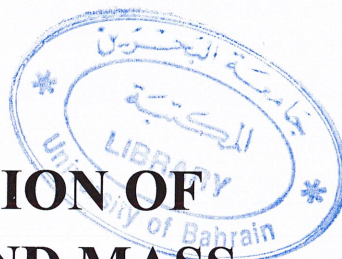


UNIVERSITY OF BAHRAIN

College of Science

Department of Mathematics



**NUMERICAL SOLUTION OF
NONLINEAR HEAT AND MASS
TRANSFER PROBLEM WITH
DIFFUSION IN THE GASEOUS PHASE**

A Thesis Submitted in Partial Fulfillment of the Requirements for the
Master Degree in Mathematics

Submitted by

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University of Bahrain
Graduate Studies Council



Thesis Discussion Committee Decision

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Thesis Title: <u>Numerical solution of nonlinear heat and mass transfer problem with diffusion in the gaseous phase</u>	
Supervisor Name: <u>Prof. Nail Sulaiman</u>	Student GPA: <u>3.27</u>

The Discussion Committee met on Monday 12/1/15 in Hall no. 541
(hall/room no.) 2000. Upon discussing the student whose details are stated above in public, the discussion committee recommends the following:

The student awarded grade A- in the thesis subject Math 599

- ☐ Without making any changes to the thesis
☒ After making the modifications attached with this decision

Discussion Committee Members Approval:

- Name
- Prof. Nail Sulaiman (Advisor)
 - Prof. ALHAMMADI (Internal Examiner)
 - Prof. Christodoulos Sophocleous (External Examiner)

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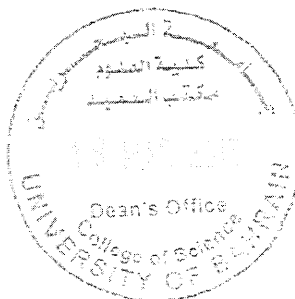
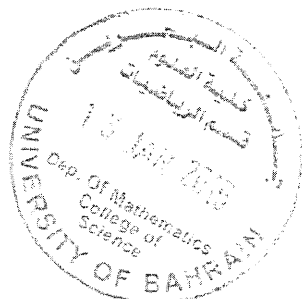
Thesis Modifications Approval Form

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We would like to report that the student has carried out all the required modifications made by the discussion committee. Upon this, we recommend awarding the student grade A- in thesis course number Math 599 based on the Discussion Committee recommendation.

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ABSTRACT

In the present thesis we describe the method of numerical solution and give the results of investigation of nonlinear heat problem with moving boundary.

We assumed the spherical symmetry of the process and the uniformity of the pressure within the bubble. We derived the basic equations of our problem. We used the variable $y = \frac{r}{R}$ freeze the moving boundary of the bubble. After that, we used the non dimensional parameters to obtain the numerical solution easily.

We transfer Rayleigh equation form the second order differential equation to a system of two first order differential equations by using the following substitution $\frac{dR}{dt} = w$.

We solved a mixed system, containing ordinary and partial differential equations, using finite-difference technique by dividing the whole system into spherical layers inside the bubble. We get a system with $n + 3$ ordinary differential equation to solve numerically, we used n between 30 to 100, since this is the value that if we will make it double result will not change.

The results of numerical calculations presented as figures showing the behavior of radius and temperature distributions inside the bubble and their evolution during the time using the power of Mathematica 8 and Maple 15 program. We used RungeKutta numerical method with other numerical methods to solve the system.

**TO MY FAMILY
AND
MY FRIENDS**

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I would like to express my deep gratitude and thanks to my adviser professor Nail Suleiman for his valuable guidance and the supervision at various stages of my work to put this thesis in its final form.

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This thesis is dedicated to my wife whose encouragement has meant so much to me during the pursuit of my graduate degree and when writing this thesis.

TABLE OF CONTENTS

Subject	Pages
THESIS DISCUSSION COMMITTEE DECISION	i
THESIS MODIFICATIONS APPROVAL FORM	ii
ABSTRACT	iii
DEDICATION	iv
ACKNOWLEDGMENTS	v
TABLE OF CONTENTS	vi
LIST OF SYMBOLS	viii
LIST OF FIGURES	x
LIST OF APPENDIXES	xi
(1) INTRODUCTION	2
(1.1) Background and history	2
(1.2) Statement of Purpose	3
(1.3) Justification of the problem	4
(1.4) Theoretical framework	5
(1.4.1) Methods and procedures	6
(1.4.2) Proposed chapter outline	8
(2) LITERATURE REVIEW	10
(3) METHODS AND PROCEDURES	14
(3.1) Introduction	14
(3.2) Governing equations for the gas-vapor bubble with diffusion in the gaseous phase	19
(3.3) Governing equations for gas bubble in liquid	23
(3.4) Non-dimensional Parameters	26
(3.5) One dimensional finite-difference technique	28

4 THE RESULTS, DISCUSSIONS AND CONCLUSIONS	32
(4.1) The results and discussions	32
(4.2) Conclusions	38
REFERENCES	39
APPENDIX:A	
(A) Programing Codes	
(A.1) Mathematica 8	
(A.2) MAPLE	
SUMMARY IN ARABIC	

LIST OF SYMBOLS

Symbol	Meaning
$a = R/R_0$	non-dimensional bubble radius
B	gas constant
c	specific heat
C	sound speed in gas
D	thermal diffusivity of gas
h	thickness of spherical layer
m	mass of the bubble
p	pressure
$P = p/p_0$	non-dimensional pressure
R	bubble radius
$\dot{R}$	time derivative of the radius
r	radial coordinate
T	temperature
τ	time
t	non-dimensional time
n	number of layers
$N = p_0 R^2 / \rho_l D^2$	non-dimensional parameter
u	specific internal energy
v	velocity
w	velocity of the bubble surface
$y = r/R(t)$	non-dimensional coordinate
k	concentration
E	inter-diffusion coefficient
j	rate of phase transitions from a unit surface

l	specific heat of vaporization
κ	accommodation coefficient
μ	viscosity
η	diffusion rate
γ	specific heat ratio
ρ	density
ν	kinematic viscosity
σ	surface tension
ω	circular frequency
θ	non-dimensional temperature
λ	thermal conductivity

Subscripts	Meaning
0	at equilibrium
g	gas
l	liquid
s	at saturation
∞	condition far away from bubble
P	at constant pressure
V	at constant volume
σ	on bubble surface
v	vapour

LIST OF FIGURES

Number	Figure	Page
3.1	The scheme of dividing the bubble interior into n spherical layers	28
4.1	Radius vs time curve for the oscillating gas bubble: $R_0 = 0.05\text{mm}$, $p_0 = 1$ bar, $p_\infty = 3$ bar	32
4.2	Radius vs time curve for the oscillating gas bubble: $R_0 = 0.05\text{mm}$, $p_0 = 1$ bar, $p_\infty = 1.2$ bar	33
4.3	Radius vs time curve for the oscillating gas bubble: $R_0 = 0.05\text{mm}$, $p_0 = 1$ bar, $p_\infty = 0.5$ bar	34
4.4	Radius vs time curve for the oscillating gas bubble: $R_0 = 0.5$ mm, $p_0 = 1$ bar, $p_\infty = 1.2$ bar	35
4.5	Characteristic temperature distribution within an oscillation gas bubble at different moments of time. Curves 1-5 correspond to the following moment of time $t = 0.1175, 0.13, 0.15, 0.187$ and 0.2 . $R_0 = 0.05$ mm, $p_0 = 1\text{bar}$, $p_\infty = 1.2$ bar	36
4.6	Characteristic temperature distribution within an oscillation gas bubble at different moments of time. Curves 1-5 correspond to the following moment of time $t = 0.1175, 0.13, 0.15, 0.187$ and 0.2 . $R_0 = 0.05$ mm, $p_0 = 1$ bar, $p_\infty = 0.5$ bar	37

LIST OF APPENDIXES

Number	Appendix
<i>A</i>	PROGRAMING CODES
<i>A.1</i>	Mathematica 8.
<i>A.2</i>	Maple15.

CHAPTER 1: INTRODUCTION

- 1.1 Background and History.**
- 1.2 Statement of Purpose.**
- 1.3 Justification of the problem.**
- 1.4 Theoretical framework.**
 - 1.4.1 Methods and procedures.**
 - 1.4.2 Proposed Chapter outline.**

Chapter: 1

INTRODUCTION

1.1 Background and history.

In vapour-liquid flows, the mass, force and energy interactions between phases originate on the interface surfaces. These interactions can significantly alter the flow velocity, pressure and temperature fields. A correct specification of the inter-phase heat and mass transfer requires the knowledge of interaction of single inclusion with the carrier phase. It has been revealed that in the case of gas bubble the heat and mass transfer dominates over other dissipation mechanisms i.e. viscosity and compressibility of the liquid. Some authors used the assumption of the temperature uniformity in the bubbles and the thinness of a thermal boundary layer in the liquid. These assumptions considerably simplify the problem but hold under certain restrictions only. This Thesis will give the results of investigation of the heat and mass transfer effect on the dynamics of gas bubbles as well as of the reverse effect of dynamics of radial bubble motion on the enhancement of heat transfer between the gas bubbles and liquid. These studies will be carried out with reference to the analysis of wave processes in vapor-liquid mixtures having a bubbly structure.

1.2 Statement of Purpose.

A nonlinear problem of thermal, mass and dynamic interaction between a gas bubble and a liquid will be considered with account of moving boundary and temperature and concentration non-uniformity in the bubble. We will consider single spherical bubble in liquid assuming spherically symmetry of the process and pressure uniformity inside the bubble. At the same time it is supposed that the gas temperature and concentration are not uniform. Such statement of the problem when the pressure uniformity and the temperature and density non-uniformities are assumed is valid for a wide range of bubble size, since the characteristic time of temperature equalization in the bubble considerably exceeds the time of pressure equalization. The numerical method will be used by dividing the whole system into spherically layers inside the bubble and employing a new variable which freeze the moving boundary of the bubble. Results of numerical simulation will be presented in the form of figures. The effect of different parameters on the process will be investigated in details.

1.3 Justification of the problem.

The study of gas bubbles in a liquid is of considerable practical interest in regards of possible use of bubble curtains for damping or amplification of shock waves and using in the industry for intensification of technological processes. It is due to the great value of transfer coefficients and large inter-phase surfaces. In order to intensify heat and mass transfer processes the possibilities of acoustic or shock wave effect on the process were studied. We will consider the behavior of a single bubble in liquid. This approach is used when the volume concentration of gas is fairly low and the influence of the neighbor bubbles is neglected. In case of small velocity of the floating bubbles we can study the process in the framework of spherically-symmetric model.

1.4 Theoretical framework.

Dynamics, heat and mass transfer of the gas bubble will be solved numerically by using finite-difference method.

The whole system will be divided into spherical layers inside the bubble. So we will solve mixed system, containing ordinary and partial differential equations (heat equations).

The calculations will be checked by means of a fit of the bubble mass predicted by integration of mass conservation equation and what obtained by direct layer to layer calculations. It is assumed that for calculation will be taken water-steam system.

Some of terms in the thesis(keywords):

Bubble radius, mass of the bubble, non-dimensional bubble radius, gas constant, sound speed in gas, thermal diffusivity of gas, pressure, time derivative of the radius, radial coordinate, temperature, time, non-dimensional time, number of layers, non-dimensional parameter, specific internal energy, velocity, velocity of the bubble surface, non-dimensional coordinate, density, kinematic viscosity, surface tension, circular frequency, non-dimensional temperature, thermal conductivity.

1.4.1 Methods and procedures.

The following steps will be done during the study of this problem:

- (1) To study the nonlinear problem of thermal mass and dynamic interaction between a gas bubble and surrounding liquid.
- (2) The system of basic equations describing this process assuming it's spherically symmetry should be constructed.
- (3) A numerical method for solving the system of equations, including partial and ordinary differential equations should be created. The new variable which freezes the moving boundary of bubble should be introduced.
- (4) The problem will be solved using finite-difference technique by dividing the whole system into spherical layers inside the bubble. The calculations will be checked by means of a fit of the bubble mass predicted by integration of mass balance equation to that obtained by direct layer to layer calculation.
- (5) Numerical calculations will be made for gas bubble in liquid with variation of initial conditions and the values of initial parameters such as initial bubble radius, temperature of the liquid and others.
- (6) On the base of numerical calculations the conclusions will be made about the effect of different parameters on the process. Calculations will be made for the case of a sudden pressure

increase in the liquid far from the bubble a situation corresponding to the behavior of bubbles in the front part of the bubble screen upon entrance of a shock wave.

- (7) The results of numerical calculations will be presented as figures showing the behavior of temperature and concentration distributions inside the bubble and their evolution during the time.

The method of RungeKutta will be used for solving the system including more than 100 ordinary differential equations (after transformation of heat and diffusion equations into the system of ordinary differential equations by using finite-difference method). Results of numerical solution of the problem will be illustrated on figures showing temperature and concentration distributions.

1.4.2 Proposed chapter outline.

The thesis contains four chapters:

Chapter one (Introduction).

It includes:

- a. Background and history of the research.
- b. Statement of Purpose.
- c. Justification of the problem.
- d. Theoretical framework and Research Overview.

Chapter two (Review of Literature).

In this chapter we present a background comprehensive review of the literature in addition to that covered in the introduction.

Chapter three (Methods and Procedures).

In this chapter we present the Derivation of Basic equations describing dynamics, heat and mass transfer of gas bubble in liquid.

Then we present the Numerical Method for solving the mixture of ordinary and partial differential equations with the powerful of Maple 15 and Mathematica 8 programs.

Chapter four (The Results, Discussions and Conclusions).

In this chapter we present the Results and discussions with all figures. Finally, we end with conclusions.

CHAPTER 2: LITERATURE REVIEW

Chapter: 2

LITERATURE REVIEW

The heat and mass transfer between gas bubble and liquid was studied in a number of publications using different simplifications. Some authors used the assumption of the temperature uniformity in the bubbles.

The assumptions adopted are those used in the Rayleigh (1917) formulation for the dynamics of a single bubble, viz. a spherical symmetry of the process and pressure uniformity within the bubble.

Chapman & Plesset (1971) found that liquid has much higher heat conductivity and much less thermal diffusivity than gas.

Nigmatulin & Khabeev (1974) found that when the pressure uniformity and the temperature and gas density nonuniformities in the bubble are assumed, is valid for a wide range of bubble sizes, since it has been estimated that the characteristic time of temperature equalization in the bubble considerably exceeds the time of pressure equalization. They also conclude that in the case of uniform pressure of the gas, the problem of heat transfer between oscillating gas bubble and liquid can be transformed to nonlinear PDE (heat equation) and two ordinary differential equations. For the first time such system of equations was desired, also they solve the system numerically by using Lagrangian coordinates, because in Lagrangian coordinates the bubble surface is not moving during oscillations.

Nigmatulin (1990) shows that, in the course of bubble oscillations, the pressure uniformity prevails when the size of the bubble is much less than the length of sound wave in the gas.

Later in Prosperetti (1991) & Sedov (1997) they use and developed this approach.

The review of latest publications in this field is done by Lauterborn (2010).

Two phase gas-liquid flow in pipes are widely spread in space application: bubble flows appear in cryogenic components transport through fuel/oxidant supply lines. Another important application is based on the fact that in liquid flows with small bubbles a close contact between the two phases occurs resulting in high rates of transfer between them. The compactness of a system makes it ideally suited to serve as a spacebased two-phase bioreactor which forms an important unit in environmental control and life support system deployed on board. Shagapov, Ginmaltdinov, Khabeev & Bailey (2003) consider the study of oscillations of gas bubbles in a liquid as a practical interest.

In the present thesis the problem will be studied with taking into account the effect of many basic parameters. This Thesis will give the results of investigation of the heat and mass transfer effect on the dynamics of gas bubbles as well as of the reverse effect of dynamics of radial bubble motion on the enhancement of heat transfer between the gas bubbles and liquid. These studies will be carried out with reference to the analysis of wave processes in gas-liquid mixtures having a bubbly structure.

We will study the nonlinear problem of thermal and dynamic interaction between a gas bubble and surrounding liquid. We should introduce a new variable which freezes the moving boundary of bubble. Numerical calculations will be made for gas bubble in liquid with variation of initial conditions and the values of initial parameters such as initial bubble radius, temperature of the liquid and others.

On the base of numerical calculations the conclusions will be made about the effect of different parameters on the process. Calculations will be made for the case of a sudden pressure increase in the liquid far from the bubble a situation corresponding to the behavior of bubbles in the front part of the bubble screen upon entrance of a shock wave.

CHAPTER 3: METHODS AND PROCEDURES

- 3.1 Introduction.**
- 3.2 Governing equations for the gas-vapor bubble with diffusion in the gaseous phase.**
- 3.3 Governing equations for gas bubble in liquid.**
- 3.4 Non-dimensional Parameters.**
- 3.5 One dimensional finite difference technique**

Chapter: 3

METHODS AND PROCEDURES

3.1 Introduction.

First, let us derive the basic equations for our problem. Let us consider a spherical bubble of radius R in an infinite domain of a liquid with pressure far from the bubble is p_∞ and liquid density is ρ_l . Rayleight (1917) analyzed the problem of cavitation and dynamics of a bubble. He derived in the absence of the surface tension and the viscosity the following equation (Brennen (1995) & Leighton (1993)):

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_R - p_\infty}{\rho_l}. \quad (3.1)$$

(3.1) was developed by Plesset in (1949) by involving the surface tension σ and the viscosity of liquid μ_l , and he found this equation:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_R - p_\infty}{\rho_l} - \frac{2\sigma}{\rho_l R} - \frac{4\mu_l \dot{R}}{\rho_l R}. \quad (3.2)$$

To derive (3.1) we will start with the continuity equation:

$$\frac{\partial \rho_l}{\partial t} + \text{div}(\rho_l \bar{v}) = 0. \quad (3.3)$$

In case of dealing with an in-compressible fluid, then the density $\rho_l = \text{constant}$, hence $\text{div}(\bar{v}) = 0$. When we consider the spherical bubble, $\text{div}(\bar{v}) = 0$ we take the following form when

we transform it to spherical coordinates:

$$\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 v) = 0. \quad (3.4)$$

Thus

$$\frac{\partial}{\partial r}(r^2 v) = 0. \quad (3.5)$$

Hence

$$r^2 v(r, t) = f(t), \quad (3.6)$$

where $f(t)$ is function depended on $R(t)$ kinematic boundary condition at the bubble surface. $f(t) = R^2 \dot{R}$.

Since we deal with in-compressible liquid then the velocity at distance r from the center of the bubble is given by:

$$v(r, t) = \frac{R^2 \dot{R}}{r^2}. \quad (3.7)$$

Using chain rule:

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r}. \quad (3.8)$$

Where the total derivative $\frac{dv}{dt}$ is the rate of change of velocity of a given fluid particle as it moves in space.

Substitute (3.7) into (3.8), and we can obtain:

$$\frac{dv}{dt} = \frac{\ddot{R} R^2}{r^2} + \frac{2R \dot{R}^2}{r^2} - \frac{2R^4 \dot{R}^2}{r^5}. \quad (3.9)$$

Substitute (3.9) into Euler's equation in one-dimensional motion

$$\rho_l \frac{d\bar{v}}{dt} = -\frac{\partial p}{\partial r}. \quad (3.10)$$

We get:

$$\rho_l \left(\frac{\ddot{R}R^2}{r^2} + \frac{2R\dot{R}^2}{r^2} - \frac{2R^4\dot{R}^2}{r^5} \right) = -\frac{\partial p}{\partial r}. \quad (3.11)$$

Now, integrate both sides of (3.11) with respect to r from R to ∞ we get:

$$\rho_l \left(\ddot{R}R + 2\dot{R}^2 - \frac{\dot{R}^2}{2} \right) = p_R - p_\infty$$

Multiplying both sides by ρ_l^{-1} we get (3.1). To get (3.2) we should construct a dynamic boundary condition on the bubble. For this purpose consider a control volume consisting of small infinitely thin lamina containing a segment of interface Brennen (1995).

Brennen (1995): the force per unit area is:

$$p_B - p_R - \frac{2\sigma}{R} - \frac{4\mu_l\dot{R}}{R}. \quad (3.12)$$

Where p_B is the pressure inside bubble. In the absence of mass transport across the boundary (evaporation or condensation) this force must be zero, so when we substitute the value of p_R in (3.1) we get the generalized Rayleigh-Plesset equation for bubble dynamics (3.2).

The continuity equations and the heat flux equations for the gas phase inside a bubble, when the motion has a spherical symmetry and when the viscosity is neglected, with taking the

heat exchange into account, takes the form:

$$\frac{d\rho}{dt} + \frac{\rho}{r^2} \frac{\partial(r^2 v)}{\partial r} = 0. \quad (3.13)$$

$$\rho \frac{di}{dt} = \frac{dp}{dt} - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 q_g), \quad (3.14)$$

where $i = u_g + \frac{p}{\rho} + \text{const.} = c_p T + \text{const.}$. Then, using the Fourier's law of heat conduction for the heat transfer in a gas, we have:

$$q_g = -\lambda \frac{\partial T}{\partial r}, \quad (3.15)$$

where λ is the coefficient of the heat conductivity in a gas. Multiply the continuity equation (3.13) by i and add the result to heat flux equation (3.14), we obtain

$$\frac{d\rho i}{dt} + \frac{\rho i}{r^2} \frac{\partial v r^2}{\partial r} = \frac{dp}{dt} + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \lambda \frac{\partial T}{\partial r} \right). \quad (3.16)$$

Then by assuming that the fluid is in-compressible and a gas in a bubble is described by the state equation of perfect gas with constant heat capacity c_p and c_v :

$$\begin{aligned} \rho &= \frac{p}{T(c_p - c_v)}, \quad \left(\frac{\partial i}{\partial p} \right)_T = 0, \\ i &= u_g + \frac{p}{\rho} + \text{const.} = c_p T + \text{const.} \end{aligned} \quad (3.17)$$

For a perfect gas, multiplying (3.16) by r^2 and integrating it with respect to r from 0 to R , we get:

$$\frac{dp(t)}{dt} = \frac{3(\gamma - 1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma p \dot{R}}{R}. \quad (3.18)$$

Using and the heat flux equation (3.14) can be rewritten in the following form:

$$\frac{\gamma}{\gamma - 1} \frac{p}{T} \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \lambda \frac{\partial T}{\partial r} \right) + \frac{dp(t)}{dt}. \quad (3.19)$$

3.2 Governing equations for the gas-vapor bubble with diffusion in the gaseous phase.

The assumptions adopted are those used in the Rayleigh (1917) formulation for the dynamics of a single bubble, viz. a spherical symmetry of the process and pressure uniformity $p_v(t)$ (homobaricity) within the bubble. In the course of bubble oscillation, the homobaricity prevails when the size of the bubble is much less than the length of a sound wave in the gas, $\omega R \ll C$. On the other hand, when the bubble radius changes monotonically (collapse, growth), the homobaricity condition can be written down (Nigmatulin (1978)) as

$$\left(\frac{v_\sigma}{C}\right) \ll 1, \quad (3.20)$$

where v_σ is velocity of the bubble surface. At the same time it is supposed according to the equation of state the gas density at each point corresponds to its temperature. This statement of the problem, when the pressure and uniformity and temperature and gas density non-uniformities in the bubble are assumed, is valid for a wide range of bubble sizes, since it has been estimated (Nigmatulin & Khabeev (1974)) that the characteristic time of temperature equalization in the bubble considerably exceeds the time of pressure equalization. By introducing the concentration of vapour, k_v and of the inner gas, k_g , in the vapour-gas mixture the thermo-physical parameters of the mixture at each point will be determined as

$$\begin{aligned}
\varphi &= k_v \varphi_v + k_g \varphi_g \\
k_v &= \frac{\rho_v}{\rho}, \quad k_g = \frac{\rho_g}{\rho}, \quad \rho = \rho_v + \rho_g \\
k_v + k_g &= 1.
\end{aligned}$$

In what follows, we shall denote, for simplicity, k_v by k and k_g by $k - 1$.

Within the framework of the assumptions made and the notation adopted, the continuity, state and heat influx equations will have the form

$$\left. \begin{aligned}
&\frac{\partial \rho_v}{\partial \tau} + \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 \rho_v (v + \eta_v)] = 0 \\
&\frac{\partial \rho_g}{\partial \tau} + \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 \rho_g (v + \eta_g)] = 0 \\
&\rho_g \eta_g = -\rho_v \eta_v = \rho E \frac{\partial k}{\partial r}, \quad [0 \leq r \leq R(t)] \\
&u_g = c_g T, \quad u_v = c_v T, \quad T_v = T_g = T, \\
&p = p_v + p_g = \rho B T \\
&\rho_g \frac{du_g}{d\tau} + \rho_v \frac{du_v}{d\tau} = \frac{p}{\rho} \frac{d\rho}{d\tau} + \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} + \rho E \frac{\partial k}{\partial r} \frac{\partial (u_v - u_g)}{\partial r} \right) \\
&r^2 v_l = R^2 w_l, \quad u_l = c_l T_l, \rho_l = \text{const.}, \quad (R < r < \infty) \\
&\rho_l \left(\frac{\partial u_l}{\partial \tau} + v_l \frac{\partial u_l}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(\lambda_l r^2 \frac{\partial T_l}{\partial r} \right) + 12 \nu_l \frac{v_l^2}{r^2}.
\end{aligned} \right\} \quad (3.21)$$

The boundary condition for the system (3.21) on the moving

boundary $R(t)$, at the center and at infinity are

$$\left. \begin{aligned} r = R(t) : \quad & T_l = T_v = T_\sigma, \quad \lambda_l \frac{\partial T}{\partial r} = jl, \\ & \rho_v \left(\dot{R} - v - \eta_v \right) = \rho_l \left(\dot{R} - v_l \right) = j, \quad \rho_g \left(\dot{R} - v - \eta_g \right) = 0. \\ r = 0 : \quad & \frac{\partial T}{\partial r} = \frac{\partial k}{\partial r} = 0, \quad T_l(\infty) = T_\infty. \end{aligned} \right\} \quad (3.22)$$

The kinetics of phase transitions is described by the Hertz-Knudsen-Langmuir equation

$$j = \kappa \frac{p_s T_\infty - p_{v\sigma}}{(2\pi B_v T_\sigma)^{\frac{1}{2}}}. \quad (3.23)$$

The equation for oscillations in a viscous in-compressible fluid in the presence of phase transitions (Nigmatulin (1978)) and that for the bubble mass change are

$$R\dot{w}_l^2 + \frac{3}{2}w_l^2 + 2w_l \frac{j}{\rho_l} = \frac{p - p_\infty - \frac{2\sigma}{R}}{\rho_l} - \frac{4\nu_l}{\rho_l R} w_l \quad (3.24)$$

$$\dot{m} = 4\pi R^2 j. \quad (3.25)$$

When the homobaricity condition is satisfied, there is an integral of the heat influx equation for the gas phase

$$\left. \begin{aligned} \frac{dp}{d\tau} &= \frac{-pR^2 w + \int_0^R Gr^2 dr}{\frac{R^3}{3} - \int_0^R \frac{B}{c+B} r^2 dr} \\ G &= \frac{(B_v - B_g) c_v T}{c + B} \frac{\partial}{r^2 \partial r} \left(\rho E r^2 \frac{\partial k}{\partial r} \right) \\ &\quad + \frac{B}{c + B} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} \right) + (c_v - c_g) \rho E \frac{\partial k}{\partial r} \frac{\partial T}{\partial r} \right]. \end{aligned} \right\} \quad (3.26)$$

The continuity equation for the vapour-gas mixture, with

the use of homobaricity condition and the boundary condition $v(0, \tau) = 0$, yields the velocity profile in the bubble

$$v(r, \tau) = \frac{1}{pr^2} \int_0^r \left(G + \frac{B}{c + B} \frac{dp}{d\tau} \right) r^2 dr - \frac{r}{3p} \frac{dp}{d\tau}. \quad (3.27)$$

For a one-component bubble, equations (3.26) and (3.27) can be considerably simplified

$$\frac{dp}{d\tau} = \frac{3(\gamma - 1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma pw}{R} \quad (3.28)$$

$$v(r, \tau) = \frac{r}{R} w + \frac{\gamma - 1}{\gamma p} \left[\lambda \frac{\partial T}{\partial r} - \frac{r}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R \right]. \quad (3.29)$$

As is seen, the temperature gradients deviate the velocity profile forms a linear one.

For the temperature dependence of the saturated vapour pressure the following approximation (Nigmatulin (1978)) has been applied

$$p_s = p_* \exp \left(-\frac{T_*}{T_s} \right), \quad (3.30)$$

where p_* and T_* are found from the condition of the best approximation of tabulated data in the specified pressure range. Ordinary exponential temperature dependences have been employed for the heat conduction and diffusion coefficients.

Here, for simplification of calculations we will consider particular case of one-component gas bubble without phase transitions. In such case the system of general equations can be simplified.

3.3 Governing equations for gas bubble in liquid.

Within the framework of the assumptions made, the continuity, state and heat equation inside the bubble will have the form:

$$\left. \begin{aligned} \frac{\partial \rho_g}{\partial \tau} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho_g v) &= 0. \\ u_g &= c_V T, \quad p_g(t) = \rho_g(r, t) B T(r, t) \\ \rho_g c_p \left(\frac{\partial T}{\partial \tau} + v \frac{\partial T}{\partial r} \right) &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} \right) + \frac{dp}{d\tau}. \end{aligned} \right\} \quad (3.31)$$

The boundary conditions for the system (3.31) on the moving boundary $R(t)$ and at the center can be written as

$$\begin{aligned} r = R(t) &: \quad T = T_0, \quad v = \dot{R}. \\ r = 0 &: \quad v = 0, \quad \frac{\partial T}{\partial r} = 0. \end{aligned} \quad (3.32)$$

The temperature at the bubble surface may be set as $T = T_0$, where T_0 is temperature of the surrounding liquid which acts as a thermostat. This is related with the fact, that liquid has much higher heat conductivity and much less thermal diffusivity than gas Chapman & Plesset (1971).

The dynamics of bubble's radial oscillation is described by Rayleigh-Plesset equation Plesset (1977) & Leighton (1997):

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p - p_\infty - \frac{2\sigma}{R}}{\rho_l} - 4\frac{\nu_l}{R}\dot{R}. \quad (3.33)$$

When the pressure uniformity takes place, there is an integral

of the heat equation for the gas phase (Nigmatulin & Khabeev (1974)):

$$\frac{dp}{d\tau} = \frac{3(\gamma - 1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma p \dot{R}}{R}. \quad (3.34)$$

Multiplying the continuity equation by r^2 and Integrating it from 0 to r taking into account (3.32) we can obtain the velocity distribution inside the bubble

$$v(r, t) = \frac{r}{R} \dot{R} + \frac{\gamma - 1}{\gamma p} \left[\lambda \frac{\partial T}{\partial r} - \frac{r}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R \right]. \quad (3.35)$$

As it seen that the temperature gradients deviate the velocity profile from linear one.

In the case of uniform pressure distribution the heat flux equation for the gas can be rewritten in the form of nonlinear heat equation

$$\frac{\gamma}{\gamma - 1} \frac{p}{T} \left(\frac{\partial T}{\partial \tau} + v \frac{\partial T}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \lambda \frac{\partial T}{\partial r} \right) + \frac{dp}{d\tau}. \quad (3.36)$$

So, in the case of uniform pressure of the gas, the problem of heat transfer between oscillating gas bubble and liquid can be transformed to nonlinear PDE "heat equation" and two ordinary differential equations. For the first time such system of equations was desired in Nigmatulin & Khabeev (1974). Later such approach was used and developed in Prosperetti (1991) & Sedov (1997). The review of latest publication in this field is done in Lauterborn (2010). In Nigmatulin & Khabeev (1974)

the system was solved numerically by dividing the whole system into spherical layers inside the bubble by using Lagrangian coordinate, because in Lagrangian coordinate the bubble surface is not moving during oscillations. In the present thesis we use the new variable $y = \frac{r}{R(t)}$ which freezes the moving boundary of the bubble.

For control of accuracy of calculation we can calculate the mass of the bubble. It should be constant.

In case of direct layer to layer calculation

$$m_0 = Pa^3 \left\{ \frac{\frac{4}{3}\pi \left(\frac{h}{2}\right)^3}{\theta_1} + \sum_{i=1}^{n-1} \frac{4\pi}{3\theta_{i+1}} \left[\left(h \left(i + \frac{1}{2} \right) \right)^3 \right] \right\} = \text{constant}. \quad (3.37)$$

In the examples considered here, the agreement was within $\pm 3\%$.

3.4 Non-dimensional Parameters.

Let us use non-dimensional parameters Sedov (1997).

$$\begin{aligned}
 P &= \frac{p}{p_0}, & \theta &= \frac{T}{T_0}, & a &= \frac{R}{R_0}, & t &= \frac{\tau}{t_0}, & t_0 &= \frac{R_0^2}{D}, \\
 D &= \frac{\lambda}{\rho c_p}, & W &= \frac{w}{v_0}, & v_0 &= \frac{D}{R_0}, & V &= \frac{v}{v_0}, & P_\infty &= \frac{p_\infty}{p_0}, \\
 y &= \frac{r}{R(t)} & 0 \leq y \leq 1.
 \end{aligned}$$

System of equations, which should be solved are given by

$$\left. \begin{aligned}
 &\frac{\gamma}{\gamma-1} \frac{p}{T} \left(\frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial y} V - y \frac{dR}{d\tau} \right) = \frac{\lambda}{R^2 y^2} \frac{\partial}{\partial y} \left(y^2 \frac{\partial T}{\partial y} \right) + \frac{dp}{d\tau}, \\
 &\frac{dp}{d\tau} = \frac{3}{R} \left[(\gamma-1) \frac{\lambda}{R} \frac{\partial T}{\partial y} \Big|_1 - \gamma p \frac{dR}{d\tau} \right], \\
 &R \frac{d^2 R}{d\tau^2} + \frac{3}{2} \left(\frac{dR}{d\tau} \right)^2 = \frac{p - p_\infty}{\rho_l}, \\
 &V(y, \tau) = y \frac{dR}{d\tau} + \frac{\gamma-1}{\gamma P} \frac{\lambda}{R} \left[\frac{\partial T}{\partial y} - y \frac{\partial T}{\partial y} \Big|_1 \right], \\
 &T(1, \tau) = T_0 = \text{constant}, \quad \frac{\partial T}{\partial y} \Big|_{y=0} = 0.
 \end{aligned} \right\} \quad (3.38)$$

Take the second order equation

$$R \frac{d^2 R}{d\tau^2} + \frac{3}{2} \left(\frac{dR}{d\tau} \right)^2 = \frac{p - p_\infty}{\rho_l}. \quad (3.39)$$

We transform (3.39) to system of first order equations

$$\left. \begin{aligned}
 &\frac{dR}{d\tau} = w, \\
 &R \frac{dw}{d\tau} = -\frac{3}{2} w^2 + \frac{p - p_\infty}{\rho_l}.
 \end{aligned} \right\} \quad (3.40)$$

Then the final system can be written in non-dimensional form:

$$\frac{da}{dt} = W, \quad (3.41)$$

$$\frac{dW}{dt} = -\frac{3}{2a}W^2 + \frac{N}{a}(P - P_\infty), \quad (3.42)$$

$$\frac{dP}{dt} = \frac{3\gamma}{a} \left[\frac{1}{a} \frac{\partial \theta}{\partial y} \Big|_1 - PW \right], \quad (3.43)$$

$$\frac{\partial \theta}{\partial t} = -\frac{1}{aP} \left(\frac{\partial \theta}{\partial y} - y \frac{\partial \theta}{\partial y} \Big|_1 \right) \frac{\partial \theta}{\partial y} + \frac{\theta}{Pa^2 y^2} \frac{\partial}{\partial y} \left(y^2 \frac{\partial \theta}{\partial y} \right) + \frac{\gamma - 1}{\gamma} \frac{\theta}{P} \frac{dP}{dt}, \quad (3.44)$$

$$V(y, t) = yW + \frac{1}{aP} \left(\frac{\partial \theta}{\partial y} - y \frac{\partial \theta}{\partial y} \Big|_1 \right), \quad (3.45)$$

where

$$N = \frac{p_0 R_0^2}{\rho_l D^2}$$

With the following boundary conditions:

$$\theta(1, t) = 1, \quad \frac{\partial \theta}{\partial y} \Big|_{y=0} = 0$$

, and we will take the initial conditions as the following:

$$t = 0 : \quad a(0) = 1, \quad P(0) = 1, \quad W(0) = 0. \quad (3.46)$$

So, we have three Ordinary Differential Equation and one Partial Differential Equation.

3.5 One dimensional finite-difference technique.

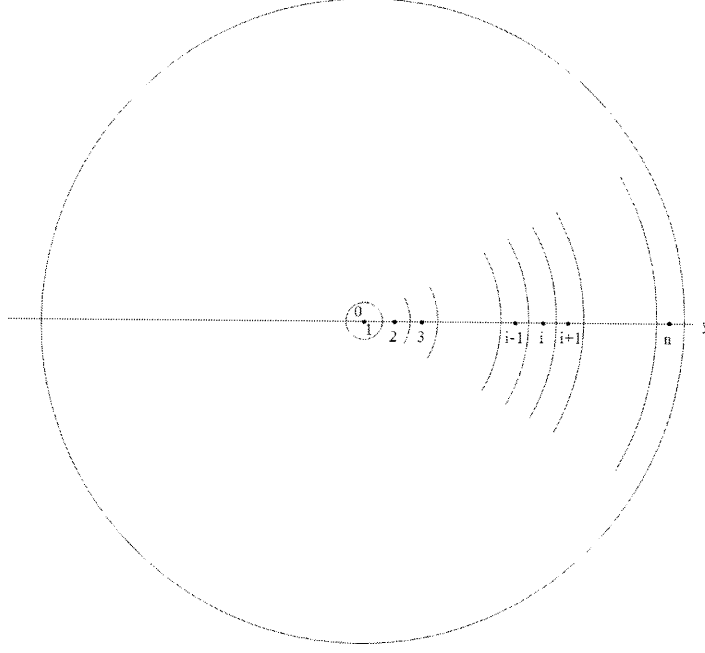


Figure (3.1)

The scheme of dividing the bubble interior into n spherical layers.

Let us transform Partial Differential Equation to n Ordinary Differential Equations by dividing spherical area to the n spherical layers as in Figure (3.1). The thickness of each layer is h , the radius of the central spherical particle is $h/2$. The total radius of the sphere is 1.

$$\text{So, } h = \frac{1}{n - \frac{1}{2}}.$$

The Partial Differential Equation (3.44) can be presented in the form of the system of Ordinary Differential Equation

$$\frac{d\theta_1}{dt} = \frac{\theta_1}{Pa^2} \frac{6(\theta_2 - \theta_1)}{h^2} + \frac{\theta_1}{P} \frac{\gamma - 1}{\gamma} \frac{dP}{dt}. \quad (3.47)$$

For $2 \leq i \leq n-1$

$$\begin{aligned} \frac{d\theta_i}{dt} = & -\frac{1}{aP} \left(\frac{\theta_{i+1} - \theta_{i-1}}{2h} - (i-1)h \frac{1-\theta_n}{\frac{h}{2}} \right) \frac{\theta_{i+1} - \theta_{i-1}}{2h} \\ & + \frac{\theta_i}{Pa^2 [(i-1)h]^2} \frac{[(i-1)h]^2 \frac{\theta_{i+1}-\theta_i}{h} - [(i-1.5)h]^2 \frac{\theta_i-\theta_{i-1}}{h}}{h} \\ & + \frac{\gamma-1}{\gamma} \frac{\theta_i}{P} \frac{dP}{dt}. \end{aligned} \quad (3.48)$$

$$\begin{aligned} \frac{d\theta_n}{dt} = & -\frac{1}{aP} \left(\frac{\theta_n - \theta_{n-1}}{h} - (n-1)h \frac{1-\theta_n}{\frac{h}{2}} \right) \frac{\theta_n - \theta_{n-1}}{h} \\ & + \frac{\theta_n}{Pa^2 [(n-1)h]^2} \frac{\frac{1-\theta_n}{\frac{h}{2}} - (1-h)^2 \frac{\theta_n-\theta_{n-1}}{h}}{h} + \frac{\gamma-1}{\gamma} \frac{\theta_n}{P} \frac{dP}{dt}. \end{aligned} \quad (3.49)$$

These equations can be simplified as

$$\frac{d\theta_1}{dt} = \frac{6\theta_1}{Pa^2} \frac{\theta_2 - \theta_1}{h^2} + \frac{\theta_1}{P} \frac{\gamma-1}{\gamma} \frac{dP}{dt}. \quad (3.50)$$

For $2 \leq i \leq n-1$

$$\begin{aligned} \frac{d\theta_i}{dt} = & -\frac{1}{aP} \left[\frac{\theta_{i+1} - \theta_{i-1}}{2h} - 2(i-1)(1-\theta_n) \right] \frac{\theta_{i+1} - \theta_{i-1}}{2h} \\ & + \frac{\theta_i}{Pa^2 [(i-1)h]^2} \left[\left(i - \frac{1}{2} \right)^2 (\theta_{i+1} - \theta_{i-1}) - (i-1.5)^2 (\theta_i - \theta_{i-1}) \right] \\ & + \frac{\gamma-1}{\gamma} \frac{\theta_i}{P} \frac{dP}{dt}. \end{aligned} \quad (3.51)$$

$$\begin{aligned} \frac{d\theta_n}{dt} = & -\frac{1}{aP} \left[\frac{\theta_n - \theta_{n-1}}{h} - 2(n-1)(1-\theta_n) \right] \frac{\theta_n - \theta_{n-1}}{h} \\ & + \frac{\theta_n}{Pa^2 [(n-1)h]^2} \frac{2(1-\theta_n) - (1-h)^2 (\theta_n - \theta_{n-1})}{h^2} + \frac{\gamma-1}{\gamma} \frac{\theta_n}{P} \frac{dP}{dt}. \end{aligned} \quad (3.52)$$

If we add ordinary differential equations (3.41)-(3.43)

$$\frac{da}{dt} = W, \quad (3.53)$$

$$\frac{dW}{dt} = -\frac{3}{2a}W^2 + \frac{N}{a}(P - P_\infty), \quad (3.54)$$

$$\frac{dP}{dt} = \frac{3\gamma}{a} \left[\frac{1}{a} \frac{1 - \theta_n}{\frac{h}{2}} - PW \right], \quad (3.55)$$

we will get $(n + 3)$ Ordinary Differential equations which can be solved by RungeKutta method. Initial conditions were given in (3.46).

Where n can be chosen from the condition that if we will make it double result will not change, usually n is between $30 - 100$.

CHAPTER 4:

THE RESULTS, DISCUSSIONS AND CONCLUSIONS

4.1 Results and discussions.

4.2 Conclusions.

Chapter: 4

THE RESULTS, DISCUSSIONS AND CONCLUSIONS

4.1 The results and discussions.

The mixture of ordinary and the partial differential equations were solved using finite-difference technique with the powerful of Mathematica 8 program.

In this chapter we will discuss the results of three cases. The first case with $p_{\infty} = 3 \text{ bar}$ which is a sudden pressure increase from the initial value $p_0 = 1 \text{ bar}$. The second with $p_{\infty} = 1.2 \text{ bar}$ which is a case of moderate shock wave. The third is a sudden pressure decrease from $p_0 = 1 \text{ bar}$ to $p_{\infty} = 0.5 \text{ bar}$. In all examples, the value of γ for air is 1.4.

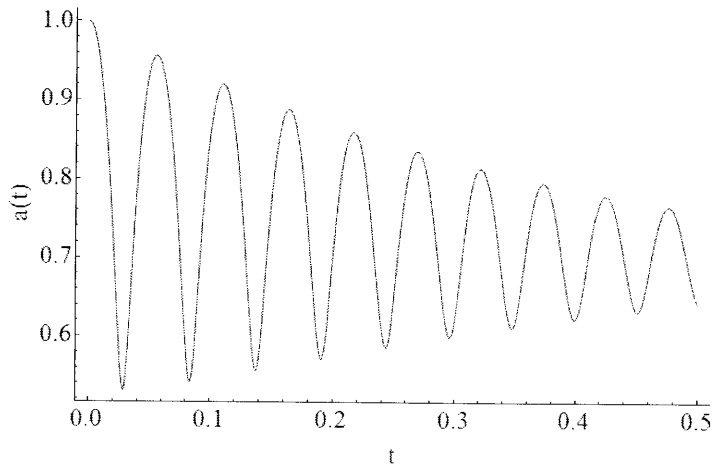


Figure (4.1)

Radius vs time curve for the oscillating gas bubble:

$$R_0 = 0.05\text{mm}, p_0 = 1 \text{ bar}, p_{\infty} = 3 \text{ bar}.$$

Figure (4.1) shows the behavior of the bubble radius oscil-

lating in consequence of a sudden pressure increase from $p_0 = 1$ bar to $p_\infty = 3$ bar in the liquid far from the bubble this is a situation of the bubble screen upon entrance of a shock wave. The initial radius is $R_0 = 0.05mm$.

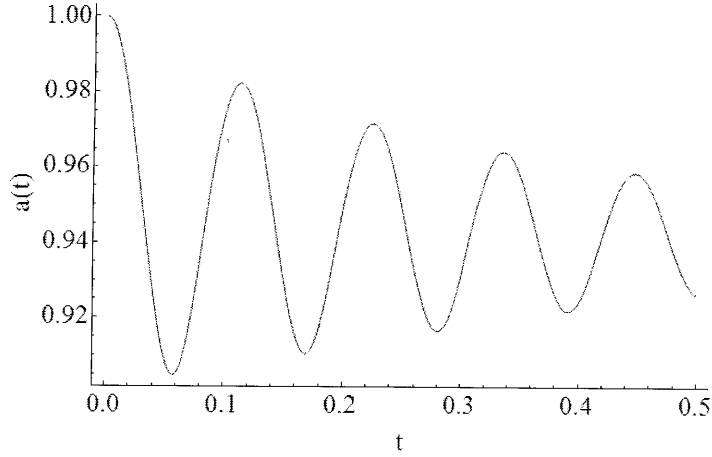


Figure (4.2)

Radius vs time curve for the oscillating gas bubble:
 $R_0 = 0.05mm$, $p_0 = 1$ bar, $p_\infty = 1.2$ bar.

Figure 4.2 shows the behavior of bubble radius in the case of moderate shock wave when $p_\infty = 1.2$ bar, $R_0 = 0.05mm$. comparing Figure (4.1) with Figure (4.2) we noticed that the radial oscillating will decrease whenever the pressure decreased.

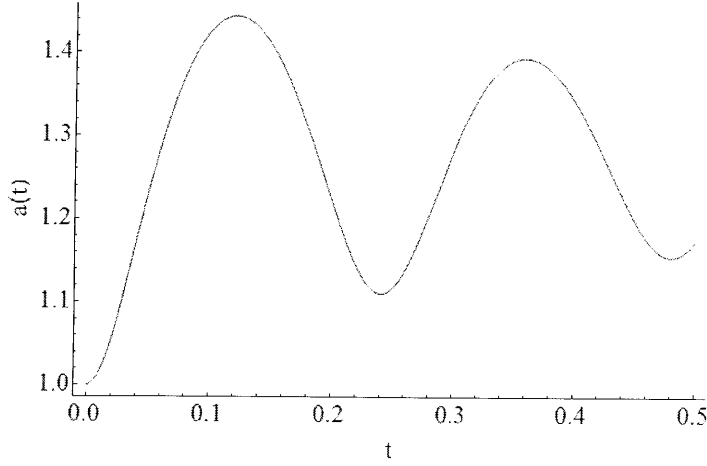


Figure (4.3)

Radius vs time curve for the oscillating gas bubble:

$$R_0 = 0.05mm, p_0 = 1 \text{ bar}, p_\infty = 0.5 \text{ bar}.$$

Figure (4.3) shows the behavior of bubble radius when the pressure in liquid suddenly decreased from $p_0 = 1 \text{ bar}$ to $p_\infty = 0.5 \text{ bar}$ with $R_0 = 0.05mm$, a situation corresponding to the behavior bubble in rarefaction wave. Here The bubble radius is naturally increasing to the equilibrium value $a_e = \left(\frac{p_0}{p_e}\right)^{\frac{1}{3}}$ AL-Mannai & Khabeev (2012). comparing with Figure 4.1 and Figure (4.2), Figure (4.3) has less radial oscillating since the pressure in liquid is less, but the radius in this case first increase instead of decreasing, that due to the sudden decreasing in the pressure in liquid far from bubble p_∞ less than 1 bar.

Consider the case of moderate shock wave with $p_0 = 1 \text{ bar}$, $p_\infty = 1.2 \text{ bar}$ Figure 4.2 if we change the radius from $R_0 = 0.05mm$ to $R_0 = 0.5mm$ we will get:

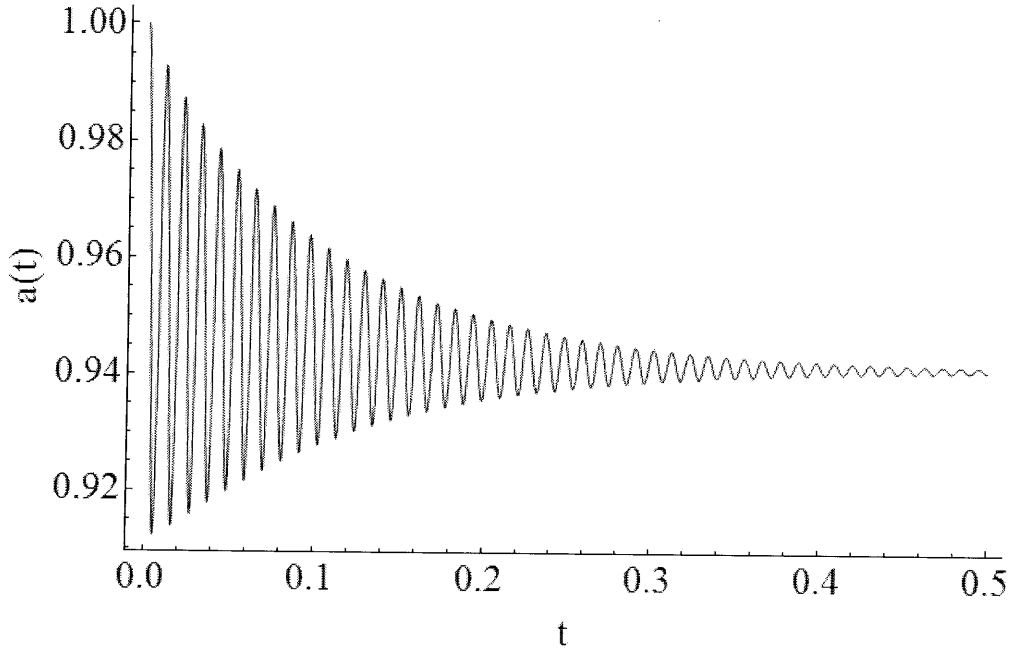


Figure (4.4)
Radius vs time curve for the oscillating gas bubble: $R_0 = 0.5$
 mm , $p_0 = 1 \text{ bar}$, $p_\infty = 1.2 \text{ bar}$.

Figure (4.4) shows that the radial oscillating increased due to this changing in initial radius to $R_0 = 0.5$.

studying the affect of the radial oscillation on the temperature distribution inside a gas bubble at different moments will be considered in the rest of this chapter. First let us study the case of moderate shock wave when $p_\infty = 1.2 \text{ bar}$.

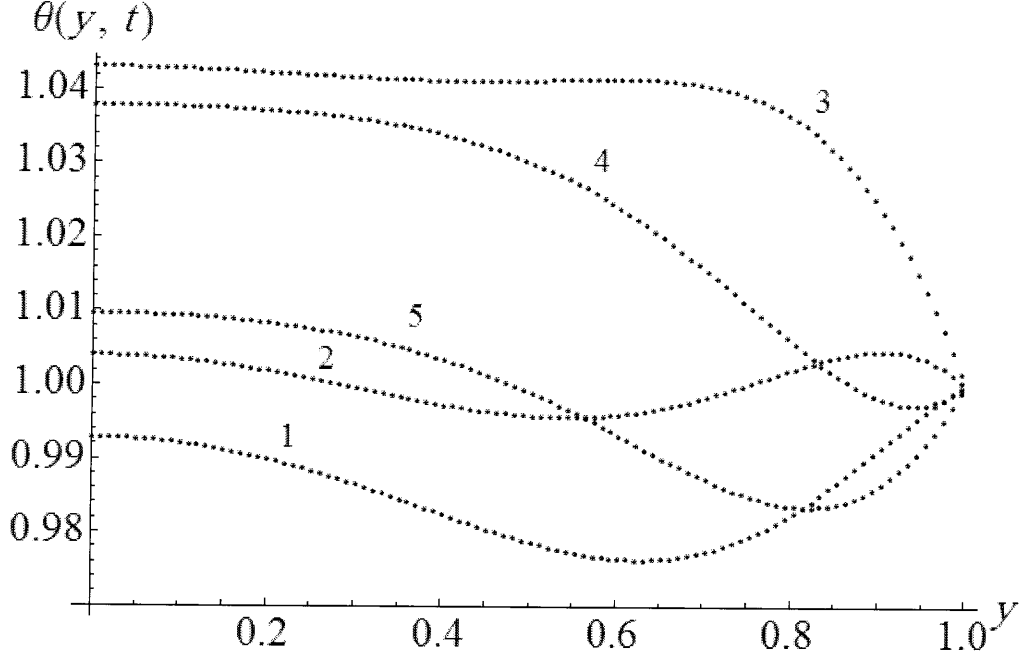


Figure (4.5)

Characteristic temperature distribution within an oscillation gas bubble at different moments of time. Curves 1-5 correspond to the following moment of time $t = 0.1175, 0.13, 0.15, 0.187$ and 0.2 .

$$R_0 = 0.05 \text{ mm}, p_0 = 1 \text{ bar}, \\ p_\infty = 1.2 \text{ bar}.$$

Figure (4.5) shows the characteristic temperature distribution inside the bubble at different moments of time during period of oscillation for the case of moderate shock wave. Curves 1-5 correspond to the following moment of time $t = 0.1175, 0.13, 0.15, 0.187$ and 0.2 . It is clear that the temperature distribution curves inside the bubble are not monotonic due to the radial oscillations.

Second we will consider the case of sudden pressure decrease from $p_0 = 1 \text{ bar}$ to $p_\infty = 0.5 \text{ bar}$ in the liquid far from bubble.

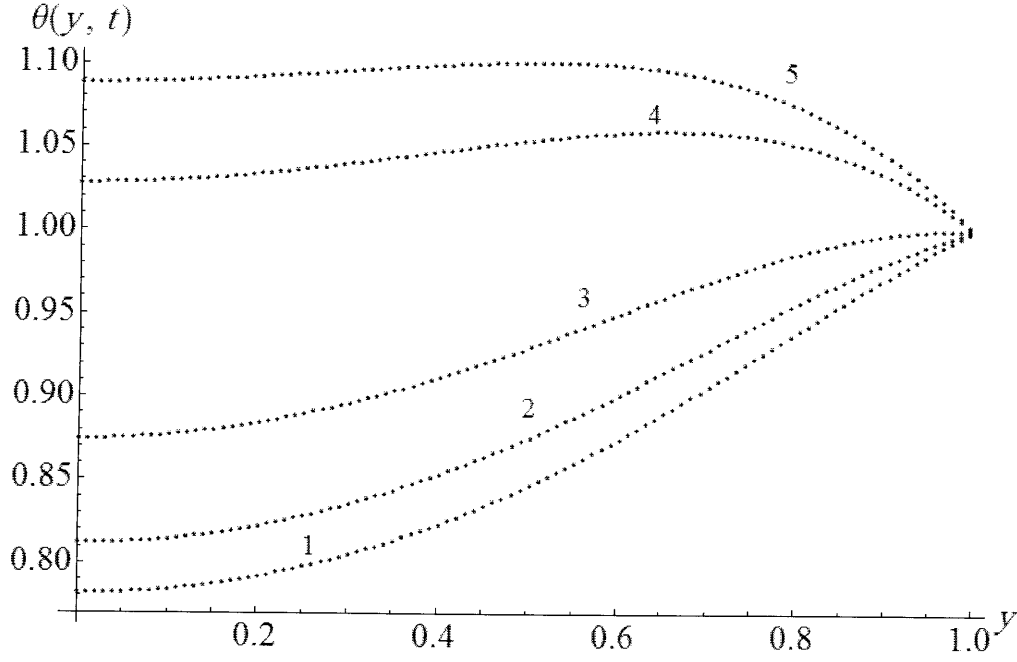


Figure (4.6)

Characteristic temperature distribution within an oscillation gas bubble at different moments of time. Curves 1-5 correspond to the following moment of time $t = 0.1175, 0.13, 0.15, 0.187$ and 0.2 .

$$R_0 = 0.05 \text{ mm}, p_0 = 1 \text{ bar}, \\ p_\infty = 0.5 \text{ bar}.$$

Figure (4.6) shows the characteristic temperature distribution inside the bubble for the case of liquid pressure decrease a situation corresponding to the behavior of bubble in rarefaction wave. Curves 1-5 correspond to the following moment of time $t = 0.1175, 0.13, 0.15, 0.187$ and 0.2 .

The comparison of the values of similar results achieved through other numerical methods grants that they practically coincide.

4.2 Conclusions.

We solved this problem numerically for inside the bubble neglecting the vapour in case of boiling water and without diffusion for simplification of the calculation. In future, we try to calculate the inside and outside distributions of temperature and concentration when we consider the vapour-gas bubble taking in account the diffusion. That will need more calculation, more time, construction new parameters, new initial conditions and Boundary condition, rewriting the programming code and may needs a special computer to solve a system of more than 200 ordinary differential equations. Then, we could study the behavior of vapor-gas bubble in acoustic field for example or compare the obtained result with experimental data and use it for analysis of accidents on Atomic Power Stations. So, we could suggest methods to minimize the risk of such accidents.

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Appendix: A

PROGRAMING CODES

A.1 Mathematica 8

$$n = 100$$

$$h = \frac{1}{n-0.5}$$

$$\rho_0 = 10^3$$

$$P_2 = 0.5$$

$$B = 2 * 10^{-5}$$

$$p_0 = 1$$

$$R_0 = 0.05$$

$$z = 1.4$$

$$N_1 = p_0 * 10^5 * \frac{(R_0 * 10^{-3})^2}{\rho_0 * B^2}$$

$$\text{eqns} =$$

$$\text{Join[}$$

$$\text{Table}[\theta[i]'[t] ==$$

$$\begin{aligned} & \frac{-1}{a[t]*P[t]} * \left(\frac{(\theta[i+1][t]-\theta[i-1][t])}{2*h} - 2 * (i-1) * (1 - \theta[n][t]) \right) \\ & * \frac{(\theta[i+1][t]-\theta[i-1][t])}{2*h} + \frac{\theta[i][t]}{P[t]*(a[t])^2*((i-1)*h)^2} \\ & * ((i-0.5)^2 * (\theta[i+1][t] - \theta[i][t]) \\ & - (i-1.5)^2 * (\theta[i][t] - \theta[i-1][t])) \\ & + \frac{z-1}{z} * \frac{\theta[i][t]}{P[t]} * P'[t], \{i, 2, n-1\} \end{aligned} \Big],$$

```

Table[{(i - 1) * h,  $\theta[i][m_3]$ }, {i, n}],
Table[{(i - 1) * h,  $\theta[i][m_4]$ }, {i, n}],
Table[{(i - 1) * h,  $\theta[i][m_5]$ }, {i, n}], {1, 1} /.sol]
Evaluate[{ $\theta[1][m_1]$ ,  $\theta[1][m_2]$ ,  $\theta[1][m_3]$ ,  $\theta[1][m_4]$ ,  $\theta[1][m_5]$ ,
 $\theta[1][m_6]$ } /.sol]

pts =
Evaluate[Join[Table[{(i - 1) * h,  $\theta[i][m_1]$ }, {i, n}],
Table[{(i - 1) * h,  $\theta[i][m_2]$ }, {i, n}],
Table[{(i - 1) * h,  $\theta[i][m_3]$ }, {i, n}],
Table[{(i - 1) * h,  $\theta[i][m_4]$ }, {i, n}],
Table[{(i - 1) * h,  $\theta[i][m_5]$ }, {i, n}], {1, 1} /.sol]
ListPlot[pts, AxesLabel -> {y, " $\theta[y,t]$ "},
AxesOrigin -> {0, 0.77},
LabelStyle -> Directive[Black, FontFamily -> "Times", Large]]

```

A.2 MAPLE

```
> n := 51;
> h := 1/(n-1/2);
> p[0] := 1;
> R[0] := .1;
> rho[e] := 10^3;
> K := 1.5;
> z := 1.4;
> B := 2*10^(-5);
> N := p[0]*10^5*(R[0]*10^(-3))^2/
(rho[e]*B^2);
> m[1] := 0.91e-1; m[2] := .1; m[3] := .1045;
m[4] := .118; m[5] := .1225; m[6] := .127;
> init2 := theta[1](0) = 1;
> for i from 2 to n do init2 := init2,
theta[i](0) = 1
end do;
> init1 := {init2, W(0) = 0, a(0) = 1,
f(0) = 1};
> for i from 2 to n-1 do
d[i] := diff(theta[i](t), t) =
-((theta[i+1](t)-theta[i-1](t))/
(2*h)-(2*(i-1))
*(1-theta[n](t)))*(theta[i+1](t)-
theta[i-1](t))
/(a(t)*f(t)*(2*h))
+theta[i](t)*((i-.5)^2
```

```

*(theta[i+1](t)-theta[i](t))-(i-1.5)^2
*(theta[i](t)-theta[i-1](t)))/(f(t)*a(t)^2
*((i-1)*h)^2)
+(z-1)*theta[i](t)*(diff(f(t), t))/
(z*f(t)) end do;
> systm2 := d[2];
> for j from 3 to n-1 do systm2 := systm2,
d[j] end do;
> sysm1 := {systm2,
diff(W(t), t) = ((-3)*(1/(2*a(t))))
*W(t)^2+N*(f(t)-K)/a(t),
diff(a(t), t) = W(t),
diff(f(t), t) = 3*z*((1-theta[n](t))/
(a(t)*((1/2)*h))
-f(t)*W(t))/a(t),
diff(theta[1](t), t) = 6*theta[1](t)
*(theta[2](t)-theta[1](t))/
(a(t)^2*f(t)*h^2)+theta[1](t)
*(z-1)*(diff(f(t), t))/(f(t)*z),
diff(theta[n](t), t) =
-((theta[n](t)-theta[n-1](t))/
h-(2*(n-1))*(1-theta[n](t)))
*(theta[n](t)-theta[n-1](t))/
(a(t)*f(t)*h)+theta[n](t)
*(2*(1-theta[n](t))-(1-h)^2*
(theta[n](t)-theta[n-1](t)))
/(f(t)*a(t)^2*((n-1)*h)^2*h^2)+

```

```

(z-1)*theta[n](t)
*(diff(f(t), t))/(z*f(t))});
> dsol := dsolve('union'(sysm1, init1),
numeric, method = rkf45,
output = listprocedure);
> for i to n do
X[i] := subs(dsol, theta[i](t))
end do;
> for i to 6 do c[i] := [0, X[1](m[i])],
[1, 1] end do;
> for i from 2 to n do c[1] := c[1],
[(i-1)*h, X[i](m[1])]
end do;
> for i from 2 to n do c[2] := c[2],
[(i-1)*h, X[i](m[2])]
end do;
> for i from 2 to n do c[3] := c[3],
[(i-1)*h, X[i](m[3])]
end do;
> for i from 2 to n do c[4] := c[4],
[(i-1)*h, X[i](m[4])]
end do;
> for i from 2 to n do c[5] := c[5],
[(i-1)*h, X[i](m[5])]
end do;
> for i from 2 to n do c[6] := c[6],
[(i-1)*h, X[i](m[6])]

```

```

end do;
> plots[odeplot](dsol,
[t, a(t)], 0 .. .5);
> plots[pointplot]([c[1], c[2], c[3],
c[4], c[5], c[6]],
labels = ["y", "&theta;"]);
> plots[odeplot](dsol, [t, f(t)],
0 .. .5);
> plots[odeplot](dsol, [t, X[1](t)]
, 0 .. 1.5);
> s := (p[0]/K)^(1/3);

```



الخلاصة

تهتم هذه الأطروحة بدراسة الحلول العددية لمشاكل النقل الحراري اللاخطي المصحوب بظاهرة الانتشار الغازي التي تحدث في الفقاعة الغازية.

حيث تم افتراض التماثل الكروي في الفقاعة الغازية واستخدام المتغير $y = \frac{r}{R(t)}$ لتجميد حركة حدود الفقاعة، ثم أُستخدمت المتغيرات اللابعدية لتسهيل التوصل إلى الحل العددي لهذه المشكلة.

تم تحويل معادلة (Rayleigh) من معادلة تفاضلية من الدرجة الثانية إلى معادلتين تفاضليتين من الدرجة الأولى وذلك باستخدام التعويض التالي $(\frac{dR}{dt} = w)$.

وقد تم تمثيل العلاقة بين نصف القطر اللابعدى والزمن اللابعدى. إضافة إلى تمثيل التوزيع الحراري اللابعدى بالنسبة للزمن اللابعدى باستخدام برنامج (Maple 15) وبرنامج (Mathematica 8) وذلك بعد تحويل معادلاته التفاضلية الجزئية إلى مجموعة من المعادلات التفاضلية الاعتيادية وذلك باستخدام طريقة الفروق المحدودة والتي أنتجت $(n + 3)$ من المعادلات التفاضلية الاعتيادية بعد إضافة المعادلات الثلاث المراد حلها في النظام.

تم استخدام طريقة (RungeKutta) في الحل العددي للمعادلات التفاضلية الاعتيادية وطرق أخرى لنتائج أكثر دقة في كلا البرنامجين. حيث تراوحت عادة القيم التي حتى لو ضوعفت فيها قيمة n لا يحدث تغيير بين $100 - 30 \sim n$.

جامعة البحرين



قسم الرياضيات

كلية العلوم



الحلول العددية لمشاكل النقل الحراري اللاخطي المصحوب بظاهرة الانتشار الغازي

أطروحة مقدمة كجزء من متطلبات الحصول على درجة الماجستير
في الرياضيات

إعداد

محمد جعفر عبدالله الدرازي

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مملكة البحرين

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