
(0.4,0.1,0.8)	1	0.831	0.696	0.585	0.495	0.42	0.358	0.305	0.261	0.222	0.19
(0.4,0.1,0.9)	1	0.778	0.616	0.495	0.402	0.33	0.273	0.227	0.19	0.159	0.133
(0.4,0.2,0.1)	1	0.82	0.669	0.545	0.443	0.359	0.291	0.235	0.189	0.151	0.12
(0.4,0.2,0.2)	1	0.943	0.863	0.772	0.681	0.592	0.511	0.437	0.37	0.312	0.26
(0.4,0.2,0.3)	1	0.962	0.906	0.838	0.765	0.69	0.617	0.546	0.479	0.417	0.36
(0.4,0.2,0.4)	1	0.965	0.914	0.854	0.79	0.723	0.656	0.592	0.529	0.47	0.415
(0.4,0.2,0.5)	1	0.965	0.916	0.858	0.796	0.732	0.669	0.608	0.549	0.492	0.439
(0.4,0.2,0.6)	1	0.965	0.916	0.858	0.797	0.734	0.672	0.612	0.554	0.499	0.447
(0.4,0.2,0.7)	1	0.965	0.915	0.856	0.794	0.73	0.667	0.606	0.548	0.493	0.441
(0.4,0.2,0.8)	1	0.96	0.903	0.837	0.768	0.7	0.633	0.57	0.511	0.456	0.406
(0.4,0.2,0.9)	1	0.892	0.792	0.702	0.62	0.547	0.483	0.425	0.374	0.328	0.287
(0.4,0.3,0.1)	1	0.833	0.698	0.587	0.494	0.415	0.349	0.293	0.245	0.204	0.168
(0.4,0.3,0.2)	1	0.962	0.906	0.838	0.765	0.69	0.617	0.546	0.479	0.417	0.36
(0.4,0.3,0.3)	1	0.982	0.953	0.913	0.865	0.809	0.748	0.685	0.622	0.559	0.499
(0.4,0.3,0.4)	1	0.985	0.963	0.932	0.894	0.849	0.799	0.745	0.689	0.632	0.575
(0.4,0.3,0.5)	1	0.985	0.964	0.936	0.902	0.861	0.816	0.767	0.715	0.662	0.609
(0.4,0.3,0.6)	1	0.985	0.965	0.937	0.903	0.863	0.819	0.772	0.722	0.672	0.621
(0.4,0.3,0.7)	1	0.985	0.963	0.934	0.899	0.858	0.813	0.764	0.714	0.663	0.613
(0.4,0.3,0.8)	1	0.98	0.95	0.913	0.869	0.822	0.771	0.719	0.666	0.614	0.563
(0.4,0.3,0.9)	1	0.909	0.831	0.762	0.699	0.64	0.586	0.535	0.487	0.442	0.4

(0.4,0.4,0.1)	1	0.835	0.703	0.596	0.508	0.434	0.371	0.317	0.27	0.23	0.195
(0.4,0.4,0.2)	1	0.965	0.914	0.854	0.79	0.723	0.656	0.592	0.529	0.47	0.415
(0.4,0.4,0.3)	1	0.985	0.963	0.932	0.894	0.849	0.799	0.745	0.689	0.632	0.575
(0.4,0.4,0.4)	1	0.988	0.973	0.952	0.925	0.893	0.855	0.812	0.765	0.715	0.664
(0.4,0.4,0.5)	1	0.989	0.974	0.956	0.934	0.906	0.873	0.836	0.795	0.751	0.705
(0.4,0.4,0.6)	1	0.989	0.975	0.957	0.935	0.908	0.877	0.842	0.803	0.762	0.718
(0.4,0.4,0.7)	1	0.988	0.973	0.954	0.931	0.903	0.87	0.834	0.794	0.752	0.709
(0.4,0.4,0.8)	1	0.983	0.96	0.932	0.9	0.864	0.825	0.783	0.74	0.696	0.651
(0.4,0.4,0.9)	1	0.912	0.839	0.777	0.722	0.672	0.626	0.582	0.541	0.501	0.463
(0.4,0.5,0.1)	1	0.835	0.704	0.598	0.511	0.439	0.377	0.325	0.28	0.24	0.206
(0.4,0.5,0.2)	1	0.965	0.916	0.858	0.796	0.732	0.669	0.608	0.549	0.492	0.439
(0.4,0.5,0.3)	1	0.985	0.964	0.936	0.902	0.861	0.816	0.767	0.715	0.662	0.609
(0.4,0.5,0.4)	1	0.989	0.974	0.956	0.934	0.906	0.873	0.836	0.795	0.751	0.705
(0.4,0.5,0.5)	1	0.989	0.976	0.961	0.942	0.919	0.892	0.861	0.827	0.789	0.748
(0.4,0.5,0.6)	1	0.989	0.977	0.961	0.943	0.922	0.897	0.868	0.836	0.801	0.763
(0.4,0.5,0.7)	1	0.989	0.975	0.959	0.939	0.916	0.889	0.859	0.826	0.791	0.753
(0.4,0.5,0.8)	1	0.983	0.961	0.936	0.907	0.876	0.842	0.807	0.77	0.731	0.692
(0.4,0.5,0.9)	1	0.912	0.84	0.78	0.728	0.681	0.639	0.599	0.562	0.526	0.492
(0.4,0.6,0.1)	1	0.835	0.704	0.598	0.512	0.439	0.379	0.327	0.282	0.243	0.21
(0.4,0.6,0.2)	1	0.965	0.916	0.858	0.797	0.734	0.672	0.612	0.554	0.499	0.447
(0.4,0.6,0.3)	1	0.985	0.965	0.937	0.903	0.863	0.819	0.772	0.722	0.672	0.621
(0.4,0.6,0.4)	1	0.989	0.975	0.957	0.935	0.908	0.877	0.842	0.803	0.762	0.718
(0.4,0.6,0.5)	1	0.989	0.977	0.961	0.943	0.922	0.897	0.868	0.836	0.801	0.763
(0.4,0.6,0.6)	1	0.989	0.977	0.962	0.945	0.924	0.901	0.874	0.845	0.813	0.778
(0.4,0.6,0.7)	1	0.989	0.975	0.959	0.941	0.919	0.894	0.866	0.835	0.803	0.768
(0.4,0.6,0.8)	1	0.983	0.962	0.937	0.909	0.879	0.847	0.813	0.778	0.742	0.705
(0.4,0.6,0.9)	1	0.912	0.841	0.781	0.729	0.683	0.642	0.603	0.568	0.534	0.501
(0.4,0.7,0.1)	1	0.835	0.704	0.597	0.51	0.437	0.376	0.324	0.279	0.24	0.207
(0.4,0.7,0.2)	1	0.965	0.915	0.856	0.794	0.73	0.667	0.606	0.548	0.493	0.441
(0.4,0.7,0.3)	1	0.985	0.963	0.934	0.899	0.858	0.813	0.764	0.714	0.663	0.613
(0.4,0.7,0.4)	1	0.988	0.973	0.954	0.931	0.903	0.87	0.834	0.794	0.752	0.709
(0.4,0.7,0.5)	1	0.989	0.975	0.959	0.939	0.916	0.889	0.859	0.826	0.791	0.753
(0.4,0.7,0.6)	1	0.989	0.975	0.959	0.941	0.919	0.894	0.866	0.835	0.803	0.768
(0.4,0.7,0.7)	1	0.988	0.974	0.957	0.936	0.913	0.886	0.857	0.826	0.793	0.758
(0.4,0.7,0.8)	1	0.983	0.96	0.934	0.905	0.873	0.84	0.805	0.769	0.733	0.696
(0.4,0.7,0.9)	1	0.912	0.84	0.779	0.726	0.679	0.637	0.598	0.561	0.527	0.495
(0.4,0.8,0.1)	1	0.831	0.696	0.585	0.495	0.42	0.358	0.305	0.261	0.222	0.19
(0.4,0.8,0.2)	1	0.96	0.903	0.837	0.768	0.7	0.633	0.57	0.511	0.456	0.406
(0.4,0.8,0.3)	1	0.98	0.95	0.913	0.869	0.822	0.771	0.719	0.666	0.614	0.563
(0.4,0.8,0.4)	1	0.983	0.96	0.932	0.9	0.864	0.825	0.783	0.74	0.696	0.651
(0.4,0.8,0.5)	1	0.983	0.961	0.936	0.907	0.876	0.842	0.807	0.77	0.731	0.692
(0.4,0.8,0.6)	1	0.983	0.962	0.937	0.909	0.879	0.847	0.813	0.778	0.742	0.705
(0.4,0.8,0.7)	1	0.983	0.96	0.934	0.905	0.873	0.84	0.805	0.769	0.733	0.696
(0.4,0.8,0.8)	1	0.977	0.947	0.912	0.875	0.836	0.796	0.756	0.717	0.678	0.64
(0.4,0.8,0.9)	1	0.907	0.829	0.761	0.702	0.65	0.604	0.562	0.523	0.487	0.454
(0.4,0.9,0.1)	1	0.778	0.616	0.495	0.402	0.33	0.273	0.227	0.19	0.159	0.133
(0.4,0.9,0.2)	1	0.892	0.792	0.702	0.62	0.547	0.483	0.425	0.374	0.328	0.287
(0.4,0.9,0.3)	1	0.909	0.831	0.762	0.699	0.64	0.586	0.535	0.487	0.442	0.4
(0.4,0.9,0.4)	1	0.912	0.839	0.777	0.722	0.672	0.626	0.582	0.541	0.501	0.463
(0.4,0.9,0.5)	1	0.912	0.84	0.78	0.728	0.681	0.639	0.599	0.562	0.526	0.492
(0.4,0.9,0.6)	1	0.912	0.841	0.781	0.729	0.683	0.642	0.603	0.568	0.534	0.501
(0.4,0.9,0.7)	1	0.912	0.84	0.779	0.726	0.679	0.637	0.598	0.561	0.527	0.495
(0.4,0.9,0.8)	1	0.907	0.829	0.761	0.702	0.65	0.604	0.562	0.523	0.487	0.454
(0.4,0.9,0.9)	1	0.845	0.728	0.638	0.566	0.508	0.459	0.417	0.381	0.35	0.321
(0.5,0.1,0.1)	1	0.721	0.528	0.392	0.294	0.222	0.169	0.129	0.099	0.075	0.056

(0.5,0.1,0.2)	1	0.82	0.67	0.546	0.445	0.363	0.296	0.241	0.196	0.158	0.127
(0.5,0.1,0.3)	1	0.833	0.699	0.588	0.497	0.42	0.355	0.3	0.253	0.213	0.178
(0.5,0.1,0.4)	1	0.835	0.704	0.598	0.511	0.439	0.377	0.325	0.28	0.24	0.206
(0.5,0.1,0.5)	1	0.836	0.705	0.6	0.515	0.444	0.384	0.333	0.29	0.252	0.218
(0.5,0.1,0.6)	1	0.836	0.705	0.601	0.515	0.445	0.385	0.335	0.292	0.255	0.222
(0.5,0.1,0.7)	1	0.835	0.705	0.599	0.513	0.442	0.383	0.332	0.289	0.252	0.219
(0.5,0.1,0.8)	1	0.832	0.696	0.587	0.498	0.425	0.364	0.313	0.27	0.233	0.201
(0.5,0.1,0.9)	1	0.778	0.617	0.496	0.405	0.334	0.278	0.233	0.197	0.166	0.141
(0.5,0.2,0.1)	1	0.82	0.67	0.546	0.445	0.363	0.296	0.241	0.196	0.158	0.127
(0.5,0.2,0.2)	1	0.944	0.864	0.775	0.685	0.599	0.52	0.447	0.383	0.325	0.275
(0.5,0.2,0.3)	1	0.962	0.907	0.842	0.771	0.699	0.628	0.56	0.496	0.436	0.381
(0.5,0.2,0.4)	1	0.965	0.916	0.858	0.796	0.732	0.669	0.608	0.549	0.492	0.439
(0.5,0.2,0.5)	1	0.966	0.917	0.861	0.802	0.742	0.682	0.625	0.569	0.516	0.465
(0.5,0.2,0.6)	1	0.966	0.917	0.862	0.803	0.744	0.685	0.629	0.574	0.523	0.474
(0.5,0.2,0.7)	1	0.965	0.916	0.86	0.8	0.739	0.68	0.623	0.568	0.516	0.468
(0.5,0.2,0.8)	1	0.96	0.904	0.841	0.774	0.709	0.646	0.586	0.53	0.478	0.43
(0.5,0.2,0.9)	1	0.893	0.794	0.704	0.625	0.554	0.492	0.437	0.387	0.344	0.304
(0.5,0.3,0.1)	1	0.833	0.699	0.588	0.497	0.42	0.355	0.3	0.253	0.213	0.178
(0.5,0.3,0.2)	1	0.962	0.907	0.842	0.771	0.699	0.628	0.56	0.496	0.436	0.381
(0.5,0.3,0.3)	1	0.982	0.955	0.917	0.872	0.82	0.763	0.704	0.645	0.586	0.528
(0.5,0.3,0.4)	1	0.985	0.964	0.936	0.902	0.861	0.816	0.767	0.715	0.662	0.609
(0.5,0.3,0.5)	1	0.986	0.966	0.941	0.909	0.873	0.833	0.789	0.743	0.695	0.646
(0.5,0.3,0.6)	1	0.986	0.966	0.941	0.911	0.876	0.837	0.795	0.75	0.705	0.659
(0.5,0.3,0.7)	1	0.986	0.965	0.939	0.907	0.87	0.83	0.787	0.742	0.696	0.65
(0.5,0.3,0.8)	1	0.98	0.952	0.917	0.877	0.833	0.787	0.74	0.692	0.644	0.597
(0.5,0.3,0.9)	1	0.91	0.833	0.765	0.704	0.649	0.598	0.55	0.505	0.464	0.424
(0.5,0.4,0.1)	1	0.835	0.704	0.598	0.511	0.439	0.377	0.325	0.28	0.24	0.206
(0.5,0.4,0.2)	1	0.965	0.916	0.858	0.796	0.732	0.669	0.608	0.549	0.492	0.439
(0.5,0.4,0.3)	1	0.985	0.964	0.936	0.902	0.861	0.816	0.767	0.715	0.662	0.609
(0.5,0.4,0.4)	1	0.989	0.974	0.956	0.934	0.906	0.873	0.836	0.795	0.751	0.705
(0.5,0.4,0.5)	1	0.989	0.976	0.961	0.942	0.919	0.892	0.861	0.827	0.789	0.748
(0.5,0.4,0.6)	1	0.989	0.977	0.961	0.943	0.922	0.897	0.868	0.836	0.801	0.763
(0.5,0.4,0.7)	1	0.989	0.975	0.959	0.939	0.916	0.889	0.859	0.826	0.791	0.753
(0.5,0.4,0.8)	1	0.983	0.961	0.936	0.907	0.876	0.842	0.807	0.77	0.731	0.692
(0.5,0.4,0.9)	1	0.912	0.84	0.78	0.728	0.681	0.639	0.599	0.562	0.526	0.492
(0.5,0.5,0.1)	1	0.836	0.705	0.6	0.515	0.444	0.384	0.333	0.29	0.252	0.218
(0.5,0.5,0.2)	1	0.966	0.917	0.861	0.802	0.742	0.682	0.625	0.569	0.516	0.465
(0.5,0.5,0.3)	1	0.986	0.966	0.941	0.909	0.873	0.833	0.789	0.743	0.695	0.646
(0.5,0.5,0.4)	1	0.989	0.976	0.961	0.942	0.919	0.892	0.861	0.827	0.789	0.748
(0.5,0.5,0.5)	1	0.99	0.978	0.965	0.951	0.933	0.912	0.888	0.86	0.83	0.796
(0.5,0.5,0.6)	1	0.99	0.978	0.966	0.952	0.936	0.917	0.895	0.87	0.842	0.812
(0.5,0.5,0.7)	1	0.989	0.977	0.963	0.948	0.93	0.909	0.886	0.86	0.832	0.801
(0.5,0.5,0.8)	1	0.984	0.963	0.94	0.916	0.889	0.861	0.832	0.801	0.769	0.736
(0.5,0.5,0.9)	1	0.913	0.842	0.784	0.734	0.691	0.652	0.617	0.584	0.553	0.523
(0.5,0.6,0.1)	1	0.836	0.705	0.601	0.515	0.445	0.385	0.335	0.292	0.255	0.222
(0.5,0.6,0.2)	1	0.966	0.917	0.862	0.803	0.744	0.685	0.629	0.574	0.523	0.474
(0.5,0.6,0.3)	1	0.986	0.966	0.941	0.911	0.876	0.837	0.795	0.75	0.705	0.659
(0.5,0.6,0.4)	1	0.989	0.977	0.961	0.943	0.922	0.897	0.868	0.836	0.801	0.763
(0.5,0.6,0.5)	1	0.99	0.978	0.966	0.952	0.936	0.917	0.895	0.87	0.842	0.812
(0.5,0.6,0.6)	1	0.99	0.979	0.967	0.954	0.939	0.921	0.902	0.88	0.855	0.828
(0.5,0.6,0.7)	1	0.989	0.977	0.964	0.949	0.933	0.914	0.893	0.87	0.845	0.817
(0.5,0.6,0.8)	1	0.984	0.964	0.941	0.917	0.892	0.865	0.838	0.81	0.781	0.751
(0.5,0.6,0.9)	1	0.913	0.842	0.784	0.735	0.693	0.655	0.622	0.59	0.561	0.533
(0.5,0.7,0.1)	1	0.835	0.705	0.599	0.513	0.442	0.383	0.332	0.289	0.252	0.219
(0.5,0.7,0.2)	1	0.965	0.916	0.86	0.8	0.739	0.68	0.623	0.568	0.516	0.468

(0.5,0.7,0.3)	1	0.986	0.965	0.939	0.907	0.87	0.83	0.787	0.742	0.696	0.65
(0.5,0.7,0.4)	1	0.989	0.975	0.959	0.939	0.916	0.889	0.859	0.826	0.791	0.753
(0.5,0.7,0.5)	1	0.989	0.977	0.963	0.948	0.93	0.909	0.886	0.86	0.832	0.801
(0.5,0.7,0.6)	1	0.989	0.977	0.964	0.949	0.933	0.914	0.893	0.87	0.845	0.817
(0.5,0.7,0.7)	1	0.989	0.976	0.961	0.945	0.927	0.906	0.884	0.86	0.834	0.807
(0.5,0.7,0.8)	1	0.983	0.962	0.938	0.913	0.886	0.858	0.83	0.801	0.771	0.741
(0.5,0.7,0.9)	1	0.912	0.841	0.782	0.732	0.689	0.65	0.616	0.584	0.554	0.526
(0.5,0.8,0.1)	1	0.832	0.696	0.587	0.498	0.425	0.364	0.313	0.27	0.233	0.201
(0.5,0.8,0.2)	1	0.96	0.904	0.841	0.774	0.709	0.646	0.586	0.53	0.478	0.43
(0.5,0.8,0.3)	1	0.98	0.952	0.917	0.877	0.833	0.787	0.74	0.692	0.644	0.597
(0.5,0.8,0.4)	1	0.983	0.961	0.936	0.907	0.876	0.842	0.807	0.77	0.731	0.692
(0.5,0.8,0.5)	1	0.984	0.963	0.94	0.916	0.889	0.861	0.832	0.801	0.769	0.736
(0.5,0.8,0.6)	1	0.984	0.964	0.941	0.917	0.892	0.865	0.838	0.81	0.781	0.751
(0.5,0.8,0.7)	1	0.983	0.962	0.938	0.913	0.886	0.858	0.83	0.801	0.771	0.741
(0.5,0.8,0.8)	1	0.978	0.949	0.916	0.882	0.848	0.813	0.779	0.746	0.713	0.68
(0.5,0.8,0.9)	1	0.908	0.83	0.764	0.708	0.659	0.616	0.578	0.544	0.512	0.483
(0.5,0.9,0.1)	1	0.778	0.617	0.496	0.405	0.334	0.278	0.233	0.197	0.166	0.141
(0.5,0.9,0.2)	1	0.893	0.794	0.704	0.625	0.554	0.492	0.437	0.387	0.344	0.304
(0.5,0.9,0.3)	1	0.91	0.833	0.765	0.704	0.649	0.598	0.55	0.505	0.464	0.424
(0.5,0.9,0.4)	1	0.912	0.84	0.78	0.728	0.681	0.639	0.599	0.562	0.526	0.492
(0.5,0.9,0.5)	1	0.913	0.842	0.784	0.734	0.691	0.652	0.617	0.584	0.553	0.523
(0.5,0.9,0.6)	1	0.913	0.842	0.784	0.735	0.693	0.655	0.622	0.59	0.561	0.533
(0.5,0.9,0.7)	1	0.912	0.841	0.782	0.732	0.689	0.65	0.616	0.584	0.554	0.526
(0.5,0.9,0.8)	1	0.908	0.83	0.764	0.708	0.659	0.616	0.578	0.544	0.512	0.483
(0.5,0.9,0.9)	1	0.846	0.729	0.64	0.57	0.514	0.468	0.429	0.396	0.368	0.342
(0.6,0.1,0.1)	1	0.721	0.528	0.392	0.294	0.223	0.17	0.13	0.099	0.075	0.057
(0.6,0.1,0.2)	1	0.82	0.67	0.546	0.446	0.364	0.297	0.242	0.197	0.16	0.129
(0.6,0.1,0.3)	1	0.833	0.699	0.589	0.497	0.421	0.357	0.302	0.255	0.216	0.181
(0.6,0.1,0.4)	1	0.835	0.704	0.598	0.512	0.439	0.379	0.327	0.282	0.243	0.21
(0.6,0.1,0.5)	1	0.836	0.705	0.601	0.515	0.445	0.385	0.335	0.292	0.255	0.222
(0.6,0.1,0.6)	1	0.836	0.705	0.601	0.516	0.446	0.387	0.337	0.295	0.258	0.226
(0.6,0.1,0.7)	1	0.835	0.705	0.6	0.514	0.443	0.384	0.334	0.292	0.255	0.223
(0.6,0.1,0.8)	1	0.832	0.697	0.587	0.499	0.426	0.365	0.315	0.272	0.236	0.205
(0.6,0.1,0.9)	1	0.778	0.617	0.496	0.405	0.335	0.279	0.235	0.198	0.168	0.143
(0.6,0.2,0.1)	1	0.82	0.67	0.546	0.446	0.364	0.297	0.242	0.197	0.16	0.129
(0.6,0.2,0.2)	1	0.944	0.864	0.776	0.686	0.6	0.521	0.45	0.386	0.329	0.279
(0.6,0.2,0.3)	1	0.962	0.907	0.842	0.772	0.701	0.63	0.563	0.5	0.442	0.388
(0.6,0.2,0.4)	1	0.965	0.916	0.858	0.797	0.734	0.672	0.612	0.554	0.499	0.447
(0.6,0.2,0.5)	1	0.966	0.917	0.862	0.803	0.744	0.685	0.629	0.574	0.523	0.474
(0.6,0.2,0.6)	1	0.966	0.918	0.862	0.804	0.746	0.688	0.633	0.58	0.53	0.483
(0.6,0.2,0.7)	1	0.965	0.917	0.86	0.801	0.741	0.683	0.627	0.574	0.523	0.476
(0.6,0.2,0.8)	1	0.96	0.904	0.841	0.775	0.711	0.648	0.59	0.535	0.484	0.438
(0.6,0.2,0.9)	1	0.893	0.794	0.705	0.625	0.555	0.494	0.439	0.391	0.348	0.31
(0.6,0.3,0.1)	1	0.833	0.699	0.589	0.497	0.421	0.357	0.302	0.255	0.216	0.181
(0.6,0.3,0.2)	1	0.962	0.907	0.842	0.772	0.701	0.63	0.563	0.5	0.442	0.388
(0.6,0.3,0.3)	1	0.982	0.955	0.918	0.873	0.822	0.766	0.709	0.651	0.593	0.537
(0.6,0.3,0.4)	1	0.985	0.965	0.937	0.903	0.863	0.819	0.772	0.722	0.672	0.621
(0.6,0.3,0.5)	1	0.986	0.966	0.941	0.911	0.876	0.837	0.795	0.75	0.705	0.659
(0.6,0.3,0.6)	1	0.986	0.967	0.942	0.912	0.878	0.84	0.8	0.758	0.715	0.671
(0.6,0.3,0.7)	1	0.986	0.965	0.939	0.908	0.873	0.834	0.793	0.75	0.706	0.662
(0.6,0.3,0.8)	1	0.98	0.952	0.917	0.878	0.835	0.79	0.745	0.699	0.653	0.609
(0.6,0.3,0.9)	1	0.91	0.833	0.766	0.705	0.65	0.6	0.554	0.51	0.47	0.432
(0.6,0.4,0.1)	1	0.835	0.704	0.598	0.512	0.439	0.379	0.327	0.282	0.243	0.21
(0.6,0.4,0.2)	1	0.965	0.916	0.858	0.797	0.734	0.672	0.612	0.554	0.499	0.447
(0.6,0.4,0.3)	1	0.985	0.965	0.937	0.903	0.863	0.819	0.772	0.722	0.672	0.621

(0.6,0.4,0.4)	1	0.989	0.975	0.957	0.935	0.908	0.877	0.842	0.803	0.762	0.718
(0.6,0.4,0.5)	1	0.989	0.977	0.961	0.943	0.922	0.897	0.868	0.836	0.801	0.763
(0.6,0.4,0.6)	1	0.989	0.977	0.962	0.945	0.924	0.901	0.874	0.845	0.813	0.778
(0.6,0.4,0.7)	1	0.989	0.975	0.959	0.941	0.919	0.894	0.866	0.835	0.803	0.768
(0.6,0.4,0.8)	1	0.983	0.962	0.937	0.909	0.879	0.847	0.813	0.778	0.742	0.705
(0.6,0.4,0.9)	1	0.912	0.841	0.781	0.729	0.683	0.642	0.603	0.568	0.534	0.501
(0.6,0.5,0.1)	1	0.836	0.705	0.601	0.515	0.445	0.385	0.335	0.292	0.255	0.222
(0.6,0.5,0.2)	1	0.966	0.917	0.862	0.803	0.744	0.685	0.629	0.574	0.523	0.474
(0.6,0.5,0.3)	1	0.986	0.966	0.941	0.911	0.876	0.837	0.795	0.75	0.705	0.659
(0.6,0.5,0.4)	1	0.989	0.977	0.961	0.943	0.922	0.897	0.868	0.836	0.801	0.763
(0.6,0.5,0.5)	1	0.99	0.978	0.966	0.952	0.936	0.917	0.895	0.87	0.842	0.812
(0.6,0.5,0.6)	1	0.99	0.979	0.967	0.954	0.939	0.921	0.902	0.88	0.855	0.828
(0.6,0.5,0.7)	1	0.989	0.977	0.964	0.949	0.933	0.914	0.893	0.87	0.845	0.817
(0.6,0.5,0.8)	1	0.984	0.964	0.941	0.917	0.892	0.865	0.838	0.81	0.781	0.751
(0.6,0.5,0.9)	1	0.913	0.842	0.784	0.735	0.693	0.655	0.622	0.59	0.561	0.533
(0.6,0.6,0.1)	1	0.836	0.705	0.601	0.516	0.446	0.387	0.337	0.295	0.258	0.226
(0.6,0.6,0.2)	1	0.966	0.918	0.862	0.804	0.746	0.688	0.633	0.58	0.53	0.483
(0.6,0.6,0.3)	1	0.986	0.967	0.942	0.912	0.878	0.84	0.8	0.758	0.715	0.671
(0.6,0.6,0.4)	1	0.989	0.977	0.962	0.945	0.924	0.901	0.874	0.845	0.813	0.778
(0.6,0.6,0.5)	1	0.99	0.979	0.967	0.954	0.939	0.921	0.902	0.88	0.855	0.828
(0.6,0.6,0.6)	1	0.99	0.979	0.967	0.955	0.941	0.926	0.909	0.89	0.868	0.845
(0.6,0.6,0.7)	1	0.989	0.978	0.965	0.951	0.935	0.918	0.9	0.88	0.858	0.834
(0.6,0.6,0.8)	1	0.984	0.964	0.942	0.918	0.894	0.87	0.844	0.819	0.793	0.766
(0.6,0.6,0.9)	1	0.913	0.842	0.784	0.736	0.695	0.659	0.626	0.597	0.57	0.544
(0.6,0.7,0.1)	1	0.835	0.705	0.6	0.514	0.443	0.384	0.334	0.292	0.255	0.223
(0.6,0.7,0.2)	1	0.965	0.917	0.86	0.801	0.741	0.683	0.627	0.574	0.523	0.476
(0.6,0.7,0.3)	1	0.986	0.965	0.939	0.908	0.873	0.834	0.793	0.75	0.706	0.662
(0.6,0.7,0.4)	1	0.989	0.975	0.959	0.941	0.919	0.894	0.866	0.835	0.803	0.768
(0.6,0.7,0.5)	1	0.989	0.977	0.964	0.949	0.933	0.914	0.893	0.87	0.845	0.817
(0.6,0.7,0.6)	1	0.989	0.978	0.965	0.951	0.935	0.918	0.9	0.88	0.858	0.834
(0.6,0.7,0.7)	1	0.989	0.976	0.962	0.946	0.929	0.911	0.891	0.87	0.847	0.823
(0.6,0.7,0.8)	1	0.983	0.962	0.939	0.914	0.889	0.862	0.836	0.81	0.783	0.756
(0.6,0.7,0.9)	1	0.912	0.841	0.782	0.733	0.69	0.653	0.62	0.59	0.563	0.537
(0.6,0.8,0.1)	1	0.832	0.697	0.587	0.499	0.426	0.365	0.315	0.272	0.236	0.205
(0.6,0.8,0.2)	1	0.96	0.904	0.841	0.775	0.711	0.648	0.59	0.535	0.484	0.438
(0.6,0.8,0.3)	1	0.98	0.952	0.917	0.878	0.835	0.79	0.745	0.699	0.653	0.609
(0.6,0.8,0.4)	1	0.983	0.962	0.937	0.909	0.879	0.847	0.813	0.778	0.742	0.705
(0.6,0.8,0.5)	1	0.984	0.964	0.941	0.917	0.892	0.865	0.838	0.81	0.781	0.751
(0.6,0.8,0.6)	1	0.984	0.964	0.942	0.918	0.894	0.87	0.844	0.819	0.793	0.766
(0.6,0.8,0.7)	1	0.983	0.962	0.939	0.914	0.889	0.862	0.836	0.81	0.783	0.756
(0.6,0.8,0.8)	1	0.978	0.949	0.917	0.884	0.85	0.817	0.785	0.754	0.723	0.694
(0.6,0.8,0.9)	1	0.908	0.83	0.765	0.709	0.661	0.619	0.582	0.55	0.52	0.493
(0.6,0.9,0.1)	1	0.778	0.617	0.496	0.405	0.335	0.279	0.235	0.198	0.168	0.143
(0.6,0.9,0.2)	1	0.893	0.794	0.705	0.625	0.555	0.494	0.439	0.391	0.348	0.31
(0.6,0.9,0.3)	1	0.91	0.833	0.766	0.705	0.65	0.6	0.554	0.51	0.47	0.432
(0.6,0.9,0.4)	1	0.912	0.841	0.781	0.729	0.683	0.642	0.603	0.568	0.534	0.501
(0.6,0.9,0.5)	1	0.913	0.842	0.784	0.735	0.693	0.655	0.622	0.59	0.561	0.533
(0.6,0.9,0.6)	1	0.913	0.842	0.784	0.736	0.695	0.659	0.626	0.597	0.57	0.544
(0.6,0.9,0.7)	1	0.912	0.841	0.782	0.733	0.69	0.653	0.62	0.59	0.563	0.537
(0.6,0.9,0.8)	1	0.908	0.83	0.765	0.709	0.661	0.619	0.582	0.55	0.52	0.493
(0.6,0.9,0.9)	1	0.846	0.73	0.641	0.571	0.515	0.47	0.432	0.401	0.373	0.349
(0.7,0.1,0.1)	1	0.721	0.528	0.391	0.293	0.222	0.169	0.129	0.098	0.074	0.056
(0.7,0.1,0.2)	1	0.82	0.669	0.545	0.444	0.362	0.295	0.24	0.195	0.158	0.127
(0.7,0.1,0.3)	1	0.833	0.698	0.587	0.496	0.419	0.354	0.299	0.253	0.213	0.179
(0.7,0.1,0.4)	1	0.835	0.704	0.597	0.51	0.437	0.376	0.324	0.279	0.24	0.207

(0.7,0.1,0.5)	1	0.835	0.705	0.599	0.513	0.442	0.383	0.332	0.289	0.252	0.219
(0.7,0.1,0.6)	1	0.835	0.705	0.6	0.514	0.443	0.384	0.334	0.292	0.255	0.223
(0.7,0.1,0.7)	1	0.835	0.704	0.598	0.512	0.441	0.381	0.331	0.289	0.252	0.22
(0.7,0.1,0.8)	1	0.831	0.696	0.586	0.497	0.423	0.363	0.312	0.269	0.233	0.202
(0.7,0.1,0.9)	1	0.778	0.616	0.495	0.404	0.333	0.277	0.232	0.196	0.166	0.141
(0.7,0.2,0.1)	1	0.82	0.669	0.545	0.444	0.362	0.295	0.24	0.195	0.158	0.127
(0.7,0.2,0.2)	1	0.944	0.863	0.774	0.683	0.597	0.518	0.446	0.382	0.325	0.276
(0.7,0.2,0.3)	1	0.962	0.906	0.84	0.769	0.697	0.626	0.558	0.495	0.436	0.383
(0.7,0.2,0.4)	1	0.965	0.915	0.856	0.794	0.73	0.667	0.606	0.548	0.493	0.441
(0.7,0.2,0.5)	1	0.965	0.916	0.86	0.8	0.739	0.68	0.623	0.568	0.516	0.468
(0.7,0.2,0.6)	1	0.965	0.917	0.86	0.801	0.741	0.683	0.627	0.574	0.523	0.476
(0.7,0.2,0.7)	1	0.965	0.915	0.858	0.798	0.737	0.678	0.621	0.567	0.517	0.47
(0.7,0.2,0.8)	1	0.96	0.903	0.839	0.772	0.706	0.643	0.584	0.529	0.478	0.432
(0.7,0.2,0.9)	1	0.892	0.793	0.703	0.623	0.552	0.49	0.435	0.387	0.344	0.306
(0.7,0.3,0.1)	1	0.833	0.698	0.587	0.496	0.419	0.354	0.299	0.253	0.213	0.179
(0.7,0.3,0.2)	1	0.962	0.906	0.84	0.769	0.697	0.626	0.558	0.495	0.436	0.383
(0.7,0.3,0.3)	1	0.982	0.954	0.916	0.869	0.817	0.76	0.702	0.644	0.586	0.53
(0.7,0.3,0.4)	1	0.985	0.963	0.934	0.899	0.858	0.813	0.764	0.714	0.663	0.613
(0.7,0.3,0.5)	1	0.986	0.965	0.939	0.907	0.87	0.83	0.787	0.742	0.696	0.65
(0.7,0.3,0.6)	1	0.986	0.965	0.939	0.908	0.873	0.834	0.793	0.75	0.706	0.662
(0.7,0.3,0.7)	1	0.985	0.964	0.937	0.904	0.867	0.827	0.785	0.742	0.698	0.654
(0.7,0.3,0.8)	1	0.98	0.951	0.915	0.874	0.83	0.784	0.738	0.691	0.645	0.601
(0.7,0.3,0.9)	1	0.909	0.832	0.764	0.702	0.647	0.596	0.548	0.505	0.464	0.427
(0.7,0.4,0.1)	1	0.835	0.704	0.597	0.51	0.437	0.376	0.324	0.279	0.24	0.207
(0.7,0.4,0.2)	1	0.965	0.915	0.856	0.794	0.73	0.667	0.606	0.548	0.493	0.441
(0.7,0.4,0.3)	1	0.985	0.963	0.934	0.899	0.858	0.813	0.764	0.714	0.663	0.613
(0.7,0.4,0.4)	1	0.988	0.973	0.954	0.931	0.903	0.87	0.834	0.794	0.752	0.709
(0.7,0.4,0.5)	1	0.989	0.975	0.959	0.939	0.916	0.889	0.859	0.826	0.791	0.753
(0.7,0.4,0.6)	1	0.989	0.975	0.959	0.941	0.919	0.894	0.866	0.835	0.803	0.768
(0.7,0.4,0.7)	1	0.988	0.974	0.957	0.936	0.913	0.886	0.857	0.826	0.793	0.758
(0.7,0.4,0.8)	1	0.983	0.96	0.934	0.905	0.873	0.84	0.805	0.769	0.733	0.696
(0.7,0.4,0.9)	1	0.912	0.84	0.779	0.726	0.679	0.637	0.598	0.561	0.527	0.495
(0.7,0.5,0.1)	1	0.835	0.705	0.599	0.513	0.442	0.383	0.332	0.289	0.252	0.219
(0.7,0.5,0.2)	1	0.965	0.916	0.86	0.8	0.739	0.68	0.623	0.568	0.516	0.468
(0.7,0.5,0.3)	1	0.986	0.965	0.939	0.907	0.87	0.83	0.787	0.742	0.696	0.65
(0.7,0.5,0.4)	1	0.989	0.975	0.959	0.939	0.916	0.889	0.859	0.826	0.791	0.753
(0.7,0.5,0.5)	1	0.989	0.977	0.963	0.948	0.93	0.909	0.886	0.86	0.832	0.801
(0.7,0.5,0.6)	1	0.989	0.977	0.964	0.949	0.933	0.914	0.893	0.87	0.845	0.817
(0.7,0.5,0.7)	1	0.989	0.976	0.961	0.945	0.927	0.906	0.884	0.86	0.834	0.807
(0.7,0.5,0.8)	1	0.983	0.962	0.938	0.913	0.886	0.858	0.83	0.801	0.771	0.741
(0.7,0.5,0.9)	1	0.912	0.841	0.782	0.732	0.689	0.65	0.616	0.584	0.554	0.526
(0.7,0.6,0.1)	1	0.835	0.705	0.6	0.514	0.443	0.384	0.334	0.292	0.255	0.223
(0.7,0.6,0.2)	1	0.965	0.917	0.86	0.801	0.741	0.683	0.627	0.574	0.523	0.476
(0.7,0.6,0.3)	1	0.986	0.965	0.939	0.908	0.873	0.834	0.793	0.75	0.706	0.662
(0.7,0.6,0.4)	1	0.989	0.975	0.959	0.941	0.919	0.894	0.866	0.835	0.803	0.768
(0.7,0.6,0.5)	1	0.989	0.977	0.964	0.949	0.933	0.914	0.893	0.87	0.845	0.817
(0.7,0.6,0.6)	1	0.989	0.978	0.965	0.951	0.935	0.918	0.9	0.88	0.858	0.834
(0.7,0.6,0.7)	1	0.989	0.976	0.962	0.946	0.929	0.911	0.891	0.87	0.847	0.823
(0.7,0.6,0.8)	1	0.983	0.962	0.939	0.914	0.889	0.862	0.836	0.81	0.783	0.756
(0.7,0.6,0.9)	1	0.912	0.841	0.782	0.733	0.69	0.653	0.62	0.59	0.563	0.537
(0.7,0.7,0.1)	1	0.835	0.704	0.598	0.512	0.441	0.381	0.331	0.289	0.252	0.22
(0.7,0.7,0.2)	1	0.965	0.915	0.858	0.798	0.737	0.678	0.621	0.567	0.517	0.47
(0.7,0.7,0.3)	1	0.985	0.964	0.937	0.904	0.867	0.827	0.785	0.742	0.698	0.654
(0.7,0.7,0.4)	1	0.988	0.974	0.957	0.936	0.913	0.886	0.857	0.826	0.793	0.758
(0.7,0.7,0.5)	1	0.989	0.976	0.961	0.945	0.927	0.906	0.884	0.86	0.834	0.807

(0.7,0.7,0.6)	1	0.989	0.976	0.962	0.946	0.929	0.911	0.891	0.87	0.847	0.823
(0.7,0.7,0.7)	1	0.988	0.975	0.959	0.942	0.923	0.903	0.882	0.86	0.837	0.813
(0.7,0.7,0.8)	1	0.983	0.961	0.937	0.91	0.883	0.855	0.828	0.8	0.773	0.746
(0.7,0.7,0.9)	1	0.912	0.84	0.78	0.73	0.686	0.648	0.614	0.584	0.556	0.53
(0.7,0.8,0.1)	1	0.831	0.696	0.586	0.497	0.423	0.363	0.312	0.269	0.233	0.202
(0.7,0.8,0.2)	1	0.96	0.903	0.839	0.772	0.706	0.643	0.584	0.529	0.478	0.432
(0.7,0.8,0.3)	1	0.98	0.951	0.915	0.874	0.83	0.784	0.738	0.691	0.645	0.601
(0.7,0.8,0.4)	1	0.983	0.96	0.934	0.905	0.873	0.84	0.805	0.769	0.733	0.696
(0.7,0.8,0.5)	1	0.983	0.962	0.938	0.913	0.886	0.858	0.83	0.801	0.771	0.741
(0.7,0.8,0.6)	1	0.983	0.962	0.939	0.914	0.889	0.862	0.836	0.81	0.783	0.756
(0.7,0.8,0.7)	1	0.983	0.961	0.937	0.91	0.883	0.855	0.828	0.8	0.773	0.746
(0.7,0.8,0.8)	1	0.977	0.948	0.915	0.88	0.845	0.81	0.777	0.745	0.715	0.685
(0.7,0.8,0.9)	1	0.907	0.829	0.763	0.706	0.657	0.614	0.577	0.543	0.513	0.486
(0.7,0.9,0.1)	1	0.778	0.616	0.495	0.404	0.333	0.277	0.232	0.196	0.166	0.141
(0.7,0.9,0.2)	1	0.892	0.793	0.703	0.623	0.552	0.49	0.435	0.387	0.344	0.306
(0.7,0.9,0.3)	1	0.909	0.832	0.764	0.702	0.647	0.596	0.548	0.505	0.464	0.427
(0.7,0.9,0.4)	1	0.912	0.84	0.779	0.726	0.679	0.637	0.598	0.561	0.527	0.495
(0.7,0.9,0.5)	1	0.912	0.841	0.782	0.732	0.689	0.65	0.616	0.584	0.554	0.526
(0.7,0.9,0.6)	1	0.912	0.841	0.782	0.733	0.69	0.653	0.62	0.59	0.563	0.537
(0.7,0.9,0.7)	1	0.912	0.84	0.78	0.73	0.686	0.648	0.614	0.584	0.556	0.53
(0.7,0.9,0.8)	1	0.907	0.829	0.763	0.706	0.657	0.614	0.577	0.543	0.513	0.486
(0.7,0.9,0.9)	1	0.845	0.729	0.639	0.569	0.512	0.467	0.428	0.396	0.368	0.344
(0.8,0.1,0.1)	1	0.718	0.523	0.384	0.286	0.214	0.161	0.122	0.092	0.069	0.051
(0.8,0.1,0.2)	1	0.816	0.662	0.535	0.432	0.348	0.281	0.226	0.182	0.146	0.116
(0.8,0.1,0.3)	1	0.83	0.69	0.576	0.481	0.403	0.337	0.282	0.236	0.197	0.164
(0.8,0.1,0.4)	1	0.831	0.696	0.585	0.495	0.42	0.358	0.305	0.261	0.222	0.19
(0.8,0.1,0.5)	1	0.832	0.696	0.587	0.498	0.425	0.364	0.313	0.27	0.233	0.201
(0.8,0.1,0.6)	1	0.832	0.697	0.587	0.499	0.426	0.365	0.315	0.272	0.236	0.205
(0.8,0.1,0.7)	1	0.831	0.696	0.586	0.497	0.423	0.363	0.312	0.269	0.233	0.202
(0.8,0.1,0.8)	1	0.828	0.688	0.574	0.482	0.407	0.345	0.294	0.251	0.215	0.185
(0.8,0.1,0.9)	1	0.775	0.61	0.486	0.392	0.32	0.264	0.219	0.183	0.154	0.129
(0.8,0.2,0.1)	1	0.816	0.662	0.535	0.432	0.348	0.281	0.226	0.182	0.146	0.116
(0.8,0.2,0.2)	1	0.939	0.853	0.758	0.663	0.574	0.493	0.42	0.357	0.301	0.253
(0.8,0.2,0.3)	1	0.957	0.895	0.822	0.745	0.668	0.595	0.526	0.462	0.404	0.352
(0.8,0.2,0.4)	1	0.96	0.903	0.837	0.768	0.7	0.633	0.57	0.511	0.456	0.406
(0.8,0.2,0.5)	1	0.96	0.904	0.841	0.774	0.709	0.646	0.586	0.53	0.478	0.43
(0.8,0.2,0.6)	1	0.96	0.904	0.841	0.775	0.711	0.648	0.59	0.535	0.484	0.438
(0.8,0.2,0.7)	1	0.96	0.903	0.839	0.772	0.706	0.643	0.584	0.529	0.478	0.432
(0.8,0.2,0.8)	1	0.955	0.892	0.82	0.748	0.677	0.611	0.55	0.494	0.443	0.397
(0.8,0.2,0.9)	1	0.888	0.783	0.688	0.604	0.53	0.466	0.41	0.361	0.318	0.281
(0.8,0.3,0.1)	1	0.83	0.69	0.576	0.481	0.403	0.337	0.282	0.236	0.197	0.164
(0.8,0.3,0.2)	1	0.957	0.895	0.822	0.745	0.668	0.595	0.526	0.462	0.404	0.352
(0.8,0.3,0.3)	1	0.977	0.941	0.895	0.841	0.783	0.722	0.661	0.601	0.543	0.488
(0.8,0.3,0.4)	1	0.98	0.95	0.913	0.869	0.822	0.771	0.719	0.666	0.614	0.563
(0.8,0.3,0.5)	1	0.98	0.952	0.917	0.877	0.833	0.787	0.74	0.692	0.644	0.597
(0.8,0.3,0.6)	1	0.98	0.952	0.917	0.878	0.835	0.79	0.745	0.699	0.653	0.609
(0.8,0.3,0.7)	1	0.98	0.951	0.915	0.874	0.83	0.784	0.738	0.691	0.645	0.601
(0.8,0.3,0.8)	1	0.974	0.938	0.894	0.845	0.795	0.744	0.693	0.644	0.597	0.552
(0.8,0.3,0.9)	1	0.905	0.821	0.747	0.68	0.62	0.565	0.516	0.471	0.429	0.392
(0.8,0.4,0.1)	1	0.831	0.696	0.585	0.495	0.42	0.358	0.305	0.261	0.222	0.19
(0.8,0.4,0.2)	1	0.96	0.903	0.837	0.768	0.7	0.633	0.57	0.511	0.456	0.406
(0.8,0.4,0.3)	1	0.98	0.95	0.913	0.869	0.822	0.771	0.719	0.666	0.614	0.563
(0.8,0.4,0.4)	1	0.983	0.96	0.932	0.9	0.864	0.825	0.783	0.74	0.696	0.651
(0.8,0.4,0.5)	1	0.983	0.961	0.936	0.907	0.876	0.842	0.807	0.77	0.731	0.692
(0.8,0.4,0.6)	1	0.983	0.962	0.937	0.909	0.879	0.847	0.813	0.778	0.742	0.705

(0.8,0.4,0.7)	1	0.983	0.96	0.934	0.905	0.873	0.84	0.805	0.769	0.733	0.696
(0.8,0.4,0.8)	1	0.977	0.947	0.912	0.875	0.836	0.796	0.756	0.717	0.678	0.64
(0.8,0.4,0.9)	1	0.907	0.829	0.761	0.702	0.65	0.604	0.562	0.523	0.487	0.454
(0.8,0.5,0.1)	1	0.832	0.696	0.587	0.498	0.425	0.364	0.313	0.27	0.233	0.201
(0.8,0.5,0.2)	1	0.96	0.904	0.841	0.774	0.709	0.646	0.586	0.53	0.478	0.43
(0.8,0.5,0.3)	1	0.98	0.952	0.917	0.877	0.833	0.787	0.74	0.692	0.644	0.597
(0.8,0.5,0.4)	1	0.983	0.961	0.936	0.907	0.876	0.842	0.807	0.77	0.731	0.692
(0.8,0.5,0.5)	1	0.984	0.963	0.94	0.916	0.889	0.861	0.832	0.801	0.769	0.736
(0.8,0.5,0.6)	1	0.984	0.964	0.941	0.917	0.892	0.865	0.838	0.81	0.781	0.751
(0.8,0.5,0.7)	1	0.983	0.962	0.938	0.913	0.886	0.858	0.83	0.801	0.771	0.741
(0.8,0.5,0.8)	1	0.978	0.949	0.916	0.882	0.848	0.813	0.779	0.746	0.713	0.68
(0.8,0.5,0.9)	1	0.908	0.83	0.764	0.708	0.659	0.616	0.578	0.544	0.512	0.483
(0.8,0.6,0.1)	1	0.832	0.697	0.587	0.499	0.426	0.365	0.315	0.272	0.236	0.205
(0.8,0.6,0.2)	1	0.96	0.904	0.841	0.775	0.711	0.648	0.59	0.535	0.484	0.438
(0.8,0.6,0.3)	1	0.98	0.952	0.917	0.878	0.835	0.79	0.745	0.699	0.653	0.609
(0.8,0.6,0.4)	1	0.983	0.962	0.937	0.909	0.879	0.847	0.813	0.778	0.742	0.705
(0.8,0.6,0.5)	1	0.984	0.964	0.941	0.917	0.892	0.865	0.838	0.81	0.781	0.751
(0.8,0.6,0.6)	1	0.984	0.964	0.942	0.918	0.894	0.87	0.844	0.819	0.793	0.766
(0.8,0.6,0.7)	1	0.983	0.962	0.939	0.914	0.889	0.862	0.836	0.81	0.783	0.756
(0.8,0.6,0.8)	1	0.978	0.949	0.917	0.884	0.85	0.817	0.785	0.754	0.723	0.694
(0.8,0.6,0.9)	1	0.908	0.83	0.765	0.709	0.661	0.619	0.582	0.55	0.52	0.493
(0.8,0.7,0.1)	1	0.831	0.696	0.586	0.497	0.423	0.363	0.312	0.269	0.233	0.202
(0.8,0.7,0.2)	1	0.96	0.903	0.839	0.772	0.706	0.643	0.584	0.529	0.478	0.432
(0.8,0.7,0.3)	1	0.98	0.951	0.915	0.874	0.83	0.784	0.738	0.691	0.645	0.601
(0.8,0.7,0.4)	1	0.983	0.96	0.934	0.905	0.873	0.84	0.805	0.769	0.733	0.696
(0.8,0.7,0.5)	1	0.983	0.962	0.938	0.913	0.886	0.858	0.83	0.801	0.771	0.741
(0.8,0.7,0.6)	1	0.983	0.962	0.939	0.914	0.889	0.862	0.836	0.81	0.783	0.756
(0.8,0.7,0.7)	1	0.983	0.961	0.937	0.91	0.883	0.855	0.828	0.8	0.773	0.746
(0.8,0.7,0.8)	1	0.977	0.948	0.915	0.88	0.845	0.81	0.777	0.745	0.715	0.685
(0.8,0.7,0.9)	1	0.907	0.829	0.763	0.706	0.657	0.614	0.577	0.543	0.513	0.486
(0.8,0.8,0.1)	1	0.828	0.688	0.574	0.482	0.407	0.345	0.294	0.251	0.215	0.185
(0.8,0.8,0.2)	1	0.955	0.892	0.82	0.748	0.677	0.611	0.55	0.494	0.443	0.397
(0.8,0.8,0.3)	1	0.974	0.938	0.894	0.845	0.795	0.744	0.693	0.644	0.597	0.552
(0.8,0.8,0.4)	1	0.977	0.947	0.912	0.875	0.836	0.796	0.756	0.717	0.678	0.64
(0.8,0.8,0.5)	1	0.978	0.949	0.916	0.882	0.848	0.813	0.779	0.746	0.713	0.68
(0.8,0.8,0.6)	1	0.978	0.949	0.917	0.884	0.85	0.817	0.785	0.754	0.723	0.694
(0.8,0.8,0.7)	1	0.977	0.948	0.915	0.88	0.845	0.81	0.777	0.745	0.715	0.685
(0.8,0.8,0.8)	1	0.972	0.935	0.893	0.85	0.808	0.768	0.73	0.694	0.661	0.629
(0.8,0.8,0.9)	1	0.903	0.819	0.746	0.683	0.63	0.583	0.542	0.506	0.475	0.446
(0.8,0.9,0.1)	1	0.775	0.61	0.486	0.392	0.32	0.264	0.219	0.183	0.154	0.129
(0.8,0.9,0.2)	1	0.888	0.783	0.688	0.604	0.53	0.466	0.41	0.361	0.318	0.281
(0.8,0.9,0.3)	1	0.905	0.821	0.747	0.68	0.62	0.565	0.516	0.471	0.429	0.392
(0.8,0.9,0.4)	1	0.907	0.829	0.761	0.702	0.65	0.604	0.562	0.523	0.487	0.454
(0.8,0.9,0.5)	1	0.908	0.83	0.764	0.708	0.659	0.616	0.578	0.544	0.512	0.483
(0.8,0.9,0.6)	1	0.908	0.83	0.765	0.709	0.661	0.619	0.582	0.55	0.52	0.493
(0.8,0.9,0.7)	1	0.907	0.829	0.763	0.706	0.657	0.614	0.577	0.543	0.513	0.486
(0.8,0.9,0.8)	1	0.903	0.819	0.746	0.683	0.63	0.583	0.542	0.506	0.475	0.446
(0.8,0.9,0.9)	1	0.841	0.72	0.626	0.551	0.491	0.443	0.403	0.369	0.34	0.316
(0.9,0.1,0.1)	1	0.677	0.468	0.329	0.235	0.17	0.124	0.09	0.066	0.047	0.033
(0.9,0.1,0.2)	1	0.765	0.588	0.454	0.353	0.275	0.216	0.169	0.132	0.103	0.08
(0.9,0.1,0.3)	1	0.777	0.612	0.487	0.392	0.317	0.258	0.211	0.172	0.14	0.114
(0.9,0.1,0.4)	1	0.778	0.616	0.495	0.402	0.33	0.273	0.227	0.19	0.159	0.133
(0.9,0.1,0.5)	1	0.778	0.617	0.496	0.405	0.334	0.278	0.233	0.197	0.166	0.141
(0.9,0.1,0.6)	1	0.778	0.617	0.496	0.405	0.335	0.279	0.235	0.198	0.168	0.143
(0.9,0.1,0.7)	1	0.778	0.616	0.495	0.404	0.333	0.277	0.232	0.196	0.166	0.141

(0.9,0.1,0.8)	1	0.775	0.61	0.486	0.392	0.32	0.264	0.219	0.183	0.154	0.129
(0.9,0.1,0.9)	1	0.728	0.542	0.413	0.32	0.253	0.202	0.163	0.133	0.109	0.09
(0.9,0.2,0.1)	1	0.765	0.588	0.454	0.353	0.275	0.216	0.169	0.132	0.103	0.08
(0.9,0.2,0.2)	1	0.875	0.751	0.638	0.538	0.451	0.377	0.314	0.261	0.216	0.178
(0.9,0.2,0.3)	1	0.89	0.786	0.689	0.602	0.524	0.454	0.392	0.338	0.291	0.249
(0.9,0.2,0.4)	1	0.892	0.792	0.702	0.62	0.547	0.483	0.425	0.374	0.328	0.287
(0.9,0.2,0.5)	1	0.893	0.794	0.704	0.625	0.554	0.492	0.437	0.387	0.344	0.304
(0.9,0.2,0.6)	1	0.893	0.794	0.705	0.625	0.555	0.494	0.439	0.391	0.348	0.31
(0.9,0.2,0.7)	1	0.892	0.793	0.703	0.623	0.552	0.49	0.435	0.387	0.344	0.306
(0.9,0.2,0.8)	1	0.888	0.783	0.688	0.604	0.53	0.466	0.41	0.361	0.318	0.281
(0.9,0.2,0.9)	1	0.829	0.691	0.58	0.489	0.416	0.355	0.305	0.263	0.228	0.198
(0.9,0.3,0.1)	1	0.777	0.612	0.487	0.392	0.317	0.258	0.211	0.172	0.14	0.114
(0.9,0.3,0.2)	1	0.89	0.786	0.689	0.602	0.524	0.454	0.392	0.338	0.291	0.249
(0.9,0.3,0.3)	1	0.907	0.824	0.748	0.677	0.611	0.549	0.492	0.439	0.391	0.346
(0.9,0.3,0.4)	1	0.909	0.831	0.762	0.699	0.64	0.586	0.535	0.487	0.442	0.4
(0.9,0.3,0.5)	1	0.91	0.833	0.765	0.704	0.649	0.598	0.55	0.505	0.464	0.424
(0.9,0.3,0.6)	1	0.91	0.833	0.766	0.705	0.65	0.6	0.554	0.51	0.47	0.432
(0.9,0.3,0.7)	1	0.909	0.832	0.764	0.702	0.647	0.596	0.548	0.505	0.464	0.427
(0.9,0.3,0.8)	1	0.905	0.821	0.747	0.68	0.62	0.565	0.516	0.471	0.429	0.392
(0.9,0.3,0.9)	1	0.843	0.722	0.627	0.549	0.485	0.43	0.384	0.343	0.308	0.277
(0.9,0.4,0.1)	1	0.778	0.616	0.495	0.402	0.33	0.273	0.227	0.19	0.159	0.133
(0.9,0.4,0.2)	1	0.892	0.792	0.702	0.62	0.547	0.483	0.425	0.374	0.328	0.287
(0.9,0.4,0.3)	1	0.909	0.831	0.762	0.699	0.64	0.586	0.535	0.487	0.442	0.4
(0.9,0.4,0.4)	1	0.912	0.839	0.777	0.722	0.672	0.626	0.582	0.541	0.501	0.463
(0.9,0.4,0.5)	1	0.912	0.84	0.78	0.728	0.681	0.639	0.599	0.562	0.526	0.492
(0.9,0.4,0.6)	1	0.912	0.841	0.781	0.729	0.683	0.642	0.603	0.568	0.534	0.501
(0.9,0.4,0.7)	1	0.912	0.84	0.779	0.726	0.679	0.637	0.598	0.561	0.527	0.495
(0.9,0.4,0.8)	1	0.907	0.829	0.761	0.702	0.65	0.604	0.562	0.523	0.487	0.454
(0.9,0.4,0.9)	1	0.845	0.728	0.638	0.566	0.508	0.459	0.417	0.381	0.35	0.321
(0.9,0.5,0.1)	1	0.778	0.617	0.496	0.405	0.334	0.278	0.233	0.197	0.166	0.141
(0.9,0.5,0.2)	1	0.893	0.794	0.704	0.625	0.554	0.492	0.437	0.387	0.344	0.304
(0.9,0.5,0.3)	1	0.91	0.833	0.765	0.704	0.649	0.598	0.55	0.505	0.464	0.424
(0.9,0.5,0.4)	1	0.912	0.84	0.78	0.728	0.681	0.639	0.599	0.562	0.526	0.492
(0.9,0.5,0.5)	1	0.913	0.842	0.784	0.734	0.691	0.652	0.617	0.584	0.553	0.523
(0.9,0.5,0.6)	1	0.913	0.842	0.784	0.735	0.693	0.655	0.622	0.59	0.561	0.533
(0.9,0.5,0.7)	1	0.912	0.841	0.782	0.732	0.689	0.65	0.616	0.584	0.554	0.526
(0.9,0.5,0.8)	1	0.908	0.83	0.764	0.708	0.659	0.616	0.578	0.544	0.512	0.483
(0.9,0.5,0.9)	1	0.846	0.729	0.64	0.57	0.514	0.468	0.429	0.396	0.368	0.342
(0.9,0.6,0.1)	1	0.778	0.617	0.496	0.405	0.335	0.279	0.235	0.198	0.168	0.143
(0.9,0.6,0.2)	1	0.893	0.794	0.705	0.625	0.555	0.494	0.439	0.391	0.348	0.31
(0.9,0.6,0.3)	1	0.91	0.833	0.766	0.705	0.65	0.6	0.554	0.51	0.47	0.432
(0.9,0.6,0.4)	1	0.912	0.841	0.781	0.729	0.683	0.642	0.603	0.568	0.534	0.501
(0.9,0.6,0.5)	1	0.913	0.842	0.784	0.735	0.693	0.655	0.622	0.59	0.561	0.533
(0.9,0.6,0.6)	1	0.913	0.842	0.784	0.736	0.695	0.659	0.626	0.597	0.57	0.544
(0.9,0.6,0.7)	1	0.912	0.841	0.782	0.733	0.69	0.653	0.62	0.59	0.563	0.537
(0.9,0.6,0.8)	1	0.908	0.83	0.765	0.709	0.661	0.619	0.582	0.55	0.52	0.493
(0.9,0.6,0.9)	1	0.846	0.73	0.641	0.571	0.515	0.47	0.432	0.401	0.373	0.349
(0.9,0.7,0.1)	1	0.778	0.616	0.495	0.404	0.333	0.277	0.232	0.196	0.166	0.141
(0.9,0.7,0.2)	1	0.892	0.793	0.703	0.623	0.552	0.49	0.435	0.387	0.344	0.306
(0.9,0.7,0.3)	1	0.909	0.832	0.764	0.702	0.647	0.596	0.548	0.505	0.464	0.427
(0.9,0.7,0.4)	1	0.912	0.84	0.779	0.726	0.679	0.637	0.598	0.561	0.527	0.495
(0.9,0.7,0.5)	1	0.912	0.841	0.782	0.732	0.689	0.65	0.616	0.584	0.554	0.526
(0.9,0.7,0.6)	1	0.912	0.841	0.782	0.733	0.69	0.653	0.62	0.59	0.563	0.537
(0.9,0.7,0.7)	1	0.912	0.84	0.78	0.73	0.686	0.648	0.614	0.584	0.556	0.53
(0.9,0.7,0.8)	1	0.907	0.829	0.763	0.706	0.657	0.614	0.577	0.543	0.513	0.486

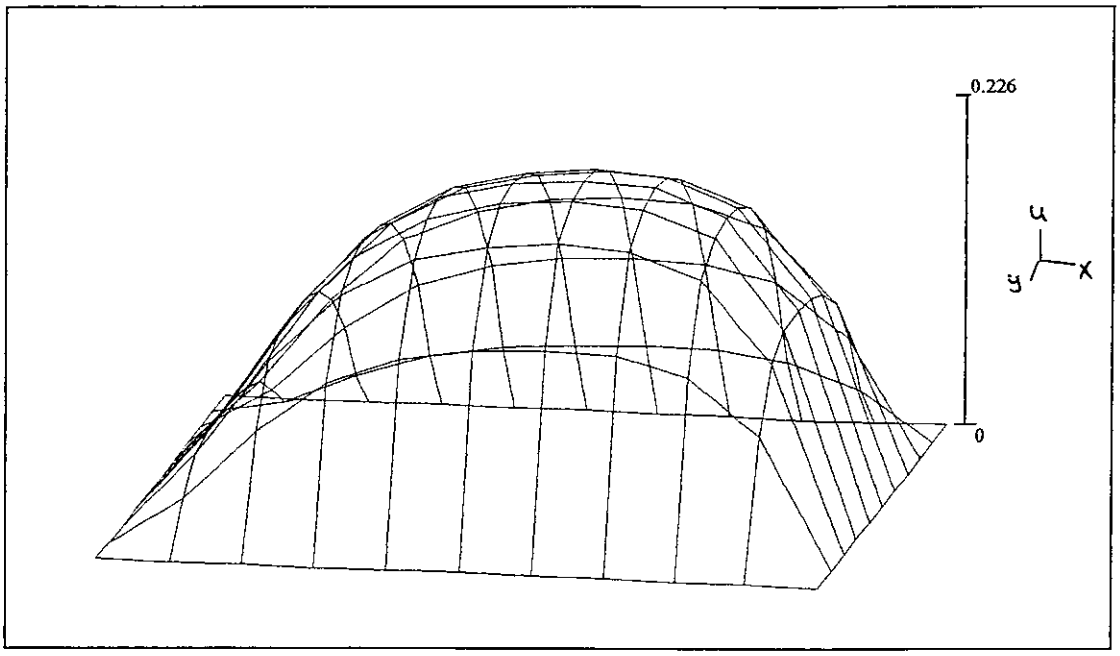
(0.9,0.7,0.9)	1	0.845	0.729	0.639	0.569	0.512	0.467	0.428	0.396	0.368	0.344
(0.9,0.8,0.1)	1	0.775	0.61	0.486	0.392	0.32	0.264	0.219	0.183	0.154	0.129
(0.9,0.8,0.2)	1	0.888	0.783	0.688	0.604	0.53	0.466	0.41	0.361	0.318	0.281
(0.9,0.8,0.3)	1	0.905	0.821	0.747	0.68	0.62	0.565	0.516	0.471	0.429	0.392
(0.9,0.8,0.4)	1	0.907	0.829	0.761	0.702	0.65	0.604	0.562	0.523	0.487	0.454
(0.9,0.8,0.5)	1	0.908	0.83	0.764	0.708	0.659	0.616	0.578	0.544	0.512	0.483
(0.9,0.8,0.6)	1	0.908	0.83	0.765	0.709	0.661	0.619	0.582	0.55	0.52	0.493
(0.9,0.8,0.7)	1	0.907	0.829	0.763	0.706	0.657	0.614	0.577	0.543	0.513	0.486
(0.9,0.8,0.8)	1	0.903	0.819	0.746	0.683	0.63	0.583	0.542	0.506	0.475	0.446
(0.9,0.8,0.9)	1	0.841	0.72	0.626	0.551	0.491	0.443	0.403	0.369	0.34	0.316
(0.9,0.9,0.1)	1	0.728	0.542	0.413	0.32	0.253	0.202	0.163	0.133	0.109	0.09
(0.9,0.9,0.2)	1	0.829	0.691	0.58	0.489	0.416	0.355	0.305	0.263	0.228	0.198
(0.9,0.9,0.3)	1	0.843	0.722	0.627	0.549	0.485	0.43	0.384	0.343	0.308	0.277
(0.9,0.9,0.4)	1	0.845	0.728	0.638	0.566	0.508	0.459	0.417	0.381	0.35	0.321
(0.9,0.9,0.5)	1	0.846	0.729	0.64	0.57	0.514	0.468	0.429	0.396	0.368	0.342
(0.9,0.9,0.6)	1	0.846	0.73	0.641	0.571	0.515	0.47	0.432	0.401	0.373	0.349
(0.9,0.9,0.7)	1	0.845	0.729	0.639	0.569	0.512	0.467	0.428	0.396	0.368	0.344
(0.9,0.9,0.8)	1	0.841	0.72	0.626	0.551	0.491	0.443	0.403	0.369	0.34	0.316
(0.9,0.9,0.9)	1	0.787	0.636	0.527	0.446	0.385	0.337	0.299	0.269	0.243	0.222

Result of 3dimension Implicit method using r=0.01

(x,y,z) n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.943	0.89	0.84	0.793	0.749	0.708	0.669	0.633	0.599	0.567
(0.1,0.1,0.2)	1	0.961	0.923	0.886	0.851	0.818	0.786	0.755	0.725	0.697	0.669
(0.1,0.1,0.3)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.68
(0.1,0.1,0.4)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.1,0.5)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.1,0.6)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.1,0.7)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.1,0.8)	1	0.961	0.923	0.888	0.853	0.821	0.79	0.76	0.731	0.704	0.678
(0.1,0.1,0.9)	1	0.952	0.907	0.864	0.823	0.785	0.749	0.715	0.683	0.653	0.624
(0.1,0.2,0.1)	1	0.961	0.923	0.886	0.851	0.818	0.786	0.755	0.725	0.697	0.669
(0.1,0.2,0.2)	1	0.979	0.958	0.937	0.915	0.894	0.873	0.853	0.832	0.812	0.792
(0.1,0.2,0.3)	1	0.979	0.959	0.938	0.918	0.899	0.879	0.86	0.841	0.823	0.805
(0.1,0.2,0.4)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.805
(0.1,0.2,0.5)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.806
(0.1,0.2,0.6)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.806
(0.1,0.2,0.7)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.823	0.805
(0.1,0.2,0.8)	1	0.979	0.958	0.938	0.918	0.898	0.878	0.859	0.839	0.821	0.802
(0.1,0.2,0.9)	1	0.97	0.941	0.912	0.885	0.858	0.833	0.808	0.783	0.76	0.737
(0.1,0.3,0.1)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.68
(0.1,0.3,0.2)	1	0.979	0.959	0.938	0.918	0.899	0.879	0.86	0.841	0.823	0.805
(0.1,0.3,0.3)	1	0.98	0.96	0.94	0.921	0.903	0.885	0.868	0.851	0.834	0.817
(0.1,0.3,0.4)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.3,0.5)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.3,0.6)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.3,0.7)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.3,0.8)	1	0.979	0.959	0.94	0.921	0.902	0.884	0.866	0.849	0.832	0.815
(0.1,0.3,0.9)	1	0.97	0.942	0.914	0.888	0.862	0.838	0.815	0.792	0.77	0.749
(0.1,0.4,0.1)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.4,0.2)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.805
(0.1,0.4,0.3)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.4,0.4)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.4,0.5)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.4,0.6)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.4,0.7)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.4,0.8)	1	0.979	0.959	0.94	0.921	0.902	0.884	0.867	0.849	0.832	0.816
(0.1,0.4,0.9)	1	0.97	0.942	0.914	0.888	0.863	0.838	0.815	0.792	0.771	0.75
(0.1,0.5,0.1)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.5,0.2)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.806
(0.1,0.5,0.3)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.5,0.4)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.5,0.5)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82
(0.1,0.5,0.6)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82
(0.1,0.5,0.7)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.5,0.8)	1	0.979	0.959	0.94	0.921	0.902	0.884	0.867	0.849	0.832	0.816
(0.1,0.5,0.9)	1	0.97	0.942	0.914	0.888	0.863	0.838	0.815	0.793	0.771	0.75
(0.1,0.6,0.1)	1	0.961	0.924	0.888	0.854	0.822	0.791	0.761	0.733	0.706	0.681
(0.1,0.6,0.2)	1	0.979	0.959	0.938	0.919	0.899	0.88	0.861	0.842	0.824	0.806
(0.1,0.6,0.3)	1	0.98	0.96	0.94	0.922	0.903	0.885	0.868	0.851	0.835	0.818
(0.1,0.6,0.4)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.868	0.852	0.835	0.819
(0.1,0.6,0.5)	1	0.98	0.96	0.94	0.922	0.903	0.886	0.869	0.852	0.835	0.82

Result of 3dimension Implicict method using r=0.001

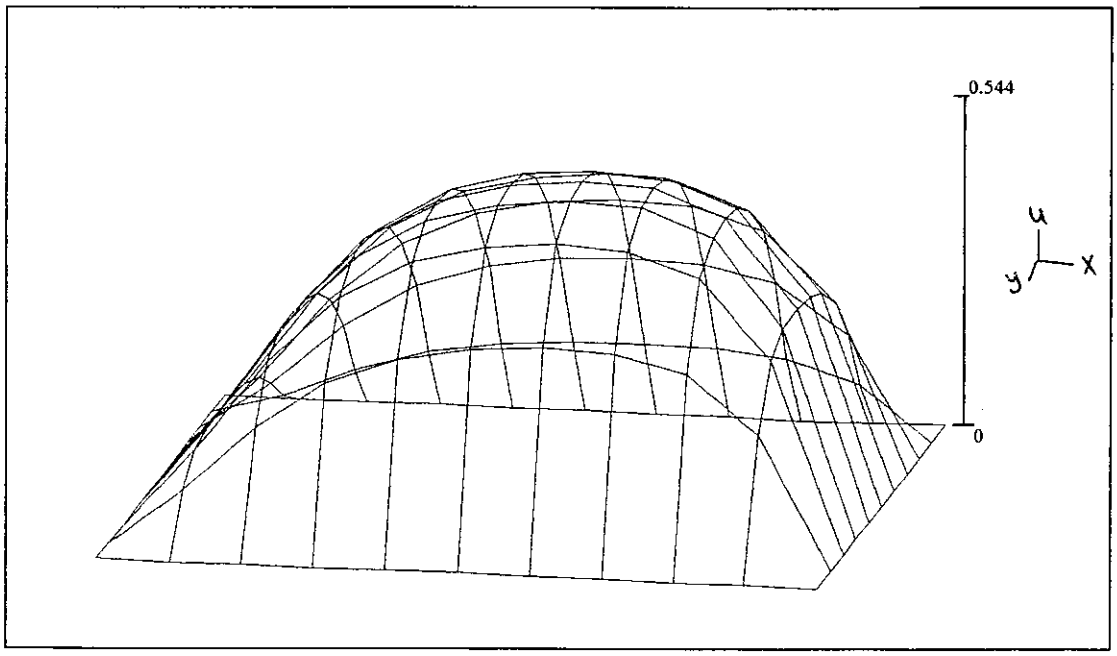
(x,y,z) n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.994	0.988	0.982	0.976	0.97	0.964	0.958	0.953	0.947	0.941
(0.1,0.1,0.2)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.3)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.4)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.5)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.6)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.7)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.8)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.1,0.9)	1	0.995	0.99	0.985	0.98	0.975	0.97	0.965	0.96	0.955	0.951
(0.1,0.2,0.1)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.2,0.2)	1	0.998	0.996	0.994	0.992	0.989	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.2,0.9)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.972	0.969
(0.1,0.3,0.1)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.3,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.3,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.3,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.3,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.3,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.3,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.3,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.3,0.9)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.4,0.1)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.4,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.4,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.4,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.4,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.4,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.4,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.4,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.4,0.9)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.5,0.1)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.5,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.5,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.5,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.5,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.5,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.5,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.5,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.5,0.9)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.6,0.1)	1	0.996	0.992	0.988	0.984	0.98	0.976	0.972	0.968	0.964	0.96
(0.1,0.6,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.987	0.985	0.983	0.981	0.979
(0.1,0.6,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.6,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.6,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979
(0.1,0.6,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.985	0.983	0.981	0.979



f

3dimension Implicit method

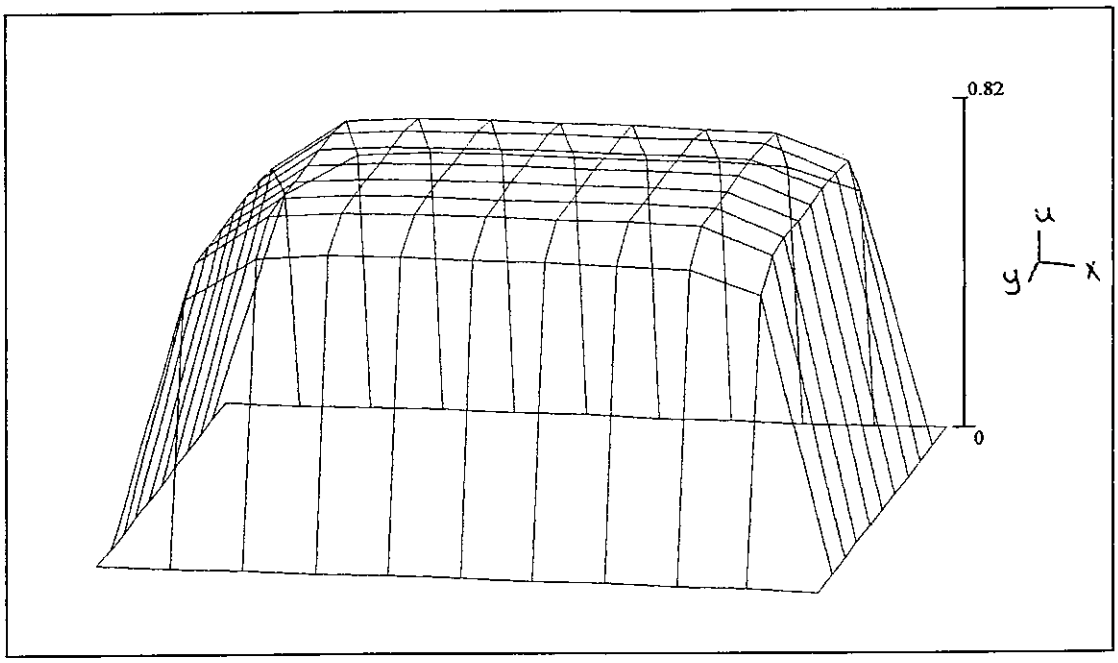
x=0.1, y & z are varible and r=0.1 at n=10



f2

3dimension Implicit method

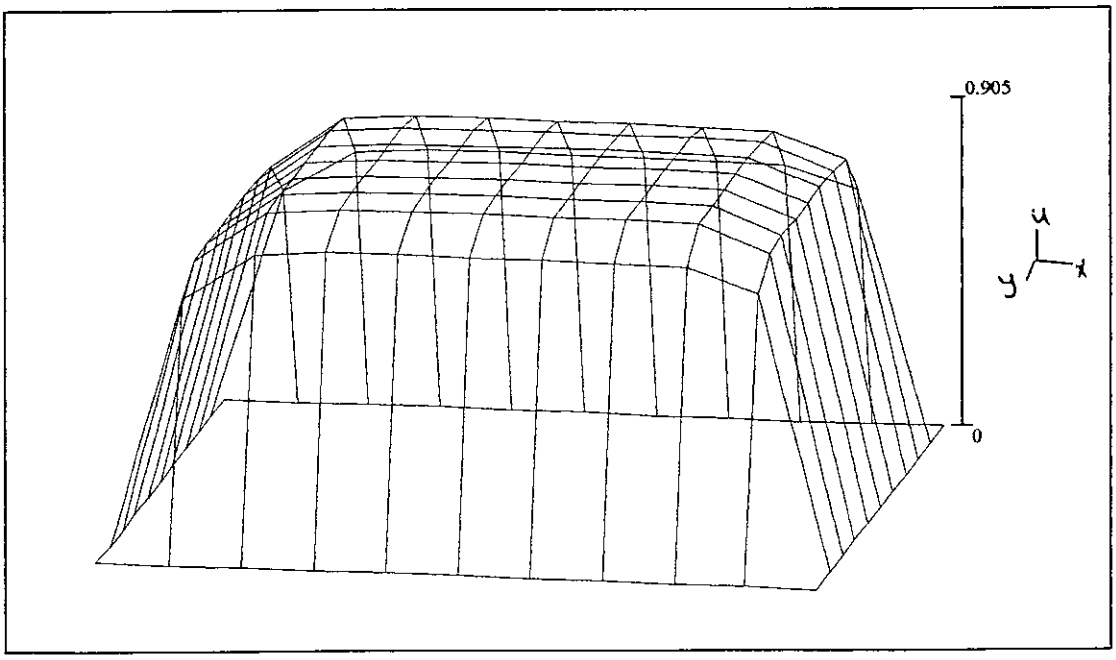
x=0.9, y & z are varible and r=0.1 at n=10



f

3dimension Implicit method

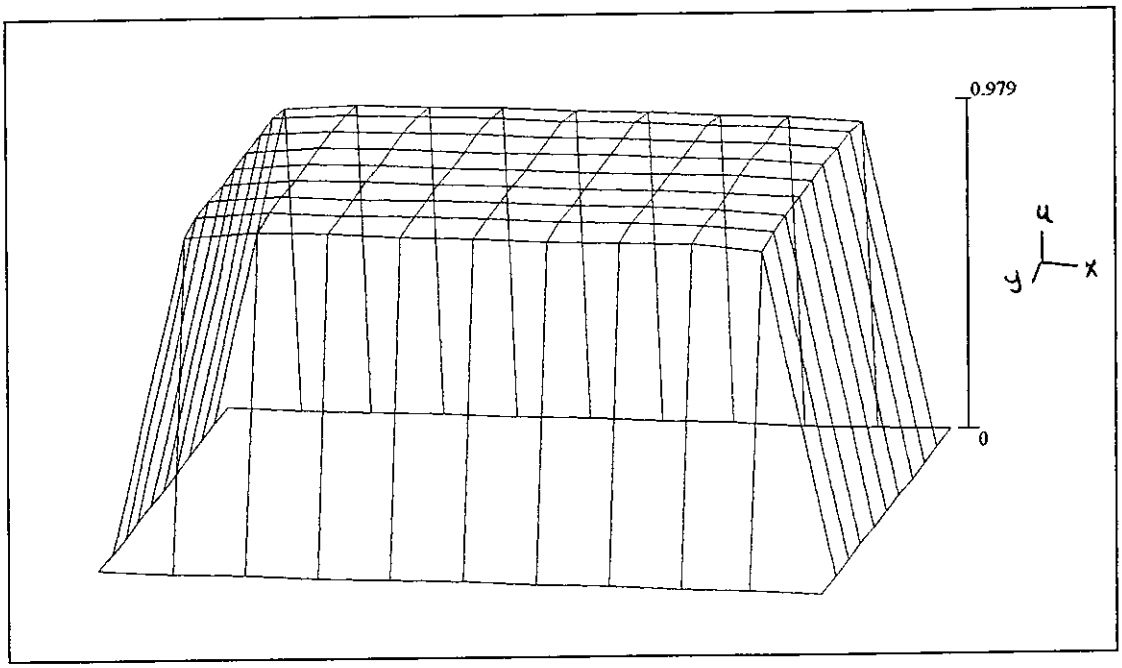
x=0.1, y & z are varible and r=0.01 at n=10



f2

3dimension Implicit method

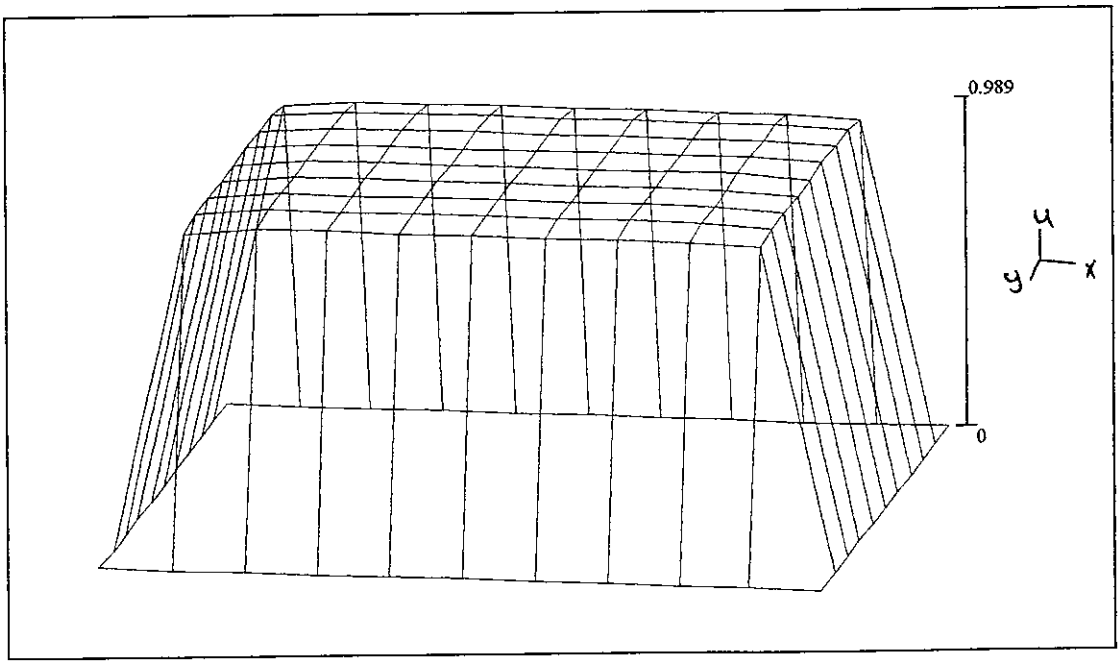
x=0.9, y & z are varible and r=0.01 at n=10



f

3dimension Implicit method

x=0.1, y & z are varible and r=0.001at n=10



f2

3dimension Implicit method

x=0.9, y & z are varible and r=0.001 at n=10

Result of 3dimension Crank-Nicolson method using r=0.1

(x,y,z) \ n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.774	0.563	0.407	0.292	0.207	0.144	0.098	0.063	0.038	0.019
(0.1,0.1,0.2)	1	0.764	0.547	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.3)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.4)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.5)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.6)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.7)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.8)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.1,0.9)	1	0.763	0.546	0.388	0.273	0.189	0.128	0.084	0.052	0.028	0.011
(0.1,0.2,0.1)	1	0.851	0.678	0.535	0.417	0.321	0.244	0.182	0.132	0.093	0.063
(0.1,0.2,0.2)	1	0.822	0.648	0.505	0.388	0.294	0.219	0.159	0.113	0.076	0.048
(0.1,0.2,0.3)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.2,0.4)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.2,0.5)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.2,0.6)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.2,0.7)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.2,0.8)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.2,0.9)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.1,0.3,0.1)	1	0.854	0.689	0.553	0.441	0.349	0.274	0.212	0.162	0.12	0.087
(0.1,0.3,0.2)	1	0.825	0.658	0.521	0.41	0.32	0.247	0.187	0.139	0.1	0.069
(0.1,0.3,0.3)	1	0.824	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.069
(0.1,0.3,0.4)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.1,0.3,0.5)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.1,0.3,0.6)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.1,0.3,0.7)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.1,0.3,0.8)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.1,0.3,0.9)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.1,0.4,0.1)	1	0.854	0.69	0.555	0.445	0.354	0.28	0.219	0.169	0.129	0.095
(0.1,0.4,0.2)	1	0.825	0.658	0.523	0.413	0.325	0.252	0.194	0.147	0.108	0.077
(0.1,0.4,0.3)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.4,0.4)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.4,0.5)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.4,0.6)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.4,0.7)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.4,0.8)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.4,0.9)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.1,0.5,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.22	0.171	0.13	0.097
(0.1,0.5,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.5,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.6,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.221	0.171	0.131	0.098
(0.1,0.6,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.6,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.1,0.6,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079

(0.1,0.6,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.1,0.6,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.1,0.6,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.1,0.6,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.1,0.6,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.1,0.7,0.1)	1	0.833	0.674	0.543	0.436	0.348	0.276	0.217	0.169	0.129	0.096
(0.1,0.7,0.2)	1	0.846	0.674	0.535	0.423	0.332	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.3)	1	0.846	0.674	0.534	0.422	0.331	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.4)	1	0.846	0.674	0.534	0.422	0.331	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.5)	1	0.846	0.674	0.534	0.422	0.331	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.6)	1	0.846	0.674	0.534	0.422	0.331	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.7)	1	0.846	0.674	0.534	0.422	0.331	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.8)	1	0.846	0.674	0.534	0.422	0.331	0.258	0.199	0.151	0.112	0.081
(0.1,0.7,0.9)	1	0.825	0.657	0.522	0.413	0.324	0.253	0.195	0.148	0.11	0.079
(0.1,0.8,0.1)	1	0.832	0.671	0.54	0.433	0.345	0.273	0.215	0.167	0.127	0.095
(0.1,0.8,0.2)	1	0.825	0.66	0.526	0.417	0.328	0.256	0.198	0.15	0.112	0.081
(0.1,0.8,0.3)	1	0.825	0.66	0.525	0.416	0.328	0.256	0.197	0.15	0.112	0.081
(0.1,0.8,0.4)	1	0.825	0.66	0.525	0.416	0.328	0.256	0.197	0.15	0.112	0.081
(0.1,0.8,0.5)	1	0.825	0.66	0.525	0.416	0.328	0.256	0.197	0.15	0.112	0.081
(0.1,0.8,0.6)	1	0.825	0.66	0.525	0.416	0.328	0.256	0.197	0.15	0.112	0.081
(0.1,0.8,0.7)	1	0.825	0.66	0.525	0.416	0.328	0.256	0.197	0.15	0.112	0.081
(0.1,0.8,0.8)	1	0.825	0.66	0.525	0.416	0.328	0.256	0.197	0.15	0.112	0.081
(0.1,0.8,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.1,0.9,0.1)	1	0.853	0.687	0.552	0.441	0.351	0.278	0.218	0.168	0.128	0.096
(0.1,0.9,0.2)	1	0.825	0.658	0.523	0.414	0.326	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.3)	1	0.824	0.657	0.523	0.414	0.325	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.4)	1	0.824	0.657	0.523	0.414	0.325	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.5)	1	0.824	0.657	0.523	0.414	0.325	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.6)	1	0.824	0.657	0.523	0.414	0.325	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.7)	1	0.824	0.657	0.523	0.414	0.325	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.8)	1	0.824	0.657	0.523	0.414	0.325	0.254	0.196	0.149	0.111	0.08
(0.1,0.9,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.2,0.1,0.1)	1	0.851	0.678	0.535	0.417	0.321	0.244	0.182	0.132	0.093	0.063
(0.2,0.1,0.2)	1	0.822	0.648	0.505	0.388	0.294	0.219	0.159	0.113	0.076	0.048
(0.2,0.1,0.3)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.1,0.4)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.1,0.5)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.1,0.6)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.1,0.7)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.1,0.8)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.1,0.9)	1	0.821	0.647	0.504	0.387	0.293	0.218	0.159	0.113	0.076	0.048
(0.2,0.2,0.1)	1	0.913	0.799	0.687	0.582	0.485	0.399	0.323	0.257	0.201	0.153
(0.2,0.2,0.2)	1	0.885	0.768	0.654	0.548	0.451	0.365	0.291	0.227	0.173	0.128
(0.2,0.2,0.3)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.2,0.4)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.2,0.5)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.2,0.6)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.2,0.7)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.2,0.8)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.2,0.9)	1	0.884	0.767	0.653	0.547	0.45	0.365	0.29	0.227	0.173	0.128
(0.2,0.3,0.1)	1	0.916	0.81	0.708	0.613	0.525	0.445	0.372	0.308	0.251	0.201
(0.2,0.3,0.2)	1	0.888	0.779	0.676	0.579	0.491	0.41	0.339	0.275	0.22	0.172
(0.2,0.3,0.3)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.2,0.3,0.4)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.2,0.3,0.5)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172

(0.2,0.3,0.6)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.2,0.3,0.7)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.2,0.3,0.8)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.2,0.3,0.9)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.2,0.4,0.1)	1	0.916	0.81	0.71	0.617	0.531	0.454	0.384	0.321	0.266	0.217
(0.2,0.4,0.2)	1	0.888	0.78	0.678	0.583	0.497	0.419	0.35	0.289	0.235	0.188
(0.2,0.4,0.3)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.4,0.4)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.4,0.5)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.4,0.6)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.4,0.7)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.4,0.8)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.4,0.9)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.2,0.5,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.269	0.221
(0.2,0.5,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.291	0.238	0.192
(0.2,0.5,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.5,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.5,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.5,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.5,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.5,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.5,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.2,0.6,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.27	0.222
(0.2,0.6,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.292	0.239	0.192
(0.2,0.6,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.6,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.6,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.6,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.6,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.6,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.6,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.7,0.1)	1	0.915	0.808	0.707	0.614	0.529	0.452	0.383	0.322	0.268	0.22
(0.2,0.7,0.2)	1	0.889	0.782	0.681	0.587	0.501	0.424	0.355	0.294	0.241	0.194
(0.2,0.7,0.3)	1	0.888	0.781	0.68	0.586	0.5	0.423	0.354	0.293	0.24	0.194
(0.2,0.7,0.4)	1	0.888	0.781	0.68	0.586	0.5	0.423	0.354	0.293	0.24	0.194
(0.2,0.7,0.5)	1	0.888	0.781	0.68	0.586	0.5	0.423	0.354	0.293	0.24	0.194
(0.2,0.7,0.6)	1	0.888	0.781	0.68	0.586	0.5	0.423	0.354	0.293	0.24	0.194
(0.2,0.7,0.7)	1	0.888	0.781	0.68	0.586	0.5	0.423	0.354	0.293	0.24	0.194
(0.2,0.7,0.8)	1	0.888	0.781	0.68	0.586	0.5	0.423	0.354	0.293	0.24	0.194
(0.2,0.7,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.2,0.8,0.1)	1	0.915	0.808	0.707	0.613	0.528	0.451	0.382	0.321	0.267	0.219
(0.2,0.8,0.2)	1	0.888	0.78	0.678	0.585	0.499	0.422	0.354	0.293	0.24	0.194
(0.2,0.8,0.3)	1	0.887	0.779	0.678	0.584	0.498	0.422	0.353	0.293	0.24	0.194
(0.2,0.8,0.4)	1	0.887	0.779	0.677	0.584	0.498	0.422	0.353	0.293	0.24	0.194
(0.2,0.8,0.5)	1	0.887	0.779	0.677	0.584	0.498	0.422	0.353	0.293	0.24	0.194
(0.2,0.8,0.6)	1	0.887	0.779	0.677	0.584	0.498	0.422	0.353	0.293	0.24	0.194
(0.2,0.8,0.7)	1	0.887	0.779	0.677	0.584	0.498	0.422	0.353	0.293	0.24	0.194
(0.2,0.8,0.8)	1	0.887	0.779	0.677	0.584	0.498	0.422	0.353	0.293	0.24	0.194
(0.2,0.8,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.239	0.192
(0.2,0.9,0.1)	1	0.916	0.81	0.709	0.616	0.531	0.453	0.384	0.322	0.268	0.22
(0.2,0.9,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.353	0.292	0.239	0.193
(0.2,0.9,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.421	0.352	0.292	0.239	0.193
(0.2,0.9,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.421	0.352	0.292	0.239	0.193
(0.2,0.9,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.421	0.352	0.292	0.239	0.193
(0.2,0.9,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.421	0.352	0.292	0.239	0.193

(0.2,0.9,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.421	0.352	0.292	0.239	0.193
(0.2,0.9,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.421	0.352	0.292	0.239	0.193
(0.2,0.9,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.239	0.192
(0.3,0.1,0.1)	1	0.854	0.689	0.553	0.441	0.349	0.274	0.212	0.162	0.12	0.087
(0.3,0.1,0.2)	1	0.825	0.658	0.521	0.41	0.32	0.247	0.187	0.139	0.1	0.069
(0.3,0.1,0.3)	1	0.824	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.069
(0.3,0.1,0.4)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.3,0.1,0.5)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.3,0.1,0.6)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.3,0.1,0.7)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.3,0.1,0.8)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.3,0.1,0.9)	1	0.823	0.656	0.52	0.41	0.32	0.246	0.187	0.139	0.101	0.07
(0.3,0.2,0.1)	1	0.916	0.81	0.708	0.613	0.525	0.445	0.372	0.308	0.251	0.201
(0.3,0.2,0.2)	1	0.888	0.779	0.676	0.579	0.491	0.41	0.339	0.275	0.22	0.172
(0.3,0.2,0.3)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.2,0.4)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.2,0.5)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.2,0.6)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.2,0.7)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.2,0.8)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.2,0.9)	1	0.887	0.778	0.674	0.578	0.49	0.41	0.338	0.275	0.22	0.172
(0.3,0.3,0.1)	1	0.919	0.82	0.729	0.645	0.567	0.494	0.428	0.366	0.31	0.26
(0.3,0.3,0.2)	1	0.891	0.79	0.698	0.612	0.533	0.461	0.394	0.333	0.278	0.228
(0.3,0.3,0.3)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.3,0.4)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.3,0.5)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.3,0.6)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.3,0.7)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.3,0.8)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.3,0.9)	1	0.89	0.789	0.697	0.611	0.532	0.46	0.393	0.332	0.277	0.228
(0.3,0.4,0.1)	1	0.919	0.821	0.731	0.649	0.574	0.504	0.44	0.382	0.328	0.279
(0.3,0.4,0.2)	1	0.891	0.791	0.7	0.617	0.541	0.471	0.407	0.349	0.296	0.248
(0.3,0.4,0.3)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.4,0.4)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.4,0.5)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.4,0.6)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.4,0.7)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.4,0.8)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.4,0.9)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.3,0.5,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.505	0.442	0.385	0.332	0.284
(0.3,0.5,0.2)	1	0.891	0.791	0.7	0.617	0.541	0.472	0.409	0.352	0.3	0.253
(0.3,0.5,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.5,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.5,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.5,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.5,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.5,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.5,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.3,0.6,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.506	0.443	0.385	0.333	0.285
(0.3,0.6,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.472	0.41	0.352	0.301	0.254
(0.3,0.6,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.6,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.6,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.6,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.6,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254

(0.3,0.6,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.6,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.7,0.1)	1	0.919	0.821	0.731	0.649	0.574	0.505	0.442	0.384	0.332	0.285
(0.3,0.7,0.2)	1	0.891	0.791	0.701	0.618	0.542	0.473	0.41	0.353	0.302	0.255
(0.3,0.7,0.3)	1	0.89	0.79	0.699	0.617	0.541	0.472	0.409	0.353	0.301	0.255
(0.3,0.7,0.4)	1	0.89	0.79	0.699	0.617	0.541	0.472	0.409	0.353	0.301	0.255
(0.3,0.7,0.5)	1	0.89	0.79	0.699	0.617	0.541	0.472	0.409	0.353	0.301	0.255
(0.3,0.7,0.6)	1	0.89	0.79	0.699	0.617	0.541	0.472	0.409	0.353	0.301	0.255
(0.3,0.7,0.7)	1	0.89	0.79	0.699	0.617	0.541	0.472	0.409	0.353	0.301	0.255
(0.3,0.7,0.8)	1	0.89	0.79	0.699	0.617	0.541	0.472	0.409	0.353	0.301	0.255
(0.3,0.7,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.8,0.1)	1	0.919	0.821	0.731	0.649	0.573	0.504	0.441	0.384	0.332	0.284
(0.3,0.8,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.473	0.41	0.353	0.301	0.255
(0.3,0.8,0.3)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.255
(0.3,0.8,0.4)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.255
(0.3,0.8,0.5)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.255
(0.3,0.8,0.6)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.255
(0.3,0.8,0.7)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.255
(0.3,0.8,0.8)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.255
(0.3,0.8,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.3,0.9,0.1)	1	0.919	0.821	0.731	0.649	0.574	0.505	0.442	0.384	0.332	0.284
(0.3,0.9,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.472	0.41	0.353	0.301	0.255
(0.3,0.9,0.3)	1	0.89	0.79	0.699	0.616	0.541	0.472	0.409	0.352	0.301	0.254
(0.3,0.9,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.472	0.409	0.352	0.301	0.254
(0.3,0.9,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.472	0.409	0.352	0.301	0.254
(0.3,0.9,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.472	0.409	0.352	0.301	0.254
(0.3,0.9,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.472	0.409	0.352	0.301	0.254
(0.3,0.9,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.472	0.409	0.352	0.301	0.254
(0.3,0.9,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.4,0.1,0.1)	1	0.854	0.69	0.555	0.445	0.354	0.28	0.219	0.169	0.129	0.095
(0.4,0.1,0.2)	1	0.825	0.658	0.523	0.413	0.325	0.252	0.194	0.147	0.108	0.077
(0.4,0.1,0.3)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.1,0.4)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.1,0.5)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.1,0.6)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.1,0.7)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.1,0.8)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.1,0.9)	1	0.824	0.657	0.522	0.413	0.324	0.252	0.194	0.146	0.108	0.077
(0.4,0.2,0.1)	1	0.916	0.81	0.71	0.617	0.531	0.454	0.384	0.321	0.266	0.217
(0.4,0.2,0.2)	1	0.888	0.78	0.678	0.583	0.497	0.419	0.35	0.289	0.235	0.188
(0.4,0.2,0.3)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.2,0.4)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.2,0.5)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.2,0.6)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.2,0.7)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.2,0.8)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.2,0.9)	1	0.887	0.779	0.677	0.582	0.496	0.419	0.349	0.288	0.234	0.187
(0.4,0.3,0.1)	1	0.919	0.821	0.731	0.649	0.574	0.504	0.44	0.382	0.328	0.279
(0.4,0.3,0.2)	1	0.891	0.791	0.7	0.617	0.541	0.471	0.407	0.349	0.296	0.248
(0.4,0.3,0.3)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.4,0.3,0.4)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.4,0.3,0.5)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.4,0.3,0.6)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.4,0.3,0.7)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.4,0.3,0.8)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248

(0.4,0.3,0.9)	1	0.89	0.79	0.699	0.616	0.539	0.47	0.406	0.348	0.295	0.248
(0.4,0.4,0.1)	1	0.919	0.822	0.733	0.653	0.58	0.514	0.453	0.397	0.347	0.3
(0.4,0.4,0.2)	1	0.891	0.792	0.702	0.621	0.548	0.481	0.42	0.365	0.315	0.269
(0.4,0.4,0.3)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.4,0.4)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.4,0.5)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.4,0.6)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.4,0.7)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.4,0.8)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.4,0.9)	1	0.89	0.791	0.701	0.62	0.547	0.48	0.419	0.364	0.314	0.269
(0.4,0.5,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.4	0.351	0.305
(0.4,0.5,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.368	0.319	0.275
(0.4,0.5,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.5,0.4)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.5,0.5)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.5,0.6)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.5,0.7)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.5,0.8)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.5,0.9)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.4,0.6,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.4,0.6,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.4,0.6,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.6,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.6,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.6,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.6,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.6,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.6,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.7,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.4,0.7,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.4,0.7,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.369	0.32	0.275
(0.4,0.7,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.369	0.32	0.275
(0.4,0.7,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.369	0.32	0.275
(0.4,0.7,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.369	0.32	0.275
(0.4,0.7,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.369	0.32	0.275
(0.4,0.7,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.369	0.32	0.275
(0.4,0.7,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.4,0.8,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.4,0.8,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.4,0.8,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.276
(0.4,0.8,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.276
(0.4,0.8,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.276
(0.4,0.8,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.276
(0.4,0.8,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.276
(0.4,0.8,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.276
(0.4,0.8,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.4,0.9,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.4,0.9,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275
(0.4,0.9,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.32	0.275

(0.5,0.1,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.22	0.171	0.13	0.097
(0.5,0.1,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.1,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.5,0.2,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.269	0.221
(0.5,0.2,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.291	0.238	0.192
(0.5,0.2,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.2,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.2,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.2,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.2,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.2,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.2,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.351	0.291	0.238	0.191
(0.5,0.3,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.505	0.442	0.385	0.332	0.284
(0.5,0.3,0.2)	1	0.891	0.791	0.7	0.617	0.541	0.472	0.409	0.352	0.3	0.253
(0.5,0.3,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.3,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.3,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.3,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.3,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.3,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.3,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.408	0.351	0.299	0.253
(0.5,0.4,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.4	0.351	0.305
(0.5,0.4,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.368	0.319	0.275
(0.5,0.4,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.4,0.4)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.4,0.5)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.4,0.6)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.4,0.7)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.4,0.8)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.4,0.9)	1	0.89	0.791	0.701	0.62	0.548	0.482	0.422	0.368	0.319	0.274
(0.5,0.5,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.457	0.404	0.355	0.31
(0.5,0.5,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.425	0.372	0.324	0.28
(0.5,0.5,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.424	0.371	0.323	0.279
(0.5,0.5,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.424	0.371	0.323	0.279
(0.5,0.5,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.424	0.371	0.323	0.279
(0.5,0.5,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.424	0.371	0.323	0.279
(0.5,0.5,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.424	0.371	0.323	0.279
(0.5,0.5,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.424	0.371	0.323	0.279
(0.5,0.5,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.483	0.424	0.371	0.323	0.279
(0.5,0.6,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.355	0.311
(0.5,0.6,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.372	0.324	0.281
(0.5,0.6,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.6,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.6,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.6,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.6,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.6,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.6,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.355	0.311

(0.5,0.7,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.373	0.325	0.281
(0.5,0.7,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.7,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.355	0.311
(0.5,0.8,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.373	0.325	0.281
(0.5,0.8,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.8,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.355	0.311
(0.5,0.9,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.372	0.325	0.281
(0.5,0.9,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.5,0.9,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.1,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.221	0.171	0.131	0.098
(0.6,0.1,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.6,0.1,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.6,0.1,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.079
(0.6,0.1,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.6,0.1,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.6,0.1,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.6,0.1,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.6,0.1,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.6,0.2,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.27	0.222
(0.6,0.2,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.292	0.239	0.192
(0.6,0.2,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.2,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.2,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.2,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.2,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.2,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.2,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.6,0.3,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.506	0.443	0.385	0.333	0.285
(0.6,0.3,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.472	0.41	0.352	0.301	0.254
(0.6,0.3,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.3,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.3,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.3,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.3,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.3,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.3,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.6,0.4,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.6,0.4,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276

(0.6,0.4,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.4,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.4,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.4,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.4,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.4,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.4,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.6,0.5,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.355	0.311
(0.6,0.5,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.372	0.324	0.281
(0.6,0.5,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.5,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.5,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.5,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.5,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.5,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.5,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.6,0.6,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.356	0.312
(0.6,0.6,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.282
(0.6,0.6,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.6,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.6,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.6,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.6,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.6,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.6,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.6,0.7,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.283
(0.6,0.7,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.7,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.6,0.8,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.283
(0.6,0.8,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.8,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.6,0.9,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.283
(0.6,0.9,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.6,0.9,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.1,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.221	0.171	0.131	0.098
(0.7,0.1,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.7,0.1,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08

(0.7,0.1,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.7,0.1,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.7,0.1,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.7,0.1,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.7,0.1,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.7,0.1,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.7,0.2,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.27	0.222
(0.7,0.2,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.292	0.239	0.193
(0.7,0.2,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.2,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.2,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.2,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.2,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.2,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.2,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.7,0.3,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.506	0.443	0.385	0.333	0.285
(0.7,0.3,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.472	0.41	0.353	0.301	0.254
(0.7,0.3,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.3,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.3,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.3,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.3,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.3,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.3,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.7,0.4,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.7,0.4,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.7,0.4,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.4,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.4,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.4,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.4,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.4,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.4,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.7,0.5,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.356	0.311
(0.7,0.5,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.372	0.325	0.281
(0.7,0.5,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.5,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.5,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.5,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.5,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.5,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.5,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.7,0.6,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.7,0.6,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.283
(0.7,0.6,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.6,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.6,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.6,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.6,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.6,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.6,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.7,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.7,0.7,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.7,0.7,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.7,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282

(0.7,0.7,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.7,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.7,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.7,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.7,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.7,0.8,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.7,0.8,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.8,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.7,0.9,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.7,0.9,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.7,0.9,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.1,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.221	0.171	0.131	0.098
(0.8,0.1,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.8,0.1,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.1,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.1,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.1,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.1,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.1,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.1,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.8,0.2,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.27	0.222
(0.8,0.2,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.292	0.239	0.193
(0.8,0.2,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.2,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.2,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.2,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.2,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.2,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.2,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.8,0.3,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.506	0.443	0.385	0.333	0.285
(0.8,0.3,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.472	0.41	0.353	0.301	0.254
(0.8,0.3,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.3,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.3,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.3,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.3,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.3,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.3,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.8,0.4,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.8,0.4,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.8,0.4,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.8,0.4,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.8,0.4,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275

(0.8,0.4,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.8,0.4,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.8,0.4,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.8,0.4,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.8,0.5,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.356	0.312
(0.8,0.5,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.372	0.325	0.281
(0.8,0.5,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.5,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.5,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.5,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.5,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.5,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.5,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.8,0.6,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.8,0.6,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.283
(0.8,0.6,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.6,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.6,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.6,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.6,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.6,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.6,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.8,0.7,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.8,0.7,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.7,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.8,0.8,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.8,0.8,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.8,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.8,0.9,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.8,0.9,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.8,0.9,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.1,0.1)	1	0.854	0.69	0.555	0.445	0.355	0.281	0.221	0.171	0.131	0.098
(0.9,0.1,0.2)	1	0.825	0.658	0.523	0.414	0.325	0.253	0.195	0.148	0.11	0.079
(0.9,0.1,0.3)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.9,0.1,0.4)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.9,0.1,0.5)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.9,0.1,0.6)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08

(0.9,0.1,0.7)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.9,0.1,0.8)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.9,0.1,0.9)	1	0.824	0.657	0.522	0.413	0.325	0.253	0.195	0.148	0.11	0.08
(0.9,0.2,0.1)	1	0.916	0.81	0.71	0.617	0.532	0.455	0.386	0.324	0.27	0.222
(0.9,0.2,0.2)	1	0.888	0.78	0.678	0.584	0.498	0.421	0.352	0.292	0.239	0.193
(0.9,0.2,0.3)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.2,0.4)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.2,0.5)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.2,0.6)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.2,0.7)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.2,0.8)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.2,0.9)	1	0.887	0.779	0.677	0.583	0.497	0.42	0.352	0.291	0.238	0.192
(0.9,0.3,0.1)	1	0.919	0.821	0.731	0.65	0.574	0.506	0.443	0.385	0.333	0.285
(0.9,0.3,0.2)	1	0.891	0.791	0.7	0.617	0.542	0.472	0.41	0.353	0.301	0.254
(0.9,0.3,0.3)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.3,0.4)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.3,0.5)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.3,0.6)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.3,0.7)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.3,0.8)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.3,0.9)	1	0.89	0.79	0.699	0.616	0.54	0.471	0.409	0.352	0.3	0.254
(0.9,0.4,0.1)	1	0.919	0.822	0.733	0.654	0.581	0.515	0.455	0.401	0.351	0.306
(0.9,0.4,0.2)	1	0.891	0.792	0.702	0.622	0.549	0.483	0.423	0.369	0.32	0.276
(0.9,0.4,0.3)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.4,0.4)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.4,0.5)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.4,0.6)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.4,0.7)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.4,0.8)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.4,0.9)	1	0.89	0.791	0.701	0.621	0.548	0.482	0.422	0.368	0.319	0.275
(0.9,0.5,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.404	0.356	0.312
(0.9,0.5,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.484	0.426	0.372	0.325	0.281
(0.9,0.5,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.5,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.5,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.5,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.5,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.5,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.5,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.483	0.425	0.372	0.324	0.281
(0.9,0.6,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.9,0.6,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.325	0.283
(0.9,0.6,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.6,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.6,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.6,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.6,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.6,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.6,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.7,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.9,0.7,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.9,0.7,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.7,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.7,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.7,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.7,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282

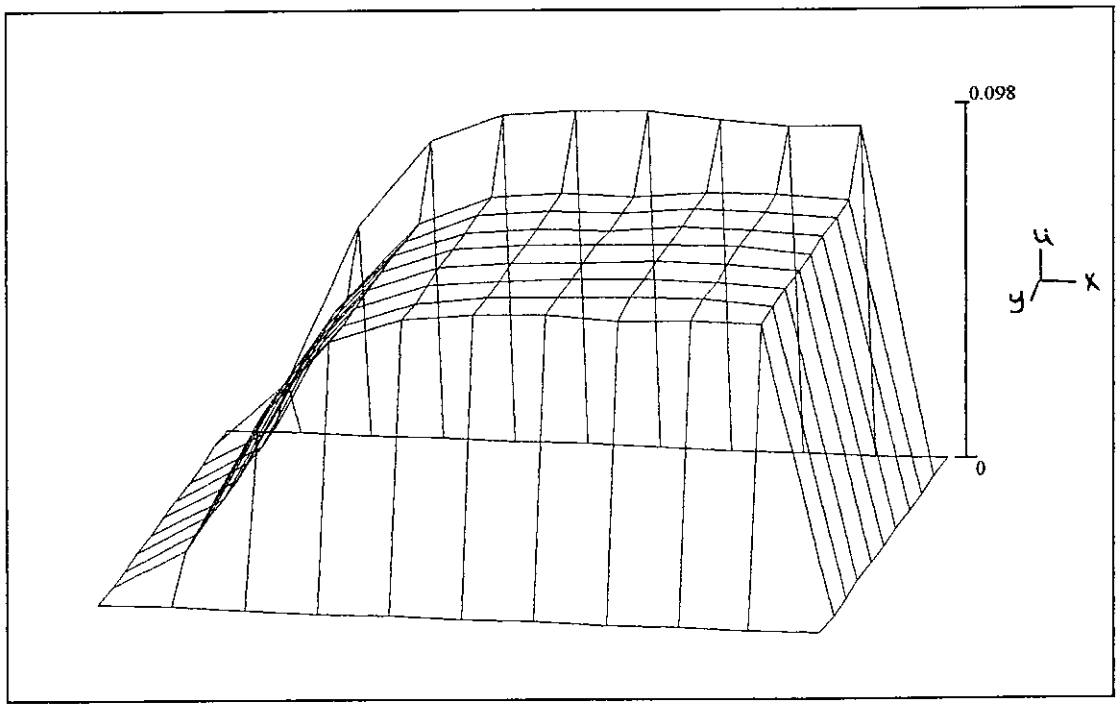
(0.9,0.7,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.7,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.9,0.8,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.9,0.8,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.8,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.1)	1	0.919	0.822	0.734	0.654	0.582	0.517	0.458	0.405	0.356	0.313
(0.9,0.9,0.2)	1	0.891	0.792	0.703	0.622	0.55	0.485	0.426	0.373	0.326	0.283
(0.9,0.9,0.3)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.4)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.5)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.6)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.7)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.8)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282
(0.9,0.9,0.9)	1	0.89	0.791	0.701	0.621	0.549	0.484	0.425	0.372	0.325	0.282

Result of 3dimension Crank-Nicolson method using r=0.01

(x,y,z) n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.974	0.945	0.916	0.888	0.86	0.834	0.808	0.783	0.759	0.736
(0.1,0.1,0.2)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.3)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.4)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.5)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.6)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.7)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.8)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.1,0.9)	1	0.974	0.944	0.915	0.887	0.86	0.833	0.807	0.782	0.758	0.735
(0.1,0.2,0.1)	1	0.984	0.963	0.943	0.923	0.903	0.884	0.865	0.846	0.828	0.81
(0.1,0.2,0.2)	1	0.982	0.961	0.941	0.921	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.3)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.4)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.5)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.6)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.7)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.8)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.2,0.9)	1	0.982	0.961	0.941	0.92	0.901	0.881	0.862	0.844	0.825	0.807
(0.1,0.3,0.1)	1	0.984	0.964	0.944	0.924	0.905	0.886	0.867	0.849	0.831	0.813
(0.1,0.3,0.2)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.3)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.4)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.5)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.6)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.7)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.8)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.3,0.9)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.1)	1	0.984	0.964	0.944	0.924	0.905	0.886	0.867	0.849	0.831	0.814
(0.1,0.4,0.2)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.3)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.4)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.5)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.6)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.7)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.8)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.4,0.9)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.1)	1	0.984	0.964	0.944	0.924	0.905	0.886	0.867	0.849	0.831	0.814
(0.1,0.5,0.2)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.3)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.4)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.5)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.6)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.7)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.8)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.5,0.9)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.6,0.1)	1	0.984	0.964	0.944	0.924	0.905	0.886	0.867	0.849	0.831	0.814
(0.1,0.6,0.2)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.6,0.3)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811
(0.1,0.6,0.4)	1	0.982	0.961	0.941	0.921	0.902	0.883	0.864	0.846	0.828	0.811

Result of 3dimension Crank-Nicolson method using r=0.001

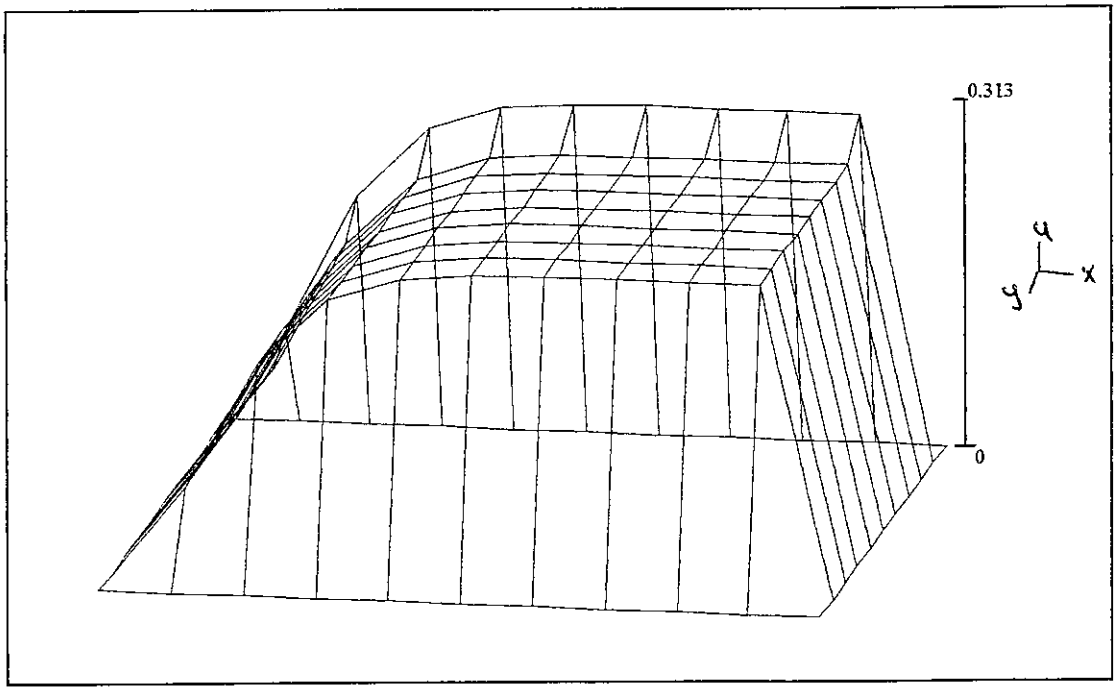
(x,y,z) n	0	1	2	3	4	5	6	7	8	9	10
(0.1,0.1,0.1)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.2)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.3)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.4)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.5)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.6)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.7)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.8)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.1,0.9)	1	0.997	0.994	0.991	0.988	0.985	0.982	0.979	0.976	0.973	0.97
(0.1,0.2,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.982	0.98
(0.1,0.2,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.2,0.9)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.982	0.98
(0.1,0.3,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.3,0.9)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.982	0.98
(0.1,0.4,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.4,0.9)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.982	0.98
(0.1,0.5,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.5)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.6)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.7)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.8)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.5,0.9)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.6,0.1)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.982	0.98
(0.1,0.6,0.2)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.6,0.3)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979
(0.1,0.6,0.4)	1	0.998	0.996	0.994	0.992	0.99	0.988	0.986	0.984	0.981	0.979



f

3dimension Crank-Niclson method

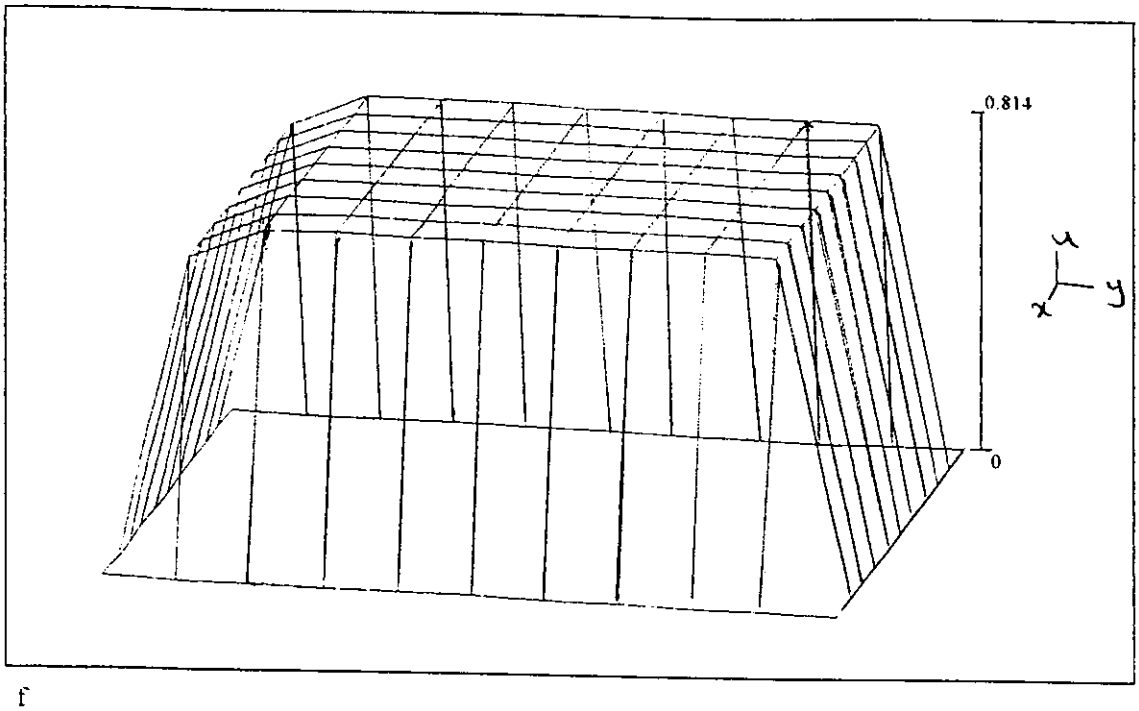
x=0.1 and y & z are varible and r=0.1 at n=10



f2

3dimension Crank-Niclson method

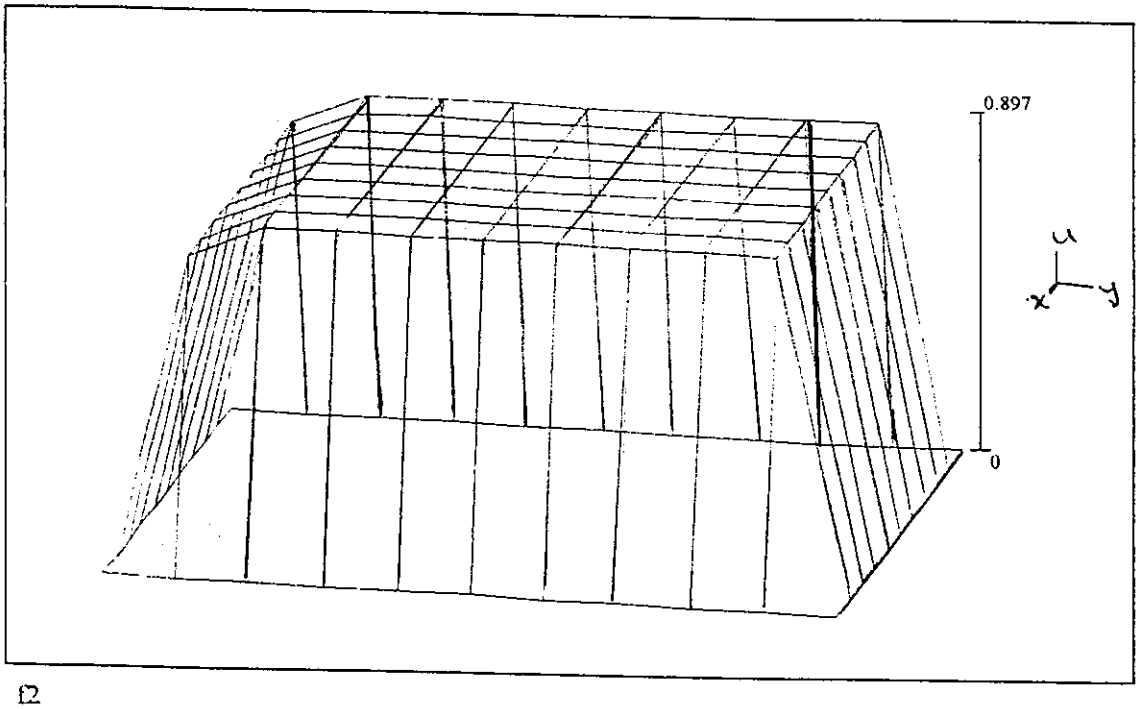
x=0.9 and y & z are varible and r=0.1 at t=10



f

3dimension Crank-Nielson method

x=0. 1, y & z are varible and r=0.01 at n=10



f2

3dimension Crank-Nielson method

x=0.9, y & z are varible and r=0.01 at n=10

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APPENDIX

A1. COMPUTER PROGRAMME

This programe solves one dimension Navier-Stokes equation [equation (3.2.4) on page21]

$h := 0.1$

$r := 0.01$

$k := r \cdot h^2$

$k = 1 \cdot 10^{-4}$

$\mu := 1$

$\rho := 1$

$U := 10$

$A := -2 \cdot \mu \cdot r + \rho \cdot U \cdot h \cdot r + \rho$

$A = 0.99$

$B := \mu \cdot r - \rho \cdot U \cdot h \cdot r$

$B = 0$

$n := 0, 0 + 1 .. 29$

the intiall condition=1
the boundary condition=0
n=number of repation

$$w^{\langle 0 \rangle} := \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$Q := \begin{bmatrix} A & B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu \cdot r & A & B & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu \cdot r & A & B & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu \cdot r & A & B & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \cdot r & A & B & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu \cdot r & A & B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \cdot r & A & B & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & A & B \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & A \end{bmatrix}$$

$$g := \begin{bmatrix} k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \\ k \cdot \rho \cdot 9.8 - h \cdot r \cdot 2 \end{bmatrix}$$

$$w^{\langle n+1 \rangle} := Q \cdot w^{\langle n \rangle} + g$$

This programe solves two dimension Navier-Stokes equation using explicit finite difference [equation (3.3.5) on page 22

$r := 0.1$

$h := 0.1$

$k := r \cdot h^2$

$k = 0.001$

$U := 10$

$V := 10$

$\mu := 1$

$\rho := 1$

$c_1 := \rho - 4 \cdot \mu \cdot r + \rho \cdot r \cdot U \cdot h + \rho \cdot h \cdot r \cdot V$

$c_2 := \mu \cdot r - \rho \cdot r \cdot U \cdot h$

$c_3 := \mu \cdot r - \rho \cdot r \cdot V \cdot h$

$$a := \begin{bmatrix} c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu \cdot r & c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu \cdot r & c_1 & c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu \cdot r & c_1 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \cdot r & c_1 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu \cdot r & c_1 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \cdot r & c_1 & c_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & c_1 & c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & c_1 \end{bmatrix}$$

$i := 0, 1 .. 80$

$g := 9.8$

$boundary_i := k \cdot \rho \cdot g - h \cdot r \cdot 2$

$$b := \begin{bmatrix} c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 \end{bmatrix}$$

$$c := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d := \begin{bmatrix} \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu \cdot r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \cdot r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r \end{bmatrix}$$

Q1 := augment(augment(augment(augment(augment(augment(augment(augment(a , b) , c) , c) , c) , c) , c) , c) , c)

Q2 := augment(augment(augment(augment(augment(augment(augment(augment(d , a) , b) , c) , c) , c) , c) , c) , c)

Q3 := augment(augment(augment(augment(augment(augment(augment(augment(c, d), a), b), c), c), c), c), c)

Q4 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), d), a), b), c), c), c), c)

Q5 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), d), a), b), c), c), c)

Q6 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), d), a), b), c), c)

Q7 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), c), d), a), b), c)

Q8 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), c), c), d), a), b)

Q9 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), c), c), c), d), a)

A := stack(stack(stack(stack(stack(stack(stack(Q1, Q2), Q3), Q4), Q5), Q6), Q7), Q8), Q9)

$A_{1,0} = 0.1$

n := 0, 0 + 1 .. 9

$U^{<0>}$:= READPRN(W)

$U^{<n+1>}$:= $A \cdot U^{<n>}$ + boundary

This programe solves two dimension Navier-Stokes using implicit finite difference [eqaution (3.4.4) on page 27]

$h := 0.1$

$r := 0.1$

$k := r \cdot h^2$

$k = 0.001$

$U := 10$

$V := 10$

$\mu := 1$

$\rho := 1$

$\mu \cdot r = 0.1$

$c_1 := \rho + 4 \cdot \mu \cdot r + \rho \cdot U \cdot h \cdot r + \rho \cdot V \cdot h \cdot r$

$c_1 = 1.6$

$c_2 := \mu \cdot r + \rho \cdot U \cdot h \cdot r$

$c_2 = 0.2$

$c_3 := \mu \cdot r + \rho \cdot V \cdot h \cdot r$

$c_3 = 0.2$

$\alpha := -c_2$

$\alpha = -0.2$

$$a := \begin{bmatrix} c_1 & -\mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_3 & c_1 & -\mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_3 & c_1 & -\mu\cdot r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_3 & c_1 & -\mu\cdot r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_3 & c_1 & -\mu\cdot r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_3 & c_1 & -\mu\cdot r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_3 & c_1 & -\mu\cdot r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_3 & c_1 & -\mu\cdot r \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_3 & c_1 \end{bmatrix}$$

$$b := \begin{bmatrix} -\mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu\cdot r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu\cdot r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu\cdot r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu\cdot r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu\cdot r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu\cdot r \end{bmatrix}$$

$$c := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d := \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha \end{bmatrix}$$

Q1 := augment(augment(augment(augment(augment(augment(augment(augment(a , b) , c) , c) , c) , c) , c) , c) , c)

Q2 := augment(augment(augment(augment(augment(augment(augment(augment(d , a) , b) , c) , c) , c) , c) , c) , c)

Q3 := augment(augment(augment(augment(augment(augment(augment(augment(c , d) , a) , b) , c) , c) , c) , c) , c)

Q4 := augment(augment(augment(augment(augment(augment(augment(augment(c , c) , d) , a) , b) , c) , c) , c) , c)

Q5 := augment(augment(augment(augment(augment(augment(augment(augment(c , c) , c) , d) , a) , b) , c) , c) , c)

Q6 := augment(augment(augment(augment(augment(augment(augment(augment(c , c) , c) , c) , d) , a) , b) , c) , c)

Q7 := augment(augment(augment(augment(augment(augment(augment(augment(c , c) , c) , c) , c) , d) , a) , b) , c)

Q8 := augment(augment(augment(augment(augment(augment(augment(augment(c , c) , c) , c) , c) , c) , d) , a) , b)

Q9 := augment(augment(augment(augment(augment(augment(augment(augment(c , c) , c) , c) , c) , c) , c) , d) , a)

A := stack(stack(stack(stack(stack(stack(stack(stack(Q1 , Q2) , Q3) , Q4) , Q5) , Q6) , Q7) , Q8) , Q9)

g := 9.8

n := 0,0 + 1 .. 9

i := 0,0 + 1 .. 80

$boundary_i := k \cdot \rho \cdot g - h \cdot r \cdot 2$

$u^{<0>} :=$ READPRN(W)

$u^{<n+1>} := A^{-1} \cdot u^{<n>} + A^{-1} \cdot boundary$

This program solves two dimension Navier-Stokes using Crank-Nicison method [equation (3.5.5) on page 30]

$h := 0.1$

$r := 0.1$

$k := r \cdot h^2$

$k = 0.001$

$U := 10$

$V := 10$

$\mu := 1$

$\rho := 1$

$\mu \cdot r = 0.1$

$$c_1 := 2 \cdot r \cdot \mu - \frac{\rho \cdot h \cdot r \cdot U}{2} - \frac{\rho \cdot h \cdot r \cdot V}{2} + \rho$$

$c_1 = 1.1$

$$c_2 := \mu \cdot \frac{r}{2} - \frac{\rho \cdot h \cdot r \cdot U}{2}$$

$c_2 = 0$

$$c_3 := \mu \cdot \frac{r}{2} - \frac{\rho \cdot h \cdot r \cdot V}{2}$$

$$c_4 := 2 \cdot \mu \cdot r - \frac{\rho \cdot h \cdot r \cdot U}{2} - \frac{\rho \cdot h \cdot r \cdot V}{2} - \rho$$

$\alpha := -1 \cdot c_4$

$\alpha = 0.9$

$\beta := -1 \cdot c_3$

$$c_4 := 2 \cdot \mu \cdot r - \rho - \frac{\rho \cdot h \cdot r \cdot U}{2} - \frac{\rho \cdot h \cdot r \cdot V}{2}$$

$\gamma := -1 \cdot c_2$

$\gamma = 0$

$$a := \begin{bmatrix} -1 \cdot c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 & c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -1 \cdot c_1 \end{bmatrix}$$

$$b := \begin{bmatrix} c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 \end{bmatrix}$$

$$c := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d := \begin{bmatrix} \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} \end{bmatrix}$$

Q1 := augment(augment(augment(augment(augment(augment(augment(augment(a, b), c), c), c), c), c), c), c)

Q2 := augment(augment(augment(augment(augment(augment(augment(augment(d, a), b), c), c), c), c), c), c)

Q3 := augment(augment(augment(augment(augment(augment(augment(augment(c, d), a), b), c), c), c), c), c)

Q4 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), d), a), b), c), c), c), c)

Q5 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), d), a), b), c), c), c)

Q6 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), d), a), b), c), c)

Q7 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), c), d), a), b), c)

Q8 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), c), c), d), a), b)

Q9 := augment(augment(augment(augment(augment(augment(augment(augment(c, c), c), c), c), c), c), d), a)

A := stack(stack(stack(stack(stack(stack(stack(stack(Q1, Q2), Q3), Q4), Q5), Q6), Q7), Q8), Q9)

g := 9.8

n := 0, 0 + 1 .. 9

i := 0, 0 + 1 .. 80

$$\text{boundary}_i := -k \cdot \rho \cdot g + 2 \cdot h \cdot r$$

$$a1 := \begin{bmatrix} -\alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-\mu \cdot r}{2} & -\alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-\mu \cdot r}{2} & -\alpha & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\mu \cdot r}{2} & -\alpha & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-\mu \cdot r}{2} & -\alpha & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-\mu \cdot r}{2} & -\alpha & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-\mu \cdot r}{2} & -\alpha & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\mu \cdot r}{2} & -\alpha & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\mu \cdot r}{2} & -\alpha \end{bmatrix}$$

$$b1 := \begin{bmatrix} \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix}$$

$$c1 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$d1 := -d$

$Q11 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(a1, b1), c1), c1), c1), c1), c1), c1), c1)$

$Q21 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(d1, a1), b1), c1), c1), c1), c1), c1), c1)$

$Q31 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, d1), a1), b1), c1), c1), c1), c1), c1)$

$Q41 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), d1), a1), b1), c1), c1), c1), c1)$

$Q51 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), d1), a1), b1), c1), c1), c1)$

$Q61 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), d1), a1), b1), c1), c1)$

$Q71 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), c1), c1), d1), a1), b1)$

$Q81 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), c1), c1), c1), d1), a1)$

$Q91 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), c1), c1), c1), c1), d1)$

$B := \text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(Q11, Q21), Q31), Q41), Q51), Q61), Q71), Q81), Q91)$

$u^{<0>} := \text{READPRN}(W)$

$u^{<n+1>} := A^{-1} \cdot B \cdot u^{<n>} + A^{-1} \cdot \text{boundary}$

This program solves two dimension Navier-Stokes using Crank-Nicison method [equation (3.5.5) on page 30]

$h := 0.1$

$r := 0.1$

$k := r \cdot h^2$

$k = 0.001$

$U := 10$

$V := 10$

$\mu := 1$

$\rho := 1$

$\mu \cdot r = 0.1$

$c_1 := \frac{\mu \cdot r}{2} - \frac{\rho \cdot h \cdot r \cdot U}{4}$

$c_1 = 0.025$

$c_2 := 2 \cdot \mu \cdot r + \rho$

$c_2 = 1.2$

$c_3 := \mu \cdot \frac{r}{2} + \frac{\rho \cdot h \cdot r \cdot U}{4}$

$c_3 = 0.075$

$c_4 := \frac{\mu \cdot r}{2} - \frac{\rho \cdot h \cdot r \cdot V}{4}$

$c_4 = 0.025$

$c_5 := \frac{\mu \cdot r}{2} + \frac{\rho \cdot h \cdot r \cdot V}{4}$

$c_5 = 0.075$

$c_6 := -\rho + 2 \cdot \mu \cdot r$

$c_6 = -0.8$

$\alpha := c_3$

$\beta := c_6$

$\gamma := -c_4$

$$a := \begin{bmatrix} -1 \cdot c_2 & c_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_5 & -1 \cdot c_2 & c_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_5 & -1 \cdot c_2 & c_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_5 & -1 \cdot c_2 & c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_5 & -1 \cdot c_2 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & -1 \cdot c_2 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_5 & -1 \cdot c_2 & c_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_5 & -1 \cdot c_2 & c_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_5 & -1 \cdot c_2 \end{bmatrix}$$

$\delta := -c_5$

$\varepsilon := -c_1$

$$b := \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 \end{bmatrix}$$

$$c := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d := \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha \end{bmatrix}$$

Q1 := augment(augment(augment(augment(augment(augment(augment(augment(a,b),c),c),c),c),c),c),c)

Q2 := augment(augment(augment(augment(augment(augment(augment(augment(d,a),b),c),c),c),c),c),c)

Q3 := augment(augment(augment(augment(augment(augment(augment(augment(c,d),a),b),c),c),c),c),c)

Q4 := augment(augment(augment(augment(augment(augment(augment(augment(c,c),d),a),b),c),c),c),c)

Q5 := augment(augment(augment(augment(augment(augment(augment(augment(c,c),c),d),a),b),c),c),c)

Q6 := augment(augment(augment(augment(augment(augment(augment(augment(c,c),c),c),d),a),b),c),c)

Q7 := augment(augment(augment(augment(augment(augment(augment(augment(c,c),c),c),c),d),a),b),c)

Q8 := augment(augment(augment(augment(augment(augment(augment(augment(c,c),c),c),c),c),d),a),b)

Q9 := augment(augment(augment(augment(augment(augment(augment(augment(c,c),c),c),c),c),c),d),a)

A := stack(stack(stack(stack(stack(stack(stack(stack(Q1,Q2),Q3),Q4),Q5),Q6),Q7),Q8),Q9)

$g := 9.8$

$n := 0,0 + 1..9$

$i := 0,0 + 1..80$

$boundary_i := -k \cdot \rho \cdot g + h \cdot r \cdot 2$

$$a1 := \begin{bmatrix} \beta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & \beta & \gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta & \beta & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta & \beta & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta & \beta & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta & \beta & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta & \beta & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta & \beta & \gamma \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta & \beta \end{bmatrix}$$

$$b1 := \begin{bmatrix} \varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon \end{bmatrix}$$

$$c1 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$d1 := -d$

$Q11 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(a1, b1), c1), c1), c1), c1), c1), c1), c1)$

$Q21 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(d1, a1), b1), c1), c1), c1), c1), c1), c1)$

$Q31 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, d1), a1), b1), c1), c1), c1), c1), c1)$

$Q41 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), d1), a1), b1), c1), c1), c1), c1)$

$Q51 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), d1), a1), b1), c1), c1), c1)$

$Q61 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), d1), a1), b1), c1), c1)$

$Q71 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), c1), d1), a1), b1), b1)$

$Q81 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), c1), c1), d1), a1), a1)$

$Q91 := \text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(\text{augment}(c1, c1), c1), c1), c1), c1), c1), c1), d1)$

$B := \text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(\text{stack}(Q11, Q21), Q31), Q41), Q51), Q61), Q71), Q81), Q91)$

$u^{\langle 0 \rangle} := \text{READPRN}(W)$

$u^{\langle n+1 \rangle} := A^{-1} \cdot B \cdot u^{\langle n \rangle} + A^{-1} \cdot \text{boundary}$

```
#include<stdio.h>
main(void)
{
int i,j,k,n,m,df,time;
float w,x,y,z,r,hf,k2,Re,p;
float a[82][163],b[90],c[90],x2[20][20],ej[10][10],bj[10][10];

   printf("insert size matrix n=");
   scanf("%d",&n);
printf("insert submatrix size m=");
scanf("%d",&m);


          Re=1;
          r=0.1;
          k2=0.001;
          hf=0.1;
/*************************************************************/
for(i=1;i<=n;++i){
b[i]=0;
c[i]=1;
tj[i]=0;
}
/*************************************************************/


for(i=1;i<=n;++i){
for(j=1;j<=2*n;++j){
a[i][j]=0;

   }
   }

  for(i=1;i<=m;++i){
  for(j=1;j<=m;++j){
  bj[i][j]=0;
  ej[i][j]=0;
  cj[i][j]=0;
  }
  }
/*****************************************************/
 /*insert the matrix a//
 /*****************************************************/
 for(i=1;i<=n;++i){
   for(j=1;j<=n;++j){
 if(i==j)
 a[i][j]=-4;
   if(i==j+1)
   a[i][j]=1;
    if(j==i+1)
      a[i][j]=1;
 if(i==j+m)
   a[i][j]=1;
    if(j==i+m)
    a[i][j]=1;

              }
              }
   for(i=1;i<=m-1;++i){
   a[i*m][i*m+1]=0;
   a[i*m+1][i*m]=0;
   }

   //check matrix a//
   /*      for(i=1;i<=n;++i){
   for(j=1;j<=n;++j){
```

```
  printf("a[%d][%d]=%f",i,j,a[i][j]);
     }}
  */




//************************************//

  for(i=1;i<=n;++i){
  for(j=n+1;j<=2*n;++j){
  if(i+n==j)
  a[i][j]=1;
  else
  a[i][j]=0;
  }
  }


  /* for(i=1;i<=n;++i){
  for(j=1;j<=n;++j){
  printf("a[%d][%d]=",i,j);
  // scanf("%f",&a[i][j]);
  }
  1  }  */

  /* inverse here */
  for(i=1;i<=n-1;++i){

  x=a[i][i];
     for(j=1;j<=2*n;++j){
     a[i][j]=a[i][j]/x;
     }

  /*     for(k=1;k<=n;++k){
  for(j=n+1;j<=2*n;++j){
  printf("**** a[%d][%d]=%f\n",k,j,a[k][j]);
  }
     } */

           // program is correct be//

  for(j=i+1;j<=n;++j){
  w=a[j][i];
  for(k=1;k<=2*n;++k){
  a[j][k]=-(w*a[i][k])+a[j][k];    //change
  }
  }

   } // end of frist part

  z=a[n][n];
  for(i=1;i<=2*n;++i){
  a[n][i]=a[n][i]/z;
  }

  printf("******** end of part one ************\n");
  for(i=1;i<=n;++i){
  for(j=n+1;j<=2*n;++j){
  printf("a[%d][%d]=%f",i,j,a[i][j]);
  }
  }

  for(i=n;i>=1;--i){
    for(j=n+1-i;j<=n-1;++j){
     y=a[n-j][i];
     for(k=1;k<=2*n;++k){
     a[n-j][k]=-y*a[i][k]+a[n-j][k];     //change hear for i
     }
```

```
    }
    }


 printf("*****final**************\n");
 for(i=1;i<=n;++i){
 for(j=n+1;j<=2*n;++j){
 printf("a[%d][%d]=%f",i,j,a[i][j]);
 }
 }
 for(i=1;i<=n;++i){
 c[i]=1;
                    }

 /*******************************************************/
 for(time=1;time<=10;++time){

   for(i=1;i<=n;++i){
   b[i]=0;
   }
 /*******************************************************/
 for(i=1;i<=n;++i){
 for(j=1;j<=n;++j){
 b[i]=b[i]+a[i][j+n]*(-hf*hf*c[j]);
 }
 }

    for(i=1;i<=n;++i){
    printf("b[%d]=%f",i,b[i]);
    }

 //*******************************************************//

 for(i=1;i<=m;++i){
 for(j=1;j<=m;++j){
 x2[i][j]=b[j+(i-1)*m];
 }
 }
 for(j=1;j<=m;++j){
 x2[10][j]=x2[9][j];
 x2[j][10]=x2[j][9];
 }
      /* for(i=1;i<=m;++i){
       for(j=1;j<=m;++j){
       printf("x2[%d][%d]=%f",i,j,x2[i][j]);
       }
       } */

/*************part two**************/

 //the matrix ej=e//
 for(i=1;i<=m;++i){
 for(j=1;j<=m-1;++j){
 ej[j][j]=1-(4*r)/Re;
 ej[9][9]=1-(4*r)/Re;
 ej[j][j+1]=(r/Re)+((k2/(2*hf*hf))*(x2[i+1][j]-x2[i][j]));
 ej[j+1][j]=(r/Re)-(k2/(2*hf))*((x2[i+1][j+1]-x2[i][j+1])/hf);
 }

//the matrix bj=b//
for(j=1;j<=9;++j){
bj[j][j]=(r/Re)-(k2/(2*hf))*((x2[i][j+1]-x2[i][j])/hf);
cj[j][j]=(r/Re)+(k2/(2*hf))*((x2[i+1][j+1]-x2[i+1][j])/hf);
}


for(j=(i-1)*9+1;j<=i*9;j++){
 aj[j][j]=ej[j-(i-1)*9][j-(i-1)*9];
 if(i<9){
 aj[j][j+9]=bj[j-(i-1)*9][j-(i-1)*9];
```

```
aj[j+9][j]=cj[j-(i-1)*9][j-(i-1)*9];}

if(j<i*9){
aj[j][j+1]=ej[j-(i-1)*9][j-(i-1)*9+1];
aj[j+1][j]=ej[j-(i-1)*9+1][j-(i-1)*9];
}
}
 }

//*************//
/* printf("************************************");
for(i=1;i<=n;++i){
for(j=1;j<=n;++j){
printf("aj[%d][%d]=%f",i,j,aj[i][j]);
}
} */

      for(i=1;i<=n;++i){
      tj[i]=0;
      }


for(i=1;i<=n;++i){
for(j=1;j<=n;++j){
p=tj[i];
tj[i]=aj[i][j]*c[j]+p;
}
}


for(i=1;i<=n;++i){
 c[i]=tj[i];
 }


/*  for(i=1;i<=n;++i){
  printf("c[%d]=%f",i,c[i]);
  }*/

scanf("asd ",&df); printf("asdasd %d",n);
}

for(i=1;i<=n;++i){
u[i]=(b[i+1]-b[i])/hf;
v[i]=-(b[i+m]-b[i])/hf;
}

          for(i=1;i<=n;++i){
          printf("u[%d]=%f",i,u[i]);
          }
          for(i=1;i<=n;++i){
          printf("v[%d]=%f",i,v[i]);
          }
    scanf("asd ",&df); printf("asdasd %d",n);


   }
```

This programe solves three dimension Navier-Stokes equation using Explicit finite difference method [equation (3.7.2) on page 49]

$h := 0.1$

$r := 0.01$

$k := r \cdot h^2$

$k = 1 \cdot 10^{-4}$

$U := 10$

$V := 10$

$W := 10$

$\mu := 1$

$\rho := 1$

$c_1 := \mu \cdot r - \rho \cdot h \cdot r \cdot U$

$c_2 := -6 \cdot \mu \cdot r + \rho + \rho \cdot h \cdot r \cdot U + \rho \cdot h \cdot r \cdot V + \rho \cdot h \cdot r \cdot W$

$c_3 := \mu \cdot r - \rho \cdot h \cdot r \cdot V$

$c_4 := \mu \cdot r - \rho \cdot h \cdot r \cdot W$

$$al := \begin{bmatrix} c_2 & c_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu \cdot r & c_2 & c_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu \cdot r & c_2 & c_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu \cdot r & c_2 & c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \cdot r & c_2 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu \cdot r & c_2 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \cdot r & c_2 & c_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & c_2 & c_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & c_2 \end{bmatrix}$$

$$I := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$bl := \begin{bmatrix} c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 \end{bmatrix}$$

$ml := \mu \cdot r \cdot I$

$$z := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$s1 := augment(augment(augment(augment(augment(augment(augment(augment(a1, b1), z), z), z), z), z), z), z), z)$

$s2 := augment(augment(augment(augment(augment(augment(augment(augment(m1, a1), b1), z), z), z), z), z), z), z)$

$s3 := augment(augment(augment(augment(augment(augment(augment(augment(z, m1), a1), b1), z), z), z), z), z), z)$

h4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), m1), a1), b1), z), z), z), z)

h5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), m1), a1), b1), z), z), z)

h6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), m1), a1), b1), z), z)

h7 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), m1), a1), b1), z)

h8 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), m1), a1), b1)

h9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), m1), a1)

λ := stack(stack(stack(stack(stack(stack(stack(stack(s1 , s2), s3), h4), h5), h6), h7), h8), h9)

$$C := \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 \end{bmatrix}$$

s4 := augment(augment(augment(augment(augment(augment(augment(augment(C , z), z), z), z), z), z), z), z)

s5 := augment(augment(augment(augment(augment(augment(augment(augment(z, C), z), z), z), z), z), z), z)

s6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), C), z), z), z), z), z), z)

s71 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), C), z), z), z), z), z)

s81 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), C), z), z), z), z)

s91 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), C), z), z), z)

s01 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), C), z), z)

s11 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), C), z)

s12 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), C)

G := stack(stack(stack(stack(stack(stack(stack(stack(s4, s5), s6), s71), s81), s91), s01), s11), s12)

s7 := augment(augment(augment(augment(augment(augment(augment(augment(m1, z), z), z), z), z), z), z), z)

s8 := augment(augment(augment(augment(augment(augment(augment(augment(z, m1), z), z), z), z), z), z), z)

s9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), m1), z), z), z), z), z), z)

v1 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), m1), z), z), z), z), z)

v2 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), m1), z), z), z), z)

v3 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), m1), z), z), z)

v4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), m1), z), z)

v5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), m1), z)

v6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), m1)

θ := stack(stack(stack(stack(stack(stack(stack(stack(s7, s8), s9), v1), v2), v3), v4), v5), v6)

e1 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), z)

E := stack(stack(stack(stack(stack(stack(stack(stack(e1, e1), e1), e1), e1), e1), e1), e1), e1)

q1 := augment(augment(augment(augment(augment(augment(augment(augment(λ, G), E), E), E), E), E), E), E)

q2 := augment(augment(augment(augment(augment(augment(augment(augment(θ, λ), G), E), E), E), E), E), E)

q3 := augment(augment(augment(augment(augment(augment(augment(augment(E, θ), λ), G), E), E), E), E), E)

q4 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), θ), λ), G), E), E), E), E)

$q5 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),\theta),\lambda),G),E),E),E)$

$q6 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),\theta),\lambda),G),E),E)$

$q7 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),E),\theta),\lambda),G),E)$

$q8 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),E),E),\theta),\lambda),G)$

$q9 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),E),E),E),\theta),\lambda)$

$\Omega := stack(stack(stack(stack(stack(stack(stack(stack(q1,q2),q3),q4),q5),q6),q7),q8),q9)$

$k0 := 0,0 + 1..728$

$Y_{k0} := 1$

$u^{<o>} := Y$

$g := 9.8$

$boundary1_0 := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5 + 0.5 + 0.5)$

$l := 1, 1 + 1..7$

$boundary1_l := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5 + 0.5)$

$boundary1_8 := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5 + 0.5) + c_4\cdot 0.5$

$boundary1_9 := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5 + 0.5)$

$lp := 10, 10 + 1..16$

$boundary1_{lp} := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5)$

$boundary1_{17} := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5) + c_4\cdot 0.5$

$boundary1_{18} := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5 + 0.5)$

$lo := 19, 19 + 1..25$

$boundary1_{lo} := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5)$

$boundary1_{26} := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5) + c_4\cdot 0.5$

$boundary1_{27} := k\cdot\rho\cdot g - h\cdot r\cdot 2 + \mu\cdot r\cdot(0.5 + 0.5)$

$lt := 28, 28 + 1..34$

$boundaryl_{lt} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5)$

$boundaryl_{35} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5$

$boundaryl_{36} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5)$

$ly := 37, 37 + 1..43$

$boundaryl_{ly} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5)$

$boundaryl_{44} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5$

$boundaryl_{45} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5)$

$lq := 46, 46 + 1..52$

$boundaryl_{lq} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5)$

$boundaryl_{53} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5$

$boundaryl_{54} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5)$

$lx := 55, 55 + 1..61$

$boundaryl_{lx} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5)$

$boundaryl_{62} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5$

$boundaryl_{63} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5)$

$la := 64, 64 + 1..70$

$boundaryl_{la} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5)$

$boundaryl_{71} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5$

$boundaryl_{72} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5) + c_3 \cdot 0.5$

$ls := 73, 73 + 1..79$

$boundaryl_{ls} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_3 \cdot 0.5$

$$boundary1_{80} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5 + c_3 \cdot 0.5$$

$$boundary2_0 := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5)$$

$$po := 1, 1 + 1 .. 7$$

$$boundary2_{po} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5)$$

$$boundary2_8 := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_4 \cdot 0.5$$

$$boundary2_9 := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$$

$$pi := 10, 10 + 1 .. 16$$

$$boundary2_{pi} := k \cdot \rho \cdot g - h \cdot r \cdot 2$$

$$boundary2_{17} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$$

$$boundary2_{18} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$$

$$pu := 19, 19 + 1 .. 25$$

$$boundary2_{pu} := k \cdot \rho \cdot g - h \cdot r \cdot 2$$

$$boundary2_{26} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$$

$$boundary2_{27} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$$

$$py := 28, 28 + 1 .. 34$$

$$boundary2_{py} := k \cdot \rho \cdot g - h \cdot r \cdot 2$$

$$boundary2_{35} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$$

$$boundary2_{36} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$$

$$pt := 37, 37 + 1 .. 43$$

$$boundary2_{pi} := k \cdot \rho \cdot g - h \cdot r \cdot 2$$

$$boundary2_{44} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$$

$$boundary2_{45} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$$

$pr := 46, 46 + 1 .. 52$

$boundary2_{pr} := k \cdot \rho \cdot g - h \cdot r \cdot 2$

$boundary2_{53} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$

$boundary2_{54} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$

$pe := 55, 55 + 1 .. 61$

$boundary2_{pe} := k \cdot \rho \cdot g - h \cdot r \cdot 2$

$boundary2_{62} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$

$boundary2_{63} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5$

$pw := 64, 64 + 1 .. 70$

$boundary2_{pw} := k \cdot \rho \cdot g - h \cdot r \cdot 2$

$boundary2_{71} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5$

$boundary2_{72} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot 0.5 + c_3 \cdot 0.5$

$pq := 73, 73 + 1 .. 79$

$boundary2_{pi} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + c_3 \cdot 0.5$

$boundary2_{80} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_3 \cdot 0.5$

$boundary3_0 := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5 + 0.5) + c_1 \cdot 0.5$

$mn := 1, 1 + 1 .. 7$

$boundary3_{mn} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + c_1 \cdot 0.5 + \mu \cdot r \cdot 0.5$

$boundary3_8 := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5 + \mu \cdot r \cdot 0.5$

$boundary3_9 := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$

$mb := 10, 10 + 1 .. 16$

$boundary3_{mb} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$

$boundary3_{17} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$

$$boundary3_{18} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$$

$$mv := 19, 19 + 1 .. 25$$

$$boundary3_{mv} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$$

$$boundary3_{26} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$$

$$boundary3_{27} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$$

$$mc := 28, 28 + 1 .. 34$$

$$boundary3_{mc} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$$

$$boundary3_{35} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$$

$$boundary3_{36} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$$

$$mx := 37, 37 + 1 .. 43$$

$$boundary3_{mx} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$$

$$boundary3_{44} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$$

$$boundary3_{45} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$$

$$mz := 46, 46 + 1 .. 52$$

$$boundary3_{mz} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$$

$$boundary3_{53} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$$

$$boundary3_{54} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$$

$$ma := 55, 55 + 1 .. 61$$

$$boundary3_{ma} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$$

$$boundary3_{62} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$$

$$boundary3_{63} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5$$

$$ms := 64, 64 + 1 .. 70$$

$$boundary3_{ms} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5$$

$boundary3_{71} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5$

$boundary3_{72} := k \cdot \rho \cdot g - h \cdot r \cdot 2 + \mu \cdot r \cdot (0.5) + c_1 \cdot 0.5 + c_3 \cdot 0.5$

$md := 73, 73 + 1 .. 79$

$boundary3_{md} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_1 \cdot 0.5 + c_3 \cdot 0.5$

$boundary3_{80} := (k \cdot \rho \cdot g - h \cdot r \cdot 2) + c_4 \cdot 0.5 + c_1 \cdot 0.5 + c_3 \cdot 0.5$

$boundary := stack(stack(stack(stack(stack(stack(stack(stack(boundary1, boundary2), boundary2), boundary2), boundary2$

$n := 1, 1 + 1 .. 9$

$u^{\langle 1 \rangle} := \frac{1}{\rho} \cdot \Omega \cdot u^{\langle 0 \rangle} + boundary$

$op := 0, 0 + 1 .. 728$

$Bound_{op} := k \cdot \rho \cdot g - h \cdot r \cdot 2$

$u^{\langle n+1 \rangle} := \frac{1}{\rho} \cdot \Omega \cdot u^{\langle n \rangle} + Bound$

This programe solves three dimension Navier-Stokes equation using Implicit finite difference method [equation(3.8.3) on page 53]

$h := 0.1$

$r := 0.1$

$k := r \cdot h^2$

$k = 0.001$

$U := 10$

$V := 10$

$W := 10$

$\mu := 1$

$\rho := 1$

$c_1 := \mu \cdot r + \rho \cdot h \cdot r \cdot U$

$c_2 := \mu \cdot r + \rho \cdot h \cdot r \cdot V$

$c_3 := \mu \cdot r + \rho \cdot h \cdot r \cdot W$

$c_4 := 6 \cdot \mu \cdot r + \rho + \rho \cdot h \cdot r \cdot U + \rho \cdot h \cdot r \cdot V + \rho \cdot h \cdot r \cdot W$

$$a1 := \begin{bmatrix} -c_4 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_3 & -c_4 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_3 & -c_4 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & -c_4 & \mu \cdot r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & -c_4 & \mu \cdot r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_3 & -c_4 & \mu \cdot r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_3 & -c_4 & \mu \cdot r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_3 & -c_4 & \mu \cdot r \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_3 & -c_4 \end{bmatrix}$$

$$I := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$b1 := \begin{bmatrix} \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \cdot r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu \cdot r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \cdot r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \cdot r \end{bmatrix}$$

$m2 := c_1 \cdot I$

$$m1 := \begin{bmatrix} c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 \end{bmatrix}$$

$$z := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$s1 := augment(augment(augment(augment(augment(augment(augment(augment(a1, b1), z), z), z), z), z), z), z)$

$s2 := augment(augment(augment(augment(augment(augment(augment(augment(m1, a1), b1), z), z), z), z), z), z)$

$s3 := augment(augment(augment(augment(augment(augment(augment(augment(z, m1), a1), b1), z), z), z), z), z)$

$h4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), m1), a1), b1), z), z), z), z)$

$h5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), m1), a1), b1), z), z), z)$

$h6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), m1), a1), b1), z), z)$

$h7 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), m1), a1), b1), z)$

$h8 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), m1), a1), b1)$

$h9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), m1), a1)$

$\lambda := stack(stack(stack(stack(stack(stack(stack(stack(s1, s2), s3), h4), h5), h6), h7), h8), h9)$

$$C := \begin{bmatrix} \mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu\cdot r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu\cdot r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu\cdot r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu\cdot r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu\cdot r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu\cdot r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu\cdot r \end{bmatrix}$$

s4 := augment(augment(augment(augment(augment(augment(augment(augment(C , z) , z) , z) , z) , z) , z) , z) , z)

s5 := augment(augment(augment(augment(augment(augment(augment(augment(z, C) , z) , z) , z) , z) , z) , z) , z)

s6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , C) , z) , z) , z) , z) , z) , z)

s71 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , C) , z) , z) , z) , z) , z)

s81 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , C) , z) , z) , z) , z)

s91 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , C) , z) , z) , z)

s01 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , z) , C) , z) , z)

s11 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , z) , z) , C) , z)

s12 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , z) , z) , z) , C)

G := stack(stack(stack(stack(stack(stack(stack(stack(s4 , s5) , s6) , s71) , s81) , s91) , s01) , s11) , s12)

s7 := augment(augment(augment(augment(augment(augment(augment(augment(m2 , z) , z) , z) , z) , z) , z) , z) , z)

s8 := augment(augment(augment(augment(augment(augment(augment(augment(z, m2) , z) , z) , z) , z) , z) , z) , z)

s9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , m2) , z) , z) , z) , z) , z) , z)

v1 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , m2) , z) , z) , z) , z) , z)

v2 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), m2), z), z), z), z)

v3 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), m2), z), z), z)

v4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), m2), z), z)

v5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), m2), z)

v6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), m2)

θ := stack(stack(stack(stack(stack(stack(stack(stack(s7, s8), s9), v1), v2), v3), v4), v5), v6)

e1 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), z)

E := stack(stack(stack(stack(stack(stack(stack(stack(e1, e1), e1), e1), e1), e1), e1), e1), e1)

q1 := augment(augment(augment(augment(augment(augment(augment(augment(λ, G), E), E), E), E), E), E), E)

q2 := augment(augment(augment(augment(augment(augment(augment(augment(θ, λ), G), E), E), E), E), E), E)

q3 := augment(augment(augment(augment(augment(augment(augment(augment(E, θ), λ), G), E), E), E), E), E)

q4 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), θ), λ), G), E), E), E), E)

q5 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), θ), λ), G), E), E), E)

q6 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), θ), λ), G), E), E)

q7 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), E), θ), λ), G), E)

q8 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), E), E), θ), λ), G)

q9 := augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), E), E), E), θ), λ)

Ω := stack(stack(stack(stack(stack(stack(stack(stack(q1, q2), q3), q4), q5), q6), q7), q8), q9)

ja := 0, 0 + 1 .. 728

Ω := stack(stack(stack(stack(stack(stack(stack(stack(q1, q2), q3), q4), q5), q6), q7), q8), q9)

$Y_{ja} := 1$

$u^{<0>} := Y$

l := 0, 1 .. 728

g := 9.8

$boundary_l := -k \cdot \rho \cdot g + h \cdot r \cdot 2$

$boundary_l := -k \cdot \rho \cdot g + 2 \cdot h \cdot r$

This programe solves three dimension Navier-Stokes equation using Crank-Nicison method [equation (3.9.4) on page 57]

$h := 0.1$

$r := 0.01$

$k := r \cdot h^2$

$k = 1 \cdot 10^{-4}$

$U := 10$

$V := 10$

$W := 10$

$\mu := 1$

$\rho := 1$

$$c_1 := \frac{\mu \cdot r - \rho \cdot h \cdot r \cdot U}{2}$$

$$c_2 := \frac{\mu \cdot r - \rho \cdot h \cdot r \cdot V}{2}$$

$$c_3 := \frac{\mu \cdot r - \rho \cdot h \cdot r \cdot W}{2}$$

$$c_5 := \rho + \frac{\rho \cdot h \cdot r \cdot U + \rho \cdot h \cdot r \cdot V + \rho \cdot h \cdot r \cdot W}{2} - 3 \cdot \mu \cdot r$$

$$c_4 := 3 \cdot \mu \cdot r + \rho - \frac{\rho \cdot h \cdot r \cdot U + \rho \cdot h \cdot r \cdot V + \rho \cdot h \cdot r \cdot W}{2}$$

$$a1 := \begin{bmatrix} -c_4 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\mu \cdot \tau}{2} & -c_4 & c_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\mu \cdot \tau}{2} & -c_4 & c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\mu \cdot \tau}{2} & -c_4 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu \cdot \tau}{2} & -c_4 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu \cdot \tau}{2} & -c_4 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot \tau}{2} & -c_4 & c_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot \tau}{2} & -c_4 & c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot \tau}{2} & -c_4 \end{bmatrix}$$

$$I := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$b1 := \begin{bmatrix} c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_2 \end{bmatrix}$$

$$m1 := \begin{bmatrix} \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} \end{bmatrix}$$

$$z := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

s1 := augment(augment(augment(augment(augment(augment(augment(augment(a1, b1), z), z), z), z), z), z), z), z)

s2 := augment(augment(augment(augment(augment(augment(augment(augment(m1, a1), b1), z), z), z), z), z), z), z)

s3 := augment(augment(augment(augment(augment(augment(augment(augment(z, m1), a1), b1), z), z), z), z), z), z)

h4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), m1), a1), b1), z), z), z), z)

h5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), m1), a1), b1), z), z), z)

h6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , m1) , a1) , b1) , z) , z)

h7 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , m1) , a1) , b1) , z)

h8 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , m1) , a1) , b1)

h9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , z) , m1) , a1)

λ := stack(stack(stack(stack(stack(stack(stack(stack(s1 , s2) , s3) , h4) , h5) , h6) , h7) , h8) , h9)

$$C := \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_1 \end{bmatrix}$$

s4 := augment(augment(augment(augment(augment(augment(augment(augment(C , z) , z) , z) , z) , z) , z) , z) , z)

s5 := augment(augment(augment(augment(augment(augment(augment(augment(z, C) , z) , z) , z) , z) , z) , z) , z)

s6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , C) , z) , z) , z) , z) , z) , z)

s71 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , C) , z) , z) , z) , z) , z)

s81 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , C) , z) , z) , z) , z)

s91 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , C) , z) , z) , z)

s01 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , z) , C) , z) , z)

s11 := augment(augment(augment(augment(augment(augment(augment(augment(z, z) , z) , z) , z) , z) , z) , C) , z)

s12 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), C)

G := stack(stack(stack(stack(stack(stack(stack(s4, s5), s6), s71), s81), s91), s01), s11), s12)

s7 := augment(augment(augment(augment(augment(augment(augment(augment(m1, z), z), z), z), z), z), z), z)

s8 := augment(augment(augment(augment(augment(augment(augment(augment(z, m1), z), z), z), z), z), z), z)

s9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), m1), z), z), z), z), z), z)

v1 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), m1), z), z), z), z), z)

v2 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), m1), z), z), z), z)

v3 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), m1), z), z), z)

v4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), m1), z), z)

v5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), m1), z)

v6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), m1)

θ := stack(stack(stack(stack(stack(stack(stack(s7, s8), s9), v1), v2), v3), v4), v5), v6)

e1 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), z), z)

E := stack(stack(stack(stack(stack(stack(stack(stack(e1, e1), e1), e1), e1), e1), e1), e1), e1)

q1 := augment(augment(augment(augment(augment(augment(augment(augment(augment(λ, G), E), E), E), E), E), E), E), E)

q2 := augment(augment(augment(augment(augment(augment(augment(augment(augment(θ, λ), G), E), E), E), E), E), E), E)

q3 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, θ), λ), G), E), E), E), E), E), E)

q4 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, E), θ), λ), G), E), E), E), E)

q5 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), θ), λ), G), E), E), E)

q6 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), θ), λ), G), E), E)

q7 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), E), θ), λ), G), E)

q8 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), E), E), θ), λ), G)

q9 := augment(augment(augment(augment(augment(augment(augment(augment(augment(E, E), E), E), E), E), E), θ), λ)

Ω := stack(stack(stack(stack(stack(stack(stack(stack(q1, q2), q3), q4), q5), q6), q7), q8), q9)

ja := 0, 0 + 1 .. 728

$Y_{ja} := 1$

$$u^{\langle 0 \rangle} := Y$$

$$a2 := \begin{bmatrix} -c_5 & -c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\mu \cdot r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\mu \cdot r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\mu \cdot r}{2} & -c_5 & -c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -c_5 & -c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -c_5 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -c_5 & -c_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -c_5 & -c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu \cdot r}{2} & -c_5 \end{bmatrix}$$

$$b2 := \begin{bmatrix} -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_2 \end{bmatrix}$$

$$a3 := \begin{bmatrix} -\frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mu \cdot r}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\mu \cdot r}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu \cdot r}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu \cdot r}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu \cdot r}{2} \end{bmatrix}$$

n1 := augment(augment(augment(augment(augment(augment(augment(augment(a2, b2), z), z), z), z), z), z), z)

n2 := augment(augment(augment(augment(augment(augment(augment(augment(a3, a2), b2), z), z), z), z), z), z)

n3 := augment(augment(augment(augment(augment(augment(augment(augment(z, a3), a2), b2), z), z), z), z), z)

n4 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), a3), a2), b2), z), z), z), z)

n5 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), a3), a2), b2), z), z), z)

n6 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), a3), a2), b2), z), z)

n7 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), a3), a2), b2), z)

n8 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), a3), a2), b2)

n9 := augment(augment(augment(augment(augment(augment(augment(augment(z, z), z), z), z), z), z), a3), a2)

R := stack(stack(stack(stack(stack(stack(stack(stack(n1, n2), n3), n4), n5), n6), n7), n8), n9)

mn1 := augment(augment(augment(augment(augment(augment(augment(augment(R, -G), E), E), E), E), E), E), E)

mn2 := augment(augment(augment(augment(augment(augment(augment(augment(-θ, R), -G), E), E), E), E), E), E)

mn3 := augment(augment(augment(augment(augment(augment(augment(augment(E, -θ), R), -G), E), E), E), E), E)

$mn4 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),-\theta),R),-G),E),E),E),E)$

$mn5 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),-\theta),R),-G),E),E),E)$

$mn6 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),-\theta),R),-G),E),E)$

$mn7 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),E),-\theta),R),-G),E)$

$mn8 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),E),E),-\theta),R),-G)$

$mn9 := augment(augment(augment(augment(augment(augment(augment(augment(E,E),E),E),E),E),E),-\theta),R)$

$B := stack(stack(stack(stack(stack(stack(stack(mn1,mn2),mn3),mn4),mn5),mn6),mn7),mn8),mn9)$

$i := 0, 1..7$

$g := 9.8$

$boundaryl_i := -k\cdot\rho\cdot g + h\cdot r\cdot 2 - \frac{\mu\cdot r}{2}\cdot(0.5 + 0.5)$

$boundaryl_8 := -k\cdot\rho\cdot g + h\cdot r\cdot 2 - \frac{\mu\cdot r}{2}\cdot(0.5 + 0.5) - c_3\cdot 0.5$

$boundaryl_9 := -k\cdot\rho\cdot g + h\cdot r\cdot 2 - \frac{\mu\cdot r}{2}\cdot(0.5 + 0.5)$

$l := 10, 10 + 1..16$

$boundaryl_l := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$

$boundaryl_{17} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$

$boundaryl_{18} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot(0.5 + 0.5)$

$x := 19, 19 + 1..25$

$boundaryl_x := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$

$boundaryl_{26} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$

$boundaryl_{27} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot(0.5 + 0.5)$

$lm := 28, 28 + 1..34$

$$boundary1_{lm} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$$

$$boundary1_{35} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$$

$$boundary1_{36} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot (0.5 + 0.5)$$

$$lk := 37, 37 + 1 .. 43$$

$$boundary1_{lk} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$$

$$boundary1_{44} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$$

$$boundary1_{45} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot (0.5 + 0.5)$$

$$lj := 46, 46 + 1 .. 52$$

$$boundary1_{lj} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$$

$$boundary1_{53} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$$

$$boundary1_{54} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$$

$$lh := 55, 55 + 1 .. 61$$

$$boundary1_{lh} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot (0.5 + 0.5)$$

$$boundary1_{62} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$$

$$boundary1_{63} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$$

$$lp := 64, 64 + 1 .. 70$$

$$boundary1_{lp} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5$$

$$boundary1_{71} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot 0.5 - c_3\cdot 0.5$$

$$boundary1_{72} := (-k\cdot\rho\cdot g + h\cdot r\cdot 2) - \frac{\mu\cdot r}{2}\cdot (0.5 + 0.5) - c_2\cdot 0.5$$

$lo := 73, 73 + 1 .. 79$

$boundary1_{lo} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_2 \cdot 0.5$

$boundary1_{80} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_2 \cdot 0.5 - c_3 \cdot 0.5$

$boundary2_0 := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5 + 0.5)$

$pl := 1, 1 + 1 .. 7$

$boundary2_{pl} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$

$boundary2_8 := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_3 \cdot 0.5$

$boundary2_9 := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$

$po := 10, 10 + 1 .. 16$

$boundary2_{po} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$

$boundary2_{17} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$

$boundary2_{18} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$

$pp := 19, 19 + 1 .. 25$

$boundary2_{pp} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$

$boundary2_{26} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$

$boundary2_{27} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$

$pu := 28, 28 + 1 .. 34$

$boundary2_{pu} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$

$boundary2_{35} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$

$boundary2_{36} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$

$$py := 37, 37 + 1 .. 43$$

$$boundary2_{py} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$$

$$boundary2_{44} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$$

$$boundary2_{45} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$$

$$pm := 46, 46 + 1 .. 52$$

$$boundary2_{pm} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$$

$$boundary2_{53} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$$

$$boundary2_{54} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$$

$$pn := 55, 55 + 1 .. 61$$

$$boundary2_{pn} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$$

$$boundary2_{62} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$$

$$boundary2_{63} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5)$$

$$pt := 64, 64 + 1 .. 70$$

$$boundary2_{pt} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2)$$

$$boundary2_{71} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5$$

$$boundary2_{72} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_2 \cdot 0.5$$

$$pa := 73, 73 + 1 .. 79$$

$$boundary2_{pa} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_2 \cdot 0.5$$

$$boundary2_{80} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_2 \cdot 0.5$$

$$boundary3_0 := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5 + 0.5) - c_1 \cdot 0.5$$

$pq := 1, 1 + 1 .. 7$

$boundary3_{pq} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5 - \frac{\mu \cdot r}{2} \cdot 0.5$

$boundary3_8 := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5 - \frac{\mu \cdot r}{2} \cdot 0.5$

$boundary3_9 := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$pw := 10, 10 + 1 .. 16$

$boundary3_{pw} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{17} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{18} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$pe := 19, 19 + 1 .. 25$

$boundary3_{pe} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{26} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{27} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$pr := 28, 28 + 1 .. 34$

$boundary3_{pr} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{35} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{36} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$pz := 37, 37 + 1 .. 43$

$boundary3_{pz} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{44} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{45} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$ps := 46, 46 + 1 .. 52$

$boundary3_{ps} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{53} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{54} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$pvb := 55, 55 + 1 .. 61$

$boundary3_{pvb} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{62} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{63} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5$

$phj := 64, 64 + 1 .. 70$

$boundary3_{phj} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5$

$boundary3_{71} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5$

$boundary3_{72} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - \frac{\mu \cdot r}{2} \cdot (0.5) - c_1 \cdot 0.5 - c_2 \cdot 0.5$

$pzx := 73, 73 + 1 .. 79$

$boundary3_{pzx} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_1 \cdot 0.5 - c_2 \cdot 0.5$

$boundary3_{80} := (-k \cdot \rho \cdot g + h \cdot r \cdot 2) - c_3 \cdot 0.5 - c_1 \cdot 0.5 - c_2 \cdot 0.5$

$boundary := stack(stack(stack(stack(stack(stack(stack(boundary1, boundary2), boundary2), boundary2), boundary2$

$n := 1, 1 + 1 .. 9$

$u^{\langle 1 \rangle} := \Omega^{-1} \cdot B \cdot u^{\langle 0 \rangle} + \Omega^{-1} \cdot boundary$

$op := 0\ 0 + 1 .. 728$

$Bound_{op} := -k \cdot \rho \cdot g + h \cdot r \cdot 2$

$u^{\langle n+1 \rangle} := \Omega^{-1} \cdot B \cdot u^{\langle n \rangle} + \Omega^{-1} \cdot Bound$

Discussions

The results which have been derived by the application of the foregoing solution procedure are presented in both tabular and graphical forms.

We first presented the results for one dimensional flow using an Explicit method. An Inspection of tables on pages (65-66) and figures on page (67-69) reveal that the maximum of the velocity occur at x=0.5, and the minimum of the velocity (u=0) occur at the boundaries x=0 and x=1.

This indicates that the rate of changing of the velocity were greater near the boundaries. It is also evident from the variation of u (x, t) with x that the velocity o f the flow decreases as time increases.

Next, we analyze the results obtained for two and three dimensional flows. For these two cases we used three methods; the explicit method, the implicit method and the Crank-Nicolson method for different r and t.

It can be shown readily from the tables and figures that our results are very close for the three methods. Figures indicate that the maximum of the velocity of the flow occurs at the middle of the region, and the minimum occurs at the boundaries.

It is also interesting to note that the velocity of the flow decresases with increasing time. For example at r=0.1, the maximum velocity of the flow is 0.949 (unit of the velocity) at n=5, while the maximum velocity of the flow is 0.898 (unit of the velocity) at n=10.

Tables on pages (106-109) shows that, u and v have different sign and that indicates that they have different directions, which formed the vortices. It also evident from tables and figures that the vorticity of the flow decreases as time increeases. For example the maximum of the vortices is about 0.996 at n=10. Whereas, the maximum of the vortices is about 0.221 at n=100.

الخلاصة

يهدف هذا البحث إلى عرض عدد من الطرق المختلفة لحل معادلات الحركة المسماة (Navier Stokes Equations) بالتحليل العددي في بعد واحد، وبعدين، وثلاثة أبعاد.

ويحتوي الفصل الأول على مقدمة لتعريف الطريقة التي اختيرت لحل تلك المعادلات وهي طريقة الفرقية المنتهية (Finite Difference)، حيث استخدمت طريقتين لربط معادلات الفرقية المنتهية بالمؤثرات التفاضلية (Differential Operators) .

وفي الفصل الثاني بينا خطوات اشتقاق معادلات الحركة (Navier Stokes Equations) باستخدام ممثل حجمي للتحكم (Control Volume) .
أما الجزء الأول من الفصل الثالث فيبين خطوات تحويل معادلات الحركة من معادلات لاخطية إلى معادلات خطية باستخدام طريقة الرجفلن (Perturbation Method) . وفي الأجزاء الأخرى استعراض للحلول المختلفة باستخدام طريقة الفرقية الضمنية و الصريحة (Implicit and Explicit) .

أما الفصل الرابع فيبين نتائج الحلول مع مقارنتها بالرسوم البيانية.