UNIVERSITY OF BAHRAIN

College of Science

Environment and Sustainable Development Program

Mathematical Modeling of COVID-19 Coronavirus Transmission and Control in the Kingdom of Bahrain

A Thesis Submitted in Partial Fulfilment of the Requirements for the Doctor of Philosophy Degree in Environment and Sustainable Development

Submitted by

Muna Abdulkarim AlSayegh

ID Number: 19962692

Supervised by

Dr. Nasser Metwally
Associate Professor
University of Bahrain

Dr. Azhar Iqbal
Senior Lecturer
University of Adelaide
Australia

Kingdom of Bahrain

February 2023

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University of Bahrain
Deanship of Graduate Studies
And scientific Research

جامعة البحرين
عمادة الدراسات العليا والبحث العلمي

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The Discussion Committee met on 25-May-23 in S41-0002. Upon discussing the student whose details are stated above in public, the discussion committee recommends the following:

The student is awarded a grade A in the thesis subject.

☐ Without making any changes to the thesis.

☒ After making the modifications attached with this decision.

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Approved at the University Council 6th meeting, 2013
In accordance with Decision No. 736/2013 dated 14/3/2013
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DEDICATION

To my parents, brothers, sisters, husband, and daughters for their infinite support and encouragement.

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ABSTRACT

The COVID-19 pandemic has been spread globally, where all countries have taken precautionary measures to reduce its spread. Therefore, we are motivated to introduce some suggested solutions that may reduce the spread of the COVID-19 pandemic.

In this context, the main aim of this dissertation is to propose a mathematical model for controlling COVID-19 transmission in the kingdom of Bahrain. The effect of precautionary measures adopted by Bahrain to limit the spread of the coronavirus during 2020 and 2021 is investigated. Statistical data and precautionary measures are obtained from the Ministry of Health (MOH) in the Kingdom of Bahrain website. The analysis of these data showed that, during both years, the number of detected infected cases was lower than expected, except for some periods when the infection number increased. The lockdown process and application of light signal system were the top precautionary measures that reduced the number of discovered infected people. On the other hand, reducing the quarantine of the arrivals, contacts, and cancelling the PCR tests, increase the risk of infection.

In this study, a $SUQCR$ model for the disease progression in Bahrain is introduced. A local equilibrium state is defined, and the conditions of its existence are established. Moreover, the reproduction number R_0 by the next-generation approach is calculated. Model parameters are used as controllers to minimize the number of infected individuals and maximize the recovered ones. According to the $SUQCR$ model, the basic reproduction number for Bahrain is $R_0 < 1$, where all the infected individuals decrease, and consequently, it is taken as a measure of increasing or decreasing the precautionary procedures.

Furthermore, the $SVEUQCHR$ model based on the game theory is presented where the payoff function is derived from the players strategies. A strategy's relative cost determines which is the most appropriate for an individual. When the cost of vaccination equals the risk of infection for un-vaccinated individuals, vaccination is selected without social distancing. If vaccination is not available, social distancing is used only when its cost equals the infection risk without it. It is more effective to use vaccination regardless of cost when both are available (social distancing and vaccination).

Additionally, vaccination and booster shots are evaluated based on the game theory approach to determine whether they are effective at containing COVID-19 in Bahrain. The payoff matrix and biometric are generated. To determine Nash equilibrium points, the maximum value is marked in the first entry of each column and the second entry of each row. Vaccination and booster shots greatly reduce the number of COVID-19, critical, and death cases. Thus, the continuous implementation of the precautionary measures in Bahrain, not reducing them, and vaccination contribute to containing the epidemic.

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Appendix (A): published

AlSayegh, M. A. K., and Iqbal, A. (2021). The impact of the precautionary measures taken in the kingdom of Bahrain to contain the outbreak of COVID-19. Arab Journal of Basic and Applied Sciences, 28 (1), 195-203, doi:org/10.1080/25765299.2021.1903143.

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Chapter 1

Introduction

1.1 General Background

The World Health Organization (WHO) informed on December 31, 2019, that a new virus COVID-19 was responsible for respiratory diseases in Wuhan, China (2020). COVID-19 has spread widely worldwide, and WHO declared it is a pandemic in March 2020 (World Health Organization, 2020a, 2020b). This pandemic has negatively affected the achievement of the Sustainable Development Goals (SDGs), specifically in the health field (SDG3) (Shulla et al., 2021), which is the focus of this research.

COVID-19 infection causes a health burden due to its rapid spread, and human behavior plays an essential role in the international spread and expansion of this disease (Miller IF, Becker AD, Grenfell BT, 2020). Therefore, observing the behavior of infection is a fundamental matter to know the best methods to combat its spread (Funk, Salathé, & Jansen, 2010). As a result, researchers used mathematical models that focused on studying the spread and development of the pandemic (Çakan, 2020; Ndaïrou, Area, Nieto, & Torres, 2020; Serhani & Labbardi, 2021). In addition, mathematical models have been used to understand the behavior of infection as it reaches the population, and conditions were studied that will lead to its elimination or persistence (Kermack WO, 1927; Nwankwo, A., Okuonghae, 2019; Egonmwan AO, 2019).

Many different mathematical models have been used to study the

behavior of the spread of COVID-19, such as *SIR*, *SEIR* and *SUQC* models (Kermack WO, 1927; Shao, Zhong, & Chen, 2020; Adekola, Adekunle, Egberongbe, Onitilo, & Abdullahi, 2020; Zimeras, S., & Diomidous, 2018). These mathematical models represent the movement of individuals between compartments, depending on the infectious state between them (Shaikh, A. S., Shaikh, I. N., & Nisar, 2020). Also, more recently, researchers used the game theory as an approach to describe quantitatively the decision-making processes of individuals with multiple choices. Game theory models have also been used in many studies focusing on vaccinations and behavioral changes as precautionary measures (Bauch & Earn, 2004; Verelst, Willem, & Beutels, 2016).

It is believed that, the virus entered Bahrain on February 24, 2020. On December 8, 2022, a total of 696,851 confirmed cases were reported, of which 694,364 have recovered, and 1,536 have died (Worldometer, 2022). At the onset of the pandemic, it has been made a priority in Bahrain to confront the coronavirus and preserve the health of its population. The country's commitment, and readiness prompted the taking of wide-ranging decisions and precautionary measures to combat the pandemic. Among the measures taken include the free vaccination of all its citizens and residents, reinforcement of social distancing guidelines, and cooperative compliance to the levels, color and traffic light schemes. As well as the free and random examination to discover infected cases and trace contacts of confirmed cases. Moreover, to set a quarantine on the infected and those in contact with confirmed cases (Bahrain.bh, 2021).

Scientific research also has many contributions in this field, as some researchers studied the COVID-19 pandemic from a medical point of view and mathematical models (Alqahtani et al., 2021; Mukherjee & Bojiah, 2020). Thus, this research tries to study the dynamics of the spread

of COVID-19 disease in Bahrain, and the impact of precautionary measures taken by the government to combat the pandemic using mathematical models based on compartmental model and game theory.

The dissertation begins with a literature review on the impact of pandemic on achieving the SDGs and research that have used mathematical models for infectious diseases such as COVID-19 pandemic. Models for spreading COVID-19 in Bahrain through this review are then proposed. Finally, conclusions based on the results and analysis, and proposals for future research, are drawn.

1.2 Research Background and Context

This research seeks to explain COVID-19's impact on the achievement of the SDGs on the global, and national levels, with a focus on health in the country. Furthermore, the impact of the measures that were used in Bahrain to combat the outbreak is investigated, as well as, the system of predicting infections. Moreover, mathematical modeling methods that are used in literature are followed by developing them in line with the health protocol applied in Bahrain. The compartmental method, and the game theory approach are used to come up with the proposed mathematical modeling of COVID-19. The precautionary measures which have been applied in Bahrain are evaluated, as well as the basic reproduction numbers of coronavirus, using the next-generation matrix approach, are calculated.

1.3 Research Problem

COVID-19 has spread worldwide, and in the absence of vaccination at the start of the pandemic, many precautionary measures have been used, such as social distancing or lockdown to limit the disease spread. Social distancing and partial closures for specific periods were used by Bahrain, like other countries. Also, Bahrain closed schools, institutes and universities, and began distance education. Furthermore, citizens and residents were required to wear masks and allowed them to work from home to reduce people's contact with each other, thus, limiting the spread of the disease. In addition, when the vaccine was made available, Bahrain made it free of charge for all.

Despite implementing the precautionary measures, the number of daily infections and deaths continued to vary since the emergence of COVID-19 in Bahrain. Therefore, the dynamics of COVID-19, the prediction of infected numbers and ending of the pandemic require them to be studied from a mathematical point of view. Mathematical models have been used to gain insight into the nature of infectious diseases. They help understand the infection spread in a population, and they describe the conditions that will either lead to the elimination or the continuation of the epidemic (Kermack WO, 1927; Okuonghae & Omame, 2020; Omame, A., Okuonghae, D., & Inyama, 2020). Therefore, mathematical models for COVID-19 in Bahrain are proposed. The research variables are the precautionary measures adopted by Bahrain to combat COVID-19, such as social distancing, vaccination, contact tracing, quarantine, random and exit tests and the basic reproduction number of COVID-19.

1.4 Research Objectives

The main aim of this dissertation is to establish a mathematical model of transmission and controlling COVID-19 in Bahrain. The objectives of this study may be summarized on the following points:

1. Clarifying the effectiveness of the precautionary measures adopted by Bahrain to combat COVID-19.

2. Developing mathematical models of infectious diseases, specifically for the COVID-19 in Bahrain.

3. Using compartmental modeling and the game theory approach to get estimates of the reproduction number R_0, based on available data from government sources on the spread of the coronavirus in Bahrain.

4. Analysing the effectiveness of the Bahrain government's measures to combat COVID-19, by using the developed mathematical models and simulations.

5. Utilizing the game theory approach in the mathematical modeling, to explain individuals' roles in taking the COVID-19 vaccine, and how this leads to a reduction in infection numbers, hospitalization of critical cases, and deaths.

1.5 Research Questions

To achieve the previously listed objectives, we try to find clear answers to the following suggested questions:

1. What is the impact of Bahrain's precautionary measures on combating the spread of COVID-19?.

2. How can the mathematical models of compartmental modeling, and game theory approach help us to:

   - understand the dynamics of virus spread ?.
   - make informed decisions on adopting precautionary measures in Bahrain?.

3. How can game theory explain the role of individual's decisions on vaccinations in reducing infections, hospital admissions of critical cases, and deaths?.

1.6 Research Significance

The significance of this research is shown the effectiveness of the precautionary measures implemented by the Bahrain government to combat the disease, such as applying social distancing and providing vaccinations for all. The importance of this research is also evident in studying the dynamics of disease spread in Bahrain using mathematical models. Given the lack of studies conducted in Bahrain using mathematical models, mathematical models that fit the health protocol applied in Bahrain are suggested. These models seek to study the dynamics in which the disease spreads and the effectiveness of the country's measures to combat it. Moreover, by using the mathematical compartmental model and the game theory, not only on how the disease spreads is shed light, but also how the basic reproduction R_0 for COVID-19 is calculated, and, ultimately, how the effectiveness of Bahrain's precautionary measures against COVID-19 is proven.

1.7 Research terminology

In this section, some terminologies that are used throughout this study are defined. The key terms are as follows:

- **Asymptomatic:** a patient who carries the disease but does not show any symptoms (Yale Medicine, 2022).
- **Booster:** is the dose that boosts the completed vaccine or set of vaccines (Queensland Government, 2021).
- **Compartmental model:** A framework for viewing a system as a set of compartments in which individuals flow according to the infection between them. (Shaikh, A. S., Shaikh, I. N., & Nisar, 2020).
- **Confirmed:** fulfils all the necessary criteria for the disease, such as the appearance of a positive result of a laboratory examination or the disease's symptoms (University of Rochester, 2022).
- **Contact Tracing:** helps stop the disease spread by detecting contacts with existing close cases (UCDAVIS, 2021).
- **Coronavirus:** a family of viruses, including seven viruses that infect humans and cause disease. Corona viruses cause colds and serious diseases such as severe acute respiratory syndrome (SARS) and east respiratory syndrome (MERS). The SARS-CoV-2 virus causes COVID-19 disease (Yale Medicine, 2022).
- **COVID-19:** a disease caused by the SARS CoV-2 virus. COVID-19 is an acronym for Coronavirus, where the number 19 indicates the year it was first discovered (UCDAVIS, 2021).

- **Disease-free equilibrium:** steady-state solution of the model in the absence of infection or disease (Nyang'inja, R., Lawi, G., Okongo, M., & Orwa, 2019).

- **Endemic disease equilibrium point:** state in which disease is entrenched in a population (Nyang'inja, R., Lawi, G., Okongo, M., & Orwa, 2019).

- **Epidemiology :** a branch of medical science that deals with the occurrence and control of disease (Mariposa Country, 2021).

- **Epidemic:** a speedy increase in the infection cases higher than expected in a particular area (Yale Medicine, 2022).

- **Epidemiologists:** are public health specialists and researchers specialized in studying patterns, causes, and methods of preventing disease in people when an epidemic spread (NPS Medicine News, 2020).

- **Exposed:** anyone who has come close to an infected person, regardless of the whether the latter has COVID-19 symptoms or not (Covid Act Now, 2021).

- **Game theory:** is a branch of applied mathematics used to analyse situations that require decisions by the parties, called players. It will provide a quantitative description of the decision-making of individuals facing multiple choices. (Shao et al., 2020).

- **Herd Immunity:** the ability of a sufficient number of people to resist spreading disease. (Mariposa Country, 2021).

- **Immunity:** the human immune system protecting a person from diseases (Queensland Government, 2021).

- **Incubation period:** the period between a person's infection with the virus and the symptoms appearing. The incubation period for COVID-19 ranges from two to fourteen days (Yale Medicine, 2022).

- **Infectious:** a person or an animal capable of transmitting diseases or a kind of disease that can be passed on through the environment (Franciscan Missionaries of Our Lady Health System, 2021).

- **Isolation:** to seclude a proven disease-infected person from others who do not show any symptoms to avoid them getting infected by the former (Johns Hopkins, 2020).

- **Mathematical modeling:** the art of transforming issues from an application area into tractable mathematical formulations where theoretical and numerical analyses provide insight, answers, and advice for the originating application (Arnold Neumaier, 2003).

- **Pandemic:** occurs when a new virus spreads quickly between individuals and people who have little or no immunity against the disease (Yale Medicine, 2022).

- **Payoff:** the reward when a player reaches a particular score of the game (Investopedia, 2021).

- **Players:** a strategic decision-maker within the framework of the game (Investopedia, 2021).

- **Quarantine:** secluding an infected person or anyone who has come in contact with such, in a home or a sanitary isolation area for 14

days (Johns Hopkins, 2020).

- **Recovered:** the passage of a whole day after the disappearance of fever without taking any medication and the improvement of symptoms, and the passage of at least ten days since the start of symptoms of the disease (Mariposa Country, 2021).

- **Reproduction number** R_0: the average number of people who will be infected with the COVID-19 virus from an infected person, which is the natural ability of the virus to spread (Covid Act Now, 2021).

- **SARS CoV-2:** the name of the coronavirus that causes COVID-19 (UCDAVIS, 2021).

- **Social distancing:** putting a physical distance between people, about 6 feet away from others (Yale Medicine, 2022).

- **Strategy:** the complete plan of action that a player takes in light of the set of circumstances that the game creates (Investopedia, 2021).

- **Susceptible:** individuals who are likely to contract a disease (Covid Act Now, 2021).

- **Sustainable development:** first coined by the 1987 Bruntland Commission, which stands for the "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (UNESCO, 2021).

- **The Nash Equilibrium:** a result that, when achieved, means that the player cannot increase the return by unilaterally changing decisions (Investopedia, 2021).

- **Transmission:** the spread of the virus between people through personal contact or by touching contaminated surfaces (Yale Medicine, 2022).

- **Utility:** an economic term that denotes the total contentment obtained from consuming a good or service (Investopedia, 2021).

- **Vaccine:** stimulates the body's immune system to produce antibodies, providing the body with weak or dead germs that are injected, consumed or inhaled (Yale Medicine, 2022).

1.8 Research outline

The research is consisted of five chapters in addition to the references and appendices. Introduction, research background and context, the problem, objectives, questions, significance, and the research terminologies are introduced on in the first chapter. Previous studies, research, and reports related to the mathematical modeling of COVID-19 are presented in the second chapter, which is divided into two axes. Some studies related to the COVID-19 pandemic and the SDGs are included in the first axis, while studies on the mathematical modeling of the COVID-19 pandemic, specifically, the compartmental modeling and the game theory approach, are included in the second axis. The methodologies that have used in the research are presented in the third chapter. Then, the data analysis, results and discussion by answering the research questions are presented in the fourth chapter. Finally, the conclusion and recommendations obtained from this dissertation are presented in the fifth chapter.

The precautionary measures which has applied by the Bahrain government are studied and the number of infections, if the use of these

measures continues, is predicted. Mathematical models that based on the compartmental model are proposed, and the basic reproduction number R_0 for COVID-19 in Bahrain is calculated. The effectiveness of the precautionary measures applied by the Bahrain government is shown by using this model. The game theory approach is used to determine the effectiveness of vaccination and social distancing in Bahrain. An expanded model to the previous model is presented, and the individuals' optimal precautionary measures are studied. Also, the basic reproduction number R_0 for the expanded model is calculated. This model clarifies the optimal method for choosing social distancing or vaccination when they are together or present in the community. Thereafter, the game theory is studied firstly, to study the effect of taking three doses compared to not taking any, in reducing the number of infections; secondly, the number of cases requiring intensive care; thirdly, reducing cases of deaths.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, the impact of the COVID-19 pandemic on the SDGs in countries around the world are reviewed. The United Nations report 2021 with some studies that show the effects of COVID-19 on achieving SDGs in different countries are presented. The effect of the pandemic on the achievement of the SDGs as well as Bahrain's effort to combat it is focused on in this chapter. In addition, some studies that showed the effect of precautionary measures to prevent the pandemic are included. Also, some studies that have used various mathematical models to show the epidemic transmission dynamics, the influence of social distancing, vaccination, and other preventive measures imposed by governments in many countries, including Bahrain, to combat disease outbreaks are presented. The compartmental method and game theory approach are used in these models. It is shown that, most of these studies focused on calculating the basic reproduction number for the spread of the COVID-19 disease. Finally, the most important results of the literature studies are reviewed, and the possibility of developing these models are summarized.

2.2 COVID-19 Pandemic and the SDGs

To improve life quality, the global community adopted the SDGs (United Nations, 2020b), and supplemented the Millennium Development Goals

(MDGs). The 2030 Agenda includes a common vision of peace and prosperity for the Planet (Suresh, A., & Jindal, 2019). The seventeen SDGs were set to shed light on human development without destroying nature, in contrast to the MDGs that were set for emerging and developing countries. The SDGs focus on the following five pillars: People, including SDGs1-6, Prosperity including SDGs7-11, Planet including SDGs12-15, Peace including SDG16; and Partnerships, including SDG17.

The 17 SDGs are SDG1: No Poverty, SDG2: Zero Hunger, SDG3: Good Health and Well-being, SDG4: Quality Education, SDG5: Gender Equality, SDG6: Clean Water and Sanitation, SDG7: Affordable and Clean Energy, SDG8: Decent Work and Economic Growth, SDG9: Industry, Innovation and Infrastructure, SDG10: Reduced Inequality, SDG11: Sustainable Cities and Communities, SDG12: Responsible Consumption and Production, SDG13: Climate Action, SDG14: Life Below Water, SDG15: Life On Land, SDG16: Peace, Justice, and Strong Institutions and SDG17: Partnerships for the Goal (United Nations, 2021b). The government allocates various industries within these major categories to achieve its objectives (Srivastava & Sharma, 2021).

During the COVID-19 pandemic, countries' progress on the 2030 Agenda was halted and their achievements of these goals were impacted. For example, the global recession during the COVID-19 pandemic caused more than 71 million additional people to live in poverty, threatening the achievement of SDG1 (United Nations, 2020c). Reducing inequality, which is SDG10, has been negatively affected because most susceptible groups are women, youth, small and medium enterprises, and low-wage workers, as they have to deal with the riskiest effects of the COVID-19 pandemic (Berchin & de Andrade Guerra, 2020). Moreover, the pandemic caused a significant economic crisis, especially in develop-

ing countries (Sirkeci & Murat Yüceşahin, 2020). Besides, the COVID-19 pandemic has a clear and unprecedented impact on human societies. For example, the Human Development Index (HDI) fell sharply in 2020 for the first time since the measure was calculated 30 years ago (Hall et al., 2020). On the other hand, one of the positive impacts of the COVID-19 pandemic in the short term is the sharp decrease in carbon dioxide emissions and pollution in many areas due to the decline in economic activities (United Nations, 2020a).

The effects of the COVID-19 pandemic on achieving the SDGs have been summarized in the report of the United Nations 2021 (United Nations, 2020a), where the report states that COVID-19 pandemic has significantly changed people's lives. It showed that, although the achievements of the SDGs was slow before the pandemic, this pandemic added an additional 119 to 124 million people into poverty in 2020. For the first time since 1998, the global extreme poverty rate increased from 8.4% in 2019 to 9.5% in 2020. In addition, nearly 255 million full-time jobs were lost. As a result, the number of people travail from hunger also increased by between 83 and 132 million. The report also mentioned that the pandemic had halted progress in the health field. About 90% of countries still report disruptions to health services. Also, the number of children and youth who have reached below the minimum reading proficiency level is 101 million. Over the next decade, almost 10 million girls will be at risk of child marriage.

The report mentions that the collapse of tourism has also disproportionately affected small island developing countries. Besides, violence against women and girls intensified, especially women who suffered from job losses and increased housework. Inequality is also evident in countries for example, as of June 17, 2021, the United States and

Europe have had nearly 68 vaccinations per 100 people compared to less than 2 in sub-Saharan Africa.

Despite the economic slowdown that swept the world in 2020, the report highlighted the negative impact to the climate, particularly with greenhouse gas concentrations that have continued to increase. The average global temperature above pre-industrial levels was around 1.2°C, dangerously short of the 1.5°C limits set in the Paris Agreement. Global flows of foreign investment decreased by 40% in 2020 compared to 2019. In addition to the rise in debt, especially for developing countries, in 2019, one third of the world's population lacked the fuel, sanitary means of cooking, and 759 million people were left without electricity. The failure of the world to achieve the 2020 goals is due to the loss of biological diversity, as the world lost about 10 million hectares of forests between 2015 and 2020.

To rebuild a sustainable, inclusive and low-carbon world, countries need to use recovery as a catalyst for reducing carbon emissions, conserving natural resources, creating higher-quality jobs, advancing gender equality, and addressing growing inequalities. According to the report, the economic recovery is being led by China and the United States, but the rest are not expected to recover before 2022 and 2023. However, among the solid foundations that can be built upon to accelerate progress in achieving the SDGs are the enormous efforts made by countries in confronting the epidemic, decisive government measures, and the rapid expansion of social protection. In addition to the speed of digital transformation of countries and the unique cooperation to develop vaccines against COVID-19.

2.3 The COVID-19 and SDGs: Previous Studies

In this section, some studies and literature that highlight the impact of the COVID-19 pandemic on SDGs are reviewed. For example, Filho et al. (2020) discussed the impact of the COVID-19 pandemic on SDGs and their implementation. The researchers analysed the literature and observations and evaluated current global trends. They used a critical contextual approach based on systems theory to examine and respond to the economic, social, and environmental effects of COVID-19. Their approach combines a composite focus on economic, social, and environmental impact with critical thinking about the concept of economic development. Their results indicated that, although COVID-19 is a priority for many health systems in developing countries, they need attention to other diseases such as malaria, yellow fever, and others. The results also showed that, the fear of COVID-19 disrupted programs to prevent other diseases, resulting in overlooking other health problems like mental health. One of their findings indicated that COVID-19 may jeopardize the implementation of the SDGs, adding caution on giving focus to them.

Shulla et al. (2021) analysed the effect of the new conditions formed by the COVID-19 pandemic on the interrelationship between the SDGs. Based on a qualitative research approach, they examined and focused on only five goals: SDG3, SDG4, SDG8, SDG12, and SDG13. Their observations revealed evidence that COVID-19 may have had consequences for the SDGs. Qualitative explanations indicated that additional repercussions could be significant obstacles to achieve SDG5, SDG9, SDG10, SDG17, and SDG11. Thus, they considered recent trends in international development and their interrelated effects as a helpful framework in the recovery era after the COVID-19 pandemic.

Hishan et al. (2021) reviewed the previous literature on the Coronavirus and provided an overview of the social, environmental, and economic impact of the COVID-19 pandemic. The researchers also sought to find the best possible solutions under different scenarios to give a future plan for policymakers to restrict the SDGs as well as used Scopus and Web of Science databases to review the literature. They also followed a rigorous screening process according to the guidelines recommended by PRISMA. They concluded that the previous literature included systematic reviews of many types of the coronavirus family and provided a framework for a strategic plan to respond to the COVID-19 pandemic.

Hörisch (2021) shed light on how sustainability, management, and policy (SAMP) research can build the post-COVID-19 era and discussed the relationship of the pandemic with the SDGs. In this study, a realistic analysis is conducted to determine the patterns of the impact of the pandemic on the SDGs and stressed the interrelationship between them. On this basis, the research agenda for SAMP is developed, where it is shown that the COVID-19 crisis is part of the research field for sustainable development. However, they found that this pandemic threatens SDG achievement. The analysis of this study confirmed that many SDGs have a role in helping mitigate the effects of the COVID-19 pandemic and possible epidemics in the future.

Besides, Pradhan et al.(2021) studied the effects of the COVID-19 pandemic on achieving the SDGs in Nepal. This study relied on the joint creativity of knowledge that included experts from different fields. They followed five methods: an online survey seven-point scale framework, 20 online workshops, an assessment of expert opinions, review and validation, and revision and synthesis. They highlighted that the impact of the COVID-19 pandemic was negative on most of the SDGs in the short

term. In particular, the targets of SDGs1, 4, 5, 8, 9, 10, 11, and 13 had weak to moderate effects. Nonetheless, some targets of SDGs2, 3, 6, and 11 can have a weak impact. These adverse effects resulted from complete closure, underemployment and unemployment, closing institutions and facilities, financing issues unrelated to the COVID- 19 pandemic, and the expected decrease in support from development partners. However, the COVID-19 pandemic has opened up opportunities for sustainable transformation, including planning, work, socio-economic recovery, use of technology, digital economy, and reverse migration.

Nundy et al. (2021) highlighted in their study the spread of COVID-19, which brings about economic, social, and environmental changes in all countries. Their study first searched through Google Scholar, Science Direct, and Web of Science to investigate the literature published in the past few decades. They used COVID-19, SARS-CoV-2, pandemic, COVID, transport, environment, economy, and energy as keywords. Then, they considered the publications between January 2020 and March 2021. Finally, they reviewed the findings from different publications where they found that many countries have used old methods of self-isolation, lockdown, and quarantine at home to stop the transmission of COVID-19 until a suitable vaccine is available. The lockdown and the total closure resulted in many economic losses, such as jobs and economic stagnation. It also changed the human mind and his social life, staying indoors for 24 hours also harmed the human mind. However, working from home and using technology has become prominent in confronting COVID-19. From an environmental point, there is a noticeable decrease in the emission of air pollutants from electricity generation due to the closure of factories. However, they also realized that it takes a long time to reach a pre-COVID-19 state, where attention must be paid to the

indoor environment. The study also predicted that it would take longer to achieve the SDGs.

Wang and Huang (2021) also conducted a comprehensive investigation into the impact of the COVID-19 pandemic on achieving the SDGs. They resorted to systematically exploring publications related to the effect of COVID-19 on sustainability in the Web of Science database using bibliometrics and a meta-analysis approach. Their results showed that the challenges of the COVID-19 pandemic to sustainable development were more significant in developing countries than in developed countries. Developed countries have committed to studying the sustainability of education, unlike developing countries, which have paid attention to economic sustainability during the COVID-19 pandemic. The cluster analysis also showed that the COVID-19 pandemic had adverse effects on all goals, while this pandemic had brought opportunities to achieve 14 SDGs.

Sarkodie and Owusu (2021) demonstrated the environmental, economic, and health dimensions of the impact of the COVID-19 pandemic. They used qualitative and empirical assessments and reported on environmental sustainability issues, fiscal and monetary measures, and exchange rates during countries' lockdown period and reported on countries' health system policies. Reports indicated a decrease in air pollution versus an increase in medical and municipal waste. They noted that the COVID-19 pandemic ranks the United Kingdom as the country with the highest level of uncertainty among 143 countries. The US also entered 100% of its GDP at the pre-pandemic level, the highest rate of policy reduction among 162 countries. They also indicated that with science, innovation, and development, the measures to combat COVID- 19 would be supported and they revealed the need for future studies in the environment,

economy and health.

2.4 Studies Conducted in Bahrain

The studies that have conducted in Bahrain, with focus on the efforts and the impact on the SDGs during the pandemic is presented in this section. Among of these is the paper introduced by Abdulla et al. (2021) which measured the effect of the virus on the environment in the country. The study's aim is to measure the effect of the COVID-19 pandemic on the environment in Bahrain. In interviews with 14 environment experts in Bahrain, who work in the public and private sectors, the following were revealed:

- The air quality was positively affected due to a decrease in air and land transportation.
- The noise pollution and carbon footprint are also reduced. Thus, there are indications of an improvement in biodiversity in Bahrain.
- There is an increase in medical waste and single-use waste in Bahrain, which leads to significant concerns for biodiversity and human health in the long run.
- The results also showed a slight increase in electricity consumption, which increases the production and consumption of oil and gas.
- In contrast, there is a positive increase in renewable energy, as its cost has decreased compared to fossil fuels. They also found increased environmental awareness among people without affecting the environment and their green behavior.

However, the pandemic also posed logistical challenges that led to a decline in recycling and waste sorting led by the private sector. Systematic allocation of resources for scholars and experts' access to environmental data is part of the recommendation, as will be seen towards the end. For children's curricula, they required to integrate environmental education into these curricula further and adopt and develop a waste management system in Bahrain. To achieve a continuous decrease in the demand for transportation requires an increase in the study of remote work permanently in the public sector. Finally, a national plan must be developed to transform Bahrain's economy into a more sustainable one.

Another study by the Bahrain Chamber of Commerce (2020) concludes on the economic impact of the COVID-19 pandemic on business owners in Bahrain. This study showed that,

- more than 70% of business owners benefited from the government's economic stimulus and support fund, and optimists foresee a recovery with two or three years.

- more than 80 % of them expressed their fear of paying salaries and were affected by the decision to close commercial and industrial stores as precautionary measures to prevent the spread of the epidemic in Bahrain.

- approximately 43% of tourism companies in Bahrain expect closure and bankruptcy as a result of stopping tourist trips during the COVID- 19 pandemic.

Abdulla et al.(2020) also discussed the social and economic impact of the COVID-19 pandemic in Bahrain. They surveyed the opinions of 3,000 individuals in Bahrain, the United States, and the United Kingdom about

the impact of the epidemic on family income, jobs, youth, gender, time use, and psychology. Four groups responded to the survey: 1,000 residing in the United States and 1,000 residing in the United Kingdom. They reached the following conclusions:

- Groups experienced economic distress due to job losses and lower incomes, with Bahraini's reporting low levels of economic hardship compared to residents in both the US and the UK.
- Bangladeshis have reported the highest levels of financial hardship. People with a lower level of education reported difficulty coping with the epidemic compared to more educated people.
- The youth group in Bahrain is the only group that got primary and secondary jobs, unlike other groups in Bahrain and other countries.
- Women in the 25-44 age group spent more time than men helping the elderly and children.

With the COVID-19 pandemic occupying the entire world, countries are paying attention to the response of this pandemic, thus, achieving the SDGs is secondary to the 2030 timeframe. However, the epidemic's persistence and endpoint are challenging to predict. Before the COVID-19 crisis, the world was far from the right track to achieve universal healthcare by 2030. Still, positive results have been achieved over the past years in reducing infant and maternal mortality rates, halving malaria deaths, and diverting the current from HIV/AIDS. But with the COVID-19 pandemic, setbacks to what has been achieved can happen during the vaccination period (United Nations, 2021a). From the seventeen goals of the SDGs, SDG 3 focuses on ensuring healthy lives and promoting health for all of all ages (United Nations, 2020c), including eliminating epidemics

such as AIDS, malaria, tuberculosis, and infectious diseases by 2030, and also providing access to medicines and vaccines for all (Fund, 2021).

Al Dallal et al. (2021) study, aimed to provide an overview of the course of COVID-19 in Bahrain. They apply a methodology to analyse the impact of the preventive measures taken by Bahrain to limit the spread of the epidemic. The researchers used the equation $POCR = \frac{14(avega-avegb)\times 100}{avegb}$ in their *SIR* model to calculate the percentage change in mean basic reproduction number (R_0) values, where a is the infection rate, and b is the recovery rate. The positive value of *POCR* indicates that, the preventive measures accelerated the disease. A negative value indicates that, the precautionary measure slows the spread of the disease. The absolute value refers to the power of the precautionary measure to accelerate or slow down the spread of the epidemic. Therefore, it can be used to know Bahrain's most successful preventive measures. The researchers also calculated the percentage change in the number of tests using the equation $POCT = \frac{14(T_a-T_b)\times 100}{T_b}$ to ensure that the mean R_0 did not arise from the number of tests conducted in Bahrain, where T_b and T_a represent the total number of tests taken during the 15 days before and followed the intervention date, respectively. The results showed that the preventive measures applied in Bahrain effectively limited the spread of the epidemic by between 20.2% and 41.8%. However, the disease by 28.7% spread after a religious event in Bahrain. Researchers did not find any sufficient evidence to conclude that the reopening of the interventions was again the cause of the spread of the disease in Bahrain.

The study of Abdulrahman et al. (2021) aimed to show the impact of quarantine for passengers on the spread of COVID-19 in Bahrain. They retrospectively examined passenger data at Bahrain International Airport and monitored infection rates upon arrival, during the quarantine period,

and after the end of the quarantine. The results showed that, the infection rates decreased among travellers upon arrival and during the quarantine period, while these rates increased after they left the quarantine. The researchers concluded that with these results, the quarantine requirement could be patched upon arrival while keeping the medical examination in addition to the precautionary measures in place to prevent disease.

Also, the study of Alqahtani et al. (2021) aimed to provide an individual historical analysis of cases vaccinated with the four available vaccines in Bahrain, namely Astra-Zeneca (AZ/Covishield), Pfizer/ BioN-Tech, Sinopharm and Sputnik V, in addition to providing an accurate description of the efficacy of these vaccines with the discovery of disease burden and risk of infection. Among the un-vaccinated compared to the vaccinated, it was concluded that hospitalization, admission to intensive care, or death was due to infection penetration among the vaccinated individuals. Vaccines available in Bahrain have also effectively reduced infection, hospitalization, access to maximum care, or death. However, those vaccinated with Sinopharm were more likely to develop the disease after vaccination, admission to hospital or intensive care, or death than those who received the Pfizer vaccine, especially those over 50 years of age. These findings support the value of vaccination against the consequences of COVID-19, even with the emergence of new mutations of the Coronavirus and the urgent need for vaccination worldwide, as vaccination is a vital tool to control the COVID-19 pandemic. The Bahrain government's decision to use these four vaccines is an unparalleled opportunity to examine and compare the effectiveness of different vaccines in preventing disease, admission to hospitals, extreme care, or death, with the emergence of new variables and investigations.

2.5 Some Efforts for Pandemic Combat in Bahrain

To combat the COVID-19 pandemic, the government of Bahrain has taken the responsibility of preserving everyone's health, so it raised the degree of preparedness to confront the epidemic and issued several decisions and precautionary measures to reduce the spread of the epidemic among people. As in the website www.bahrain.bh (Bahrain.bh, 2021), Bahrain's efforts aim to enhance the pandemic response on several levels, including education, work, economy, and health. For example, Bahrain used information and communication technology in the educational process and brought back Bahraini students stranded abroad to avoid getting sick. It also presented a virtual laboratory project that enables students to perform practical experiences are as close to reality as possible.

In health, Bahrain's efforts have emerged in ensuring the protection of all citizens and residents by providing safe and free vaccines for all. As a first step, Bahrain launched a conscious society platform (Be Aware), which combines instructions, preventive instructions, official and national statistics for existing cases and contacts, and tracking contacts and supporting precautionary quarantine. The cases subject to home quarantine or self-isolation were also provided with a smart bracelet linked to the Be Aware application. The application also made it possible to register, to make an appointment for vaccination or report any vaccination problems, in addition to viewing the result of the Coronavirus examination, the vaccination certificate, and registration to take the booster dose.

Moreover, Bahrain provided free preventive and health services and laboratory tests to citizens and residents. Furthermore, Bahrain launched a vehicle inspection service to facilitate procedures, avoid mixing and

maintain social distance. It also launched mobile examination campaigns that covered most areas of Bahrain.

As for travellers coming to Bahrain, health centres have been allocated. Bahrain facilitated the import of medicines and medical devices from international companies, secure the storage of medical equipment, and developed a home delivery service to maintain the health and safety of patients. The country also has activated a telemedicine service for simple medical consultations, requesting medical analyses and results, and renewing prescriptions via video technology. In addition, Bahrain has successfully implemented a community immunization policy, expanding the scope of tests and vaccines.

The effects of the COVID-19 pandemic on achieving SDGs in Bahrain and other countries were reviewed, focusing on health and the country's effort to confront the pandemic. In the next section of this chapter, previous studies are reviewed, showing the impact of government-issued precautionary measures to combat the pandemic and after that, highlighting the role of mathematical models (compartmental method and game theory method) in showing the most important precautionary measures to be applied to prevent the spread of COVID-19 disease.

2.6 Mathematical modeling of COVID-19 Pandemic

In the absence of empirical data on the effectiveness of preventive measures introduced by governments, policymakers should use epidemiological modeling to evaluate options for responding to a pandemic. Mathematical models help understand the behavior of infection as it enters a community and investigates conditions that will be eliminated or persisted (Zeb, Alzahrani, Erturk, & Zaman, 2020). Mathematical mod-

els are important because they allow rapid assessment and may be applied within dynamic frameworks that predict the progression of disease spread. These models play an essential role in defining strategies that control the spread of infectious diseases (Close Rachah A., 2016). For example, in the UK, mathematical models have been used to inform policy responses, particularly simulation models to predict the effects of different non-pharmaceutical interventions (Ferguson et al., 2020).

Here, some previous studies that have used mathematical models are reviewed to demonstrate the importance of the preventive measures adopted by countries to limit and control the spread of COVID-19 disease, such as:

Almutari et al. (2020) assessed people's confidence in the Kingdom of Saudi Arabia in the preventive measures imposed by the Saudi government to combat the COVID- 19 disease. The researchers used the cross-sectional study method of a sample of Saudi families by using an electronic questionnaire during the spread of the epidemic. Pooled data were analysed using the general confidence and compliance tool, by using the chi-square test, t-test and binary logistic regression. The results showed that, the participants had a high degree of confidence in the precautions imposed by the government. The people's confidence in these measures slowed the spread of the epidemic and lowered the death rate in Saudi Arabia. The groups most respectful of these precautions were females, the elderly and the most educated.

Alumran (2020), evaluated the impact of precautionary and preventive measures to combat the COVID-19 disease in the Kingdom of Saudi Arabia. The data was collected from the World Health Organization website and other websites such as Worldometer. The expected number of new infections was calculated by using a mathematical formula to assess

the difference between the observed and expected number of new cases, which indicates the impact of the precautionary measures approved by Saudi Arabia to combat COVID-19 disease. It was shown that the precautionary measures adopted by Saudi Arabia were effective and greatly affected the reduction of new cases. Thus, the number of deaths decreased than the expected number.

Jentsch et al. (2021) developed a socio-epidemiological model associated with the transmission of COVID-19, where they compared the expected deaths from COVID-19 under different strategies for prioritizing a COVID-19 vaccine. Moreover, they modelled how the population responds to preventive measures with closing and reopening schools and workplaces according to the number of informed disease cases.

Okuonghe and Omame (2020) tested the effect of precautionary measures on combating COVID-19 in Lagos State, Nigeria, where, they used a mathematical model with the available data. Moreover, they developed a predictive tool for the cumulative number of informed and active cases and estimated the baseline reproduction number for disease prevalence in this state. Then, using numerical simulation, the researchers demonstrated the effect of precautionary measures such as social distancing, wearing face masks, detection of infected cases by contact tracing and subsequent testing on the spread of COVID- 19. Their results showed that, when least 55% of the residents of Lagos State comply with the rules of social distancing and wear face masks, the disease will end. Likewise, if they can increase the detection of infected cases among people who show symptoms, this will reduce the spread of the epidemic among people.

A proposed mathematical model to study the impact of preventive measures in Wuhan, China, to limit COVID-19 disease is conducted by

Lin et al. (2020). This model included an individual's behavior with government measures and used estimates of the 1918 avian influenza pandemic in London, UK. Also, they included zoonotic introductions and migration and calculated future trends and cases. They found that this model has succeeded in shedding light on understanding the epidemiological trends of COVID-19 in China.

Yang and Wang (2020) proposed a mathematical model to investigate the spread of COVID-19 in Wuhan, China. In the dynamics of infection, their model specified several transmission channels. In addition, their model emphasized the role of the environmental reservoir in the transmission of this disease. Their analytical and numerical results indicated that COVID-19 infection would remain endemic, requiring preventive programs and preventative measures to halt the spread of the disease.

In Morocco, Serhani and Labbardi (2021) proposed a modified mathematical model of COVID-19 transmission, where it considered asymptomatic people and preventive measures such as hospital isolation for infected people, quarantine of people who have contacted them, and home containment of all residents to restrict movement. They established a relationship between the control coefficients for different values with the basic reproduction number. The disease-free equilibrium point is related to the condition connecting these two values. They found that the lack of respect for confinement leads to several setbacks in developing the epidemic and that home containment if strictly applied, plays a crucial role in containing the disease and controlling it.

Zeb et al. (2020) developed a mathematical model to present the dynamic behavior of COVID-19 infection by integrating the quarantine category. The main task of this model is estimating the spread of the COVID-19 epidemic in the future. The suggested model is solved by

using the nonstandard finite difference scheme and Runge-Kutta fourth-order method. As a result, researchers have concluded that the main reason for the spread of the COVID-19 disease is the direct contact between people, so isolating the infected can reduce the spread of the epidemic in the future.

The possibility of transmission of the epidemic between individuals is discussed in a segmented model introduced by Ndaïrou et al. (2020) of COVID-19, where they studied the local stability of disease-free equilibrium. The numerical simulations showed the suitability of the proposed model for the outbreak of COVID-19 that occurred in Wuhan, China, as their results were adapted to accurate data and reflected reality in the city. Moreover, it is shown that this model may be used to study the reality of attempts made by countries where the spread of the epidemic is increasing.

Zhao and Chen (2020) proposed a $SUQC$ model, which is Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected, to describe the dynamics of COVID-19 disease and the applied precautionary measures in China. They showed that, the basic reproduction number is less than 1, which means that the preventive measures imposed by the Chinese government effectively limit the spread of the epidemic. Therefore, they suggested maintaining these strict quarantine and monitoring measures. Also, this model can be helpful to provide sufficient evidence of the outbreak in other countries such as South Korea, Italy and Japan.

Chen et al. (2020) developed a Bats-Hosts-Reservoir-People transmission network model to calculate the transmissibility of the COVID-19 virus in China, where the transmission of the virus from the main source of infection, which may be bats to human infection. They calculated the

basic reproduction number using the next-generation matrix. The results showed that, the transmissibility of the COVID-19 virus is higher than Middle East respiratory syndrome in the Republic (MERS-CoV) in the Middle East countries but less than MERS-CoV in the Republic of Korea.

Kouidere et al. (2020) proposed mathematical modeling (SIWICHR) of the transmission of the COVID-19 virus by using six compartments of susceptible persons, infected persons without symptoms, infected persons with symptoms, infected persons in critical cases, those subject to quarantine and health monitoring, in addition to those recovering from the disease. The researchers suggested an optimal strategy with practical measures to limit the spread of the virus by conducting awareness campaigns for citizens, diagnosing and quarantining the infected, and monitoring airports.

A study is introduced by Sameni (2020) to study the patterns of spread of the COVID-19 disease from the perspective of mathematical modeling. The researcher used an extension of the *SEIR* model. This model depended on the average behavior of the population, better to understand the importance of mathematical modeling of epidemics. This model displayed the impact of social distancing, regional closures, and public health vigilance on the model's parameters, affecting mortality rates and active pollutant cases over time.

An expanded model of *SEIR* in Saudi Arabia is introduced by Ghostine et al. (2021), where the vaccination compartment is added to the SEIR model for simulating the spread of COVID-19 inside the kingdom. They performed a mathematical analysis to clarify the negativity, limitations, epidemiological equilibrium, existence and uniqueness of endemic equilibrium, and the basic reproductive number is calculated for the suggested model. The numerical results showed the ability of the proposed

model to achieve an accurate prediction of the evolution of the epidemic on time scales of up to two weeks. Moreover, it is shown that the intensification of the vaccination campaign significantly reduces the number of confirmed cases and deaths.

In Jakarta, Aldila et al. (2021) suggested a mathematical model to predict the spread of COVID-19, where they considered undetected cases, asymptomatic cases, and several non-pharmaceutical interventions. From sensitivity analysis, it is found that rapid testing has a significant role in detecting cases and thus reducing the basic reproduction number instead of contact tracing. In addition, the vaccination strategy also plays an essential role in easing the rules of social distancing. The basic reproduction number can be maintained as low as possible through vaccination, and the disease is eliminated from the population who have a higher vaccination.

In India, Shaikh et al.(2020) proposed a mathematical model, Bats-Hosts-Reservoir-People, for the spread of the COVID-19 disease, to simulate a potential transition while considering individual social response and control measures by the government. The Picard approximation technique and Banach's fixed point theorem are used to check the existence and stability criteria of the model. They proved that, in the absence of vaccination, complete closure and physical distancing effectively measure the control of the spread of the COVID-19 disease.

Also, there are some studies that are focused on finding the basic reproduction number. For example, Al-Raeei paper (Al-Raeei, 2021) estimated the basic reproduction number for the Middle East region. The researcher used the Runge–Kutta method on the SEIR model and used the data recorded for the disease in two stages. The exposure, infection, recovery, and death rates due to COVID-19 are estimated for the Middle

Eastern countries, including Bahrain. The results showed that, the basic reproduction number of COVID-19 in the Middle East countries starts from 1.5583 to 3.0395 in the first stage and from 1.0001 to 4.5757 in the last stage. Based on the values of basic reproduction numbers, it is shown that, is worse in Lebanon and Syria compared to the rest of the Middle East countries in the last stage.

Even though there is an excellent level of uncertainty in epidemiological models, they are practical tools in public health management programs. The work of Anirudh (2020) described the results and challenges of mathematical models such as *SIR*, *SEIR*, *SIERU*, *SIRD*, *SLIAR*, *ARIMA*, *SIDARTHE*, and others used in predicting and containing the spread of COVID-19 disease. The results showed that, many countries succeeded with the help of these models in planning and mobilizing enormous resources at an early stage and the ability to combat the epidemic by implementing containment policies and measures.

2.7 Mathematical Models Based on Game Theory

Some models that used the game theory approach, a branch of applied mathematics used to analyse situations that require decisions by the parties, called players are listed in this section. It is also suitable for modeling the decision-making process of individuals with many choices, such as applying social distancing or taking a vaccination (Chang, Piraveenan, Pattison, & Prokopenko, 2019).

For example, Timothy C. Reluga (2010) used the differential game model to determine the best methods of social distancing and protection behaviors preferred by individuals during the epidemic. The researcher also used numerical methods to calculate the total costs of an epidemic as

a function of the time of mass vaccination. The results showed that social distancing is the most preferred behavior for individuals when the basic reproduction number is around 2. However, optimal social distancing does not recover more than 30% of the infection cost without immunization.

Kabir et al. (2020) combined partial epidemiological models with behavioral dynamics using game theory. The players in the game were individuals exposed to the virus and must choose whether to stay at home or not, to stop the spread of the epidemic, even though staying at home will cost them economically. When selecting a stay-at-home strategy or not, the payoff depends on balancing the perceived gain of complying versus the risk of non-compliance. Also, they expanded the *SEIR* model by adding the quarantine and hospitalized compartment. They showed that compliance with economic shutdowns could reduce over time as individuals balance the risk of infection against the certainty of the financial cost while staying home. Also, they showed that the shield's immunity takes place in the population more efficiently when the closing costs are low for the individuals.

Jinyu Wei et al. (2020) analysed an evolutionary game model for the actions taken by the government and individuals in the early days of the epidemic and combined it with interaction strategies in the natural transmission mechanism epidemic model. They used SPSS to perform logistic curve fitting and obtained transmission frequency equations for cumulative confirmed cases under different scenarios. Their results stated that the strategy adopted by the government to control the epidemic is effective. Furthermore, they analysed the government measures taken by the government, judged each measure's effect on the frequency variable, and got its impact on the spread of the epidemic.

Choi and Shim (2020) developed a theoretical epidemiological model for the game that considers vaccination and social distancing as necessary strategies to prevent the spread of the epidemic. This model studied three scenarios: the presence of vaccination without applying social distancing, the presence of social distancing in the absence of vaccination, and their presence together. They determined the best strategy based on Nash's strategy in each of the three scenarios. Furthermore, the relative costs of vaccination versus social distancing.

Bairagi et al. (2020) formulated an optimization problem that accommodated individuals' social distancing features and resolved it by implementing a non-cooperative game that provides an incentive for individuals applying social distancing to avoid the spread of COVID-19. With the help of the proposed game-theoretical payoff to preserve social distancing with a Nash equilibrium, they explained the sustainability of the lockdown policy and the proposed approach's effectiveness in applying the desired social distancing to prevent the spread of COVID-19. Their results showed that, individual motivation increased by more than 85%, with home isolation rising 75% for all scenarios. Moreover, as the number of individuals increases, there is also an increase in home isolation as motivation decreases.

Considering social distancing as a social dilemma that each individual plays against their people, Zhijun Wu (2021) provided a theoretical basis for understanding the nature of social distancing in his suggested proposal. From this perspective, where it is shown that each individual, as a player in this game, must either apply social distancing to protect himself from disease risks or not access the necessary social activities. Players' decisions depend on the actions of the rest of the population and vice versa. Reaching a balance is the best strategy for social distancing.

The paper explained how to determine the optimal strategy for the people with social activities linked to the other population. It also clarified how social distancing behaviors could be simulated as a distance in the game. The group simulation results show that if each individual made rational spacing decisions, the individual actions converged with the optimal distance strategy in silico and perhaps also in nature.

Martcheva et al. (2021) proposed a game-theoretical model for studying human behavior associated with the disease. Their game assumed two strategies: the first, to strictly enforce social distancing and the second, not to implement it strictly. Their results were as follows: If the cost of strict social distancing is less than the cost of following health standards, the epidemic can be eliminated by applying strict social distancing. But if this cost is greater than the cost of following health standards, then eliminating the epidemic through social distancing will be impossible, and the disease will persist at a certain level. In addition to their results, they found that the spread of the epidemic decreases as more residents apply social distancing.

2.8 Chapter Summary

This chapter may be summarized as:

- The effects of the COVID-19 pandemic on achieving the SDGs that the world's countries have come a long way to achieve are reviewed. As well as that, how COVID-19 affects SDG3 progress is explained.

- From the perspective of these effects on the health side, countries' efforts, including Bahrain, to combat the epidemic are presented.

The studies reviewed in this chapter showed that the COVID-19 pandemic has negatively affected social, economic, environmental and health aspects. On the other hand, other studies have confirmed that these goals have a key role in mitigating the effects of COVID-19 and potential epidemics in the future. Furthermore, other studies concluded that this pandemic provided opportunities for sustainable transformation through planning, action and recovery in various fields. However, reaching the pre-COVID-19 state, as some studies have found, takes a long time, and they expect that achieving the SDGs will take a long time.

- Mathematical modeling is used to predict disease progression and spread and helps to identify effective strategies for controlling infectious diseases and mitigating their potential effects. By reviewing the mathematical models, it was found that most studies used a compartmental model such as the *SIR* model or an expanded model by adding more compartments to their models. For example, some researchers added the exposed E, vaccination V, un-quarantined U, or quarantine Q compartments in their models to study the importance of government preventive measures, such as applying social distancing or vaccination in preventing the spread of disease. The mathematical models successfully demonstrated the importance of precautionary measures in combating the epidemic. The model showed that the main cause of the spread of COVID-19 disease is direct contact, so isolating the infected person reduces the spread of the disease. However, other results indicated that the COVID-19 pandemic would remain epidemic, which requires more precautionary measures. Other researchers, by using the mathematical

model, also found that the COVID-19 virus is more transmissible than MERS in Middle Eastern countries. Different results showed that these mathematical models have helped many countries plan and mobilize enormous resources to combat COVID-19 by implementing containment policies and measures.

- The view of the game theory had a prominent role in proving the importance of precautionary measures in combating the COVID-19 pandemic. They showed by the game theory that, compliance with lockdowns can decline over time as people balance the risk of contracting COVID-19 with the financial cost of staying home. They also proved that the most applied measure among individuals is the application of social distancing. Also, through game theory, other researchers determined the best strategy based on Nash's strategy regarding vaccination and social distancing. Using game theory, researchers also found that choosing strict social distancing depends on its cost versus following health standards. Therefore, the epidemic can be eliminated by strict social distancing if its cost is less than the cost of following health standards. They also found that the spread of the epidemic is reduced by applying social distancing by many people.

- One of the results of studies conducted in Bahrain was that the precautionary measures applied had an effective role in limiting the spread of the epidemic by between 20.2% and 41.8%. Another study conducted in Bahrain showed that infection rates among those coming to Bahrain are low during quarantine, while infection rates rise after exiting from quarantine. Another study investigated the effectiveness of vaccinations adopted by Bahrain. Also, a model

assumed the natural pattern of COVID-19 spread in Bahrain, Saudi Arabia and Egypt, indicated that the number of infections will decrease in the tenth week from the beginning of the pandemic.

As part of Bahrain's efforts to combat COVID-19 disease, mathematical models are proposed to show the dynamics of the epidemic and help show the most effective precautionary measures in preventing the epidemic in Bahrain. It is benefited from the method used in previous studies to propose mathematical models and calculate the basic reproduction number based on Bahrain statistics.

Chapter 3

Methodology

3.1 Introduction

The methodologies, the suggested mathematical models, and the data collection methods are introduced in this chapter. In Sec.(3.2), the precautionary measures during the COVID-19 pandemic in the Kingdom of Bahrain are presented. Then a mathematical model, that may be used to describe the behavior of COVID-19 pandemic is discussed. A modified version of this model with vaccination and social distance factors is considered. Finally, the modified model is discussed on the context of a game theory. In Sec.(3.3), the tools used to perform the calculations are reviewed. Finally, the collection of the used data of this study is shown in Sec.(3.4).

3.2 Methodologies

The methods that have used to achieve the research's main goals are described in this section, where different methods have been used.

3.2.1 The precautionary measures and the COVID-19 pandemic in Bahrain

In this subsection, the confirmed infected individuals' number is compared to the expected infected individuals' number when the precautionary measure was continued, where the average rate of decrease or

increase is depicted. First, the preventive measures taken by Bahrain between February 24, 2020, and September 18, 2020, and from January 31,2021 to February 4, 2022 are reviewed and collected as stated in Table (4.1) and Table (4.3) respectively. The periods are split into five stages based on the preventive measures taken throughout each period, as seen in Tables (4.2) and (4.4). Then, the observed number of infected cases are plotted for each period, where their best-fit lines are obtained by using Microsoft Excel. The parameters x and y refer to the number of days and the number of infected cases, respectively. The polynomial functions which describe the best-fit lines using the polynomial approximation are obtained. The expected number of the infected cases is estimated by using the derived polynomial functions. Finally, the average % rate of decrease or increase between the confirmed cases of a certain period and the projected number of infected cases of its previous period is calculated by using the formula:

$$Average\%\ rate = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{O_i - E_i}{E_i} \times 100\%\right), \tag{3.1}$$

where O_i is the confirmed number of cases on the day i, E_i is the projected number of cases on day i and n is the number of days. The summary of method applied to find the effect of precautionary measures in combating the COVID-19 pandemic in Bahrain is given in Fig. (3.1)

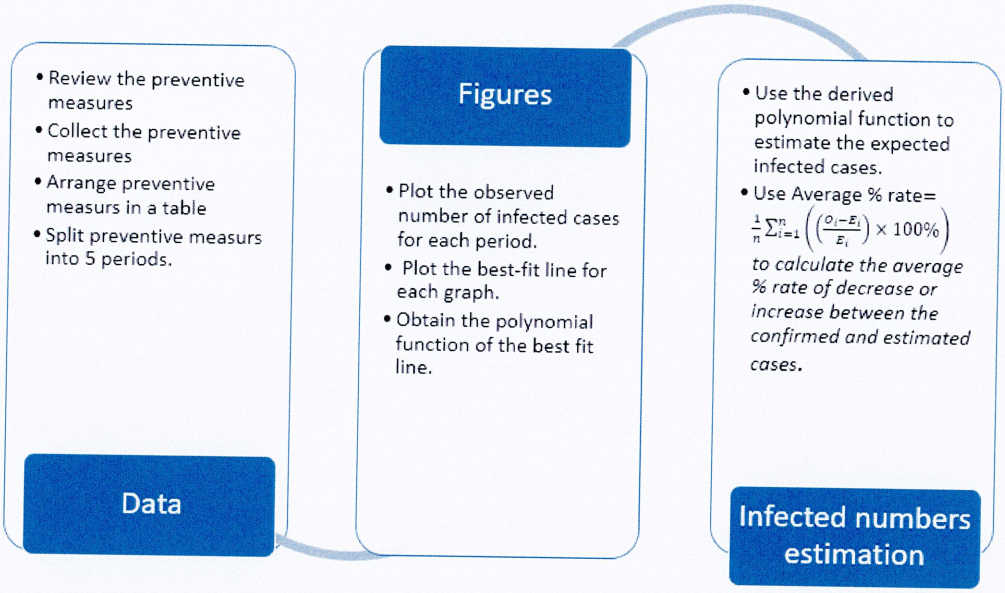


Figure 3.1: A summary of the method applied to determine the impact of precautionary measures on the COVID-19 pandemic in Bahrain.

In the next subsections, the proposed $SUQCR$ model, and the $SVEUQCHR$ model for the COVID-19 epidemic in Bahrain are introduced. In addition, the basic reproduction numbers in both models are calculated. Furthermore, the precautionary measures and their impact on combating COVID-19 in Bahrain are included (AlSayegh & Iqbal, 2021).

3.2.2 The $SUQCR$ Model for COVID-19 in Bahrain

The $SUQCR$ model for disease development in Bahrain is a time-dependent Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected, and Recovered transmission model, which is a modified version of the Zhao and Chen (2020) $SUQC$ model. Due to the importance of understanding the impact of COVID-19 immunization and natural immunity, the recovered number referred to as R is included in the model. The influence of patients' disease histories λ and the effect of early detection of COVID-19 by the random test δ on the recovered numbers are

investigated in this model. Also, the death rate μ in all compartments of the model is included. In addition, the impact of quarantine procedures on disease propagation is explicitly depicted in the $SUQC$ model. The $SUQCR$ model, on the other hand, shows how un-quarantine and quarantine procedures affect disease progression in Bahrain. Furthermore, in greater detail, it reveals the influence of random testing and contact tracing of COVID-19 patients on detecting infected cases.

The $SUQCR$ model contains five variables and several parameters. The variables of the $SUQCR$ model are: Susceptible $S = S(t)$, individuals susceptible to disease without disease resistance at time t. Unquarantined infected $U = U(t)$, infected individuals without symptoms at time t. Quarantined infected $Q = Q(t)$, people in contact with Corona patients whether they show symptoms or not at time t. Confirmed infected $C = C(t)$, the number of actual infected cases at time t. Recovered $R = R(t)$, the number of individuals who survive the infection and who have developed a natural immunity to the virus at time t (Ivorra et al., 2020). And the total population $N = N(t) = S(t) + U(t) + Q(t) + C(t) + R(t)$ is non-negative for all $t \geq 0$. The $SUQCR$ model is presented in Fig. (3.2).

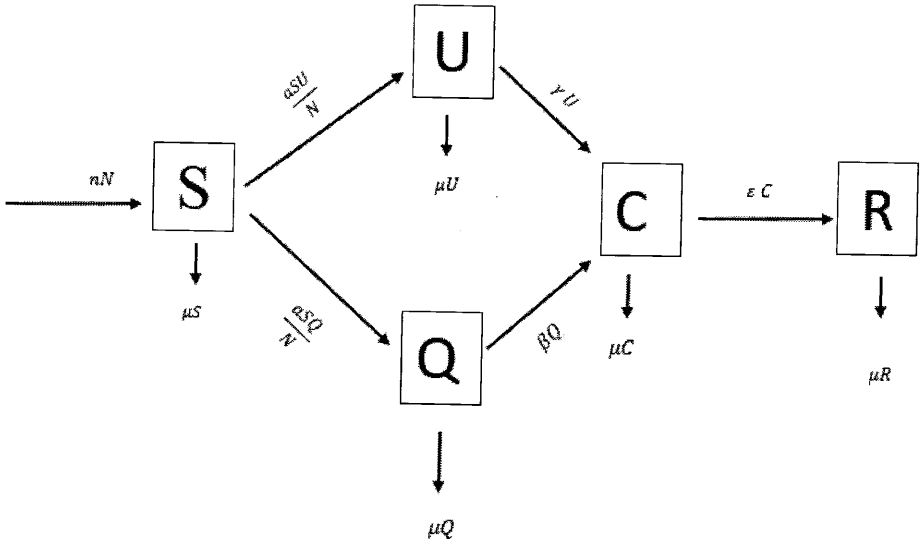


Figure 3.2: The *SUQCR* model diagram of COVID-19 in Bahrain.

The *SUQCR* model of COVID-19 in Bahrain may be presented by the following system of differential equations,

$$\begin{aligned}
\frac{dS(t)}{dt} &= nN(t) - \mu S(t) - a\frac{S(t)}{N(t)}U(t) - a\frac{S(t)}{N(t)}Q(t),\\
\frac{dU(t)}{dt} &= a\frac{S(t)}{N(t)}U(t) - (\mu+\delta+\sigma+\eta)U(t),\\
\frac{dQ(t)}{dt} &= a\frac{S(t)}{N(t)}Q(t) - (\mu+\beta)Q(t) + \eta U(t),\\
\frac{dC(t)}{dt} &= (\delta+\sigma)U(t) + \beta Q(t) - (\mu+\tau-\lambda)C(t),\\
\frac{dR(t)}{dt} &= (\tau-\lambda)C(t) - \mu R(t),
\end{aligned} \tag{3.2}$$

where $\frac{S(t)}{N(t)}$ gives the probability that the contact is made with a susceptible individual. Also, a is the influential contacts average number produced by each infectious individual (M. Y. Li, 2018). In the *SUQCR* model, it is assumed that $\gamma = \delta+\sigma+\eta$ and $\varepsilon = \tau-\lambda$, and consequently the system (3.2) reduces to be,

$$\begin{aligned}
\frac{dS(t)}{dt} &= nN(t) - \mu S(t) - a\frac{S(t)}{N(t)}U(t) - a\frac{S(t)}{N(t)}Q(t),\\
\frac{dU(t)}{dt} &= a\frac{S(t)}{N(t)}U(t) - (\mu+\gamma)U(t),\\
\frac{dQ(t)}{dt} &= a\frac{S(t)}{N(t)}Q(t) - (\mu+\beta)Q(t) + \eta U(t),\\
\frac{dC(t)}{dt} &= (\gamma-\eta)U(t) + \beta Q(t) - (\mu+\varepsilon)C(t),\\
\frac{dR(t)}{dt} &= \varepsilon C(t) - \mu R(t),
\end{aligned} \tag{3.3}$$

where $N(t)$ is a constant, namely $n = \mu$. The model limitation will be the positive values for S, U, Q, C, and R, with a birth rate and death rate equal, ensuring a constant population size. The parameters used in the system (3.3) are presented in Table (3.1).

Sitting $s(t) = \frac{S(t)}{N(t)}$, $u(t) = \frac{U(t)}{N(t)}$, $q(t) = \frac{Q(t)}{N(t)}$, $c(t) = \frac{C(t)}{N(t)}$, and $r(t) = \frac{R(t)}{N(t)}$, then the system (3.3) reduces to its normalized form as,

$$\begin{aligned}
\frac{ds(t)}{dt} &= \mu - \mu s(t) - as(t)u(t) - as(t)q(t),\\
\frac{du(t)}{dt} &= as(t)u(t) - (\mu+\gamma)u(t),\\
\frac{dq(t)}{dt} &= as(t)q(t) - (\mu+\beta)q(t) + \eta u(t),\\
\frac{dc(t)}{dt} &= (\gamma-\eta)u(t) + \beta q(t) - (\mu+\varepsilon)c(t),\\
\frac{dr(t)}{dt} &= \varepsilon c(t) - \mu r(t).
\end{aligned} \tag{3.4}$$

Now, let us consider that, the initial conditions of the system (3.4) are: $s(0) = s_0 \geq 0$, $u(0) = u_0 \geq 0$, $q(0) = q_0 \geq 0$, $c(0) = c_0 \geq 0$, $r(0) = r_0 \geq 0$, then all the solutions $s(t)$, $u(t)$, $q(t)$, $c(t)$, $r(t)$ are non-negative. In an explicit form,

$$\begin{aligned}
&\frac{ds(t)}{dt}|_{s=0} = \mu \geq 0, \quad \frac{du(t)}{dt}|_{u=0} = 0, \quad \frac{dq(t)}{dt}|_{q=0} = \eta u(t) \geq 0,\\
&\frac{dc(t)}{dt}|_{c=0} = (\gamma-\eta)u(t) + \beta q(t) \geq 0, \quad \frac{dr(t)}{dt}|_{r=0} = \varepsilon c(t) \geq 0.
\end{aligned} \tag{3.5}$$

Table 3.1: The *SUQCR* model Parameters of COVID-19 in Bahrain.

Parameters	Description	Units
a	Infection rate	Day^{-1}
N	Total population	Day
n	Birth rate	Per 1000 people
μ	Death rate	Per 1000 people
γ	Non-isolated infected people rate	Day^{-1}
δ	Random tests rate	Day^{-1}
σ	COVID-19 symptoms appearing rate	Day^{-1}
β	Exit tests rate	Day^{-1}
ε	Recovery rate	Day^{-1}
η	Contact tracing rate	Day^{-1}
τ	Immunization including the natural and through vaccination rate	Day^{-1}
λ	Medical history rate	Day^{-1}

3.2.2.1 Equilibrium points, Local Stability and the basic reproduction number R_0

The stability of the system (3.4) and the existence of a single non-negative equilibrium depends on the basic reproduction number R_0 – which is determined by the next-generation matrix method (Van den Driessche, P., & Watmough, 2002) – on the free equilibrium point P_0. Thus, if $R_0 > 1$, the system (3.4) is unstable and the pandemic continue spread occurs; and if $R_0 < 1$, the system (3.4) exhibits asymptotically high stability, and the disease disappears (Serhani & Labbardi, 2021). The disease-free equilibrium point of system (3.4) is the point at which there is no disease in

the population is found as

$$\left(\frac{n}{\mu},0,0,0,0\right). \tag{3.6}$$

The positive disease-free equilibrium point for all the equilibrium points $U = Q = C = 0$ for the system (3.4) is $s(t) = 1$. Thus, the disease-free equilibrium point is $P_0 = (1,0,0,0,0)$.

To give a clear definition of the basic reproduction number R_0 and the next-generation matrix approach, suppose X and Y are the vectors of infected and uninfected classes, respectively. Suppose $f(X;Y)$ be the vector of new infection rates which flows from Y to X, and $v(X;Y)$ be the other rates which are not new infections. In v, the inflow is negative, and the outflow is positive for each compartment. Assume that the matrix G, which connects the number of newly infected individuals of the distinct groups with subsequent generations (Diekmann, Heesterbeek, & Roberts, 2010) is defined by $G = FV^{-1}$where $F = \left[\frac{df}{dx}\right]\Big|_{P_0}$ and $V = \left[\frac{dv}{dx}\right]\Big|_{P_0}$. Van den Driessche (2002) defined the reproduction number R_0 as the dominant eigenvalue of the generation matrix $G = FV^{-1}$ i.e, $R_0 = \rho(FV^{-1})$ where $\rho(G)$ is the spectra radius of the generation matrix G. Now, the details to calculate the reproduction number R_0 of the *SUQCR* model for COVID-19 in Bahrain as follow:

The vectors f and v can be written as,

$$f = (as(t)u(t) + as(t)q(t)), \tag{3.7}$$

$$v = \begin{pmatrix} \mu u(t) + \gamma u(t) \\ \mu q(t) + \beta q(t) + \eta u(t) \\ -(\gamma - \eta)u(t) + \mu c(t) - \beta q(t) + \varepsilon c(t) \end{pmatrix}.$$

At the equilibrium point $P_0 = (1,0,0,0,0)$ one obtains

$$F = \begin{bmatrix} a & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \mu+\gamma & 0 & 0 \\ \eta & \mu+\beta & 0 \\ -(\gamma-\eta) & -\beta & \mu+\varepsilon \end{bmatrix}. \tag{3.8}$$

The inverse matrix of V is given by,

$$V^{-1} = \frac{1}{|V|}(c_{ij})^T, \tag{3.9}$$

where, $|V| = (\mu+\gamma)(\mu+\beta)(\mu+\varepsilon)$, and

$$c_{ij} = \begin{bmatrix} (\mu+\beta)(\mu+\varepsilon) & -\eta(\mu+\varepsilon) & -\beta\eta+(\gamma-\eta)(\mu+\beta) \\ 0 & (\mu+\gamma)(\mu+\varepsilon) & \beta(\mu+\gamma) \\ 0 & 0 & (\mu+\beta)(\mu+\gamma) \end{bmatrix}, \tag{3.10}$$

The matrix G may be written as,

$$G = \begin{bmatrix} \frac{a}{\mu+\gamma} + \frac{-\eta a}{(\mu+\gamma)(\mu+\beta)} & \frac{a}{\mu+\beta} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.11}$$

and consequently, R_0 of system (3.4) will be,

$$R_0 = \frac{a(\mu+\beta-\eta)}{(\mu+\delta+\sigma+\eta)(\mu+\beta)}. \tag{3.12}$$

Theorem 1 *System (3.4) is locally stable related to the disease-free equilibrium point P_0 if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. To prove this theorem, all the eigenvalues of the Jacobian matrix of the system (3.4) are verified as negative when $R_0 < 1$, which means that the system is stable at $P_0 = (1,0,0,0,0)$. However, if at least one eigenvalue is non-negative at $R_0 > 1$, then the system is unstable at P_0. The Jacobian matrix J of the system (3.4) is given by,

$$J=\begin{bmatrix} -\mu-au(t)-aq(t) & -as(t) & -as(t) & 0 & 0 \\ au(t) & as(t)-(\mu+\gamma) & 0 & 0 & 0 \\ aq(t) & \eta & as(t)-(\mu+\beta) & 0 & 0 \\ 0 & \gamma-\eta & \beta & -(\mu+\varepsilon) & 0 \\ 0 & 0 & 0 & \varepsilon & -\mu \end{bmatrix}.$$

The eigenvalues of the matrix J at P_0 are given by,

$$\begin{aligned} &\lambda_1=-\mu, \quad \lambda_2=\frac{(\mu+\gamma)\left[(R_0-1)(\mu+\beta)+\eta\right]}{\mu+\beta-\eta}, \\ &\lambda_3=\frac{(\mu+\beta)\left[(\mu+\gamma)(R_0-1)+(\gamma-\beta+\eta)\right]}{\mu+\beta-\eta}, \\ &\lambda_4=-(\mu+\varepsilon), \quad \lambda_5=-\mu. \end{aligned} \tag{3.13}$$

It is clear that, the eigenvalues $\lambda_4<0$, $\lambda_5<0$ are always negative. However, if $R_0<1$ then all the eigenvalues are negative, and consequently the disease-free equilibrium point P_0 is asymptotically stable localized. Furthermore, P_0 is asymptotically unstable if $R_0>1$, since it has two roots with positive real parts. Thus, related to the disease-free equilibrium point, system (3.4) is locally stable at P_0 if $R_0<1$ and unstable if $R_0>1$. ■

3.2.3 The *SVEUQCHR* Model for COVID-19 in Bahrain

The *SVEUQCHR* model for disease development in Bahrain is a time-dependent Susceptible, Vaccinated, Exposed, Un-quarantined infected, Quarantined infected, Confirmed infected, Hospitalized infected and Recovered transmission model. It is an expanded model of the *SUQCR* model. The diagram of the *SVEUQCHR* model of COVID-19 in Bahrain is shown in Fig (3.3), where, Susceptible $S=S(t)$, individuals exposed to the disease at time t. Vaccinated $V=V(t)$, individuals getting vaccinated at time t. Exposed $E=E(t)$, individuals in the incubation period

after being infected with the virus but not displaying symptoms at time t. Un-quarantined infected $U = U(t)$, infected asymptomatic individuals at time t who were not quarantined. Quarantined infected $Q = Q(t)$, infected individuals with or without symptoms at time t. Confirmed infected $C = C(t)$, confirmed infected individuals at time t. Hospitalized infected $H = H(t)$, infected individuals who need hospitalization at time t. And recovered $R = R(t)$, individuals who have survived at time t.

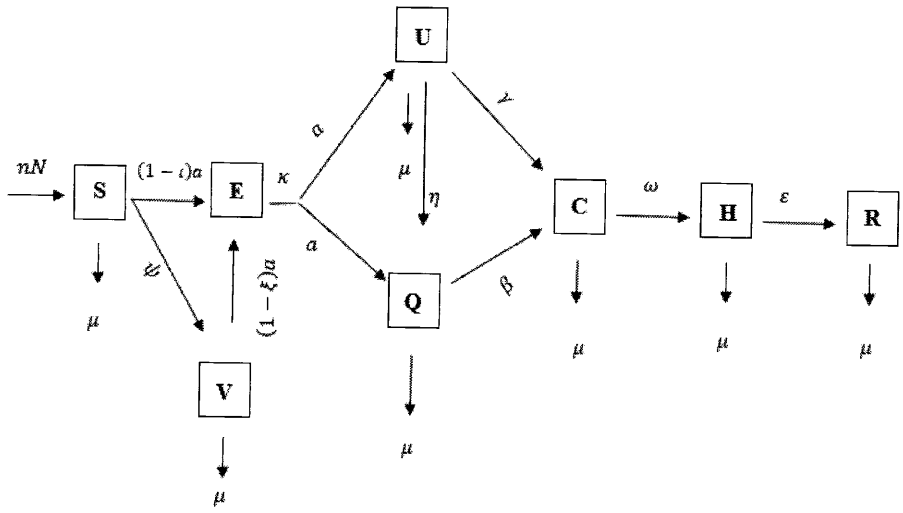


Figure 3.3: The diagram of the *SVEUQCHR* Model.

The total population is given by,

$$N(t) = S(t) + V(t) + E(t) + U(t) + Q(t) + C(t) + H(t) + R(t), \quad (3.14)$$

where it is non-negative for all $t \geq 0$. Now, the *SVEUQCHR* model may be described by the following system of differential equations,

$$
\begin{aligned}
\frac{dS(t)}{dt} &= nN(t) - \mu S(t) - (1-\iota)a\frac{S(t)}{N(t)}E(t) - \psi S(t),\\
\frac{dV(t)}{dt} &= \psi S(t) - (1-\xi)a\frac{V(t)}{N(t)}E(t) - \mu V(t),\\
\frac{dE(t)}{dt} &= (1-\iota)a\frac{S(t)}{N(t)}E(t) + (1-\xi)a\frac{V(t)}{N(t)}E(t)\\
&\quad - \mu E(t) - \kappa a\frac{E(t)}{N(t)}U(t) - \kappa a\frac{E(t)}{N(t)}Q(t),\\
\frac{dU(t)}{dt} &= \kappa a\frac{E(t)}{N(t)}U(t) - (\mu+\gamma+\eta)U(t),\\
\frac{dQ(t)}{dt} &= \kappa a\frac{E(t)}{N(t)}Q(t) - (\mu+\beta)Q(t) + \eta U(t),\\
\frac{dC(t)}{dt} &= \gamma U(t) + \beta Q(t) - (\mu+\omega)C(t),\\
\frac{dH(t)}{dt} &= \omega C(t) - (\mu+\varepsilon)H(t),\\
\frac{dR(t)}{dt} &= \varepsilon H(t) - \mu R(t).
\end{aligned}
\tag{3.15}
$$

Assuming that, $N(t)$ is a constant, then one can write $S(t)$ as,

$$
S(t) = N - V(t) - E(t) - U(t) - Q(t) - C(t) - H(t) - R(t), \tag{3.16}
$$

and, consequently, system (3.15) can be described by,

$$\begin{aligned}
\frac{dV(t)}{dt} &= \psi S(t) - (1-\xi)a\frac{V(t)}{N(t)}E(t) - \mu V(t),\\
\frac{dE(t)}{dt} &= (1-\iota)a\frac{S(t)}{N(t)}E(t) + (1-\xi)a\frac{V(t)}{N(t)}E(t)\\
&\quad - \mu E(t) - \kappa a\frac{E(t)}{N(t)}U(t) - \kappa a\frac{E(t)}{N(t)}Q(t),\\
\frac{dU(t)}{dt} &= \kappa a\frac{E(t)}{N(t)}U(t) - (\mu+\gamma+\eta)U(t),\\
\frac{dQ(t)}{dt} &= \kappa a\frac{E(t)}{N(t)}Q(t) - (\mu+\beta)Q(t) + \eta U(t),\\
\frac{dC(t)}{dt} &= \gamma U(t) + \beta Q(t) - (\mu+\omega)C(t),\\
\frac{dH(t)}{dt} &= \omega C(t) - (\mu+\varepsilon)H(t),\\
\frac{dR(t)}{dt} &= \varepsilon H(t) - \mu R(t).
\end{aligned} \tag{3.17}$$

Table (3.2) describes the parameters and the variables which appear on the system (3.17),

Table 3.2: The *SEVUQCHR* model parameters.

Parameters	Description	Units
a	Infection rate	Day^{-1}
N	Total population	Day
n	Birth rate	Per 1000 people
μ	Death rate	Per 1000 people
γ	Non-isolated infected people rate	Day^{-1}
β	Exit tests rate	Day^{-1}
ε	Recovery rate	Day^{-1}
η	Contact tracing rate	Day^{-1}
ι	Social distancing rate	Day^{-1}
ψ	Vaccination rate	Day^{-1}
ω	Hospitalization rate	Day^{-1}
ξ	Efficacy of vaccination	Per infected people
$\frac{1}{\kappa}$	Latent period duration	Day

To normalize the system (3.17), one sets

$$v(t)=\frac{V(t)}{N},\quad e(t)=\frac{E(t)}{N},\quad u(t)=\frac{U(t)}{N(t)},\quad q(t)=\frac{Q(t))}{N(t)},$$
$$c(t)=\frac{C(t)}{N(t)},\quad h(t)=\frac{H(t)}{N(t)},\quad r(t)=\frac{R(t)}{N(t)}.$$

Now, the normalized system is described by,

$$\begin{aligned}
\frac{dv(t)}{dt} &= \psi(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t)) \\
&\quad -(1-\xi)av(t)e(t)-\mu v(t), \\
\frac{de(t)}{dt} &= (1-\iota)a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t) \\
&\quad +(1-\xi)av(t)e(t)-\mu e(t)-\kappa av(t)e(t)u(t)-\kappa av(t)e(t)q(t), \\
\frac{du(t)}{dt} &= \kappa av(t)e(t)u(t)-(\mu+\gamma+\eta)u(t), \\
\frac{dq(t)}{dt} &= \kappa av(t)e(t)q(t)-(\mu+\beta)q(t)+\eta u(t), \\
\frac{dc(t)}{dt} &= \gamma u(t)+\beta q(t)-(\mu+\omega)c(t), \\
\frac{dh(t)}{dt} &= \omega c(t)-(\mu+\varepsilon)h(t), \\
\frac{dr(t)}{dt} &= \varepsilon h(t)-\mu r(t).
\end{aligned} \tag{3.18}$$

To solve this system, one considers that, the initial conditions are given by,

$$\begin{aligned}
&v(0)=v_0\geq 0;\ e(0)=e_0\geq 0;\ u(0)=u_0\geq 0;\ q(0)=q_0\geq 0; \\
&c(0)=c_0\geq 0;\ h(0)=h_0\geq 0;\ r(0)=r_0\geq 0.
\end{aligned} \tag{3.19}$$

3.2.3.1 The Equilibrium point, the Control and Basic Reproduction number of the $SVEUQCHR$ model

In this context, it is important to analyse the stability of the dynamics of the system (3.18). Therefore, the equilibrium point of this system, which is called the disease-free equilibrium point is evaluated. This point indicates that there is no disease exists in the population. Based on the next-generation matrix method, the basic reproduction number R_0 at the free equilibrium point P_0, which determines whether or not there is a

single non-negative equilibrium and stability (Van den Driessche, P., & Watmough, 2002) is determined. Moreover, the number R_0 may be used to check the stability of the system (3.18). However, if the basic reproduction number $R_0 > 1$, then the system (3.18) is unstable, resulting in continued pandemic spread; while, if $R_0 < 1$, the system (3.18) is asymptotically stable and the disease disappears (Serhani & Labbardi, 2021). Thus, the disease-free equilibrium point of the system (3.18) is given by,

$$\left(\frac{\psi}{\psi+\mu}, 0, 0, 0, 0, 0, 0\right). \tag{3.20}$$

The control reproduction number R_c and the basic reproduction number R_0 are calculated for the *SVEUQCHR* model by using the next-generation approach. The vectors f and v are given by,

$$\begin{aligned} f = &((1-\iota)a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t) \\ &+ (1-\xi)av(t)e(t)), \end{aligned}$$

$$v = \begin{pmatrix} \mu e(t) + \kappa a e(t)u(t) + \kappa a e(t)q(t) \\ -\kappa a e(t)u(t) + (\mu+\gamma+\eta)u(t) \\ -\kappa a e(t)q(t) + (\mu+\beta)q(t) - \eta u(t) \\ -\gamma u(t) - \beta q(t) + (\mu+\omega)c(t) \end{pmatrix}, \tag{3.21}$$

so,

$$F = \begin{bmatrix} X & -(1-\iota)ae(t) & -(1-\iota)ae(t) & -(1-\iota)ae(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.22}$$

where,

$$X = (1-\iota)a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t)) \\ +(1-\iota)(t)ae+(1-\xi)av(t),$$

and

$$V = \begin{bmatrix} \mu+\kappa au(t)+\kappa aq(t) & \kappa ae(t) & \kappa ae(t) & 0 \\ -\kappa au(t) & -\kappa ae(t)+\mu+\gamma+\eta & 0 & 0 \\ -\kappa aq(t) & -\eta & -\kappa ae(t)+\mu+\beta & 0 \\ 0 & -\gamma & -\beta & \mu+\omega \end{bmatrix}.$$

At the equilibrium point, $\left(\frac{\psi}{\psi+\mu},0,0,0,0,0,0\right)$, the matrices F and V reduce to be,

$$F = \begin{bmatrix} (1-\iota)a(1-\frac{\psi}{\psi+\mu})+(1-\xi)a\frac{\psi}{\psi+\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} \mu & 0 & 0 & 0 \\ 0 & \mu+\gamma+\eta & 0 & 0 \\ 0 & -\eta & \mu+\beta & 0 \\ 0 & -\gamma & -\beta & \mu+\omega \end{bmatrix},$$

The inverse of the matrix V is given by,

$$V^{-1} = \frac{1}{|V|}\left(c_{ij}\right)^T, \tag{3.23}$$

where, $|V| = \mu(\mu+\gamma+\eta)(\mu+\beta)(\mu+\omega)$, and

$$c_{ij} = \begin{bmatrix} (\mu+\gamma+\eta)(\mu+\beta)(\mu+\omega) & 0 & 0 & 0 \\ 0 & \mu(\mu+\beta)(\mu+\omega) & \mu\gamma(\mu+\omega) & \mu(\eta\beta+\gamma(\mu+\beta)) \\ 0 & 0 & \mu(\mu+\gamma+\eta)(\mu+\omega) & \mu(\mu+\gamma+\eta)\beta \\ 0 & 0 & 0 & \mu(\mu+\gamma+\eta)(\mu+\beta) \end{bmatrix},$$

Now, the generation matrix G can be calculated as,

$$G = FV^{-1} \begin{bmatrix} \frac{1}{\mu}(1-\iota)a(1-\frac{\psi}{\psi+\mu}) + (1-\xi)a\frac{\psi}{\mu(\psi+\mu)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3.24)$$

Then, the control reproduction number $R_c(\psi, \iota)$ of system (3.18) will be

$$R_c(\psi, \iota) = \frac{(1-\iota)a}{\mu} + \frac{(\iota-\xi)a\psi}{\mu(\psi+\mu)}. \quad (3.25)$$

By supposing there are no control measures, i.e. $\psi = 0$ and $\iota = 0$, then the basic reproduction number $R_0 = \frac{a}{\mu}$, and $\frac{dR_c(\psi,\iota)}{d\psi} = \frac{a(\iota-\xi)}{(\psi+\mu)^2}$. Therefore, $\frac{dR_c(\psi,\iota)}{d\psi} < 0$ if $\iota < \xi$. This means that, if the vaccination efficacy is more than the social distancing rate ($\iota < \xi$), then the control reproduction number R_c will decrease. This means that, the R_0 drops with a social level in general, but only with vaccination if the vaccine's efficacy is greater than the social distance. On the other hand, if the vaccine efficacy is less than the social distancing rate ($\iota > \xi$), then R_c will increase, and the epidemic will be uncontrolled.

To achieve the last objective of this research, which is to use the game theory approach to explain the role of individuals' decisions when taking vaccinations against COVID-19 in reducing the number of infections and critical cases and reducing the number of deaths, the game theory approach is used as shown in the following section.

3.2.4 Game theory approach for vaccination against COVID-19 in Bahrain

By using the game theory approach, the effect of complete vaccination and the booster shots compared to non-vaccination on reducing the num-

ber of infected cases, critical cases, and death cases due to COVID-19 in Bahrain are examined. The game is divided into three parts (VI, VC, VD), and in each part, the effect of booster shots by repeating the game (basic stage) is examined. In both stages, individuals will choose whether to take the vaccine or not. For choosing the Nash equilibrium point, defined as the point with the best response (Prisner, 2014), no one can benefit from individually changing their strategy, the highest value of the first entry in each column and the highest value of the second entry in each row (Straffin Jr Ph.D, 1993) are marked. The binary with two marks will be the Nash equilibrium point.

3.2.4.1 The payoff matrix for vaccination and getting infected (VI)

A payoff matrix is created to compare the rewards of vaccination and infection. This game has two elements in its player strategy set $S = (V_I, V_I^{'})$, where V_I is getting vaccinated, and $V_I^{'}$ is not getting vaccinated. The probability that has been obtained from the Ministry of Health (MOH) statistics (MOH, 2021b) is used to find the probability of a vaccinated and infected person, which is 0.06, while the probability of un-vaccinated person and getting infected is 0.94. Hence the payoff for V_I strategy is $I^{'}$ and the payoff of $V_I^{'}$ strategy is I. To represent the infection effect reflected in players' utilities, a negative sign is used. Therefore, when the two players select the action, get vaccinated V_I, the strategy (V_I, V_I) appears, and the payoff for both of them is $I^{'}$. But when one player selects the action, not get vaccinated $V_I^{'}$, then there is an opportunity to get infected and the strategies $(V_I, V_I^{'})$ or $(V_I^{'}, V_I)$ appear so that the payoff either $(I^{'}, -I)$ or $(-I, I^{'})$. However, when the two players select the action,

not get vaccinated $V_I^{'}$, then the strategy $(V_I^{'}, V_I^{'})$ appears, and the payoff for both is $-I$.

The following is the basic payoff matrix for the first round of vaccination and becoming infected V_I. The payoff matrix for this game is given by,

$$VI = \begin{bmatrix} & V_I & V_I^{'} \\ V_I & (I^{'},I^{'}) & (I^{'},-I) \\ V_I^{'} & (-I,I^{'}) & (-I,-I) \end{bmatrix}. \tag{3.26}$$

However, by using the payoff values from the MOH, one can write (3.26) as,

$$VI = \begin{bmatrix} & V_I & V_I^{'} \\ V_I & (\overline{0.94},\overline{0.94}) & (\overline{0.94},-0.06) \\ V_I^{'} & (-0.06,\overline{0.94}) & (-0.06,-0.06) \end{bmatrix}. \tag{3.27}$$

To see the gain or loss of this game, one marks the highest value of the first entry in each column and the highest value of the second entry in each row. According to this method, the Nash equilibrium point is $(0.94,0.94)$, which is $(I^{'},I^{'})$ if the two players choose to get the vaccine, where a low chance of infection is depicted. On the other hand, the worst choice is $(-0.06,-0.06)$ when the players choose not to be vaccinated, and there is a chance to be infected. From this game, it is clear that the strategy $(-I,-I)$ is not worth playing while the strategy $(I^{'},I^{'})$ is the best to play. The basic stage is repeated, if the players decide to get the booster shots after completing the COVID-19 doses or not to get the vaccine. The tree diagram of the repeated stage is shown in Fig. (3.4).

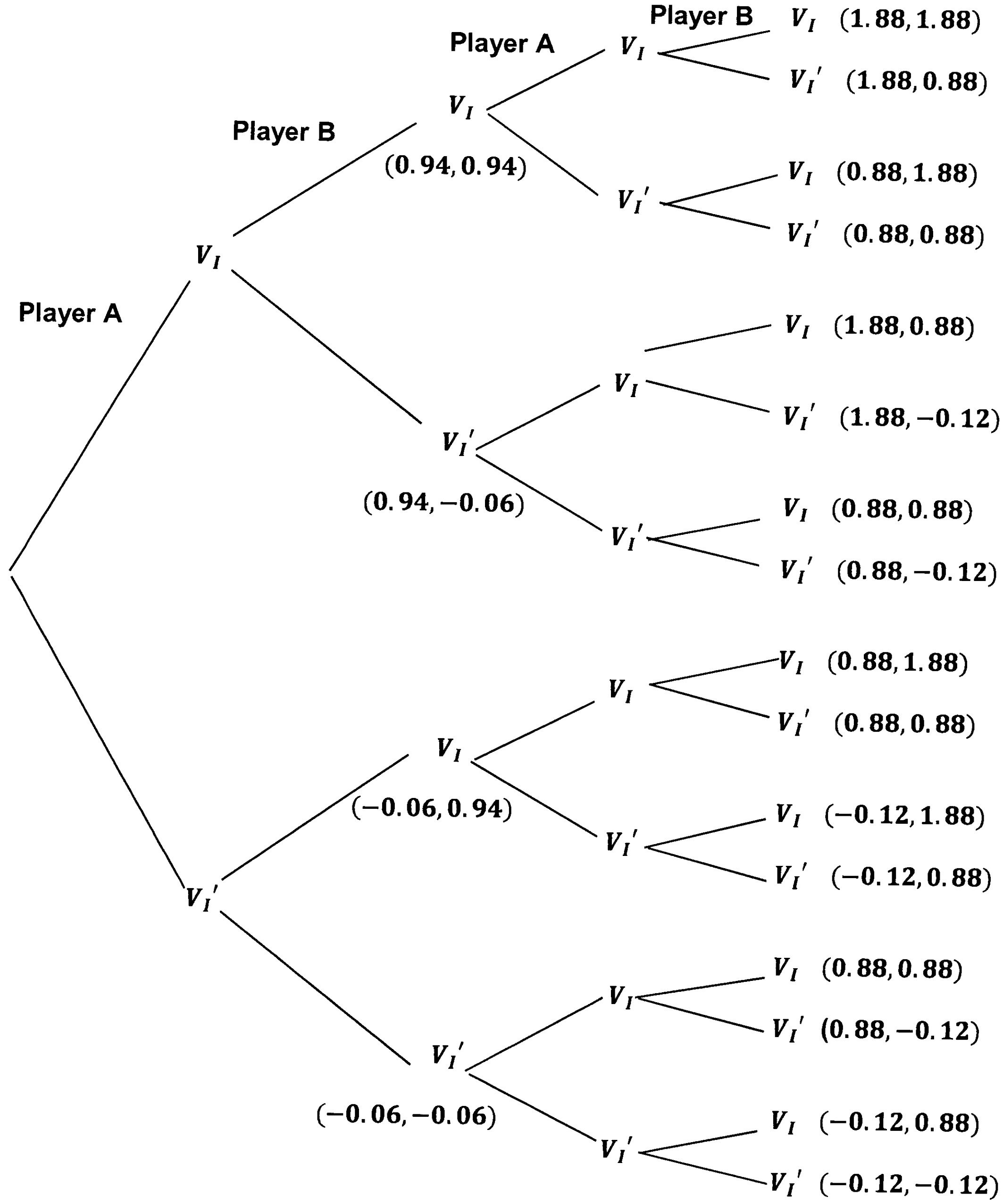


Figure 3.4: Tree diagram of the two rounds of the game (Sec.3.2.4.1).

The payoff biometric of the two rounds is given by,

$$\begin{bmatrix} (\overline{1.88},\overline{1.88}) & (\overline{1.88},0.88) & (\overline{1.88},0.88) & (\overline{1.88},-0.12) \\ (0.88,\overline{1.88}) & (0.88,0.88) & (0.88,0.88) & (0.88,-0.12) \\ (0.88,\overline{1.88}) & (0.88,0.88) & (0.88,0.88) & (0.88,-0.12) \\ (-0.12,\overline{1.88}) & (-0.12,0.88) & (-0.12,0.88) & (-0.12,-0.12) \end{bmatrix}. \quad (3.28)$$

From this payoff matrix, the Nash equilibrium point is $(1.88, 1.88)$, where the same method is used. Therefore, to reduce the chance of infection, the best choice of players is to get the booster dose. In contrast, the worst strategy is $(-0.12, -0.12)$ which means that the chance of being infected is doubled when players decide not to get vaccinated. To find out the effect of full vaccination and booster shots in reducing critical cases or deaths due to COVID-19, the same procedure is followed in VC and VD parts.

3.2.4.2 The payoff matrix of vaccination and being as critical cases due to COVID-19 (VC)

The player strategy set of this game is consisted of two elements $S = (V_C, V_C^{'})$, where V_C is getting vaccinated and $V_C^{'}$ is not getting vaccinated. The probability obtained by the Ministry of Health (MOH) statistics (MOH, 2021a) is used to form the payoff for the strategy V_C as $C^{'}$ and the payoff of the strategy $V_C^{'}$ as C. The probability of an infected person not being in a critical case due to COVID-19, $C^{'}$ is 0.83, and the probability of an infected person being in a critical case due to COVID-19, C is 0.17. The negative sign shows the effect of infection reflected in players' utilities. Thus, in this game, if the two players select the action, get vaccinated V_C, then the strategy (V_C, V_C) appears, and $C^{'}$ is the payoff for both of them. If one player selects the action, not get vaccinated $V_C^{'}$, then there is a probability to be in a critical case from COVID-19 and

the strategy $(V_C, V_C^{'})$ or $(V_C^{'}, V_C)$ appear, so the payoff is either $(C^{'}, -C)$ or $(-C, C^{'})$. However, if the two players select the action, not get vaccinated $V_C^{'}$, then the strategy $(V_C^{'}, V_C^{'})$ appears, and the payoff for both of them is $-C$. The following is a basic payoff matrix for the first round of vaccination and being in a critical situation due to COVID-19, VC,

$$VC = \begin{bmatrix} & V_C & V_C^{'} \\ Vc & (C^{'}, C^{'}) & (C^{'}, -C) \\ V_C^{'} & (-C, C^{'}) & (-C, -C) \end{bmatrix}. \tag{3.29}$$

However, by using the payoff values from the MOH, one can write (3.29) as,

$$VC = \begin{bmatrix} & V_C & V_C^{'} \\ V_C & (\overline{0.83}, \overline{0.83}) & (\overline{0.83}, -0.17) \\ V_C^{'} & (-0.17, \overline{0.83}) & (-0.17, -0.17) \end{bmatrix}. \tag{3.30}$$

Following the same method of finding the Nash equilibrium point, it is found to be $(0.83, 0.83)$, which is $(C^{'}, C^{'})$ if the two players select to be vaccinated, the chance of being in a critical case due to COVID-19 is low. The worst option is $(-0.17, -0.17)$ when the players select not to be vaccinated, there is a probability of being in a critical case from COVID-19. From this game, the strategy $(-C, -C)$ is not worth playing, and the strategy $(C^{'}, C^{'})$ is the best to play. The basic stage of this game is repeated if the players decide to get the booster shots after completing the COVID-19 doses or not to get the vaccine. The tree diagram of the repeated stage is shown in Fig (3.5).

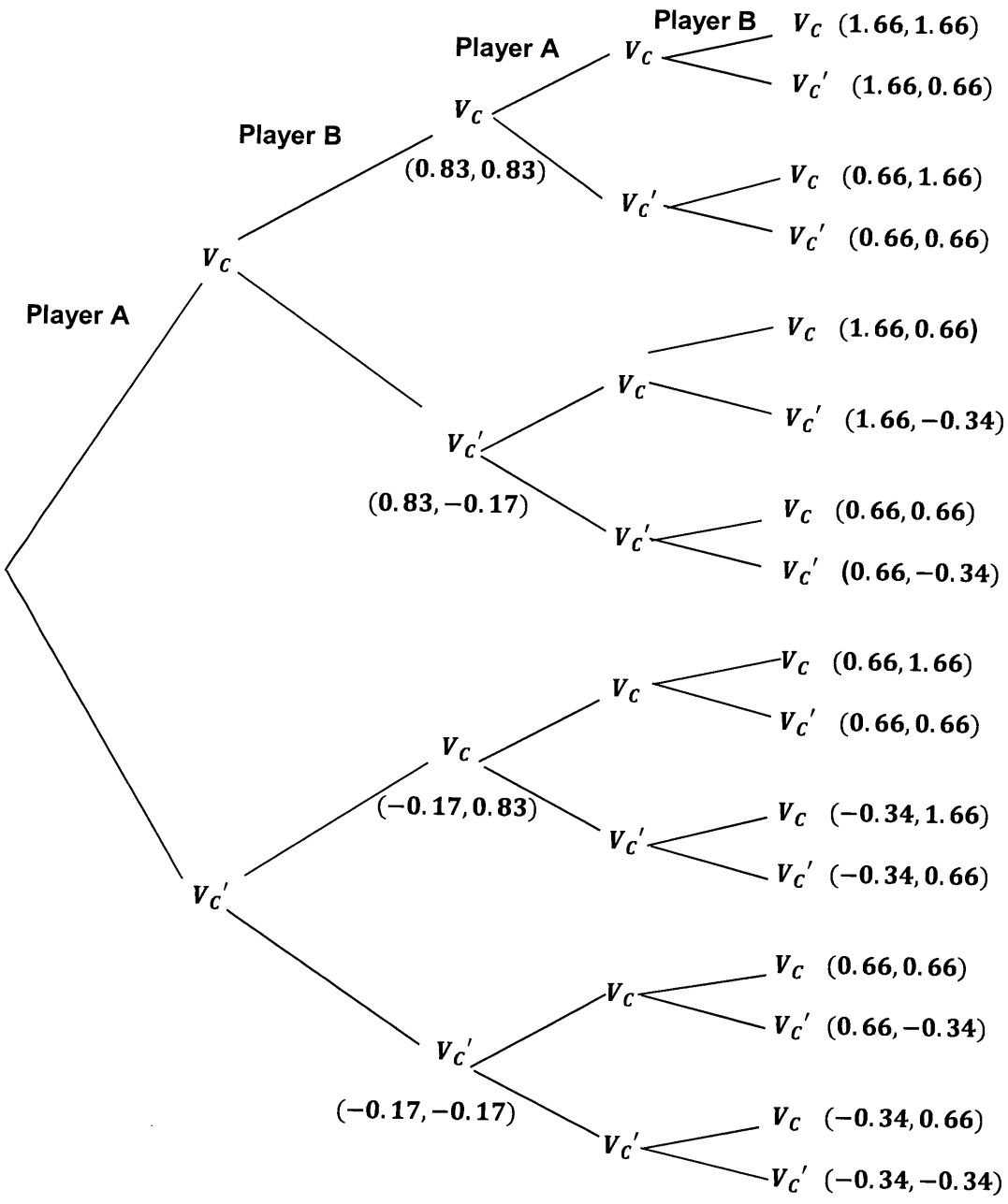


Figure 3.5: Tree diagram of the two rounds of the game (Sec.3.2.4.2).

The payoff biometric of the two rounds is given by,

$$\begin{bmatrix} (\overline{1.66},\overline{1.66}) & (\overline{1.66},0.66) & (\overline{1.66},0.66) & (\overline{1.66},-0.34) \\ (0.66,\overline{1.66}) & (0.66,0.66) & (0.66,0.66) & (0.66,-0.34) \\ (0.66,\overline{1.66}) & (0.66,0.66) & (0.66,0.66) & (0.66,-0.34) \\ (-0.34,\overline{1.66}) & (-0.34,0.66) & (-0.34,0.66) & (-0.34,-0.34) \end{bmatrix}. \quad (3.31)$$

The same method to find the Nash equilibrium point is followed and it is found to be $(1.66, 1.66)$. So, to reduce the chance of being in critical cases, the best choice for players is to get the booster dose. In contrast, the worst strategy is $(-0.34, -0.34)$, which means that the chance of being in a critical case is doubled when players decide not to get vaccinated.

3.2.4.3 The payoff matrix of vaccination and dying due to COVID-19 (VD)

The player strategy set of this game is consisted of two elements as $S = (V_D, V_D')$, where V_D is getting vaccinated and V_D' is not getting vaccinated. The probability obtained by the Ministry of Health (MOH) statistics, (MOH, 2021a) is used to form the payoff strategy of V_D as D' and the payoff strategy of V_D' as D (MOH, 2021d), where D' is the probability of a person not dying due to COVID-19 is 0.9, and D is the probability of dying due to COVID-19 is 0.1. Also, as in the previous parts, the negative sign represents the infection effect reflected in players' utilities. If the two players select the action, get vaccinated V_D, then the strategy (V_D, V_D) appears, and the payoff for both of them is D'. If one player selects the action, not get vaccinated V_D', then there is a probability of dying due to COVID-19 and the strategy (V_D, V_D') or (V_D', V_D) appear, so the payoff is either $(D', -D)$ or $(-D, D')$. However, if the two players select the action, not get vaccinated V_D', then the strategy (V_D', V_D') appears, and the payoff for both of them is $-D$. Therefore, the basic payoff matrix VD

of the first round of vaccination and dying due to COVID-19, is given by,

$$VD = \begin{bmatrix} & V_D & V_D' \\ V_D & (D',D') & (D',-D) \\ V_D' & (-D,D') & (-D,-D) \end{bmatrix}. \tag{3.32}$$

However, by using the payoff values from the MOH, one can write (3.32) as,

$$VD = \begin{bmatrix} & V_D & V_D' \\ V_D & (\overline{0.9},\overline{0.9}) & (\overline{0.9},-0.1) \\ V_D' & (-0.1,\overline{0.9}) & (-0.1,-0.1) \end{bmatrix}. \tag{3.33}$$

If the two players decided to get vaccinated, the Nash equilibrium point of this game is $(0.9,0.9)$, which is (D',D'). However, the worst option is $(-0.1,-0.1)$ when the two players select not to get vaccinated; there is a probability of dying due to COVID-19. It is evident that, the strategy $(-D,-D)$ is not worth playing, and the optimum approach is (D',D'). The basic stage of this game is repeated if the players decide to get the booster shots after completing the COVID-19 doses or not to get the vaccine. The tree diagram of the repeated stage is shown in Fig (3.6).

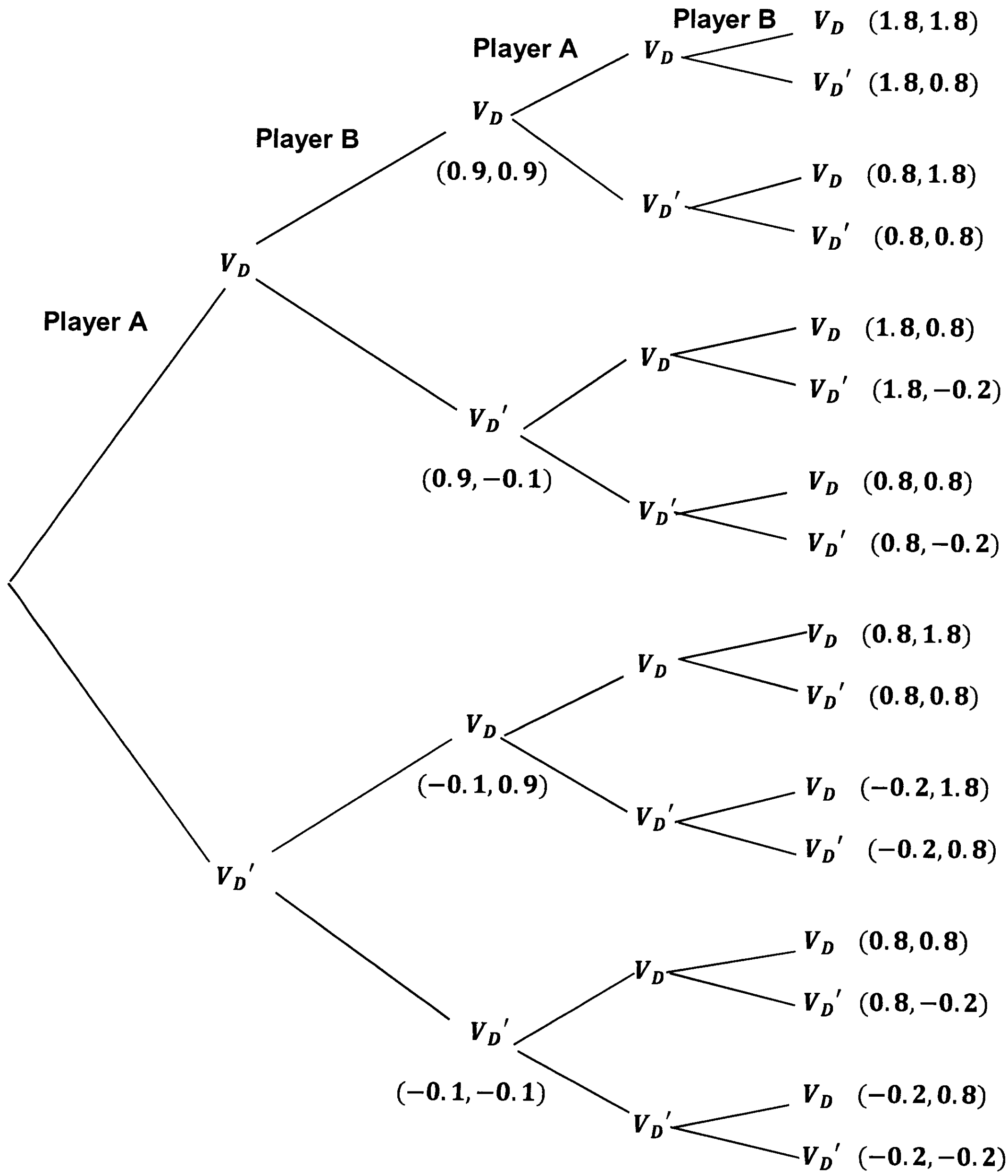


Figure 3.6: Tree diagram of the two rounds of the game (Sec.3.2.4.3).

The payoff biometric of the two rounds of this game is given by,

$$\begin{bmatrix} (\overline{1.8},\overline{1.8}) & (\overline{1.8},0.8) & (\overline{1.8},0.8) & (\overline{1.8},-0.2) \\ (0.8,\overline{1.8}) & (0.8,0.8) & (0.8,0.8) & (0.8,-0.2) \\ (0.8,\overline{1.8}) & (0.8,0.8) & (0.8,0.8) & (0.8,-0.2) \\ (-0.2,\overline{1.8}) & (-0.2,0.8) & (-0.2,0.8) & (-0.2,-0.2) \end{bmatrix}. \quad (3.34)$$

The Nash equilibrium point is $(1.8, 1.8)$ from the repeated round generated matrix. Thus, the best option is if players decide to get the vaccine's booster doses. This is because the booster doses will reduce twice the chance of dying from COVID-19. On the other hand, the worst strategy at $(-0.2, -0.2)$ means that, there is a chance of dying due to COVID-19 when they choose not to be vaccinated, and this chance is doubled in the second round of the game.

3.3 Research tools

To obtain the results, Microsoft Excel 2016 is used to plot graphs, estimate the infected numbers, calculate the % rate of increase and decrease, and calculate the parameters used in the mathematical models and game theory approach. Besides, Mathematica Worlform 12.1 is used to solve the system of differential equations of the $SUQCR$ and $SVEUQCHR$ models by using the 'NDSolve' function and plotting these models by using the plot function.

3.4 Data collection/procedures

To achieve the first goal, the statistics of COVID-19 are collected from the website (OWID, 2020), and on a daily basis of implementing precautionary measures from the Bahrain Ministry of Health (MOH) website

(Ministry of Health, 2020b) from February 24, 2020, to September 18, 2020, and from January 31, 2021, to February 4, 2022. The data is arranged in Microsoft Excel and the number of real infected cases in each period is plotted with the best-fit line of each plot by using the polynomial approximation. The increasing or decreasing rate between the actual data of each period and the estimated number of infected cases of the preceding period is calculated.

For the $SUQCR$ model, the model is solved by using the Wolfram Mathematica 12.1 with code "NDSolve" and the initial conditions $S(0) = 1$, $U(0) = 0.5$, $Q(0) = 0.3$, $C(0) = 0.1$ and $R(0) = 0.1$. Where the data is taken from the website Our World in Data and the Ministry of Health (OWID, 2020; Ministry of Health, 2020a) from September 24, 2020, to December 9, 2020. Some parameters from these data are calculated such as the rate of recovery $\varepsilon = 0.978$, rate of appearing symptoms $\sigma = 0.09$, rate of contact tracing $\eta = 0.09$, rate of random tests $\delta = 0.09$ and exit tests $\beta = 0.09$. Besides, the death rate $\mu = 0.00000671$ per day is from the website (Macrotrends, 2020) and the rate of infection $a = 0.2425$ is from the study (Rahman, Aziz, Khdhr, & Mahmood, 2020).

For the $SVEUQCHR$ model, the values of the required parameters are collected from different sites, such as the effectiveness of the Sinopharm vaccine in preventing infection is 73% (IHME, 2021), the daily recovery rate is 99.1% of the total cases, the death rate is 0.5% of the total cases, and hospitalization rate is 0.4% of the total cases (CoronaTracker, 2021). Besides, the birth rate is 13.163 (Macrotrends, 2020), and the reproduction number in Bahrain is estimated between $0.55 - 1.3$ (Epiforecasts, 2021). Furthermore, the interval between probable latent infection and emergence was observed to range from 1 to 5 days, with an average of two days (Liu, Chu, Gong, Su, & Wu, 2020), so $\kappa = 0.5$.

For the last objective, the data is obtained from the MOH statistics. For the first part (VI), the statistics were from January 2021 to May 19, 2021. The percentage of vaccinated individuals with two doses who got infected is 6%. In contrast, the percentage of individuals who did not get vaccinated and infected was 94% (MOH, 2021b). For the second part (VC), according to Bahrain's Ministry of Health, the overall number of critical cases reached 326.

On June 1, 2021, with 270 cases not receiving the COVID-19 vaccine (MOH, 2021a). Besides, for the last part (VD), there were 665 death instances between January 2021 and May 2021. However, just 10% of people received vaccinations, while 90% did not. 94 % of those who have already been vaccinated have a chronic illness (MOH, 2021d).

Chapter 4

Results and Discussion

4.1 Introduction

In chapter three, the methods were used for drawing results and building mathematical models to combat COVID-19 in Bahrain have been derived. This chapter is devoted to introduce and discuss the obtained results. The outline of this chapter is organized as follows: In Sec.(4.2), the impact of precautionary measures to combat COVID-19 taken in Bahrain during the years 2020 and 2021 on the number of infections, is analysed. The most significant precautionary measures to combat the spread of the disease based on the *SUQCR* model for COVID-19 in Bahrain is discussed in Sec.(4.3). The modification of the *SUQCR* model is described in Sec.(4.4), where the vaccination and the social distancing strategies are considered by using the game theory approach. In Sec.(4.5), the vaccination strategy is investigated in the context of the game theory approach. Finally, a summary of this chapter is introduced in Sec.(4.6).

4.2 The precautionary measures and the COVID-19 pandemic in Bahrain

The precautionary measures approved and implemented by Bahrain to combat COVID-19 during the years 2020 and 2021 are studied. The following are the obtained results:

4.2.1 The precautionary measures implemented by Bahrain in 2020

From the beginning of the pandemic in Bahrain until September 18, 2020, the infected and deaths numbers are expected to increase without precautionary measures. The estimated numbers are given by the following linear system of equations,

$$y_i = 1.4858x^2 - 12.116x + 2024.6,$$
$$y_d = 0.0073x^2 - 1.1581x + 43.106, \tag{4.1}$$

where y_i, and y_d represent the infected and the death numbers in this period, respectively (AlSayegh & Iqbal, 2021).

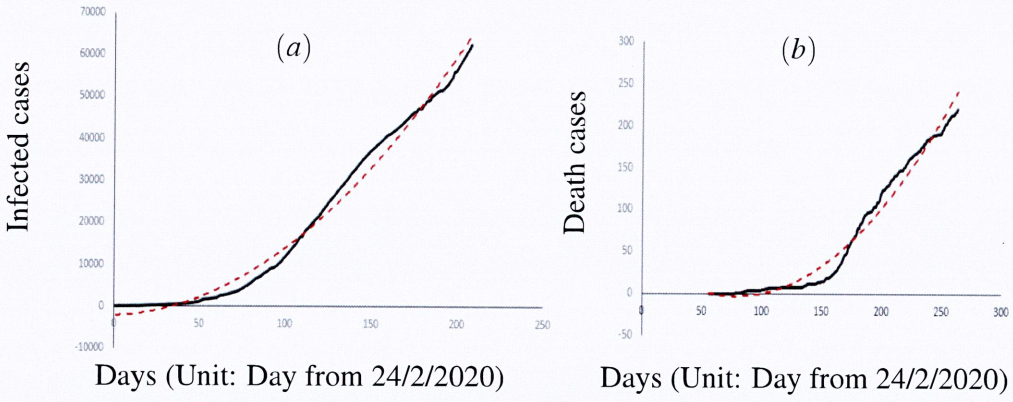


Figure 4.1: Total infected cases of COVID-19, where the black and red curves represent the observed and estimated number, respectively, from February 24, 2020 (a) the infected cases and (b) the total death cases.

Fig. (4.1 a) shows the observed cases of COVID-19 in Bahrain, compared to the estimated numbers without implementing protection measures. Fig. (4.1 b) shows the number of deaths observed in Bahrain caused by COVID-19, compared to the estimated number without implementing protection measures. There is a significant difference between the number of detected infections and deaths and the number of estimated cases, except for some periods when the estimated cases are less

than those detected. The Government Executive Committee in Bahrain has continued to work hard since the first case of COVID-19 to control the spread of the virus in Bahrain, and it has taken many precautions, as shown in Table (4.1). These approaches are based on the World Health Organization (WHO) recommendations (AlSayegh & Iqbal, 2021).

Table 4.1: Precautionary measures applied in Bahrain to combat COVID-19 pandemic from February 24, 2020, to September 18, 2020.

Date	Precautionary measures
February 24, 2020-March 14, 2020	-Applying social distance. -Quarantine and home isolation for travellers returning home. -Travel bans to and from Coronavirus affected countries
March 15, 2020-March 24, 2020	-Reducing the number of upcoming trips. -Distance working Restriction of restaurants to requests serve only external. -Closing of all educational institutions, cinemas, salons, and sports centres
March 26,2020 -April 9, 2020	Closing commercial and industrial shops.
March 29, 2020- Farther notice	Random COVID-19 tests.
April 10, 2020- April 23, 2020	Reopening commercial and industrial shops.
April 23, 2020 - Farther notice	Wearing a face mask is mandatory.
April 24, 2020 -May 7, 2020	Closing commercial and industrial shops.
May 8, 2020 - Farther notice	Reopening commercial and industrial shops.
June 3, 2020 - Farther notice	Masks are mandatory, and violators are fined.
August 6, 2020	Reopening gyms, sports halls, swimming pools, and outdoor sports.
September 3, 2020	Dining outdoors in restaurants and cafes.

Table 4.2: Periods of precautionary measures applied in Bahrain in 2020.

Study periods	Precautionary measures in 2020	Average % rate of change of infected cases
Period 1	Social distance	5%
Period 2	Distance working and closing all educational institutions	-18.68%
Period 3	Closing commercial and industrial shops	12.14%
Period 4	Reopening commercial and industrial shops	36%
Period 5	Closing commercial and industrial shops	13.4%

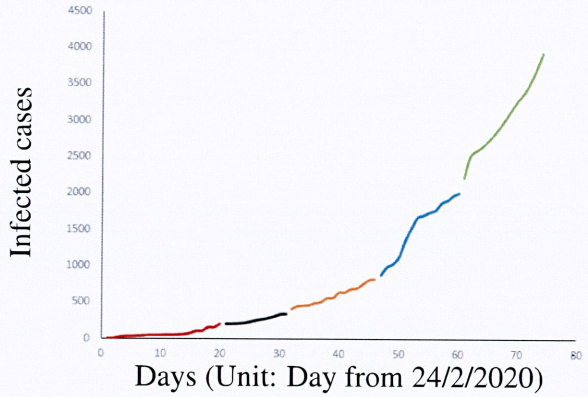


Figure 4.2: Total number of infected cases, when precautionary measures were taken from February 24, 2020. The red, black, orange, blue and green curves represent the total infected cases for periods 1-5 (social distance, distance working and closing all educational institutions, closing commercial and industrial shops, reopening commercial and industrial shops, closing commercial and industrial shops), respectively.

Fig (4.2) represents the total number of the infected cases during applying precautionary measures. Data is broken down into five categories: social distance, distance work, closing schools, closing shops that sell goods directly to the public for two weeks, reopening these shops for another two weeks, and closing them again for two weeks as seen in Table (4.2).

Fig. (4.3) shows the number of infected and expected cases when applying precautionary measures compared to the number of infected cases when applying the new precautionary measures. Fig. (4.3 a) indicates that social distancing and the travel ban did not significantly reduce the expected number of infected cases between February 24 and March 13, 2020. In this period, the average % rate of change is increased by 5% compared with the expected rate of infected cases and the number of infected cases is forecasted by

$$y = 0.6067x^2 - 4.3201x + 26.752. \tag{4.2}$$

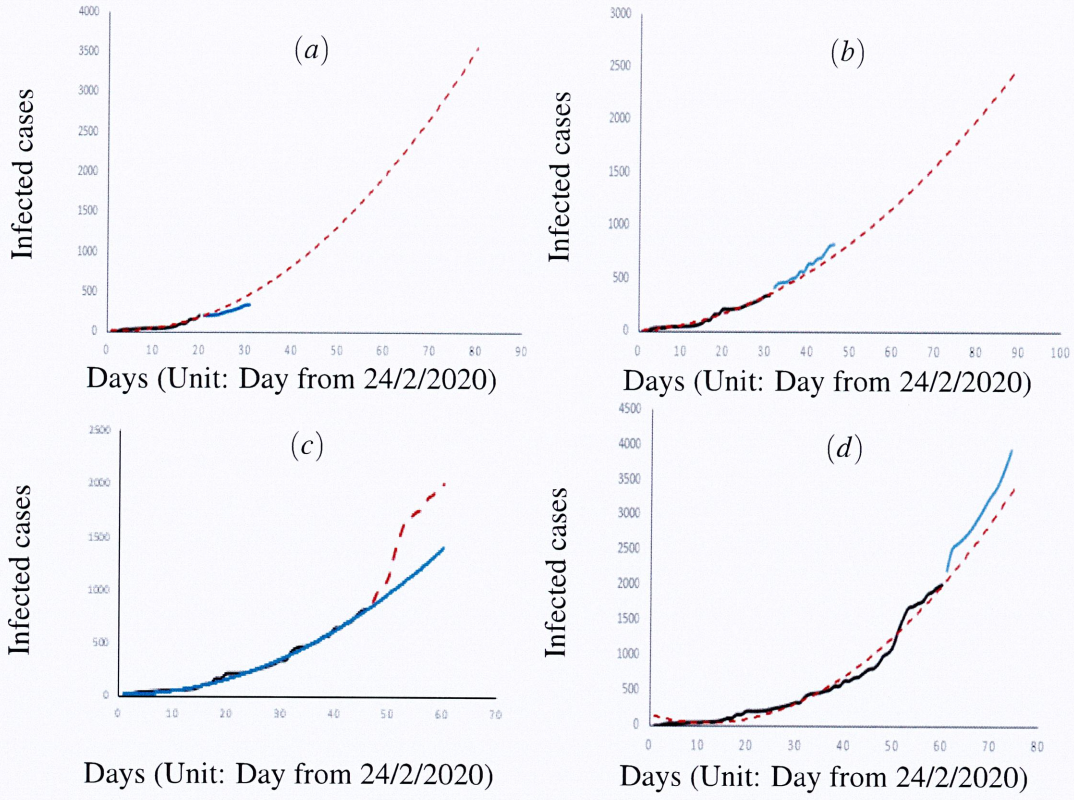


Figure 4.3: Measures taken in Bahrain to reduce the number of infected cases from February 24, 2020, to May 7, 2020. The black, blue and red curves represent a) period 1 (social distance) and period 2 (distance working and closing all educational institutions) and the expected infected cases of period 1, respectively. b) periods 1-2 and period 3 (closing commercial and industrial shops) and the expected infected cases of periods 1-2, respectively. c) periods 1-3 and period 4(reopening commercial and industrial shops) and the expected infected cases of periods 1- 3, respectively. d) periods 1-4 and period 5 (closing commercial and industrial shops) and the expected infected cases of periods 1- 4, respectively.

In order to combat COVID-19, the Bahrain National Medical Team issued directives for transitioning into distance work, suspending all studies at educational institutions, closing cinemas and sports centres, and limiting activity in restaurants to outdoor requests only in the following ten days (Ministry of Health, 2020b). Accordingly, the average % rate of changing the infected cases decreased by 18.68% compared to social distancing and travel bans alone, as seen in Fig. (4.3 a). Fig. (4.3 b) indicates

that, when shops and industrial stores that provide direct services to individuals were closed from March 26 to April 9, 2020, the average % rate of changing the infected numbers increased by 12.4% than the expected infected numbers when applying the previous precautionary measures. This behavior may be described by,

$$y = 0.2858x^2 - 2.3039x + 3.8004. \tag{4.3}$$

After that, a decision was issued to open shops, which increased the average % rate of infected cases by 36% compared to the estimated numbers when applying the previous precautions, as shown in Fig. (4.3 c). The best fit curve for this period is given by,

$$y = 0.3967x^2 - 0.7884x + 18.291. \tag{4.4}$$

As it is presented in Fig. (4.3 d), the average % rate of change of infected cases rose 13.4% when the shops were closed again two weeks later. The best fit curve used during this period is given by,

$$y = 0.8488x^2 - 20.152x + 151.85. \tag{4.5}$$

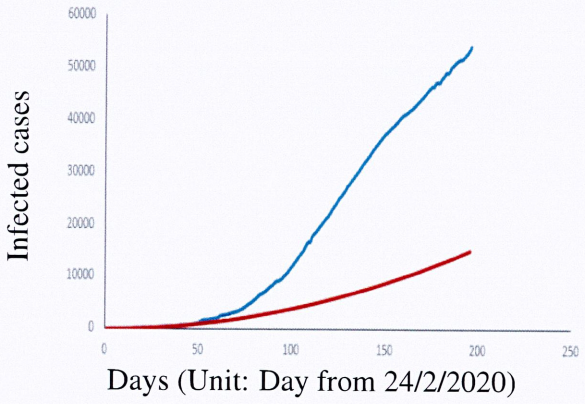


Figure 4.4: Comparing the number of expected infected cases (red) with the actual number of infected cases (blue) after the lockdown is implemented.

Fig. (4.4) shows the number of infected cases when schools, educational institutions, shops, and sports centres continue to be closed compared to the recorded infected numbers. If the lockdown period continues, the number of infections will decrease with an average rate of 64% than the estimated infected cases.

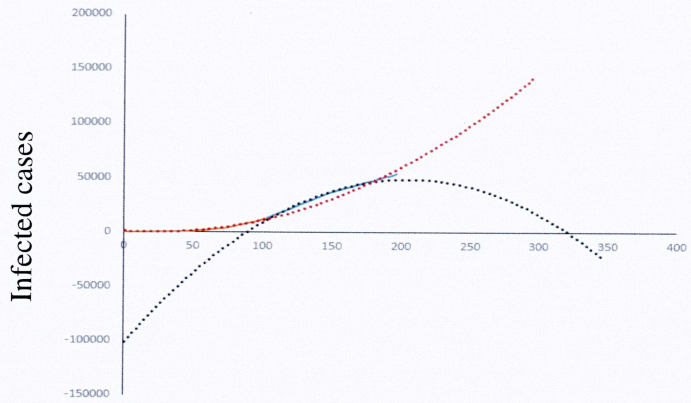


Figure 4.5: The number of infected cases in Bahrain when face masks are mandatory versus the number of infected cases when face masks are not mandatory. The blue and black dash curves represent the infected cases and expected infected cases when face masks are mandatory, respectively. The red solid and dash curves represent the infected and expected cases when face masks are not mandatory in Bahrain, respectively.

People in Bahrain have been wearing masks since the outbreak began, which contributed to the reduction in the number of infections, as shown in Fig. (4.5). The number of infections predicted when face masks became mandatory was 14% less than the number predicted before face masks became mandatory.

Fig. (4.6) shows more than 700 cases after Eid-al-Fitr, and after Ashura, the highest number of cases was seen on September 16, 2020, when more than 800 people were infected daily. It also represents that, the estimated number of infected cases will slowly decrease during March, April, and May. From this approximation, the epidemic in Bahrain is expected to

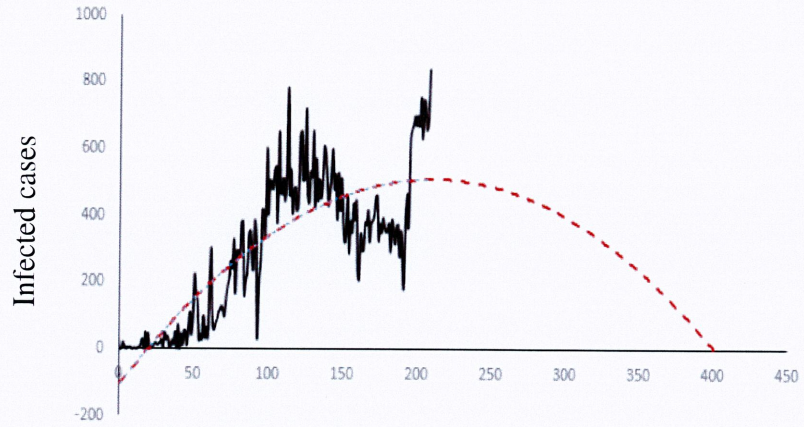


Figure 4.6: The infected cases where the black and dash curves represent the daily infected and expected cases from February 24, 2020.

end after 403 days from the initial day, which agrees to late March 2021. The best-fit curve is described by,

$$y = 0.014x^2 + 5.8968x - 110.47. \tag{4.6}$$

4.2.1.1 Discussion

This section investigates the impact of precautionary measures in Bahrain to halt the spread of COVID-19 during 2020. The results support the clear effect of precautionary measures on combating the pandemic.

Bahrain has started applying social distancing as a first measure since the first case appeared. Unfortunately, this measure increased the expected number of infected cases by 5% instead of reducing it. However, a study conducted in Spain between February 24 and April 5, 2020, indicated that physical distancing measures effectively control the spread of disease (Saez, Tobias, Varga, & Barceló, 2020), especially when applied correctly and for the right amount of time (Reis et al., 2020; Ambikapathy & Krishnamurthy, 2020). Therefore, the Ministry of Health called for implementing some other measures that help reduce infections, as

shown in Table (4.1). In the early stages of the pandemic, travellers were quarantined at shown in Table (4.1) and Fig. (4.3 a), which significantly limited the spread of the virus, so the number of infections matched the number expected. According to Mayr et al.(2020), integrating home isolation with other preventive measures had a greater impact than applying quarantine alone. In addition, Li et al.(2020) argue that, quarantines in Hubei Province have effectively slowed the spread of COVID-19. As well, Tang et al.(2020) suggested that quarantine and isolation strategies should be strengthened further for China to improve COVID-19 detection rates.

Several precautionary measures have been taken, including closing educational institutions, applying social distances, reducing upcoming trips, distance working, closing sports centres and cinemas, restricting restaurant activities to external requests, and closing salons. As a result of implementing these measures within 21 days of discovering the first COVID-19 case in Bahrain, cases dropped by 18%. These results are consistent with a study conducted by Kim et al. in Korea (2020), which found that earlier school closures could reduce the number of infected cases by at least 200. After that, shops offering direct services to residents should close for two weeks, then reopen for two weeks, and then close for the next two weeks. Unfortunately, implementing these measures did not decrease the number of infected cases but instead increased them by 11.14 %, 36%, and 13.4 %, respectively.

Since March 29, 2020, the number of infected cases has increased than expected during the first closure period due to random testing (Ministry of Health, 2020c). Moreover, the number of infections increased during the second closure period because shops opened between the two closure periods (see Fig. (4.3 d)). As shown in Fig. (4.3 c), the confirmed cases

also rise when shops reopen. This increase is due to people not adhering to preventive measures when leaving their homes, especially since the holy months of Ramadan and Eid al-Fitr are approaching when people buy supplies, clothes, and so on. Those reasons, along with the evacuation of stranded individuals in some countries, increased the number of cases.

Along with the results obtained, a clear decrease in expected infections can also be seen when closed shops. These results are consistent with the research that conducted in Italy, which demonstrated the importance of lockdown in disease control (Chintalapudi, Battineni, & Amenta, 2020). Tobias (2020) and Saez et al.(2020) also analysed trends in confirmed COVID-19 cases and deaths in Spain and Italy before and after the national lockdown, using time-series statistics. Their results show that COVID-19 infections have decreased significantly after the implementation of lockdown. Furthermore, lockdowns allow the health system in the country to increase its capacity to help reduce disease transmission by approximately 60%, given that it affects asymptomatic patients (Anderson, Heesterbeek, Klinkenberg, & Hollingsworth, 2020). On the other hand, Xiao and Torok (2020) suggested that traffic bans and village closures are of no importance for preventing and controlling COVID-19.

As shown in Fig. (4.5), wearing face masks has led to a significant decrease in infected cases since June 3, 2020. Similarly, a United States study suggested that states where masks have been mandated, experienced a considerable reduction in the number of infected cases, reducing the number of infected cases and infection rates of COVID-19, in contrast to cases where mask use was not mandatory (Lyu & Wehby, 2020). In addition, Barasheed et al. (2016) analysed the effectiveness of wearing face masks by participating 12,710 samples from more than 50 countries.

They concluded that, wearing face masks reduces the risk of infection by 20%. Also, South Korea has successfully prevented COVID-19 spread during the ongoing pandemic by applying comprehensive social distancing policies, wearing facial masks, and delaying school openings (Choi & Shim, 2020).

During the year 2020, the application of these precautionary measures helped decrease the infected number in Bahrain. Therefore, according to the reduction in cases, it was assumed that, the epidemic would disappear in late March 2021 (see Fig. (4.6)) (AlSayegh & Iqbal, 2021).

4.2.2 The precautionary measures implemented by Bahrain in 2021

In 2021, Bahrain continued to implement precautionary measures to combat COVID-19. Despite the emergence of several mutations of the Coronavirus during this period, precautionary measures have a prominent role in combating the disease and limiting its spread. Table (4.3) shows the most important precautionary measures that were implemented in Bahrain during 2021. Table (4.4) shows the periods of these measures from January 31, 2021, to January 13, 2022 (Ministry of Health, 2022).

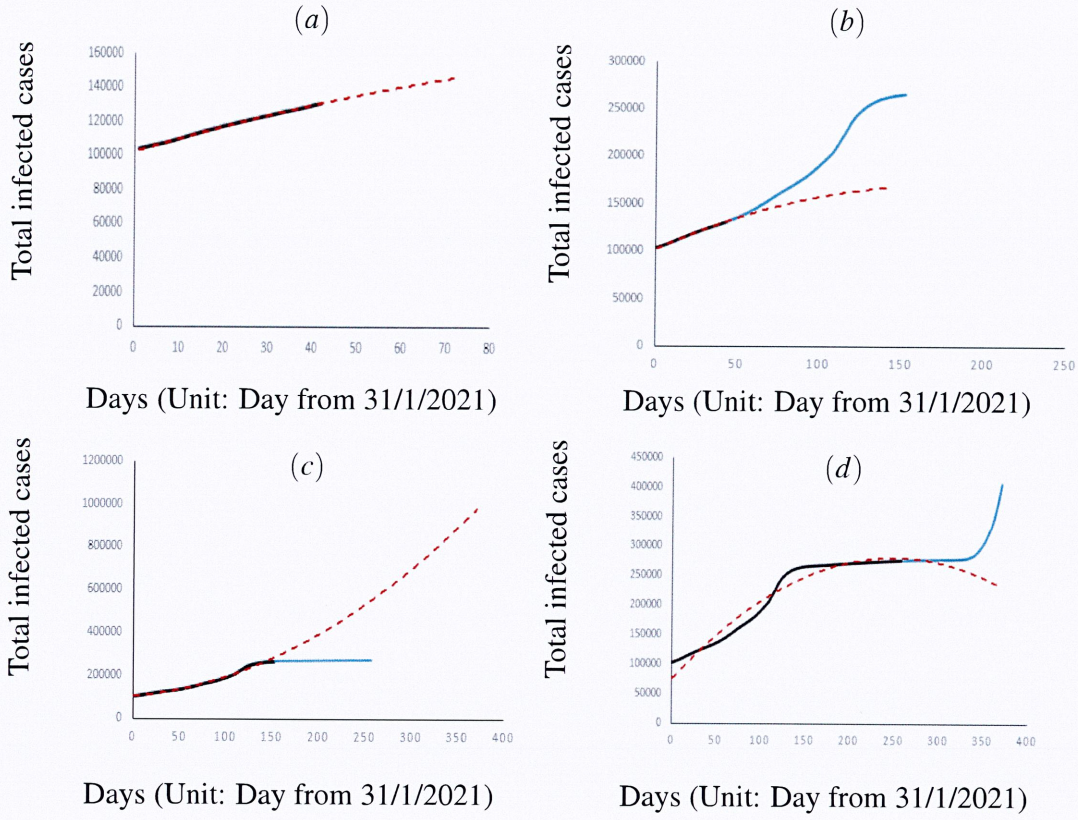


Figure 4.7: Measures taken in Bahrain to reduce the number of infected cases from January 31, 2021, to February 4, 2022. The black, blue and red curves represent a) period 1 (restricting some precautionary measures and allowing work from home by 70%) and the expected infected cases of period 1 respectively. b) periods 1- 2 (opening different sectors and updating quarantine procedures and PCR testing) and the expected infected cases of period 1, respectively. c) periods 1-2 and period 3 (implementation of the traffic light system for COVID-19 disease) and the expected infected cases of periods 1-2, respectively. d) periods 1-3 and period 4 (update the procedures for contacts and quarantine tracking and travellers) and the expected infected cases of periods 1-3, respectively.

Table 4.3: Precautionary measures in Bahrain from January 31, 2021, to January 13, 2022.

Date	Precautionary measures
January 31, 2021-February 21,2021	-Distance learning in educational institutions -Only allow external services in restaurants and cafes.
February 7, 2021-February 20,2021	-Allow 70% of employees to work from home. -Temporary closure of gyms, swimming pools, and indoor exercise classes.
February 21,2021-March 14, 2021	-70% of government employees may work from home -Suspension of personal activities in government rehabilitation centres and training institutes. -Temporarily limiting outdoor dining services and closing gyms -Social events or gatherings in private spaces with more than 30 person
March 14, 2021-Farther notice	-Reopen of different sectors and internal services in restaurants and gyms -Resume of learning in educational institutions: in person or at a distance
May 7, 2021- Farther notice	Update the arrival procedures for vaccinated passengers, allowing them to enter Bahrain without the PCR test and self-isolating
May 7, 2021-July 2, 2021	Restrict some activities, such as malls, gyms, salons,family gatherings, attendance events and conferences
July 2, 2021-Farther notice	Adoption of the yellow alert level in COVID-19
August 18, 2021-August 19, 2021	Adoption of the orange alert level in COVID-19
August 20, 2021- Farther notice	Adoption of the yellow alert level in COVID-19
August 29, 2021-Farther notice	All arrivals from the red list countries must self-quarantine for 10 days.
October 15, 2021-Farther notice	Update the procedures for contacts and quarantine tracking as follows: -Green shield holders: not required to be quarantined -Non-green shield: required to be quarantined for seven days.
January 13, 2022-Farther notice	Update the procedures for active cases, travellers and contacts as follow: Active cases: -Green shield holders: required to quarantine for seven days -Non-holders of the green shield: required to be quarantined for ten days Travellers: -Green shield holders: not required to quarantine. -Non-holders of the green shield: required to be quarantined for seven days after undergoing a PCR examination upon arrival Contact of active cases: -Green shield holder: not required to be quarantined. -Non-holder of green shield: quarantined for seven days and PCR test when symptoms appear.

Restricting some precautionary measures and allowing work from home by 70% between January 31, 2021, and March 14, 2021, helped to reduce the number of infections. Therefore, the average % rate of change in this period decreased by 4.06% compared to the previous period if they continued implementing the precautionary measures followed, as seen in

Table 4.4: Periods of precautionary measures applied in Bahrain in 2021.

Periods	Precautionary measures in 2021	Average % rate of change of infected cases
Period 1	Restricting some precautionary measures and allowing work from home by 70% (January 31, 2021 –March 14, 2021)	-4.06%
Period 2	Opening different sectors and updating quarantine procedures and PCR testing (March 15, 2021 – July 2, 2021)	25.93%
Period 3	Implementation of the traffic light system for COVID-19 disease (July 3, 2021-October14,2021)	-31.75%
Period 4	Update the procedures for contacts and quarantine tracking and travellers (October 15, 2021- February 4, 2022)	53.24%

Fig. (4.7 a). In this period, Eq. (4.7) is used to estimate the number of infected cases

$$y = -2.4508x^2 + 783.39x + 102334. \tag{4.7}$$

As a result of opening different sectors, updating quarantine procedures, and PCR testing, the number of infections increased more than expected when previous precautionary measures were utilized. As a result, the average % rate of change increased by 25.93% compared to the last period, as shown in Fig. (4.7 b). Also, Eq. (4.7) is used to estimate the infected cases in this period. It was evident that the light signalling system in Bahrain was effective in reducing the number of infections caused by COVID-19, as the average % change decreased by 31.75% in comparison to the expected number of infections when continuing previous measures, as shown in Fig. (4.7 c). The infected cases in this period are estimated by,

$$y = -2.1962x^2 + 774.79x + 101700. \tag{4.8}$$

However, it was noted in the fourth period that, updating some procedures to track contacts, quarantine, and travellers had the opposite result. As a result, the average % rate of change in this period increased by

53.24% compared to the expected number of cases when the previous measures were continued, as seen in Fig. (4.7d). In this case, the infected cases in this period are estimated by,

$$y = 5.4059x^2 + 365.44x + 104874. \tag{4.9}$$

4.2.2.1 Discussion

In 2021, Bahrain continued to implement precautionary measures to combat the virus. All government and private institutions in Bahrain continued to implement distance learning with limited attendance, including administrative, educational, and technical bodies. In addition, Bahrain allowed restaurants and cafes to provide only external services, with a maximum of 30 reservation personnel. Furthermore, government agencies and institutions allowed 70% of their employees to work remotely. In addition, gyms, swimming pools, and indoor exercise classes were temporarily closed as well as a temporary suspension of personal activities in government rehabilitation centres, nurseries, and private training institutes. According to preliminary precautionary measures, it reduced the infections by 4.06% compared to expected, as seen in Fig. (4.7 a). These findings agreed with a study done by Frdinger and Schymik (2020) that working from home effectively reduces infection.

Another measure that helped control the spread of the virus in Bahrain during this period was applying the traffic light system, which was based on the average % of positive cases. If the average % of test-positive cases is less than 2% for 14 days, the level is green. If the average % of test-positive cases is less than 2%-5% for seven days, the level is yellow. The orange level applies when the average % of test-positive cases is between 5% and less than 8% for four days. While the red level is applied when

the average % of test-positive cases is 8% or more for three days. According to the average of positive cases since July 7, 2021, the yellow level was adopted in Bahrain. It effectively reduced infections equivalent to 31.75% of what would be expected if the previous measures were continued, as seen in Fig. (4.7 c).

As shown in Fig. (4.7 b), opening shops in 2021 increased infected cases. In contrast, reducing procedures, such as quarantine for arrivals and contacts, and canceling PCR tests for contact individuals, contributed to higher infection rates, which were 53.24% higher than expected when previous procedures were followed, as shown in Fig. (4.7 d). Quarantine has an essential role in reducing infections, as shown in the study conducted by Kucharski et al. (Kucharski et al., 2020). They show in their research that individuals at risk of recent infection are being quarantined as one of the non-pharmacological interventions to combat the ongoing SARS-CoV-2 epidemic.

4.3 The *SUQCR* Model for COVID-19 in Bahrain

The *SUQCR* model is solved by using the code "NDSolve" in the Wolfram Mathematica 12.1 with the initial conditions $S(0) = 1$, $U(0) = 0.5$, $Q(0) = 0.3$, $C(0) = 0.1$ and $R(0) = 0.1$ and the plots of the model are shown in Fig. (4.8- 4.11). In the *SUQCR* model, different parameters are used, such as the death rate $\mu = 2.450$ per 1000 people, which means $\mu = 0.00000671$ per day (Macrotrends, 2020) and the infection rate $a = 0.2425$ (Rahman et al., 2020). Furthermore, some parameters obtained from Bahrain COVID-19 data (OWID, 2020; Ministry of Health, 2020a) starting from September 24, 2020, to December 9, 2020 such as, the recovery rate $\varepsilon = 0.978$, symptoms appearing rate $\sigma = 0.09$,

contact tracing rate $\eta = 0.09$, random tests $\delta = 0.09$ and exit tests rate $\beta = 0.09$ are calculated. According to the $SUQCR$ model and Table (4.5), the basic reproduction number of Bahrain is $R_0 < 1$. However, R_0 will gradually increase as the contact tracing rate, random test and exit test rates all fall from 0.09, which is the rate currently calculated for Bahrain in the period of this study (OWID, 2020; Ministry of Health, 2020a). R_0 in Bahrain depends on six parameters, the infection rate, the death rate, the symptoms appearing rate, the random, and exit test rates, in addition to the contact tracing rate.

Table 4.5: Values of reproduction numbers R_0 with different parameters.

a	μ	δ	σ	β	η	R_0
0.2425	0.00000671	0.09	0.09	0.09	0.09	0.00007
0.2425	0.00000671	0.09	0.09	0.3	0.6	0.3
0.2425	0.00000671	0.09	0.09	0.3	0.3	0.6
0.2425	0.00000671	0.09	0.09	0.11	0.02	1
0.2425	0.00000671	0.09	0.09	0.09	0.55	1.7
0.2425	0.00000671	0	0.09	0	0	2.7

In Fig. (4.8), the effect of the random test number is investigated, where $\delta = 0.05$ and 1.55 and all the other parameters are fixed. A general pattern of this behavior indicates that, the individual recovery increases gradually over time. At a later time, it behaves continuously. However, the stabilized behavior appears faster at smaller values of δ, as shown in Fig. (4.8 a- 4.8 b). In a short period of time, the quarantined individual numbers Q increase gradually to reach their maximum values, which are predicted at small values of δ. As time goes on, the Q numbers gradually decrease until they disappear completely, depending on the random test number δ. With increasing time, the number of un-quarantined U

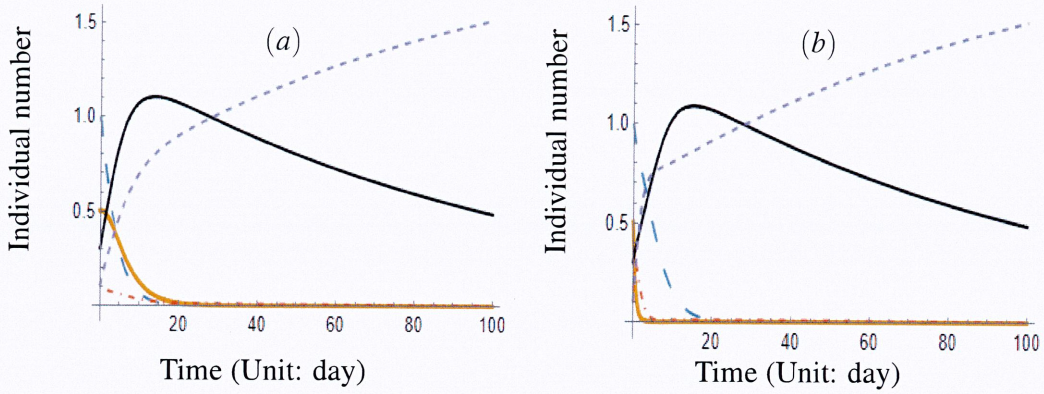


Figure 4.8: The susceptible (long blue dash), un-quarantined infected (orange), quarantined infected (black), confirmed infected (red dot-dash) and recovers (small blue dash) when $a = 0.2425$, $\mu = 0.00000671, \varepsilon = 0.978, \sigma = 0.09, \beta = 0.01, \eta = 0.09$ and (a) $\delta = 0.05, (b)\delta = 1.55$.

decreases gradually, and suddenly at large δ values. In addition, the numbers of confirmed infected individuals and susceptible individuals are expected to decrease abruptly/ gradually at low/ high values of δ.

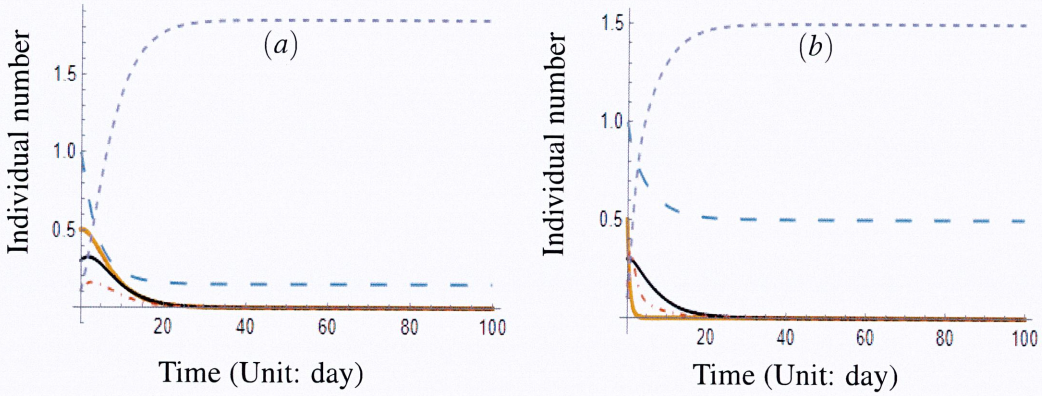


Figure 4.9: Same as Fig. (4.8) with $\beta = 0.3$.

In Fig. (4.9), one increases the exit test rate on the behavior of all individuals' number. It is clear that, the behaviors are almost the same as those shown in Fig. (4.8), that is, all individual numbers decrease except the recovered number, namely it increases. Besides, for a short time, the susceptible individual numbers decrease, then it stabilizes at a further time. The gradual decay of the U, Q, C, S is shown at a small rate of random test, while their sudden decay is shown at larger values of δ.

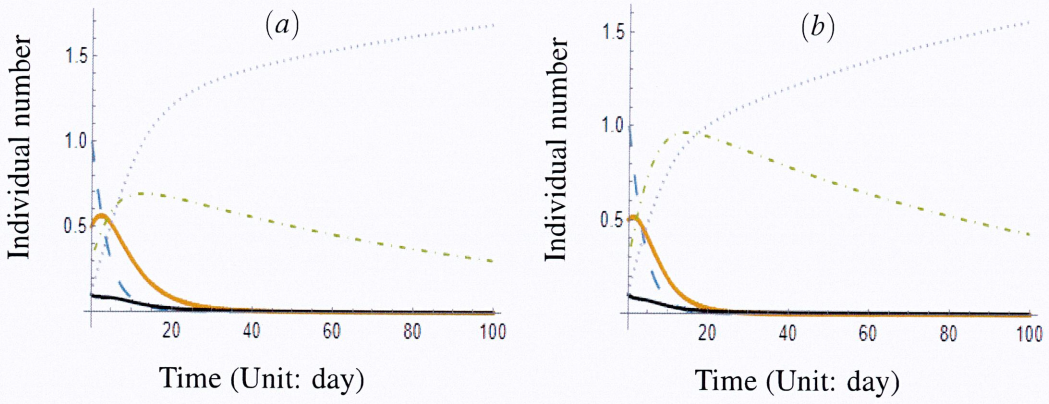


Figure 4.10: The susceptible (long blue dash), un-quarantined infected (orange), quarantined infected (black), confirmed infected (green dot dash) and recovers (small blue dot) when $a = 0.2425, \mu = 0.00000671, \varepsilon = 0.978, \sigma = 0.09, \beta = 0.01, \delta = 0.05$ and (a) $\eta = 0$, (b) $\eta = 0.05$.

Fig. (4.10) shows the effect of the contact tracing rate η on the behavior of all the individual numbers. Generally, all the individual numbers predicted a similar behavior to that shown in Fig. (4.8- 4.9), but only the quarantined infected number Q and the confirmed infected number C increase in a short period of time. As one increases the time, it gradually decays to its non-vanishing minimum value. However, as one increases η (see Fig. (4.10 b)), the upper bounds of the quarantined infected numbers Q and the confirmed infected number C are greater than that shown in Fig. (4.10 a). It is clear that, the recovered number R increases gradually on the expanses of the other decreased individual numbers.

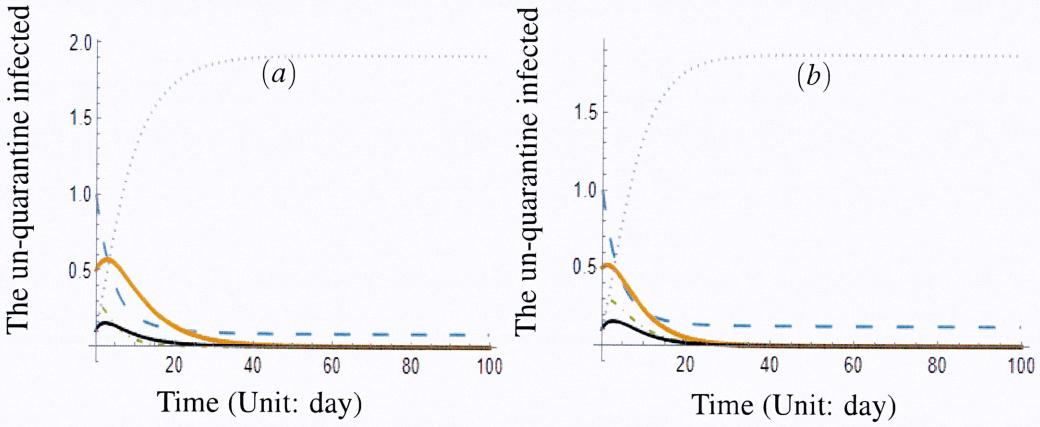


Figure 4.11: Same as Fig. (4.10), but the exit test rate $\beta = 0.3$.

In Fig. (4.11), the effect of large values of the exit test rate, β is discussed. This behavior indicates that, the recovered individual number R increases at the expense of decreasing all the remaining individual numbers. Fig. (4.11) shows, the upper bounds of the recovered number R that is displayed at $\beta = 0.3$ are larger than those displayed at $\beta = 0.01$ as represented in Fig. (4.10). In contrast, at large values of β, the susceptible individual number S behaves as a constant, while at small values, it vanishes faster. Fig. (4.8- 4.11) illustrate that, by controlling the random test parameter δ, the contact tracing rate η, and the exit test rate β, one can increase the recovery individual numbers and decrease the other individual numbers.

The possibility of increasing the recovered numbers R is discussed based on the $SUQCR$ model. Fig. (4.12 a) shows the effect of the medical history rate, λ on the behavior of the individual recovered numbers R. The behavior of R indicates that, as one increases the time, the recovered rate number increases gradually. Assuming $\lambda = 0$, namely, the patient does not have a medical history, the possibility of increasing the recovered numbers is greater than that at $\lambda > 0$, lies the patient has a medical history.

The initial immunity of the patient should be investigated in this context, since it may influence the possibility of increasing the number of the recovered patient. According to Fig. (4.12 b), the recovery rate increases as immunity increases. Nevertheless, the results indicate that, at small immunity values, such as $\tau = 0.2$, the recovered numbers R cannot be increased. Furthermore, the large value of immunity increases R slightly.

Fig. (4.12 c) shows the impact of η on the behavior of individual recovered numbers. Although R increases generally, the predicted numbers increase at small values of η are greater than that displayed at large

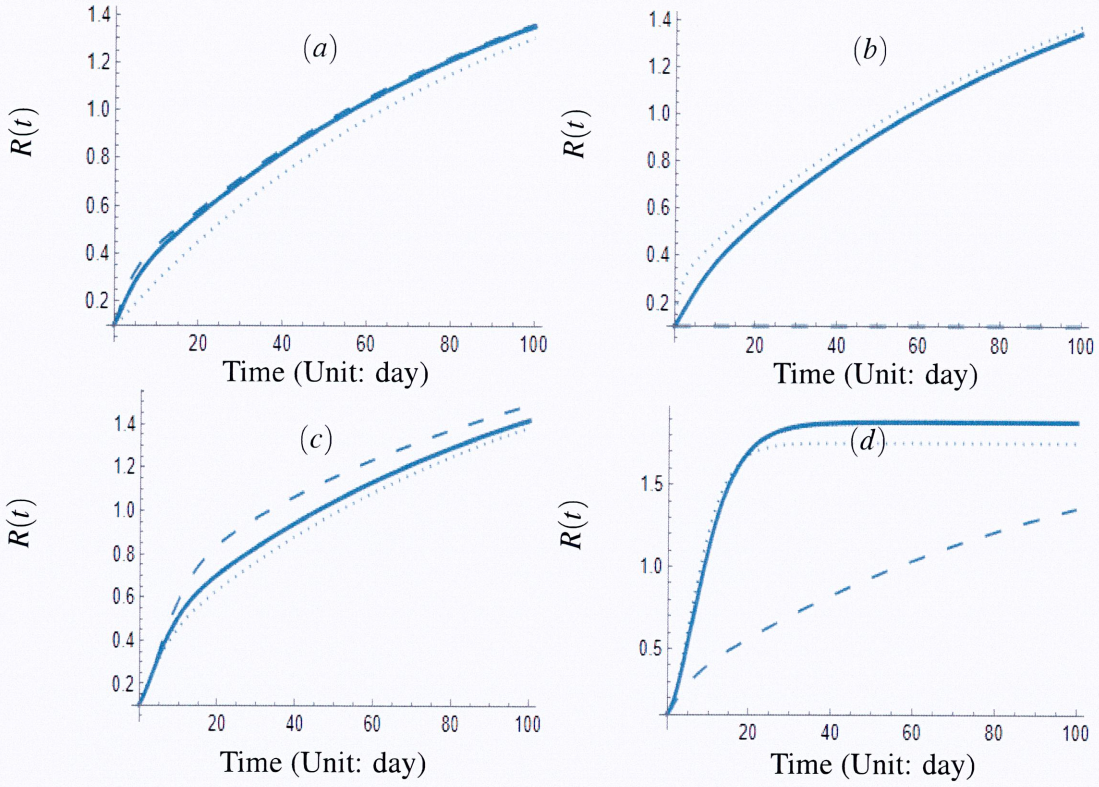


Figure 4.12: The recovered individuals when $a = 0.2425, \mu = 0.00000671, \sigma = 0.09$ a) shows the effect of medical history λ with, $\beta = 0.01$, $\tau = 0.5$, $\eta = 0.5$, where the dash, solid and dot curves are evaluated at $\lambda = 0, 0.2$ and 0.4, respectively, (b) shows the effect of the immunity τ with $\beta = 0.01$, $\eta = 0.5$, $\lambda = 0.2$, where the dash, solid and dot curves are evaluated at $\tau = 0.2, 0.4, 2$, respectively, (c) shows the effect of the contact tracing rate η with $\beta = 0.01, \lambda = 0.2$ and $\tau = 0.5$, where the dash, solid and dot curves are evaluated at $\eta = 0.1, 0.2$ and 0.3, respectively, (d) shows the effect of exit test rate β, with $\eta = 0.2, \tau = 0.5, \lambda = 0.2$, where the dash, solid and dot curves are evaluated at $\beta = 0.01, 0.2$ and 0.3, respectively.

values of η. This result is logic, since the greater the η, the greater the chances of discovering the infected people and consequently the infected sample increases.

A major procedure has taken by the Ministry of Health is the exit tests. Therefore, it is worthy to discuss the impact of β on the proposed model. Fig. (4.12 d) shows how β affects the behavior of recovered numbers R. As one increases β, the recovers R increases. Nevertheless, R

appears to increase suddenly at large values of β, while it increases gradually at small values. In addition, the constant values of β are displayed at large values of $R(t)$, where the large constant values are displayed at large values of β.

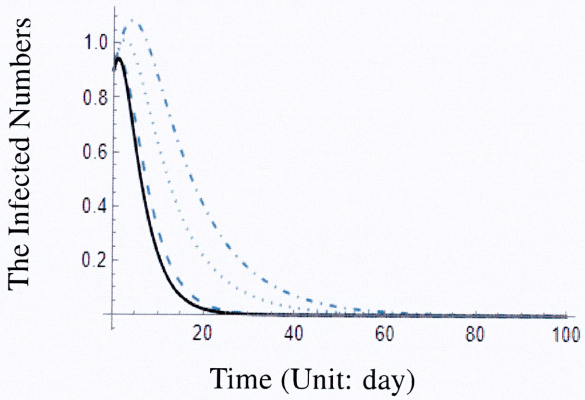


Figure 4.13: Infected individuals for different reproduction numbers R_0. The dot-dash, dot, dash and the solid curves are evaluated at $R_0 = 1.7, 1, 0.6, 0.3$, respectively.

In Fig. (4.13), the number of infected individuals in Bahrain is shown at different values of reproduction number, namely the behavior of the total sample of U, Q and C. Generally, the infected sample decreases gradually as time increases. However, by decreasing R_0, the infected sample decreases rapidly and vanishes completely within a short period of time. In contrast, when the reproduction number is large, the infected sample vanishes longer. Due to this, Bahrain's Ministry of Health has taken precautionary measures in accordance with the increasing or decreasing reproduction number.

4.3.1 Discussion

Based on Bahrain's daily data, the $SUQCR$ model is developed to analyse the dynamics of the COVID-19 outbreak, leading to the best preventive measures to limit its spread. $SUQCR$ model uses five variables and few parameters represented in Table (3.1). Mathematically, the model is rep-

resented by a set of differential equations dependent on some parameters, which play a crucial role in the epidemic dynamics. An equilibrium point for the mathematical model is determined based on its local stability and existence. Furthermore, the reproduction number R_0 was calculated by the next-generation approach. The $SUQCR$ model can be used to measure quarantine-related parameters and variables, conducting random and exit tests as well as contact tracing to detect confirmed cases.

It appears from the behavior of the individual numbers that, some parameters play a significant role in controlling the spread of COVID-19 disease in Bahrain. The results are based on COVID-19 data in Bahrain for the period from September 24 to December 9, 2020. This model has the greatest priority of reducing the un-quarantined infected individuals, reducing the reproduction number R_0, but increasing the number of recovery individuals. Among the most important parameters are the random test δ, the exit test β, the contact tracing rate η, and the immunity rate, which includes natural immunity and the vaccine immunity rate τ, and the medical history rate λ.

The Bahrain government played a vital role in tracing contacts and conducting random checks. A targeted and strict random test system was followed: daily random units visited seven to eight locations, including health centres and sports clubs. A total of 300-400 tests are conducted at each site, while 2000-2300 random checks are conducted on a daily basis (Ministry of Health, 2020c). The results show that, all the individual numbers decrease over time as the random test parameter increases. In spite of this, only quarantined infections increase over time when exit tests are set to a small value. Further, the number of infected individuals who were not quarantined decreased as the rate of random testing increased. These results are consistent with those obtained by Crook et

al.(2017) who found that contact tracing may reduce the number of cases in the event of an epidemic along with other control methods.

The immunity parameter may be used as a control parameter to increase the recovered numbers. This immunity includes the natural immunity of individuals or vaccine-mediated immunity. The history of the individual's disease also plays an important role in the healing process, so that the number of people recovering decreases when the individual has a history of the disease.

Reproduction number is an important tool for disease control, where disease spreads rapidly and infections increase with an increase in reproduction number. Therefore, reducing it is important. It is possible to achieve this goal by increasing the number of random tests, exit tests, and contact tracing. As it is determined from Table (4.5), the reproduction number of the $SUQCR$ model is 0.00007 when $a = 0.2425$, $\mu = 0.0000671$, $\sigma = 0.09$, $\delta = 0.09$, and $\eta = 0.09$, which means that, the disease will not spread rapidly and will disappear in Bahrain. When the number of contact tracing, random, and exit tests is reduced compared to its current level, the reproduction number will gradually increase. The estimated reproduction number in March 2020 was 3.85 (French Chamber of Commerce and Industry in Bahrain, 2020) and from September to November 2020, it was between 0.86 to 0.97 (OWID, 2020).

According to $SUQCR$ model, $R_0 < 1$, thus, Bahrain's government has effectively controlled the spread of COVID-19 by following precautionary measures, such as increasing random testing to detect un-quarantined infected individuals and increasing contact tracing and individual immunity. In order to prevent disease spread among people, Bahrain's government imposes a number of restrictions and preventive measures that must be followed and adhered to by its citizens.

4.4 The $SVEUQCHR$ Model for COVID-19 in Bahrain

The $SVEUQCHR$ model is summarized to $VEUQCHR$ model as presented in equation (3.4). Then the $VEUQCHR$ model is solved by using "NDSolve" code in the Wolfram Mathematica 12.1 with the initial conditions $V(0) = 0.2$, $E(0) = 1$, $U(0) = 0.3$, $Q(0) = 0.2$, $C(0) = 0.1$, $H(0) = 0.1$ and $R(0) = 0.1$ and the plot of the model is shown in Fig. (4.14). Also, a lot of parameters are used such as, Sinopharm vaccine efficacy of preventing infection was 73% (IHME, 2021). The daily recovery rate was 99.1% of total cases, the death cases rate was 0.5% of total cases and the hospitalization rate was 0.4% of total cases (CoronaTracker, 2021), the birth rate was 13.163 (Macrotrends, 2020). The estimated reproduction number in Bahrain was between 0.55-1.3 and it was found to be 0.99 (Epiforecasts, 2021). The duration of the latent period is $\frac{1}{\kappa}$, so, the period prior to the latent infection appearing ranged from 1 to 5 days, with 2 days being the average time frame (Liu et al., 2020), thus, $\kappa = 0.5$.

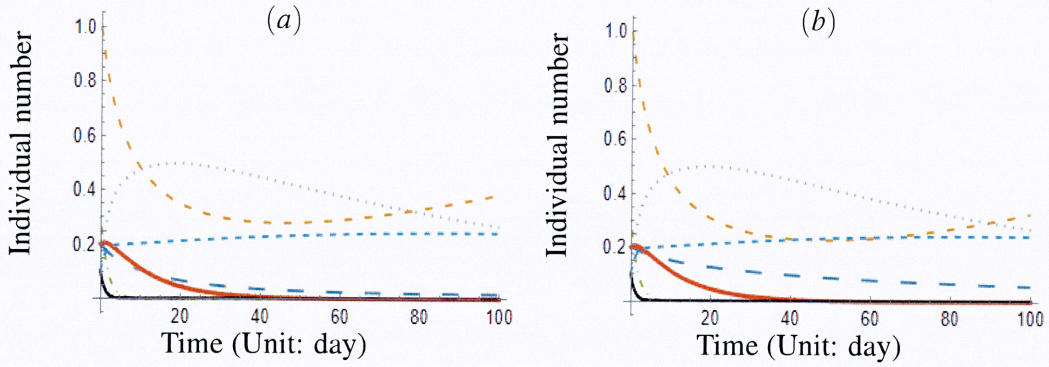


Figure 4.14: The normalized model of $SVEUQCHR$ model of COVID-19 in Bahrain with $a = 0.2425, N = 1701583$, $\mu = 0.005$, $\varepsilon = 0.978$, $n = 0.0135$, $\delta = 0.05$, $\sigma = 0.09$, $\beta = 0.09$, $\eta = 0.09$, $\omega = 0.004$, $\kappa = 0.5$, $\gamma = 0.05$, $\psi = 0.004$ and $\iota. = 0.5$ and different ξ a) $\xi = 0.73$ b) $\xi = 0.9$. The large blue dashed, orange dashed, green dot dashed, red, blue dot, black and blue dashed curves represent the vaccinated, exposed, un-quarantined infected, quarantined infected, confirmed infected, hospitalized and recovered, respectively.

Fig. (4.14) represents the normalized model of $SVEUQCHR$ model for COVID-19 in Bahrain with vaccination efficacy rates of 0.73 and 0.9. It shows that, the behavior of all individuals decreases continuously, except for recovering individuals, which increases over time. Even though the number of those exposed to infection decreases in a short time period, it increases when the vaccinated people number decreases. Moreover, the confirmed numbers increase in a short period of time and then begin to decrease as time goes on. Fig. (4.14 a) and Fig. (4.14 b) illustrate how vaccination effectiveness affects the behavior of vaccinated and exposed individuals, where vaccinated individuals number decreases and exposed number increases with decreasing vaccination effectiveness.

Fig. (4.15) shows the effect of vaccination and social distancing on confirmed cases in Bahrain. According to Fig. (4.15 a), increasing vaccination rates without social distancing reduces confirmed cases. However, according to Fig. (4.15 b), the absence of vaccination and increased social distancing rates have no apparent effect on reducing confirmed cases.

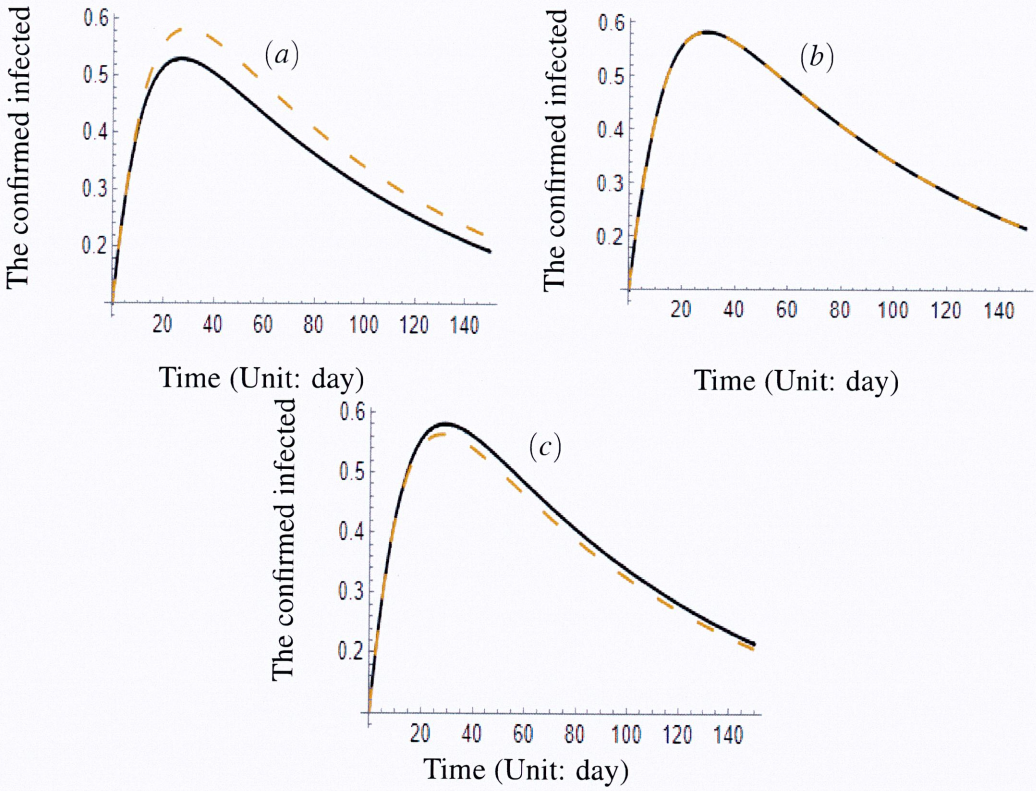


Figure 4.15: The confirmed infected individuals C of the $SVEUQCHR$ model of COVID-19 in Bahrain with $a = 0.2425, N = 1701583$, $\mu = 0.005$, $\varepsilon = 0.978$, $n = 0.0135$, $\delta = 0.05$, $\sigma = 0.09$, $\beta = 0.09$, $\eta = 0.09$, $\omega = 0.004$, $\kappa = 0.5$, $\gamma = 0.05$ and $\xi = 0.73$ and different ψ and ι. a) the solid curve represents C when $\psi = 0.1, \iota = 0$ and the dash curve represents C when $\psi = 0.0001, \iota = 0$. b) the solid curve represents C when $\psi = 0, \iota = 0.1$ and the dash curve represents C when $\psi = 0, \iota = 0.9$. c) the solid curve represents C when $\psi = 0.001, \iota = 0.1$ and the dash curve represents C when $\psi = 0.01, \iota = 0.9$.

Contrary to this, increasing both vaccination and social distancing rates will reduce confirmed cases.

4.4.1 Vaccination as an individual strategy

In this subsection, vaccination and social distancing are considered as the only personal protection against COVID-19. Therefore, first, it is assumed that vaccination is the only personal protection against COVID-19, and let $\iota = 0$ in the system (3.18) to get the following system (Al-

Sayegh, Iqbal, & Metwally, 2022).

$$\begin{aligned}
\frac{dv(t)}{dt} &= \psi(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t)) \\
&\quad -(1-\xi)av(t)e(t)-\mu v(t), \\
\frac{de(t)}{dt} &= a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t) \\
&\quad +(1-\xi)av(t)e(t)-\mu e(t)-\kappa av(t)e(t)u(t)-\kappa av(t)e(t)q(t), \\
\frac{du(t)}{dt} &= \kappa av(t)e(t)u(t)-(\mu+\gamma+\eta)u(t), \\
\frac{dq(t)}{dt} &= \kappa av(t)e(t)q(t)-(\mu+\beta)q(t)+\eta u(t), \\
\frac{dc(t)}{dt} &= \gamma u(t)+\beta q(t)-(\mu+\omega)c(t), \\
\frac{dh(t)}{dt} &= \omega c(t)-(\mu+\varepsilon)h(t), \\
\frac{dr(t)}{dt} &= \varepsilon h(t)-\mu r(t).
\end{aligned} \tag{4.10}$$

Let $\iota = 0$ in the Eq. (3.25), the vaccination control reproduction number will be,

$$\begin{aligned}
R_c(\psi) &= \frac{a}{\mu}+\frac{(\iota-\xi)a\psi}{\mu(\psi+\mu)}, \\
&= R_0(1-\frac{\xi\psi}{\psi+\mu}).
\end{aligned} \tag{4.11}$$

To find the threshold value of the vaccination denoted as ψ_{thr} one solves $R_c(\psi) = 1$ for ψ. Explicitly one can write ψ_{thr} as,

$$\psi_{thr} = \frac{\mu(R_0-1)}{R_0(1-\xi)-1}, \tag{4.12}$$

by assuming that $1 < R_0 < \frac{1}{1-\xi} = 3.7$ ($\xi = 0.73$). Thus, if $R_0 > \frac{1}{1-\xi}$, then the diseases cannot be controlled solely by vaccination. If the vaccination rate is lower than the vaccination threshold ($\psi < \psi_{thr}$), then the disease remains endemic. Otherwise, it is possible to eradicate the disease. To find the endemic point, let the L.H.S of the system (4.10) equal to zero and solve for v, e, u, q, c, h, r (Choi & Shim, 2020).

$$
\begin{aligned}
0 &= \psi(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t)) \\
&\quad -(1-\xi)av(t)e(t)-\mu v(t), \\
0 &= a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t) \\
&\quad +(1-\xi)av(t)e(t)-\mu e(t)-\kappa av(t)e(t)u(t)-\kappa av(t)e(t)q(t), \\
0 &= \kappa av(t)e(t)u(t)-(\mu+\gamma+\eta)u(t), \\
0 &= \kappa av(t)e(t)q(t)-(\mu+\beta)q(t)+\eta u(t), \\
0 &= \gamma u(t)+\beta q(t)-(\mu+\omega)c(t), \\
0 &= \omega c(t)-(\mu+\varepsilon)h(t), \\
0 &= \varepsilon h(t)-\mu r(t). \qquad (4.13)
\end{aligned}
$$

Thus, from system (4.13) the endemic point will be,

$$
P_{\psi}^{*}=(v_{\psi}^{*},e_{\psi}^{*},u_{\psi}^{*},q_{\psi}^{*},c_{\psi}^{*},h_{\psi}^{*},r_{\psi}^{*}), \qquad (4.14)
$$

where,

$$
\begin{aligned}
e_{\psi}^{*} &= \frac{\mu+\gamma+\eta}{\kappa a}, \quad u_{\psi}^{*}=\frac{(\mu+\omega)(\gamma+\eta-\beta)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c_{\psi}^{*}, \\
q_{\psi}^{*} &= \frac{-\eta(\mu+\omega)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c_{\psi}^{*}, \quad h_{\psi}^{*}=\frac{\omega}{\mu+\varepsilon}c_{\psi}^{*}, \quad r_{\psi}^{*}=\frac{\varepsilon\omega}{\mu(\mu+\varepsilon)}c_{\psi}^{*}, \\
v_{\psi}^{*} &= \frac{(R_0-1)\kappa a-(\mu+\gamma+\eta)}{\xi R_0\kappa a} \\
&\quad -\frac{(\kappa+1)((\mu+\omega)(\gamma-\beta-\gamma(\gamma+\eta-\beta)-\eta\beta))}{\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)}c_{\psi}^{*}, \\
c_{\psi}^{*} &= \frac{Y}{Z},
\end{aligned}
$$

and,

$$
\begin{aligned}
Y = {} & (\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta))(\xi R_0\kappa\psi \\
& (\kappa a-\mu+\gamma+\eta)-((R_0-1)\kappa a-(\mu+\gamma+\eta))(\kappa\psi+(1-\xi) \\
& (\mu+\gamma+\eta)+\mu\kappa)), \\
Z = {} & \xi R_0\kappa^2 a(\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)((\kappa+1)((\mu+\omega)(\gamma+\eta-\beta) \\
& -\eta(\mu+\omega)-(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))(\kappa\psi+(1-\xi) \\
& (\mu+\gamma+\eta)+\mu\kappa))+\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)(\psi((\mu+\omega) \\
& (\gamma+\eta-\beta)-\eta(\mu+\omega)+(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))).
\end{aligned}
$$

The changing rate of the confirmed infected endpoint c_ψ^* with respect to the vaccination rate is written as,

$$
\frac{dc_\psi^*}{d\psi} = \frac{k_1}{k_2} - \frac{L_1}{L_2}, \tag{4.15}
$$

where,

$$
\begin{aligned}
K_1 = {} & \mu(\gamma(\gamma+\eta-\beta)-\beta\eta)(\mu\gamma(\gamma+\eta-\beta)-\beta\eta)(\kappa R_0\xi(\gamma-\mu \\
& +a\kappa+\eta)-\kappa(-\gamma+a\kappa(R_0-1)-\mu-\eta)),
\end{aligned}
$$

$$
\begin{aligned}
K_2 = {} & a\kappa R_0((\kappa+1)\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)((-\omega-\mu)(\gamma(\gamma+\eta \\
& -\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))(\kappa\psi+(1-\xi) \\
& (\gamma+\mu+\eta)+\kappa\mu)+\xi\kappa(\mu\gamma(\gamma+\eta-\beta)-\beta\eta)((\omega+\mu) \\
& (\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))\psi),
\end{aligned}
$$

$$
\begin{aligned}
L_1 = ((&\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)(\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)(\kappa\xi\mu \\
&(\gamma(\gamma+\eta-\beta)-\beta\eta)\omega+\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu) \\
&(\gamma+\eta-\beta)-\eta(\omega+\mu))+\kappa(\kappa+1)\mu(\gamma(\gamma+\eta-\beta)-\beta\eta) \\
&((-\omega-\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta) \\
&-\eta(\omega+\mu)))(\kappa R_0\xi(\gamma-\mu+a\kappa+\eta)\psi-(-\gamma+a\kappa(R_0-1) \\
&-\mu-\eta)(\kappa\psi+(1-\xi)(\gamma+\mu+\eta)+\kappa\mu),
\end{aligned}
$$

$$
\begin{aligned}
L_2 = a\kappa R_0((&\kappa+1)\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)((-\omega-\mu)(\gamma(\gamma+\eta-\beta) \\
&-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))(\kappa\psi+(1-\xi) \\
&(\gamma+\mu+\eta)+\kappa\mu)+\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\beta\eta)((\omega+\mu) \\
&(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta) \\
&-\eta(\omega+\mu))\psi)^2, \qquad (4.16)
\end{aligned}
$$

and by using (4.16), one can show that $\frac{dc^*_\psi}{d\psi} < 0$ if $\psi < \frac{\eta-\beta+\gamma}{\eta}$.

4.4.1.1 Applying game theory

Here, the vaccination strategy, (VS), is discussed by using game theory. This can be achieved by considering two groups: let V be the first group which represents vaccine recipients and the $V^{'}$ be the second group which represents vaccine refusals. Let π_V be the probability of infection between vaccinated individuals and $\pi_{V'}$ be the probability of infection between un-vaccinated individuals. π_V and $\pi_{V'}$ can be calculated as the number of susceptible individuals becoming infected over the number of individuals dying through any unit of time (Bauch & Earn, 2004). Thus

the probabilities π_V and $\pi_{V'}$ are given by,

$$\pi_V = \frac{(1-\xi)a(\gamma u_\psi^* + \beta q_\psi^*)}{(1-\xi)a(\gamma u_\psi^* + \beta q_\psi^*) + \mu},$$

$$\pi_{V'} = \frac{a(\gamma u_\psi^* + \beta q_\psi^*)}{a(\gamma u_\psi^* + \beta q_\psi^*) + \mu}. \tag{4.17}$$

Therefore, using the endemic point $P_\psi^* = (v_\psi^*, e_\psi^*, u_\psi^*, q_\psi^*, c_\psi^*, h_\psi^*, r_\psi^*)$ calculated in (4.14), it is found that the probabilities π_V and $\pi_{V'}$ may be written as,

$$\pi_V = \frac{\pi_{V1}}{\pi_{V2}} \quad \text{and} \quad \pi_{V'} = \frac{\pi_{V3}}{\pi_{V4}}, \tag{4.18}$$

where,

$$\begin{aligned}
\pi_{V1} &= (1-\xi)ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta)),\\
\pi_{V2} &= (1-\xi)ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))\\
&\quad +\mu(\gamma(\gamma+\eta-\beta)-\eta\beta),\\
\pi_{V3} &= ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta)),\\
\pi_{V4} &= ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))\\
&\quad +\mu(\gamma(\gamma+\eta-\beta)-\eta\beta).
\end{aligned} \tag{4.19}$$

Let $\pi_{VV'}$ be the increased risk of infection for un-vaccinated individuals compared to vaccinated individuals

$$\pi_{VV'}(c_\psi^*) = \frac{\pi_1}{\pi_2}, \tag{4.20}$$

where,

$$\pi_1 = -a\xi\mu c^*_\psi(\eta+\gamma)(\mu+\omega)(\beta-\gamma)(\gamma^2+\eta\gamma-\beta\gamma-a\eta\beta),$$
$$\pi_2 = (ac^*_\psi((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)$$
$$-\eta\beta))(((1-\xi)ac^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))$$
$$+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)). \tag{4.21}$$

By using the general game-theoretical framework (Bauch & Earn, 2004), the expected payoff related to each strategy can be defined as:

$$E_V = -C^\psi_V - \pi_V C^\psi_i, \quad E_{V'} = -\pi_{V'} C^\psi_i, \tag{4.22}$$

where C^ψ_V and C^ψ_i represent the cost of vaccination and the infection, respectively. Dividing by C^ψ_i, E_V and $E_{V'}$ will be

$$E_V = -C^\psi - \pi_V, \quad E_{V'} = -\pi_{V'}, \tag{4.23}$$

where $C^\psi = \frac{C^\psi_V}{C^\psi_i}$ is the relative vaccination cost of infection.

Let ψ_{pop} be the population vaccination rate. To analyse the predictable payoffs for individuals, if $\psi_{pop} > \psi_{thr}$ then $R_c(\psi) < 1$ and the population will reach a disease-free equilibrium. This means that the probability of infection between vaccinated and un-vaccinated individuals are null ($\pi_{V'} = \pi_V = 0$) and thus, there is no benefit for individuals to take the vaccine. If $\psi_{pop} < \psi_{thr}$ then $R_c(\psi) > 1$ and the epidemic will reach endemic equilibrium P^*_ψ. The expected payoff of changing the strategies from un-vaccinated to vaccinated is defined as,

$$\Delta E = E_V - E_{V'}, \tag{4.24}$$
$$= -C^\psi + \pi_{VV'}(c^*_\psi).$$

Individuals now will select the vaccination strategy (VS) if $\Delta E > 0$, which means that, $\pi_{VV'}(c^*_\psi) > C^\psi$. They will choose the non-vaccination

strategy when $\Delta E < 0$, namely $\pi_{VV'}(c^*_\psi) < C^\psi$. The Nash equilibrium for vaccination ψ_{Nash} is defined when the risk of infection and the cost of infection are equal ($\Delta E = 0$), i.e. when $\pi_{VV'}(c^*_\psi) = C^\psi$. By substituting the endemic level c^*_ψ in to equation $\pi_{VV'}(c^*_\psi) = C^\psi$, the Nash equilibrium equation $J(\psi)$ is obtained given that, $J(\psi) = 0$ (Choi & Shim, 2020) where,

$$\begin{aligned} J(\psi) = C^\psi((a((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta) \\ -\beta\eta))((1-\xi)((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma \\ +\eta-\beta)-\beta\eta))(\kappa\xi\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)\mu(\gamma(\gamma+\eta \\ -\beta)-\eta\beta))(\xi R_0\kappa\psi(\kappa a-\mu+\gamma+\eta)-((R_0-1)\kappa a \\ -(\mu+\gamma+\eta))(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)))-(\xi R_0 \\ a\kappa^2(-\mu\xi(\eta+\gamma\gamma)(\mu+\omega)(\beta-\gamma)(\gamma^2+\eta\gamma-\beta\gamma-a\eta\beta)) \\ (\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)((\kappa+1)(\mu+\omega)(\gamma+\eta-\beta)-\eta \\ (\mu+\omega)-(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\beta\eta))(\kappa\psi+(1-\xi) \\ (\mu+\gamma+\eta)+\mu\kappa))+\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)((\psi((\mu \\ +\omega)\gamma(\gamma+\eta-\beta)-\eta(\mu+\omega)+(\mu+\omega)(\gamma(\gamma+\eta-\beta) \\ -\eta\beta))))) \\ = 0. \end{aligned} \tag{4.25}$$

To analyse the existence of the vaccination Nash equilibrium, the roots of Eq. (4.25) are defined as ψ_{root}. By using Mathematica Wolfram 12.1, it is found that there is one root of $J(\psi) = 0$, which is $\psi_{root} \to 0$.

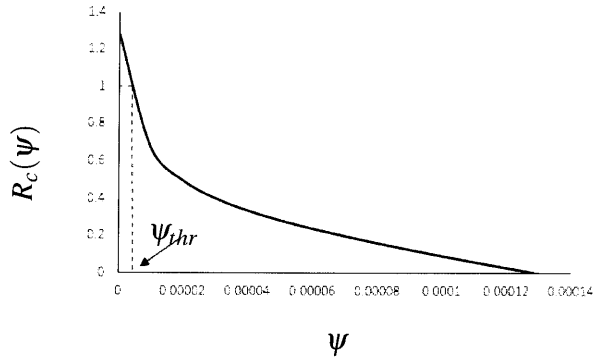


Figure 4.16: Vaccination control reproduction number $R_c(\psi)$ as a function of vaccination rate ψ.

Fig. (4.16) illustrates the behaviour of the vaccination control reproduction number when applying only the vaccination strategy. As soon as the vaccination process is initiated, $R(\psi,t)$ drops suddenly. Nevertheless, the vaccination rate increases linearly. In addition, at higher vaccination levels, the reproduction number disappears completely.

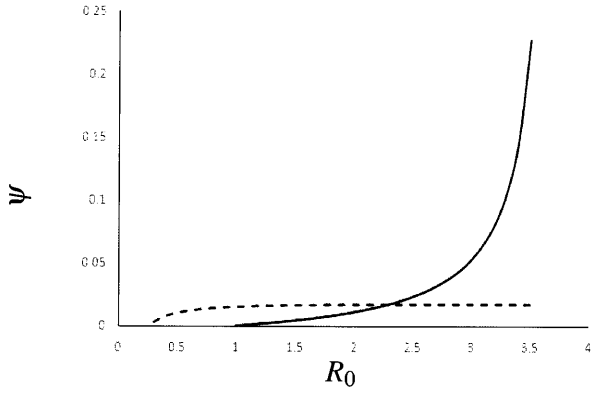


Figure 4.17: The vaccination rate ψ, where the solid and dash curves are $\psi_{thr}(R_0)$ and $\psi_{Nash}(R_0)$, respectively.

Fig. (4.17) represents the equilibrium vaccination ψ_{thr} in comparison with predictions based on game theory ψ_{Nash}. At a small reproduction number, ψ_{Nash} increases slightly and gradually. However, one gets a linear response as $R(\psi,t)$ increases. Conversely, ψ_{thr} is shown at large

$R(\psi,t)$ and increases gradually at $R(\psi,t) < 3$, while it increases suddenly as $R(\psi,t)$ increase. From these behaviors, one can conclude that, $\psi_{thr} < \psi_{Nash}$ for all $0.2 < R_0 < 2.3$ and $\psi_{thr} > \psi_{Nash}$ for all $R_0 > 2.3$. If the basic reproduction number is less than 2.3, Bahrain can control the epidemic by vaccination only with a vaccination efficacy rate of not more than 0.565. Otherwise, vaccination efficacy needs to be improved.

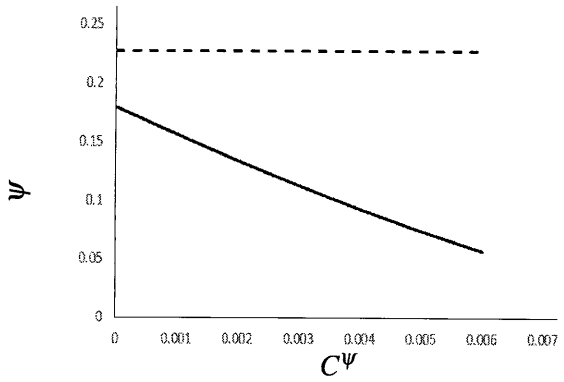


Figure 4.18: Vaccination rate ψ as a function of the relative cost of infection C^ψ, where the solid and dash lines represent ψ_{pop} and ψ_{thr}, respectively.

Fig. (4.18) shows as the relative vaccination cost of infection increases, the population vaccination rate decreases.

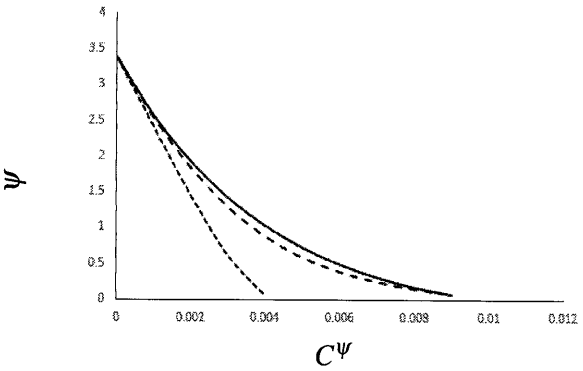


Figure 4.19: Vaccination rate ψ as a function of the relative vaccination cost of infection C^ψ. The solid, dash and dot curves are evaluated at $R_0 = 2.9, 1.3$, and 0.5, respectively.

Fig. (4.19) shows the best population vaccination rate as a function of the relative vaccination cost of infection with different reproduction numbers R_0 is $0.9, 1.3$ and 2.9. Generally, vaccination rates decrease as the relative vaccination cost of infection increases. Nevertheless, the decreasing rate is dependent on the initial of the reproduction number, where it decreases gradually at large values of R_0, and consequently disappears at large values of the relative vaccination cost.

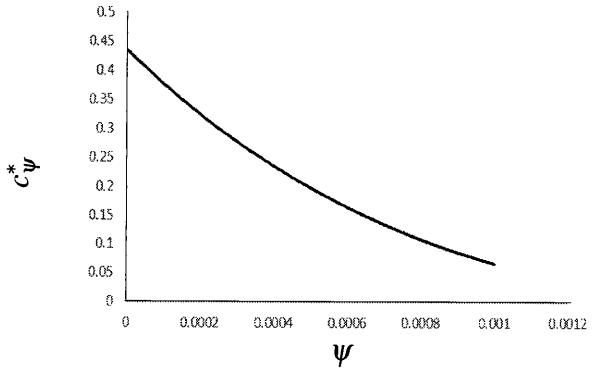


Figure 4.20: Endemic level c^*_ψ as a function of vaccination rate ψ.

Fig. (4.20) shows a decrease in the endemic level c^*_ψ with the vaccination rate ψ as $\frac{dc^*_\psi}{d\psi} < 0$. Thus, the endemic level c^*_ψ attends its maximum when $\psi = 0$, and $c^*_\psi = \frac{A_1}{A_2}$ where,

$$\begin{aligned} A_1 &= -((R_0-1)\kappa a-(\mu+\gamma+\eta))((1-\xi)(\mu+\gamma+\eta)+\mu\kappa)) \\ &\quad (\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)), \\ A_2 &= \xi R_0\kappa^2 a(\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)((\kappa+1)((\mu+\omega)(\gamma+\eta \\ &\quad -\beta)-\eta(\mu+\omega)-(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))((1-\xi) \\ &\quad (\mu+\gamma+\eta)+\mu\kappa)). \end{aligned} \tag{4.26}$$

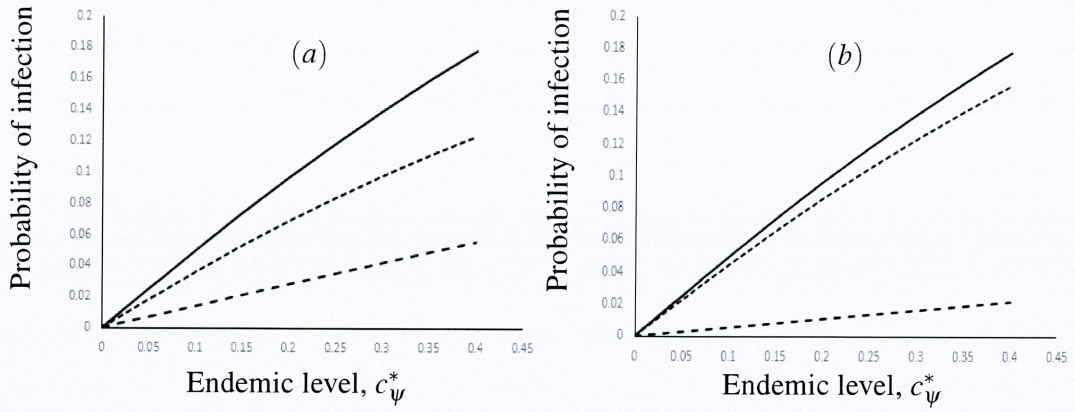


Figure 4.21: The probability of infection in individuals who accept or reject vaccination. The solid, dot and dash curves represent $\pi_{V'}$, $\pi_{VV'}$ and π_V, respectively, with vaccination efficacy a) $\xi = 0.73$ b) $\xi = 0.9$.

As shown in Fig. (4.21), the efficacy ξ significantly impacts the infection probability, $\pi_V(c^*_\psi)$ between individuals accepting vaccinations. As the endemic level c^*_ψ increases linearly, the probability of infection increases. In contrast, $\pi_V(c^*_\psi)$ decreases as one increase ξ, depending on the degree of vaccination efficacy. The results are confirmed in Fig. (4.21), which investigates how the risk of infection varies as a function of the endemic point c^*_ψ at several values of ξ. Behaviorally, the infection risk of vaccinated individuals decreases as ξ increases.

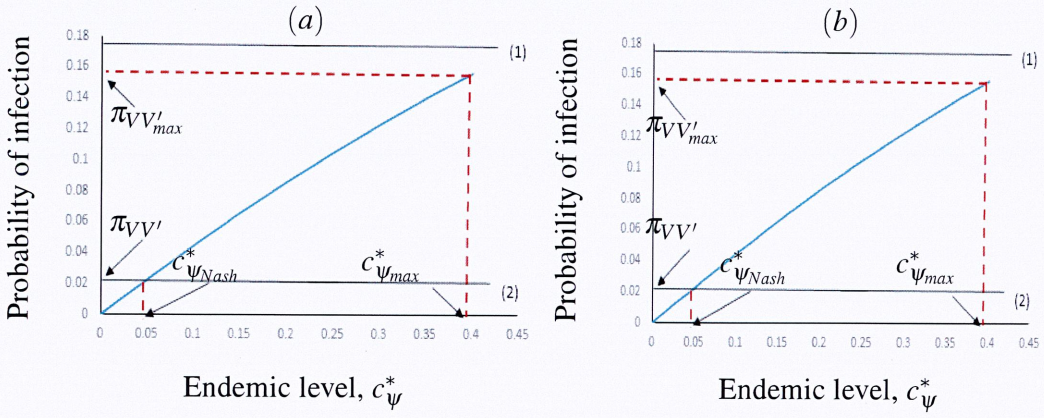


Figure 4.22: The probability of the increased risk of infection for vaccinated/ unvaccinated individuals with vaccination efficacy a) $\xi = 0.73$ b) $\xi = 0.9$. Line(1), when $C^\psi > \pi_{VV'}$ there is no Nash vaccination strategy and in line(2) when $C^\psi = \pi_{VV'}$, then there is a unique Nash vaccination strategy.

Based on various values of vaccination efficacy, ΔE is represented in Fig. (4.22). Two cases are considered, either $C^{\psi} > \pi_{VV'}$ or $C^{\psi} = \pi_{VV'}$. In the first case (line (1)), the expected payoff is $\Delta E < 0$, meaning fewer individuals accept vaccination than reject it, and there is, therefore, no Nash vaccination strategy. In the second case (line (2)), a Nash vaccination strategy is displayed, where $\Delta E = 0$, namely, the number of accepted and rejected vaccine recipients are equal. As vaccination increases, the upper bounds of $\pi_{VV'}$ increase, i.e. the probability of infection increases. Moreover, $\pi_{VV'}(c^*_{\psi})$ increases when $0 < c^*_{\psi} < c^*_{\psi_{\max}}$ and reaches its maximum when $c^*_{\psi} = c^*_{\psi_{\max}}$. From Eq. (4.25), one can write vaccination ψ as a function of the relative cost C^{ψ} as,

$$\psi = \frac{FE - AG}{a\kappa\mu - B + FD}, \tag{4.27}$$

where,

$$\begin{aligned}
A &= aR_0(\beta-\gamma)(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))\kappa\xi(\mu+\omega)^2(-\gamma-\eta)\\
&\quad(-\gamma^2+\beta(-1+\gamma+\eta-\kappa)+\eta\kappa+\gamma(1-\eta+\kappa)),\\
B &= aR_0(\beta-\gamma)(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))\kappa\xi(\mu+\omega)^2\xi(1+\gamma+\eta)\\
&\quad\kappa(\gamma(\gamma+\eta)\mu-\beta(\eta+\gamma\mu)),\\
F &= -C^{\psi}(\gamma(\gamma+\eta)\mu-\beta(\eta+\gamma\mu))(-(\beta-\gamma)(\gamma+\eta)\mu+a(\gamma(\gamma\\
&\quad+\eta)-\beta(\gamma+a\eta))(\mu+\omega))(-(\beta-\gamma)(\gamma+\eta)\mu\\
&\quad+(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))(1-\xi)(\mu+\omega)),\\
D &= (-(-\gamma-\eta+a(-1+R_0)\kappa-\mu)\kappa+R_0\kappa\xi[\gamma+\eta+a\kappa-\mu]),\\
G &= (\kappa\mu^2-\mu(\gamma+\eta+\mu)(-1+\xi)),\\
E &= -(-\gamma-\eta+a(-1+R_0)\kappa-\mu)(\kappa\mu-(\gamma+\eta+\mu)(-1+\xi)).
\end{aligned}$$

4.4.2 Social distancing as an individual strategy

A study of social distancing is presented in this subsection as the only method of preventing infectious diseases. The most effective means of reducing the spread of the epidemic without vaccination is to apply the social distancing. It is imperative to note that, social distance does not eliminate the epidemic, but plays an important role in flattening it (Bauch & Earn, 2004). To analyse the best social distancing at the individual level, it is assumed that, there is no vaccination i.e ($\psi = 0$), and consequently the system (3.18) reduces to be,

$$\begin{aligned}
\frac{dv(t)}{dt} &= \psi(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t)) \\
&\quad -(1-\xi)av(t)e(t)-v(t), \\
\frac{de(t)}{dt} &= a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t) \\
&\quad +(1-\xi)av(t)e(t)-\mu e(t)-\kappa av(t)e(t)u(t)-\kappa av(t)e(t)q(t), \\
\frac{du(t)}{dt} &= \kappa av(t)e(t)u(t)-(\mu+\gamma+\eta)u(t), \\
\frac{dq(t)}{dt} &= \kappa av(t)e(t)q(t)-(\mu+\beta)q(t)+\eta u(t), \\
\frac{dc(t)}{dt} &= \gamma u(t)+\beta q(t)-(\mu+\omega)c(t), \\
\frac{dh(t)}{dt} &= \omega c(t)-(\mu+\varepsilon)h(t), \\
\frac{dr(t)}{dt} &= \varepsilon h(t)-\mu r(t). \qquad (4.28)
\end{aligned}$$

In this case the social control reproduction number will be,

$$R_c(\iota) = (1-\iota)R_0.$$

To find the threshold value of the social distance ι_{thr}, one solves $R_c(\iota) = 1$ for ι. In an explicit form,

$$\iota_{thr} = 1 - \frac{1}{R_0}.$$

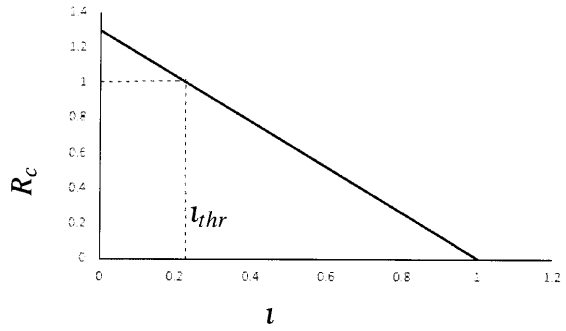


Figure 4.23: The level of social distancing required to achieve herd immunity ι_{thr}.

Fig. (4.23) displays the control reproduction number $R_c(\iota)$ when the only method of personal protection is social distancing and the social distancing threshold needed to achieved herd immunity ι_{thr}. It could be seen that if the $\iota < \iota_{thr}$ then $R_c(\iota) > 1$ and the disease continues to spread. Fig. (4.23) represents that the social control reproduction number decreases as the social rate increases. The endemic point of system (4.28) can be written as,

$$P_\iota^* = (v_\iota^*, e_\iota^*, u_\iota^*, q_\iota^*, c_\iota^*, h_\iota^*, r_\iota^*), \tag{4.29}$$

where,

$$\begin{aligned}
e_\iota^* &= \frac{\mu+\gamma+\eta}{\kappa a}, \qquad u_\iota^* = \frac{(\mu+\omega)(-\kappa a e_\iota^* + \mu - \beta)}{\gamma(-\kappa a e_\iota^* + \mu + 2\beta)} c_\iota^*, \\
q_\iota^* &= \frac{\eta(\mu+\omega)}{\gamma(-\kappa a e_\iota^* + \mu - \beta)} c_\iota^*, \qquad h_\iota^* = \frac{\omega}{\mu+\varepsilon} c_\iota^*, \\
r_\iota^* &= \frac{\varepsilon\omega}{\mu(\mu+\varepsilon)} c_\iota^*, \qquad v_\iota^* = \frac{((1-\iota)(1-\iota)e_\iota^* - \frac{\mu}{a})}{(\xi-1)} - \frac{V_1}{V_2} c_\iota^*, \\
c_\iota^* &= \frac{Y_1}{Z_1}.
\end{aligned}$$

with,

$$
\begin{aligned}
V_1 &= (1-\iota)(\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega) \\
&\quad \gamma(-\kappa a e_\iota^* + \mu + 2\beta)) + (\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) \\
&\quad + \eta(\mu+\omega)), \\
V_2 &= (\xi - 1)\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta), \\
Y_1 &= ((1-\iota)(1-\iota)e_\iota^* - (1-\iota)(1-e_\iota^*))\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta), \\
Z_1 &= (1-\iota)(\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega) \\
&\quad \gamma(-\kappa a e_\iota^* + \mu + 2\beta)) + (\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) \\
&\quad + \eta(\mu+\omega)) + (((1-\iota)(\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) \\
&\quad + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega))(\kappa + (1-\iota))).
\end{aligned}
$$

4.4.2.1 Applying game theory

Let SD denotes the individuals who choose to use the social distance method and $SD^{'}$ denote individuals who refuse to use it. Let π_{SD} be the probability of infection between individuals who apply social distancing and $\pi_{SD'}$ be the probability of infection between individuals who do not apply social distancing. π_{SD} and $\pi_{SD'}$ can be calculated as the amount of susceptible individuals becoming infected over the number of individuals dying through any unit of time (Bauch & Earn, 2004). Thus the probabilities π_{SD} and $\pi_{SD'}$ are given by

$$
\pi_{SD} = \frac{(1-\iota)a(\gamma u_\iota^* + \beta q_\iota^*)}{(1-\iota)a(\gamma u_\iota^* + \beta q_\iota^*) + \mu},
$$

$$
\pi_{SD'} = \frac{a(\gamma u_\iota^* + \beta q_\iota^*)}{a(\gamma u_\iota^* + \beta q_\iota^*) + \mu}. \tag{4.30}
$$

By using Eq (4.29), π_{SD} and $\pi_{SD'}$, take the following forms:

$$
\pi_{SD} = \frac{\pi_{SD1}}{\pi_{SD2}}, \quad \text{and} \quad \pi_{SD'} = \frac{\pi_{SD3}}{\pi_{SD4}}, \tag{4.31}
$$

where,

$$\begin{aligned}
\pi_{SD1} &= (1-\iota)ac_\iota^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)+\beta\eta(\mu+\omega)),\\
\pi_{SD2} &= (1-\iota)ac_\iota^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)+\beta\eta(\mu+\omega)),\\
&\quad +\mu\gamma(-\kappa ae_\iota^*+\mu+2\beta),\\
\pi_{SD3} &= ac_\iota^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)+\beta\eta(\mu+\omega)),\\
\pi_{SD4} &= ac_\iota^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)+\beta\eta(\mu+\omega))\\
&\quad +\mu\gamma(-\kappa ae_\iota^*+\mu+2\beta).
\end{aligned} \tag{4.32}$$

Let $\pi_{SDSD'}$ represents the individual's increased risk of infection

$$\pi_{SDSD'}(c_\iota^*) = \frac{\pi_1}{\pi_2}, \tag{4.33}$$

where,

$$\begin{aligned}
\pi_1 &= \iota(ac_\iota^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)+\beta\eta(\mu+\omega)))(\mu\gamma(-\kappa ae_\iota^*\\
&\quad +\mu+2\beta)),\\
\pi_2 &= (ac_\iota^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)+\beta\eta(\mu+\omega)))+\mu\gamma(-\kappa ae_\iota^*\\
&\quad +\mu+2\beta))(a(1-\iota)c^*(\gamma(\mu+\omega)(-\kappa ae_\iota^*+\mu+\beta)\\
&\quad +\beta\eta(\mu+\omega))+\mu\gamma(-\kappa ae_\iota^*+\mu+2\beta)).
\end{aligned} \tag{4.34}$$

By using the general framework of game theory (Bauch & Earn, 2004), the payoff associated with each strategy can be expressed as follows:

$$\Delta E_{SD} = -C_{SD}^\iota - \pi_{SD}C_i^\iota, \quad \text{and} \quad \Delta E_{SD'} = -\pi_{SD'}C_i^\iota, \tag{4.35}$$

where C_{SD}^ι denote the cost of applying social distancing and C_i^ι denotes the cost of infection. Dividing by C_i^ι one gets

$$\Delta E_{SD} = -C^\iota - \pi_{SD}, \quad \text{and} \quad \Delta E_{SD'} = -\pi_{SD'}, \tag{4.36}$$

where $C^{\iota} = \frac{C^{\iota}_{SD}}{C^{\iota}_{i}}$ represents the relative cost of using social distancing. Assume that ι_{pop} is the population's distancing rate. To analyse the expected payoffs for individuals, if $\iota_{pop} > \iota_{thr}$ then $R_c(\iota) < 1$ and the population reaches a disease-free equilibrium, which indicates that the individuals who used the social distancing have no chance of contracting a disease ($\pi_{SD'} = \pi_{SD} = 0$), and there is no benefit for individuals to use social distancing. If $\iota_{pop} < \iota_{thr}$ then $R_c(\iota) > 1$ and the pandemic reach endemic equilibrium P^*_{ι}. Changing from not using social distancing to using social distancing will have the following payoff:

$$\Delta E = -C^{\iota} + \pi_{SDSD'}(c^*_{\iota}). \tag{4.37}$$

Social distancing will now be chosen by individuals if $\Delta E > 0$, which means that $\pi_{SDSD'}(c^*_{\iota}) > C^{\iota}$. They will choose the non-social distancing strategy when $\Delta E < 0$, namely that $\pi_{SDSD'}(c^*_{\iota}) < C^{\iota}$. The Nash equilibrium for social distancing ι_{Nash} is defined when the risk of infection and the cost of infection are equal ($\Delta E = 0$), i.e. when $\pi_{SDSD'}(c^*_{\iota}) = C^{\iota}$. The Nash equilibrium must satisfy,

$$J(\iota) = C^{\iota}\{[A_{\iota} + B_{\iota}][(1-\iota)A_{\iota} + B_{\iota}]\} - \iota A_{\iota} B_{\iota}, \tag{4.38}$$

where $A_{\iota} = \frac{A_{\iota 1}}{A_{\iota 2}}$, and

$$\begin{aligned}
A_{\iota 1} &= ((1-\iota)(1-\iota)e^*_{\iota} - (1-\iota)(1-e^*_{\iota}))\mu\gamma(-\kappa a e^*_{\iota} + \mu + 2\beta) \\
&\quad a((\beta - a e^*_{\iota}\kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega)), \\
A_{\iota 2} &= (1-\iota)(\mu(\mu+\omega)(-\kappa a e^*_{\iota} + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega) \\
&\quad \gamma(-\kappa a e^*_{\iota} + \mu + 2\beta)) + (\mu\kappa(\mu+\omega)(-\kappa a e^*_{\iota} + \mu + \beta) \\
&\quad + \eta(\mu+\omega)) + (((1-\iota)(\mu+\omega)\gamma(-\kappa a e^*_{\iota} + \mu + 2\beta)) \\
&\quad + (\mu+\omega)\gamma(-\kappa a e^*_{\iota} + \mu + \beta) + \eta(\mu+\omega))(\kappa + (1-\iota))), \\
B_{\iota} &= \mu\gamma(-\kappa a e^*_{\iota} + \mu + 2\beta).
\end{aligned} \tag{4.39}$$

by using Wolfram Mathematica 12.1, the root of $J(\iota) = 0$ is $\iota \to 0$.

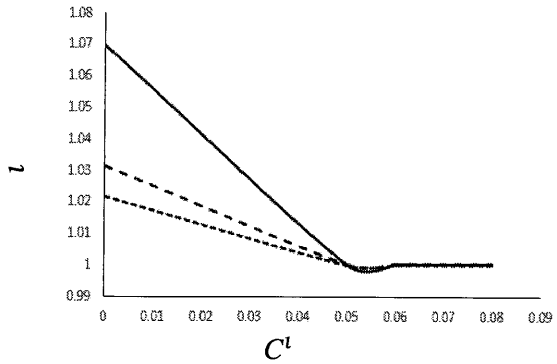


Figure 4.24: The population social distance rate ι as a function of the relative social cost of infection C^ι with different reproduction numbers. The solid, dash and dot curves are evaluated at $R_0 = 2.9, 1.3, 0.9$, respectively.

Fig. (4.24) shows how the social distance rate changes with the relative social cost of infection, when different reproduction numbers are taken into account. Generally, this rate decreases linearly by increasing the relative infection costs. The upper bounds of the social distance rate increase as R_0 increases. In contrast, the equilibrium behavior of the distance rate can be observed at high levels of the relative cost distance.

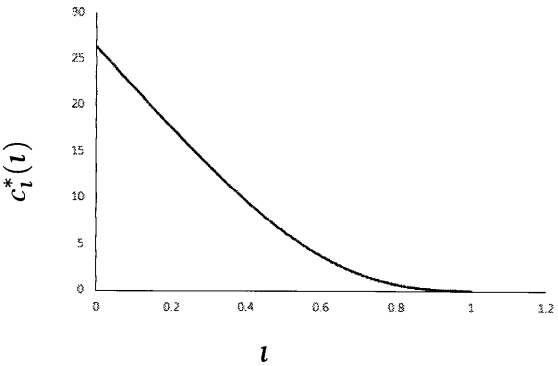


Figure 4.25: Endemic level behavior $c_\iota^*(\iota)$

Fig. (4.25), illustrates the effect of the social distance ι on the endemic level, which decreases with increasing social distance ι. Furthermore, $\iota = 0$ is predicted to be the maximum value of the endemic level. The endemic level c_ι^* decreases with the social distancing ι, where $\frac{dc_\iota^*}{d\iota} < 0$. Thus, the endemic level c_ι^* attends its maximum at $\iota = 0$, where,

$$c^*_{\iota\,\max} = \frac{Y_{1\,\max}}{Z_{1\,\max}}, \tag{4.40}$$

with,

$$\begin{aligned}
Y_{1\,\max} &= (e_\iota^* - (1-e_\iota^*))\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta), \\
Z_{1\,\max} &= (\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega) \\
&\quad \gamma(-\kappa a e_\iota^* + \mu + 2\beta)) + (\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) \\
&\quad + \eta(\mu+\omega)) + (((\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) \\
&\quad + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega))(\kappa+1)).
\end{aligned}$$

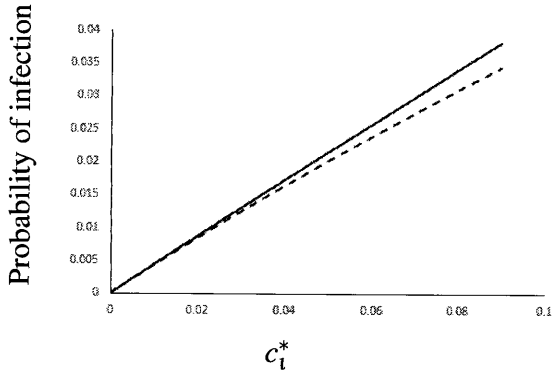


Figure 4.26: Probability of infection among individuals who apply/ don't apply social distancing. The dash and solid curves represent $\pi_{SD}(c_\iota^*)$ and $\pi_{SD'}(c_\iota^*)$, respectively.

The effect of social distance on the probabilities π_{SD} and $\pi_{SD'}$ can be observed in Fig. (4.26). Both probabilities increase linearly as one increases the endemic level c_ι^*. However, when the social distance is not applied, the predicted increasing rate is greater than that displayed if the social distance is applied.

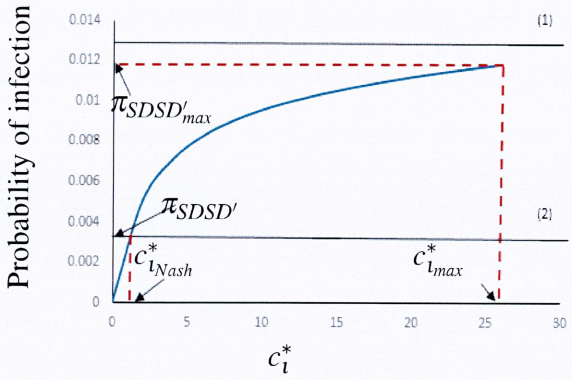


Figure 4.27: The probability of increased risk of infection for individuals who use/ don't use the social distancing. Line (1) when $C^\iota > \pi_{SDSD'}$, there is no Nash social strategy. Line (2) when $C^\iota = \pi_{SDSD'}$, there is a unique Nash social strategy.

Fig. (4.27) shows the probability of the increased risk of the infection for individuals who don't use the social distancing compared to individuals who use social distancing. Line (1) when $C^\iota > \pi_{SDSD'}$, then $\Delta E < 0$, individuals will choose not to use social distancing strategy, and there is no Nash social strategy. However, line (2) when $C^\iota = \pi_{SDSD'}$, then $\Delta E = 0$ and shows there is a unique Nash social strategy. Thus, ι can be written as function of C^ι as,

$$\iota \to 1 - \frac{1}{a}\left(\frac{\iota_1}{\iota_2}\right), \tag{4.41}$$

where,

$$\begin{aligned}
\iota_1 = &-\mu a c_\iota^*((\beta - a e_\iota^* \kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega))\gamma(2\beta - a e_\iota^* \kappa + \mu) \\
&-\mu^2\gamma(2\beta - a e_\iota^* \kappa + \mu)^2 + C^{\iota-1}(\mu\iota(a c_\iota^*((\beta - a e_\iota^* \kappa + \mu)\gamma(\mu+\omega) \\
&+\beta\eta(\mu+\omega)))\gamma(2\beta - a e_\iota^* \kappa + \mu)), \\
\iota_2 = &\, e_\iota^*((\beta - a e_\iota^* \kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega))(a c_\iota^*((\beta - a e_\iota^* \kappa + \mu) \\
&\gamma(\mu+\omega) + \beta\eta(\mu+\omega)) + \mu\gamma(2\beta - a e_\iota^* \kappa + \mu)).
\end{aligned} \tag{4.42}$$

As the endemic level increases, there is an increase in the probability risk when $c_\iota^* \in [0, c_{\ell Nash}^*]$, whereas it increases gradually as $c_\iota^* \in [c_{\ell Nash}^*, c_{\ell max}^*]$ increases.

4.4.3 Combining social distancing and vaccination strategies

In this subsection, vaccination and social distancing strategies are used, For this case. the control reproduction number $R_c(\psi, \iota)$ of system (3.18) will be,

$$R_c(\psi, \iota) = \frac{(1-\iota)a}{\mu} + \frac{a\psi(\iota - \xi)}{\mu(\psi+\mu)}. \tag{4.43}$$

By solving (4.43) with respected to the social distance rate ι, one gets,

$$\iota = \frac{a(\psi+\mu-\psi\xi) - \mu(\psi+\mu)R_c(\psi,\iota)}{a\mu}. \tag{4.44}$$

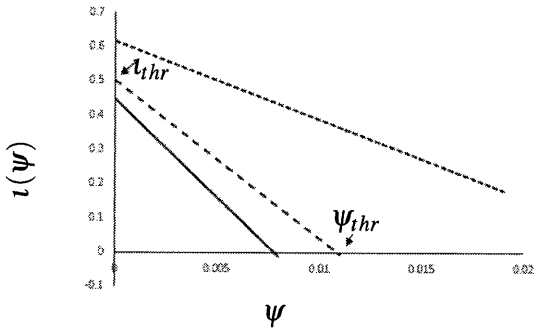


Figure 4.28: The social distance rate $\iota(\psi)$ as a function of vaccination rate ψ. The solid, dash and dot curves are evaluated at $R_0 = 0.9, 1, 1.3$, respectively.

Fig. (4.28), indicates that $\iota(\psi)$ behaves differently depending on the initial reproduction number. At small values of R_0, small vaccination and distance rates are clearly visible. When R_0 increase, one must also increase ψ and ι as well. To examine system stability that describes this strategy, it is necessary to discuss the endemic point of the system (3.18), which is given by

$$P_{\psi,\iota}^* = (v^*, e^*, u^*, q^*, c^*, h^*, r^*), \tag{4.45}$$

where,

$$e^* = \frac{\mu+\gamma+\eta}{\kappa a}, \quad u^* = \frac{(\mu+\omega)(\gamma+\eta-\beta)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c^*, \quad q^* = \frac{-\eta(\mu+\omega)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c^*,$$
$$h^* = \frac{\omega}{\mu+\varepsilon}c^*, \quad r^* = \frac{\varepsilon\omega}{\mu(\mu+\varepsilon)}c^*,$$
$$v^* = \frac{(1-\iota)(1-e^*)}{(\xi-\iota)} - \frac{1}{(\xi-\iota)R_0} - \frac{V_3}{V_4}c^*, \quad c^* = \frac{Y_2}{Z_2},$$

with,

$$\begin{aligned}
V_3 &= (1-\iota)(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)+\mu(\kappa+1+\iota)((\mu+\omega)\\
&\quad (\gamma+\eta-\beta)-\eta(\mu+\omega))),\\
V_4 &= (\xi-\iota)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta),\\
Y_2 &= (\xi-\iota)\mu(-\eta\beta+\gamma(\gamma+\eta-\beta))\kappa(\frac{\psi(\kappa a-\mu+\gamma+\eta)}{\kappa a}\\
&\quad -\frac{\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa}{(\xi-1)R_0\kappa}\\
&\quad -\frac{(1-\iota)(1-e^*)(\kappa\mu+(1-\xi)(\mu+\gamma+\eta)+\kappa\psi)}{(\xi-1)\kappa}),\\
Z_2 &= (1-\iota)(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)+\mu(\kappa+1+\iota)((\mu+\omega)\\
&\quad (\gamma+\eta-\beta)-\eta(\mu+\omega)))(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)\\
&\quad +(\xi-\iota)\kappa(\psi((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)\\
&\quad +(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)).
\end{aligned}$$

If social distancing and vaccination are both possible strategies to individuals, then the Nash equilibrium of vaccination will be a social function $\psi_{Nash}(\iota)$, and the Nash equilibrium of social will be a function of vaccination $\iota_{Nash}(\psi)$. By substituting c^* from Eq. (4.45) in equation $\pi_{VV'}(c^*) = C^{\psi}$, then the Nash equilibrium of vaccination strategy

$\psi_{Nash}(\iota)$ satisfies,

$$\begin{aligned}\psi_{Nash}(\iota) = C^{\psi}((((1-\iota)(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)+\mu(\kappa+1+\iota)\\ ((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)))(\kappa\psi+(1-\xi)(\mu+\gamma\\ +\eta)+\mu\kappa)+(\xi-\iota)\kappa(\psi((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)\\ +(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))))(a((\mu+\omega)(\gamma(\gamma+\eta-\beta)\\ -a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta))((1-\xi)((\mu+\omega)\\ (\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)))\\ -(((\xi-\iota)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)\kappa(\frac{\psi(\kappa a-\mu+\gamma+\eta)}{\kappa a}\\ -\frac{(1-\iota)(1-e^*)(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)}{(\xi-\iota)\kappa}\\ -\frac{\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa}{(\xi-\iota)R_0\kappa}))[-\mu\xi(\eta+\gamma)(\mu+\omega)\\ (\beta-\gamma)(\gamma^2+\eta\gamma-\beta\gamma-a\eta\beta)]). \qquad (4.46)\end{aligned}$$

Substituting $\psi = 0$ in equation (4.46) and solve for ι the solution will be $\iota \longrightarrow 0$. Also, by using c^* from Eq. (4.45), the increased risk of the individuals becomes $\pi_{SDSD'}(c^*) = C^{\iota}$. According to this strategy, the Nash equilibrium of social distancing strategy $\iota_{Nash}(\psi)$ is given by,

$$\iota_{Nash}(\psi) = C^{\iota}((A+B)((1-\iota)A+B)) - \iota AB, \qquad (4.47)$$

where,

$$\begin{aligned}A &= ac^*\gamma(\mu+\omega)(-\kappa ae^*+\mu+\beta)+\beta\eta(\mu+\omega)),\\ B &= \mu\gamma(-\kappa ae^*+\mu+2\beta). \qquad (4.48)\end{aligned}$$

Substituting $\iota = 0$ in Eq. (4.47) and solve for ψ , the solution will be $\psi_0 = 0$, thus, vaccination is the best strategy when $\iota_0 > \iota_{Nash}$.

As shown in Fig. (4.29), the relative cost of function is used to investigate the best applicable strategies for Bahrain, where three cases are

considered; $C^{\iota} > C^{\psi}$, $C^{\iota} = C^{\psi}$ and $C^{\iota} < C^{\psi}$. When vaccination costs exceed social costs, the upper bounds of the social distance increase.

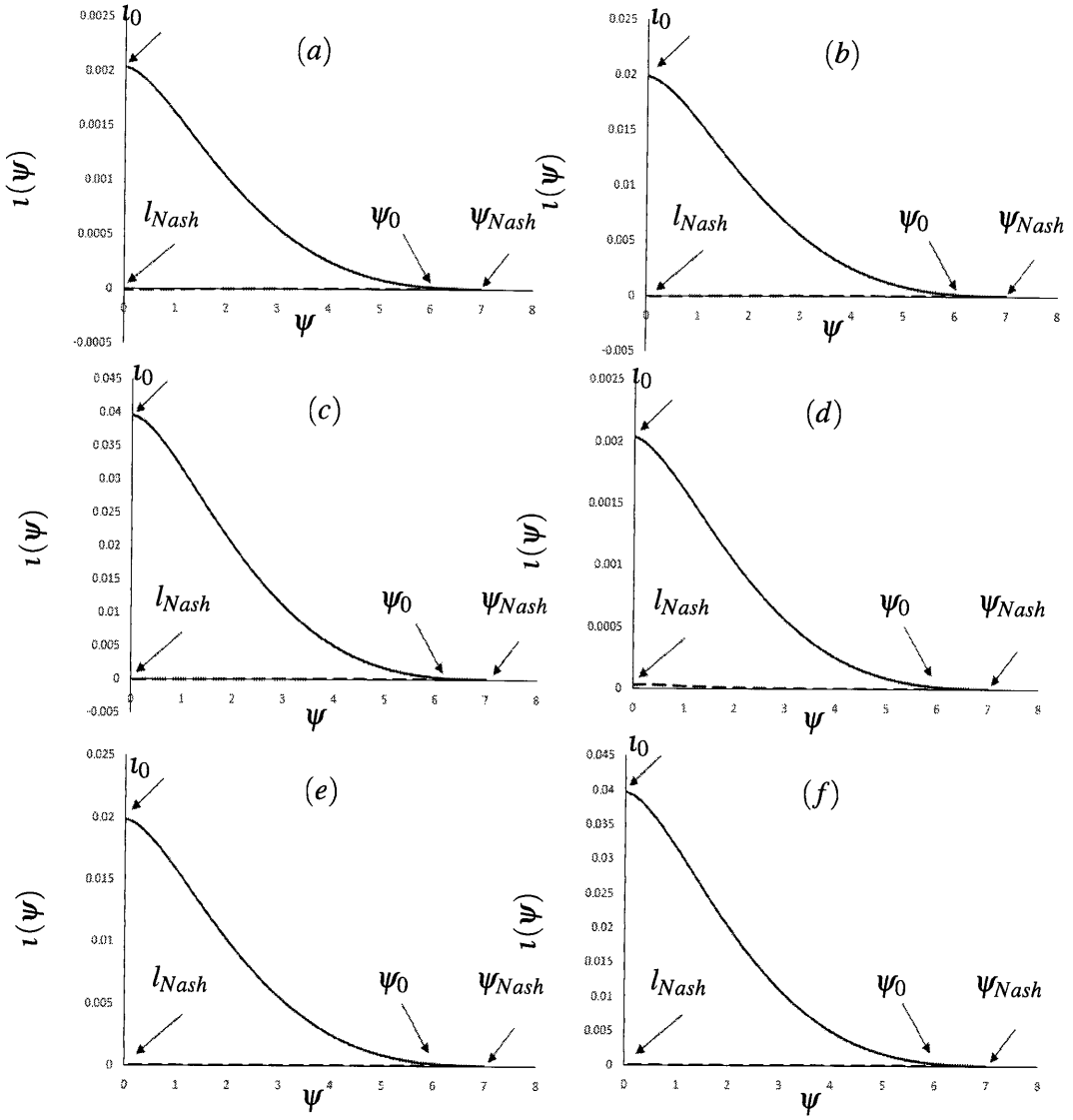


Figure 4.29: The social distance rate $\iota(\psi)$ as a function of ψ at different relative vaccination cost of infection C^{ψ} and relative social cost of infection C^{ι}. The best vaccination rate (solid) and the best social distancing rate (dash) a) $C^{\iota} = 0.1, C^{\psi} = 0.01$. b) $C^{\iota} = 0.1$, $C^{\psi} = 0.1$. c) $C^{\iota} = 0.1, C^{\psi} = 0.2$. d) $C^{\iota} = 0.2, C^{\psi} = 0.01$. e) $C^{\iota} = 0.2, C^{\psi} = 0.1$. f) $C^{\iota} = 0.2, C^{\psi} = 0.2$.

4.4.4 Discussion

In this subsection, the previous *SUQCR* model is expanded to a game-theoretic *SVEUQCHR* model. Based on the model results, the number of individuals exposed to infections increases as vaccination effectiveness decreases. Additionally, vaccination and social distancing reduce the number of confirmed infected individuals. Vaccination and social distancing were also incorporated into the developed model as control measures. Furthermore, the return from each available strategy is calculated by constructing a payoff function. An individual's preference for a strategy depends on its relative costs. According to the calculation, both vaccination and social distancing are necessary to prevent COVID-19 from spreading in Bahrain. Vaccination only reduces the control reproduction number if the vaccination efficacy outweighs social distancing rates.

Three scenarios are highlighted in the results section: in the absence of social distancing, vaccination is applied as an individual strategy and in the absence of vaccination, social distancing is applied as an individual strategy. While the last scenario is the application of both vaccination and social distancing as an individual strategy.

In the first scenario, Bahrain can control the epidemic with vaccination if the basic reproduction number is lower than 2.3 with an efficiency of 0.565 at most. In such a case, more vaccination efficacy is needed (Fig. 4.17). The effectiveness of a vaccine also reduces the risk of infection among individuals. Additionally, it was found that un-vaccinated individuals were more likely to become infected when vaccination efficacy increased (Fig. 4.21). When the relative cost of infection increases, the population vaccination rate decreases in fear of a rise in vaccine-related

infections (Fig. 4.18- 4.20). The cost of vaccination can also make individuals decide not to vaccinate when the risk of infection is greater for people who aren't vaccinated (Fig. 4.22). As a general rule in this scenario, the most appropriate strategy is when the relative cost of infection equals the probability of increased infection risk for un-vaccinated individuals. Choi and Shim obtained similar results in their study (Choi & Shim, 2020).

In the case of the second scenario, the social distancing plays a critical role in minimizing contact between individuals. Therefore, social distancing alone is not sufficient to control the epidemic (Fig. 4.23). The severe social distancing between individuals can reduce infection effectively, but it can also cause economic damage to individuals and society. In addition, everyone must participate in social distancing in order to achieve the herd immunity threshold, resulting in economic losses. Social distancing rates have also been observed to be affected by the rising relative cost of it. This rate is also affected by the basic reproduction number. According to (Fig. 4.24), social distancing decreases as the basic reproduction number decreases.

As social distance increases, the endemic level decreases. People who do not practice social distancing are at a greater risk of infection as it grows, as shown in (Fig. 4.25- 4.26). The best strategy in the second scenario involves using social distancing when the relative social cost of infection is equal to the increased risk of infection for individuals who don't use social distancing, as shown in (Fig. 4.27).

According to the last scenario, when R_0 increases, one must also increase ψ and ι as well (Fig. 4.28). Furthermore, vaccination is the most effective strategy, regardless of the cost (Fig. 4.29). In contrast, Choi and Shim (2020) found that vaccination will be the best strategy if so-

cial distancing is more than vaccination. As long as vaccination costs are the same as social distancing costs, social distancing will be the best approach.

4.5 Game theory approach for vaccination against COVID-19 in Bahrain

This section presents the results of the game theory approach for vaccination against COVID-19 based on statistics provided by the Ministry of Health in Bahrain from January to June 2021:

- The payoff matrix for vaccination and getting infected (VI):
  For the basic and repeated rounds, when players decide to take the COVID-19 vaccine and the booster doses, the Nash equilibrium points are $(0.94, 0.94)$ and $(1.88, 1.88)$, respectively. It is therefore possible to minimize the risk of contracting COVID-19 by taking the vaccine and booster doses. The infection risk for un-vaccinated individuals is 0.06 in the basic round, and if they do not take the vaccine, the risk doubles, while others, choose to take a booster shot.

- The payoff matrix of vaccination and being as critical cases due to COVID-19 (VC):
  The COVID-19 vaccine and booster doses lead to Nash equilibrium points of $(0.83, 0.83)$ and $(1.66, 1.66)$ in the second part. It is possible to reduce the number of infected individuals reaching the critical level by taking the vaccine. The probability of not being in critical cases is increased with the booster doses. However, un-vaccinated individuals' chance to be in critical cases in the basic

round is 0.17. As a result, their chance of contracting this disease will double if they do not take the vaccine and, at the same time, other individuals decide to take the booster shot. These results agreed with Johnston, Pell, and Nelson (2022), who found that fully vaccinated individuals had an 89.8% lower risk of being infected with COVID-19.

- The payoff matrix of vaccination and dying due to COVID-19 (VD): As for the last part, players who choose to take the COVID-19 vaccine with booster doses are found to have Nash equilibrium points of $(0.9, 0.9)$ and $(1.8, 1.8)$. Therefore, vaccine booster doses help protect vaccinated individuals against death. In contrast, the death risk for un-vaccinated individuals is 0.1. The probability of dying due to COVID-19 will double if they refuse to take the vaccine while others take the booster dose. Regarding vaccinations and booster doses, this results confirms the Bahrain Ministry of Health's announcement. For example, on July 27 2021, the Ministry of Health in Bahrain announced that $131,192$ people were given booster shots. Two weeks later, however, only 71 of these individuals (0.05%) had contracted COVID-19, and no one with the virus required hospitalization. These findings demonstrate that booster injections effectively reduce symptoms. Bahrain also recorded no hospital admissions the week after booster shots, indicating a significant reduction in mortality (MOH, 2021c).

4.6 Summary

Following is a summary of the research findings:

1. The impact of precautionary measures on infected cases in Bahrain is reviewed over two years (2020 and 2021). The results may be summarized as follows:

   (a) The number of infected cases was lower than expected, except for some periods when the number increased.

   (b) Among the precautions implemented, which resulted in a significant reduction of infections, are remote work, the closure of schools and shops, and wearing face masks. For example, continuing the closure reduces the number of infections by 64% while wearing face masks minimises the number of infections by 14%.

   (c) In addition, using the light signal system has also played a critical role in reducing the epidemic spread in Bahrain, where the infection rate decreased by 31% from the expected numbers.

   (d) In contrast, reducing or changing these procedures, such as lowering quarantine days for travellers and those in contact with infected cases and cancelling the PCR tests for them, led to an increase in the number of infections.

2. According to $SUQCR$ model, the local stability and existence equilibrium point were found, and the basic reproduction number R_0 was calculated by the next-generation approach. The findings from this model were as follows:

(a) *SUQCR* can be used to measure quarantine-related parameters and detect confirmed cases through random and exit testing.

(b) The model's sensitivity analysis recommended increasing the number of random and exit tests and contact tracing rates.

(c) Bahrain will have fewer un-quarantined infections and reduce the basic reproduction number while controlling the spread of COVID-19.

(d) Furthermore, by early detecting the disease and boosting people's immunity through vaccination, the recovery people will increase.

3. Regard the *SVEUQCHR* model, it is shown that decreasing vaccination effectiveness affects the behavior of vaccinated individuals and exposed to infection, where the vaccinated number decreases and the exposed to infected increases. In addition, this model showed that:

(a) vaccination alone or in combination with social distancing reduces the number of confirmed infected people.

(b) Also, using the game theory approach, three different strategies were discussed to overcome or reduce the COVID-19 endemic in Bahrain, with the assumption that individuals would either accept vaccination and refuse social distancing or refuse vaccination and employ social distancing. In the last scenario, individuals accept vaccination and apply social distancing.

(c) The three scenarios found that the vaccination is the best strategy in the absence of social distancing, whereas social

distancing was the best strategy in the absence of vaccination. In contrast, when vaccinations and social distancing are present, the best strategy can be applied.

(d) The analysis shows that, the vaccination should be applied when its cost equals the risks of infection for un-vaccinated individuals versus vaccinated individuals. The optimal strategy for individuals in the absence of vaccination, however, is to use social distancing whenever the cost is equal to the increased risk of infection. In the presence of vaccination and social distancing, vaccination is the best decision, regardless of its cost. Controlling the spread of disease requires vaccination as a precautionary and preventative measure.

4. In the context of the game theory approach, the three scenarios were used to illustrate how the full doses of the COVID-19 vaccine and the booster dose would affect disease spread, entered the hospital as critical cases, or the death of those infected. Results showed that:

   (a) The vaccination and booster doses reduced the COVID-19 spread and the patients number arriving at the hospital as critical cases.

   (b) It reduces the deaths number compared to un-vaccination individuals.

   (c) For un-vaccinated individuals, the risk of infection, critical patients, or death from COVID-19 was 0.06, 0.17, and 0.1, respectively. The risk for them will be doubled when others choose to take a booster dose.

Chapter 5

Conclusion and Recommendations

5.1 Introduction

The 17 SDGs emphasized human development without destroying nature, and the 2030 Agenda confirmed global peace and prosperity. Unfortunately, the COVID-19 pandemic halted this progress and stopped countries from achieving their SDGs. Moreover, the epidemic had a significant impact on countries in economic, social, and environmental terms as detailed in the second chapter of this study.

In Bahrain, the government took some measures and precautions to prevent the spread of the epidemic and provided health care to those affected. This study is a contribution which mainly focuses on developing mathematical models for COVID-19 transmission and control in Bahrain.

A summary of how the research goals and objectives were met throughout this dissertation is presented in Sec.(5.2). The main conclusions are drawn in Sec.(5.3). The research limitation is introduced in Sec.(5.4). Finally, the recommendations of this dissertation are suggested in Sec.(5.5).

5.2 The Research's Objectives

In order to achieve the purpose of this study, several objectives, as defined in chapter One, were fulfilled, which affected the literature review, the research methodology, and the findings presented in chapter four. Table (5.1) summarizes these objectives.

Table 5.1: Research Objective's Achievement.

Objectives	Chapter
Clarifying the effectiveness of the precautionary measures adopted by Bahrain to combat COVID-19.	Chapter 3, Sec. (3.2.1) Chapter 4, Sec. (4.2)
Developing mathematical models of infectious diseases, specifically for the COVID-19 in Bahrain.	Chapter 3, Sec. (3.2.1) Chapter 4, Sec. (4.3,4.4))
Using compartmental modeling and the game theory approach to get estimates of the reproduction number R_0, based on available data from government sources on the spread of the coronavirus in Bahrain.	Chapter 3, Sec. (3.2.1) Chapter 4, Sec. (4.3,4.4)
Analysing the effectiveness of the Bahrain government's measures to combat COVID-19, by using the developed mathematical models and simulations.	Chapter 4, Sec. (4.3,4.4)
Utilizing the game theory approach in the mathematical modeling, to explain individuals' roles in taking the COVID-19 vaccine, and how this leads to reduction in infection numbers, hospitilization of critical cases, and death.	Chapter 4, Sec. (4.4,4.5)

5.3 Research findings

This study aims to develop some mathematical models that may be applied to analyse the dynamics of COVID-19 outbreaks in Bahrain. The precautionary measures approved by the Bahraini authorities in 2020 and 2021 are reviewed, and their impact on preventing the epidemic is discussed. In addition, two mathematical models are proposed, which examine the dynamics of disease spread in Bahrain and the main precautionary measures. The game theory approach shows the optimal strategies to prevent epidemic spread.

First, the total number of infected cases in Bahrain during 2020 and 2021 in response to precautionary measures is examined. In both years, the number of infected patients was less than expected, except for some

periods when the number of infected patients increased. A lockdown was one of the precautionary measures used to reduce the number of infected. The number of infected in Bahrain decreased significantly after schools, salons, sports venues, and entertainment centers were closed. Additionally, the number of cases was reduced by social distancing and people wearing masks. As a result of continuing the lockdown phase, the number of infections will be reduced by 64% compared to the actual observed cases. According to statistics, the epidemic will end in late March 2021 based on the daily number of patients infected until September 18, 2020. One of the measures that effectively reduced infections was applying the light signal system, which relied on the average percentage of positive cases. In contrast, reducing precautionary measures, such as reducing quarantine for arrivals and contacts, and cancelling the PCR tests for contact individuals, has increased infections.

After reviewing the precautionary measures, a $SUQCR$ model is developed with five variables and a few parameters for Bahrain's COVID-19 epidemic dynamics. Also, the local stability and existence equilibrium points are determined. Additionally, the next-generation approach is used to calculate the reproduction number R_0. By using the $SUQCR$ model, one can measure quarantine-related parameters and variables, conduct random and exit tests, and trace contacts to detect confirmed cases. Reducing un-quarantined infected individuals, reducing reproduction number R_0, and increasing recovery number are the highest priorities for this model. Random tests, exit tests, contact tracing rates, immunity rates, such as natural immunity and vaccine immunity, as well as medical history rates λ, are also discussed. To increase recovered numbers, the immunity parameter may be used as a control parameter. People who have a history of the disease are less likely to recover. In order to reduce re-

production numbers, random tests, exit tests, and contact tracing have to be increased. Based on the results of this model, $R_0 < 1$, which indicates that the disease can be contained and controlled. Reproduction numbers gradually increase when contact tracing, random and exit tests are reduced.

Then, the $SUQCR$ model is extended to $SVEUQCHR$ model, where the results showed that, as vaccine effectiveness decreases, the number of individuals exposed to infections increases. Furthermore, vaccination together with social distancing reduces the number of confirmed infected people.

Also, the game theory approach is used to examine three different scenarios in order to overcome or reduce the spread of COVID-19 in Bahrain. The assumption is that, people either accept vaccinations in the absence of social distancing or refuse vaccinations and apply social distancing instead. The last scenario involves individuals accepting vaccinations and applying social distancing. Additionally, these three dynamical systems are investigated using game theory. Based on these scenarios, vaccination is supposed to be the best strategy in the absence of social distancing, and social distancing is supposed to be the best strategy in the absence of vaccination. In contrast, if vaccination and social distancing are available, the best strategy is applied. The obtained results show that, in the absence of social distancing, vaccination is more effective when its cost is equal to the risk of infection for un-vaccinated individuals compared to vaccinated individuals. However, in the absence of vaccination, the best strategy is social distancing when the cost is equal to the infection risk for those who don't use it. When vaccination and social distancing are both present, vaccination should be chosen over social distancing. In order to curb the spread of disease throughout the country,

vaccinations should be viewed as precautions and preventive measures.

Finally, the game theory approach is used to study the importance of vaccination in reducing the spread of the pandemic. The game theory can be seen as evidence of the efforts and studies conducted by Bahrain's Ministry of Health, which has proven that vaccination and booster doses are effective in reducing the number of infected people and critical cases as well as deaths caused by COVID-19. By using the game theory, it is demonstrated that vaccination with booster doses reduces the risk of becoming infected with COVID-19 and the risk of infected individuals entering hospitals as critical cases or dying from the disease compared to those who do not receive the vaccine.

5.4 Research Limitation

The models $SUQCR$ and $SVEUQCHR$ are limited to the assumption that the system is endemic. The age of individuals is not taken into account, as is their susceptibility to infection. In real life, the prevalence, severity, and age of an individual affect susceptibility to contracting the disease. Vaccination rates differ between age groups, and at the beginning of a seasonal influenza epidemic, the best vaccination rates occur for all age groups (Choi & Shim, 2020). Moreover, due to the severity and spread of the disease, the government needs to intervene more, for example, by closing schools, allowing remote work, and partially closing stores. Social distancing, for example, is chosen as the most appropriate method to prevent infection exposure. Therefore, appropriate to consider vaccinations and social distancing as case variables.

5.5 Future Research Recommendations

As a result of this study, the following recommendations are made:

- At the government level:
  - Precautionary measures must be continued as mitigating or limiting them leads to an increase in the spread of the disease.
  - It is important to increase random tests, exit tests, and contact tracing rates in order to better detect infections and reduce reproduction rates.
- At the family level:
  - Bahrain's precautionary measures must be followed both individually and collectively to control COVID-19.
  - It is essential that all residents receive a vaccine to combat the outbreak.
- Additionally, it is recommended to prepare more studies and mathematical models to control the spread of infectious diseases in Bahrain.

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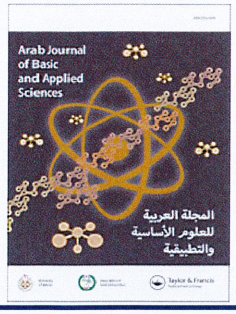
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Appendix (A)



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The impact of the precautionary measures taken in the kingdom of Bahrain to contain the outbreak of COVID-19

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The impact of the precautionary measures taken in the kingdom of Bahrain to contain the outbreak of COVID-19

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ABSTRACT

As a worldwide pandemic is unavoidable, its spread must be observed to reduce and contain it. The Kingdom of Bahrain sought to implement the World Health Organization (WHO) directives to take precautionary measures and appropriate preventive steps to limit the spread of the coronavirus among people and developed an integrated plan to contain it. This study purposes to investigate the effect of precautionary and preventive measures in limiting the spread of coronavirus in Bahrain. Statistics and the details of precautionary measures described in this study are obtained from websites maintained by the Ministry of Health (MOH) in Bahrain. The calculated number of infected and death cases is determined by using polynomial functions that fit the data. The outcomes obtained showed that the number of cumulative infected and death cases were less than the expected numbers except for certain periods. Reopening stores and businesses that provide direct services to people after their two-week closing resulted in a 36% increased in infected cases than expected in the closing period. When the lockdown continued, the number of infected people dropped to 64% of the total number of detected infections. In accordance with our results, it is expected that the number of infected cases will gradually decrease till late March 2021. With continuing lockdown and their compliance in implementing the precautionary measures with consciousness, it is anticipated that Bahrain will soon be in the final state of the epidemic.

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1. Introduction

On December 31, 2019, the World Health Organization (WHO) informed about a new virus, the COVID-19, responsible for respiratory disease in Wuhan, China (Chen et al., 2020). It stimulated great concern inside China as well as universally resulting in the virus' spread and improvement patterns (Shao, Zhong, & Chen, 2020). COVID-19 has now spread widely across the world and WHO declared it as a pandemic in March 2020 (World Health Organization, 2020a; World Health Organization, 2020b).

COVID-19 has an incubation period of three to seven days and may last up to fourteen days, and this period varies from patient to patient (Wang, C et al. 2020). It is believed that the coronavirus is contagious during incubation period when patients do not have symptoms, and can be spread from one person to another by direct contact and through sneezing, coughing, or speaking (Ministry of Health, 2020a).

COVID-19 is caused by Respiratory Coronavirus Syndrome (SARS-CoV-2). The virus is believed to have entered Bahrain on February 24, 2020. On September 18, 2020, a total of 63189 confirmed cases have been reported, of which 56700 have recovered, and 220 have died, whereas 157 cases have been receiving treatment, 43 of which are in critical condition. Out of 6959 active cases, 6916 are in a satisfactory condition (Ministry of Health, 2020b; WHO, 2020).

Since the infection started to spread universally, Bahrain has shown its sincere commitment to keep up the health and well-being of its population. Additionally, it has demonstrated a high degree of readiness to stand up to this infection since its appearance in China. As a result, a wide range of decisions, precautionary measures, and preventive measures have been taken in Bahrain.

The government of Bahrain had made efforts to contain the virus and to reduce its spread to the broader population. Since the start of the pandemic, it has followed the WHO's guidelines, implementing

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a strategic response plan that included tracking the contacts, increasing the preparedness of healthcare facilities, and finding ways to communicate with the target groups. The government also used many advanced technologies to follow up on the infected and quarantined and trace contacts (E Government, 2020).

People also participated in the National Campaign to Support Fight Against COVID-19 by staying in their homes and not going outside except for necessity, distancing themselves from society and avoiding interaction with others, not spreading rumors and misleading facts. Besides, respecting hygiene, hand washing, they adhered to the commitment to home quarantine after their travel (Ministry of Health, 2020c). The Ministry of Health (MOH) requires all citizens and residents to adhere to all the directives issued by the official authorities, represented in wearing masks, adhering to social distancing steps, washing hands regularly, and always using hand sanitizer. It also urges not to leave the house except when necessary to meet basic living needs.

A comprehensive national plan was developed for surveillance to confront the virus and to control its spread. A surveillance and control committee was formed in the Military Hospital to control the virus' spread at the beginning of February 2020 to decide the protocols and to set strategies that must be followed by the citizens and residents of Bahrain.

Alumran (2020) showed that the COVID-19 pandemic can be controlled if drastic precautions are taken in the early phases of an outbreak in the affected area. Following the introduction of many preventive steps, the spread of COVID-19 has been brought under control in the neighboring Kingdom of Saudi Arabia.

In another study, Dowd et al. (2020) agreed that governments worldwide should take strict precautionary steps to prevent the pandemic of COVID-19. In addition, Xiao and Torok (2020) emphasized in their article that in the face of the rapidly spreading disease and many infected people, effective infection prevention and control measures are urgently needed.

Al Nasser (2020a) stated in his study that according to the reported models and assumptions, the number of actual cases of the COVID-19 in Bahrain is less than the expected cases in Wuhan by approximately 16.3 times. These results could be attributed to the National Coronavirus Response Team's arrangements to tackle the COVID-19 epidemic in Bahrain.

This study tries to assess the effect of precautionary measures taken in Bahrain to contain the virus and determine the best of these measures in controlling the virus's spread.

2. Method

The statistics of COVID-19 and the data for implementing precautionary measures were collected from the Ministry of Health (MOH) website in Bahrain (Ministry of Health, 2020c) and the website (Our world in data, 2020). The first case of COVID-19 appeared on February 24, 2020, and from that date to September 18, 2020, the data on the spreading epidemic was collected daily.

Statistics of the number of infected cases in Bahrain during this study are divided into five periods based on the precautionary measures that were followed during each period (see Table 2). The observed number of infected cases for each period—and their best-fit lines— are plotted using Microsoft Excel, where x refers to the number of days and y refers to the number of infected and the death cases. The polynomial functions defining the best-fit lines are obtained using the polynomial approximation available in Microsoft Excel. Using the derived polynomial functions, the expected number of infected and death cases are calculated. The average % rate of increase or decrease between the observed data of a certain period and the expected number of infected cases of its previous period is calculated using the formula:

$$\text{Average \% rate} = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{O_i - E_i}{E_i}\right) \times 100\%\right)$$

where O_i is the observed number of cases on day i, E_i is the expected number of cases on day i and n is the number of days.

3. Results

Up to September 18, 2020, the expected number of infected and death cases is expected to increase without precautionary measures. The polynomial function used for estimating the infected and death cases are given as

$$y = 1.4858x^2 - 12.116x + 2024.6 \quad (1)$$

and

$$y = 0.0073x^2 - 1.1581x + 43.106 \quad (2)$$

with R^2=0.9871 and R^2= 0.977, respectively.

Figure 1 displays the observed number of cases infected with COVID-19 in Bahrain versus the estimated number when the protective measures are not implemented. Figure 2 shows the monitored number of death cases from COVID-19 in Bahrain versus the estimated number if the protective measures are not implemented. It is noticed that the numbers of detected infections and deaths are less than the estimated cases, except in some periods in

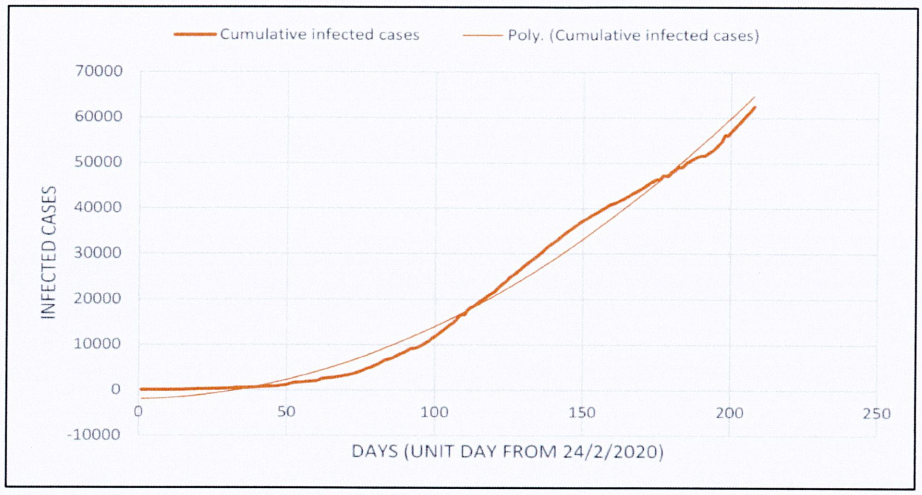


Figure 1. Number of total observed infected individuals of COVID-19 vs the estimated number of infected individuals from 24/2/2020.

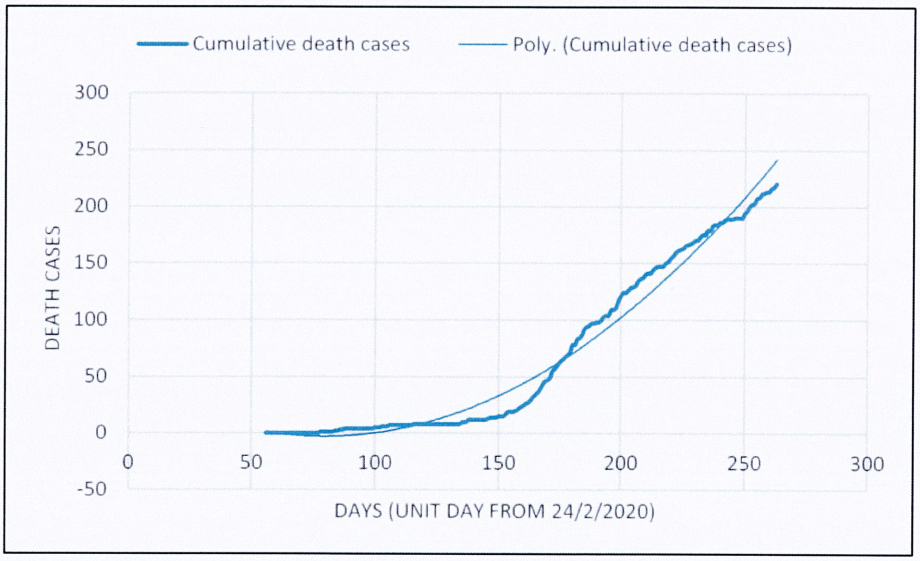


Figure 2. The number of total observed deaths from COVID-19 vs the estimated number of deaths from 24/2/2020.

which the estimated cases are less than the detected ones.

From the first case of COVID-19, the Government Executive Committee has endeavored to work hard to control the extent of the virus in the people, and it imposed many precautions shown in Table 1. These strategies are based on the recommendations of the WHO.

Figure 3 shows the number of total infected cases during the periods of applying precautionary measures. The data is splitting into five categories: using social distancing, distance working and closing educational schools, closing commercial and industrial shops that provide goods directly to people for two weeks, reopening these shops for another two weeks, and re-closing them again for two weeks.

On the other hand, Figure 4 represents the number of infected and expected cases when using precautionary measures separately. It is noticed in Figure 4a that applying social distancing and the travel ban to some affected countries, between February 24 and March 13, had an unnoticeable impact on the expected numbers of infected cases. The average % rate of change in this period increased by 5% compared to the expected number of infected cases. The quadratic equation (3) was used to estimate the number of infected cases in this period:

$$y = 0.6067x^2 - 4.3201x + 26.752 \quad (3)$$

During the next ten days, the National Medical Team for Responding to coronavirus issued directives for transitioning to the distance work, the suspension of studies in all educational institutions, closing of cinemas and sports centers and restricting the activities of restaurants to external requests only

Table 1. The Precautionary measures applied to stop the spread of COVID-19 in Bahrain from the beginning of the pandemic until September 18, 2020 (Ministry of Health, 2020c).

Date	Precautionary measures
February 24-March 14	• Social distance • Travel ban to and from some affected countries by coronavirus • Quarantine and home isolation for people returning from travel.
March 15-March 24	• Reducing the number of upcoming trips • Distance working • Closing all educational institutions. • Closing of cinemas and sports centers • Restrictions on restaurants to entertain only the external requests • Closing salons
March 26-April 9	Closing commercial and industrial shops that provide goods directly to people
March 29- Farther notice	Random COVID-19 tests
April 10-April 23	Reopening commercial and industrial shops that provide goods directly to people
April 23 - Farther notice	Wearing face masks is declared mandatory for everyone
April 24-May 7	Closing commercial and industrial shops that provide goods directly to people
May 8- Farther notice	Reopening commercial and industrial shops that provide goods directly to people
June 3- Farther notice	Mandatory wearing masks and imposing a fine on violators
August 6	Reopening gyms and sports halls, swimming pools, and outdoor sports
September 3	Outdoor dining at restaurants, coffee shops, and training institutions

Table 2. The average % rate of change in infected cases at the different periods of applying the precautionary measures.

Periods of the study	Precautionary measures	Average % rate of increase or decrease in infected cases
Period 1	Social distance	5%
Period 2	Distance working and closing all educational institutions	−18.68%
Period 3	Closing commercial and industrial shops that provide goods directly to people	12.14%
Period 4	Reopening commercial and industrial shops that provide goods directly to people	36%
Period 5	Closing commercial and industrial shops that provide goods directly to people	13.4%



Figure 3. The number of total infected cases when the precautionary measures were applied from 24/2/2020.

(Ministry of Health, 2020c). During this period, the average % rate of change of infected cases decreased by 18.68% compared to the expected infected cases when only social distancing and travel bans were implemented, as shown in Figure 4a.

In Figure 4b, from March 26 to April 9, a decision was issued to close commercial and industrial shops that provide direct services to the people. During this period, the average % rate of change of the expected number of infected cases increased by 12.4% compared to the expected numbers in case continuing with the previous precautionary measures. The quadratic equation that fits the curve of this period is given by

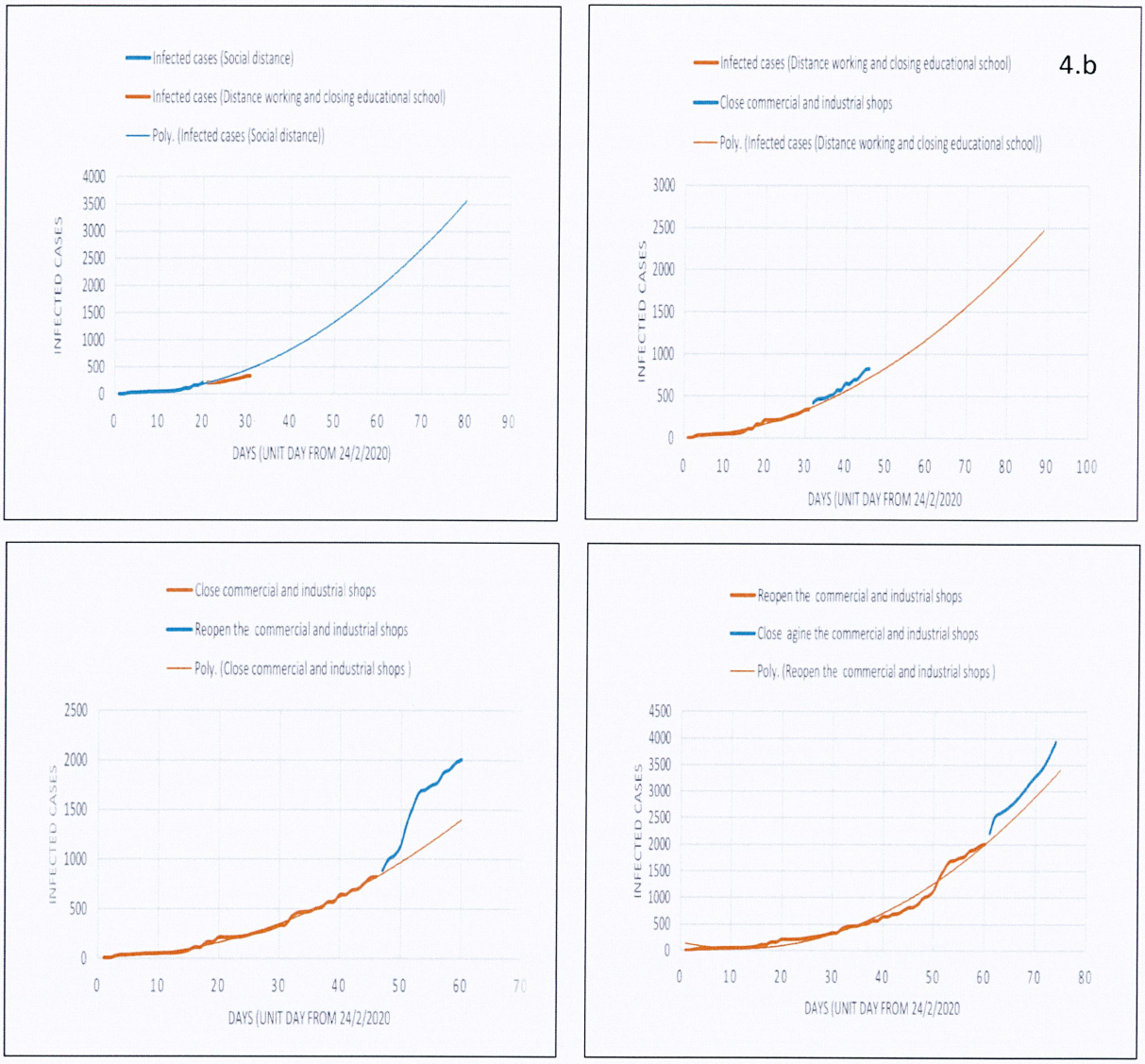


Figure 4. The impact of effective measures on the number of infected cases during the period of February 24, 2020, to May 7, 2020.

$$y = 0.2858x^2 - 2.3039x + 3.8004. \quad (4)$$

Later, a decision was issued to open these shops, which led to an increase in the average % rate of change of COVID-19 cases by 36% compared to the numbers expected when applying the previous precautions, as shown in Figure 4c. The best fit curve used to compare the infected patients in the reopening period is given as

$$y = 0.3967x^2 - 0.7884x + 18.291. \quad (5)$$

Two weeks later, the shops closed again, which led to a 13.4% increased in the average % rate of change in the infected cases than the expected infection in case continuing with the previous measures as shown in Figure 4d. The best fit curve used in this period is given as

$$y = 0.8488x^2 - 20.152x + 151.85 \quad (6)$$

Figure 5 displays the number of infected cases when continuing with closing schools, educational institutions, shops, and sports centers, compared to the number of recorded infectious. If the lockdown phase continues, the number of infections will decrease with an average % rate of 64% than the expected observed infected cases.

The adherence shown by the people to wear masks had a significant impact in limiting the spread of the epidemic in Bahrain, as the number of expected infections when wearing masks was 14% less than the numbers estimated before the wearing of masks was made compulsory as shown in Figure 6.

Figure 7 shows that above 700 cases were observed after Eid-Al-Fitr, and after Ashura, the highest observed cases were on September 16, 2020, with more than 800 cases. The best fit curve for the daily infected cases is given as

$$y = 0.014x^2 + 5.8968x - 110.47. \quad (7)$$

Figure 7 also shows that the expected number of infected cases will gradually decrease during March,

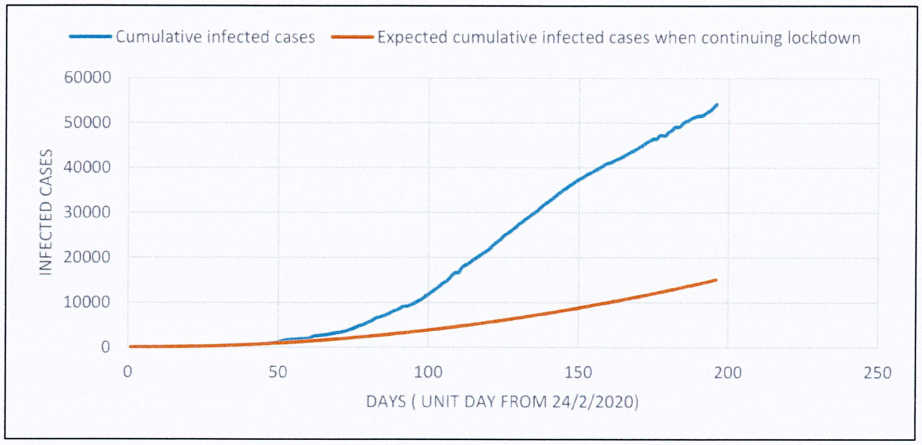


Figure 5. The expected number of infected cases upon continuing the implementation of the lockdown compared to the number of real infected.

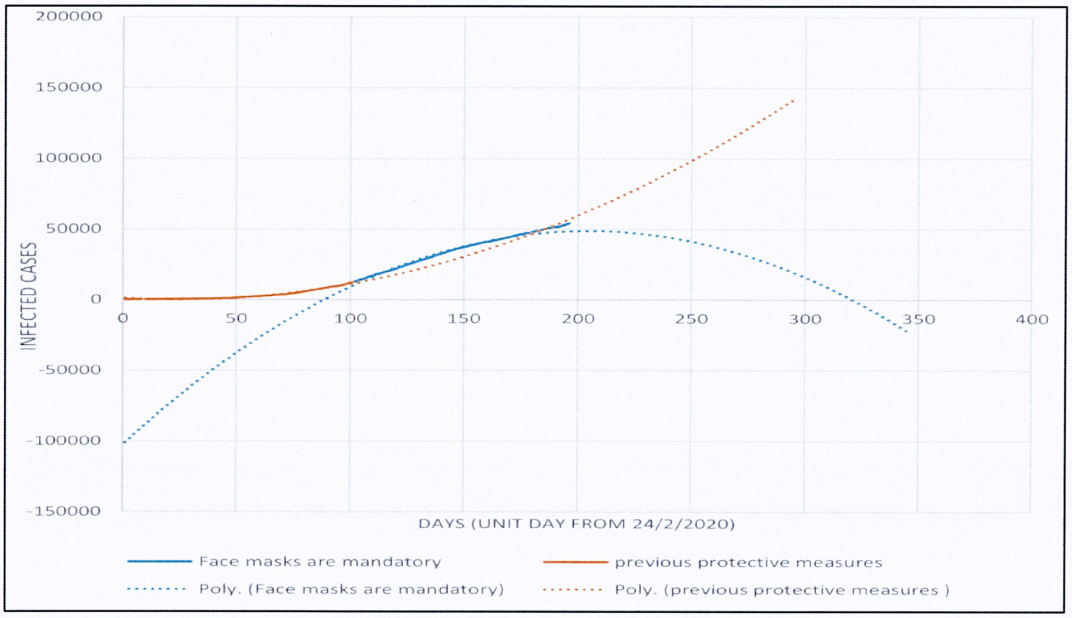


Figure 6. Number of infected cases when the masks are mandatory for citizens and residuals compared to the number of infected patients when the masks are not mandatory.

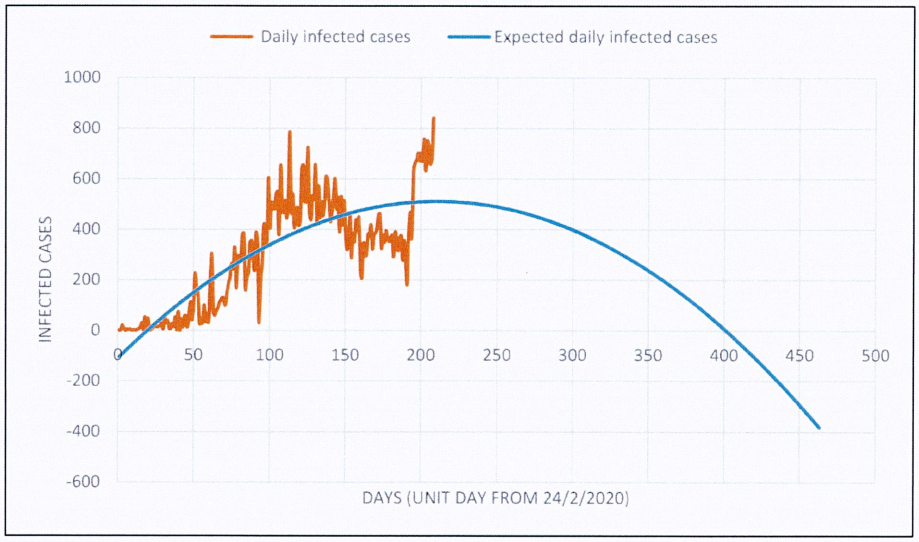


Figure 7. Number of daily infected cases and the expected number of cases.

April, and May. From this estimation, the epidemic in Bahrain is projected to end after 403 days from the starting day, which corresponds to late March 2021.

4. Discussion

This study aims to investigate the effect of Bahrain's precautionary measures to stop the spread of COVID-19. The results support the tremendous effect of precautionary measures during the pandemic outbreak.

Since the first case of Covid-19 virus appeared in Bahrain, the Ministry of Health has called on all citizens and residents to implement social distancing. This measure had the effect of increasing the expected number of infected cases by 5% rather than decreasing it. A study that evaluated epidemic data in Spain, between February 24 and April 5, 2020, determined that physical distancing measures effectively controlled the spread of the disease (Saez, Tobias, Varga, & Barceló, 2020; Reis et al., 2020), especially when applied correctly and for an appropriate duration (Ambikapathy & Krishnamurthy, 2020). This increase in the infected cases had to be reduced, so the Ministry of Health called for the application of other measures (see Table 1).

Applying home quarantine to travelers at the beginning of the pandemic (see Table 1 and Figure 4a) has had a significant effect on controlling the virus's spread, so the number of infections seemed to be roughly equivalent to the number predicted in case the measures were implemented. Similar to these results, Nussbaumer-Streit et al. (2020) reported in their review article that a combination of isolation and other prevention and control measures had a more significant impact than the quarantine alone. Also, Yuan, Li, Lv, and Lu (2020) reported that quarantine in Hubei Province has been effective in slowing the quick increase in new cases of COVID-19. Besides, Tang et al. (2020) proposed that it is essential to strengthen the quarantine and isolation strategy further in order to improve the detection rate in mainland China.

The closing of educational institutions, applying social distances, reducing the number of upcoming trips, distance working, closing cinemas and sports centers, restriction on restaurant activities to external requests, and closing salons is among the precautionary measures, which have had a significant impact on the reduction of infectious from the expected number. These steps were introduced after 21 days following the discovery of the first case of COVID-19 in Bahrain, leading to a decrease of 18% in the number of cases than expected. A study conducted in Korea (Kim, Kim, Peck, & Jung, 2020) showed that the Korean government would be able to reduce the number of infected cases by at least 200 if schools were closed earlier.

The policy requiring shops that provide direct services to residents to be closed for two weeks, then reopening them for another two weeks, and re-closing them for the next two weeks of the opening period. The implementation of these measures did not reduce the number of infected cases but instead increased them than the expected number by 12.14%, 36% and 13.4%, respectively.

During the first closure period, the number of infected cases was increased than what was expected when the previous measures continued to be applied due to the application of random testing, which revealed more cases since March 29 (Ministry of Health, 2020d). Besides, during the second closure period, the number of infections increased more than the first closure period due to shops opening between these two closure periods (see Figure 4d). The results also show a rise in the observed cases at the time when shops are reopened (see Figure 4c). This increase could be due to people not adhering to preventive measures when leaving their homes, especially since this time precedes the holy month of Ramadan and Eid al-Fitr when people prepare to buy supplies, clothes, and so on. In addition to these reasons, people stranded in some affected countries were evacuated, which caused a growth in the number of cases.

In addition to the results obtained, it was found that the continued closure of shops leads to an apparent decrease in the number of projected infections. These results are consistent with the research in Italy, which showed the importance of closures in disease control (Chintalapudi, Battineni, & Amenta, 2020). Tobías (2020) and Saez et al. (2020) analyzed trends in Covid-19 confirmed and deaths cases in Spain and Italy before and after the national lockdown, using time-series statistics. Their results show that the trends in COVID-19 infection have decreased dramatically after applying the lockdown. The closures enable the health system to increase its ability to help and reduce disease transmission by about 60%, considering that it affects patients with or without symptoms (Anderson, Heesterbeek, Klinkenberg, & Hollingsworth, 2020). On the other hand, Xiao and Torok (2020) suggested that blocking traffic and villages lockdown is of no significance for the prevention and control of COVID-19.

The imposition of wearing face masks starting from June 3 has led to a decrease in the number of infected cases more than expected as in Figure 6. Similar to these result, a U.S. study provided evidence that states, where the use of masks was imposed, saw a significant drop in rates of Covid-19 infection, unlike other states where masks were not

set (Lyu & Wehby, 2020). Besides, Barasheed et al. (2016) systematically analyzed the use and efficiency of masks by integrating 12,710 samples from more than 50 countries. They discovered that wearing masks in crowded locations could reduce the risk of respiratory infection by 20%.

By adhering to the precautionary measures and not being complacent in their application, besides taking the vaccine against Covid-19 provided by Bahrain's government, the number of infected cases will disappear at the late March 2021 (see Figure 7). On the other hand, Al Nasser (2020b) proposed that the end date of the COVID-19 pandemic in Bahrain, according to mathematical models, is August's end. In his opinion, it will end in December 2020.

5. Conclusion

The Kingdom of Bahrain has taken several precautionary measures to curb the spread of COVID-19, starting with some lenient measures and ending with drastic measures. This study showed the impact of the precautionary measures on the cumulative number of infected cases in Bahrain. Detected infected cases were lower than expected throughout the study period, except for some periods, in which the number of infected were increased.

One of the precautionary measures that were applied and had a significant effect in reducing the number of detected infected was the lockdown. Bahrain witnessed a decrease in the number of infected during the closing of schools, salons, sports, and entertainment centers, as well as the distance work. The application of social distancing and the compulsory wearing of masks for all people resulted in a decrease in the number of cases. Continuing the lockdown phase will significantly impact in reducing the number of infections by 64% than the real observed cases. According to the statistics of the daily number of infected patients until September 18, 2020, the end of the epidemic will be on late March 2021.

COVID-19 can be controlled by applying and adhering to the precautionary measures. Everyone needs to join with others to wipe out the epidemic and take part with greater consciousness and dedication than ever before reducing the coronavirus spread.

Data availability statement

The data that support the findings of this study are openly available in "Our World in Data" at https://github.com/owid/covid-19-data/tree/master/public/data, (Our world in data, 2020).

Disclosure statement

No potential conflict of interest was provided by the author(s).

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The impact of the vaccination and booster shots in containing the COVID-19 epidemic in Bahrain: a game theory approach

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Abstract— **In this study, game theory is used to determine the effect of vaccination and booster shots in containing COVID-19 in Bahrain. The study is divided into three parts and in each part, the payoff matrix and the biometric is generated. The Nash equilibrium points are determined by marking the maximum value in the first entry of each column and the maximum value in the second entry of each row. The results show that the vaccination and the booster shots significantly reduce the number of infected individuals, the number of patients reaching the ICU, and the number of deaths due to COVID-19.**

Keywords— Game theory, vaccination, booster doses, COVID-19, Kingdom of Bahrain

I. INTRODUCTION

Fair access to safe and effective vaccines is critical to ending the COVID-19 pandemic, so it is very encouraging to see many vaccines being developed and approved for use. The World Health Organization (WHO) works tirelessly with partners to develop, manufacture and deploy safe and effective vaccines [1] that prevent people from becoming seriously ill or dying from COVID-19.

In addition to the main preventive measures of staying at least one meter away from others, covering your face while coughing or sneezing, cleaning hands frequently, wearing a mask, and avoiding poorly ventilated rooms, vaccination is part of COVID-19 management [2]. However, vaccination does not mean that we can put ourselves and others at risk, especially since research is still ongoing on how much vaccines protect from disease against infection and transmission [1].

As of June 4, 2021, 1,900,955,505 vaccine doses were administered worldwide [3] with 1,813,734 vaccine doses administered in Bahrain [4]. Bahrain has started its free vaccination campaign against COVID-19 for citizens and residents in Dec 2020. Vaccination started for age groups from 18 years and above, and on May 18, 2021, the vaccination included age groups from 12 years and above.

Bahrain provided the vaccine and made it free for citizens and residents. Among the types of vaccines available in Bahrain are AstraZeneca, Pfizer, Sinopharm and Sputnik [5]. In addition, Bahrain announced the opening of registration for a booster dose of the COVID-19 vaccine for the most vulnerable groups at least six months after taking the second dose of the Sinopharm vaccine. The priority in booster doses is for citizens and residents over fifty and those who suffer from obesity, low immunity, or other underlying health conditions [6]. In addition, National Medical Taskforce for Combating the Coronavirus (COVID-19) (NMTC) agreed on booster shots for those 60 and older who are immune-compromised and have been fully vaccinated for six months with Pfizer-BioNTech and AstraZeneca [7]. Later, in September 2021, the NMTC announced that eligible individuals might receive booster shots 12 months after the date of infection [8]. Bahrain also provided the opportunity to obtain the booster dose for citizens and residents who completed six months of taking their second dose [9].

Combining game theory with epidemiological models is the best way to predict the expected level of vaccination coverage during the pandemic season [10]. Many researchers have studied the impact of vaccination on the containment of COVID-19 with mathematical models and game theory. For instance. Stone et al. [11] had used delay differential equations to model immune vulnerability. Schimita [12] suggest a vaccination game based on probabilistic cellular robotics, according to public health procedures and personal decisions. The players are the government and the vulnerable newborns. In their simulation results, Han and Sun [13] showed that convergence might effectively weaken both the spread of epidemic and immunization. In addition, recent studies such as Moore et al. [14] find that vaccination alone is insufficient to contain the pandemic. Aldila et al. [15] suggested a mathematical model that the Jakarta government could use to ease social distancing by building on the potential of a future COVID-19 vaccine. Besides, Ghostine et al. [16] proposed an extended SEIR model with a vaccination compartment to simulate the spread of COVID-19 in Saudi Arabia. They showed the effect of vaccination on the spread of the epidemic using their model.

Game theory is a branch of applied mathematics used to analyze situations that require decisions by parties called players. Game theory is also well suited to model the decision making of individuals facing several intervention options,

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such as vaccination and social distancing, to protect themselves from an epidemic spread. In this game, each player tries to improve his profit which is called payoffs and he has to take a strategy among the strategies available to him. Choosing the best strategy for a player depends on what the other player will decide [17].

According to the NMTC [18], medical indicators and statistics confirmed that the booster shots had been effective. From this point of view, and the possibility of taking booster doses, the significance of this study will be made evident. Through the game theory approach, the importance of taking the vaccine and the booster shots will be demonstrated in reducing the chances of contracting Covid-19 disease, reducing hospital admissions, reaching critical conditions and dying due to Covid-19.

II. METHOD

Game theory approach for COVID-19 in Bahrain

Our model consists of three parts. In these parts, we will examine the effect of taking a full dose and a booster dose against COVID-19 and not getting vaccinated on the chance of getting sick, reaching a hospital in a critical condition and dying from COVID-19. Also, in each part, we will repeat the first game (basic stage) to study the effect of the booster shots. In the first round, individuals will choose either to take the vaccine or not. Then, in the repeated round, the vaccinated individuals will decide to take the booster doses, while others still do not choose to take the vaccine.

We took the probabilities of the vaccinated individuals contracting COVID-19 disease, entering the hospital in the ICU, and dying from COVID-19 from the data published on the website of the Ministry of Health (MOH) of Bahrain.

To choose the Nash equilibrium point, which is the point that has the best strategic response [19] and no player can benefit from unilaterally changing their strategy. First, we marked the highest value of each column in the first entry of the binary and the same for the value in each row of the second entry in the binary [20]. The binary with the marks on each entry becomes Nash equilibrium points.

Part 1: Procedure to create a payoff matrix for vaccination and getting infected (VI)

1. The player strategy set consists of two elements as $S = (V_I, V_I')$, where
   V_I: Get vaccinated
   V_I': Not get vaccinated
2. According to the MOH statistics in Bahrain, from January 2021 to May 19, 2021, the number of vaccinated individuals who completed the two doses of the COVID-19 vaccine and got infected was only 6%. In comparison, the percentage of individuals who didn't vaccinate and got infected was 94% [21]. Thus, in this game, we can create the payoff matrix by using the quantities obtained by the MOH statistics. Let the payoff for the strategy V_I to be I' and the payoff of the strategy V_I' to be I, where
   I': The probability of a person not be infected is 0.94
   I: The probability of a person being infected is 0.06. We will use a negative sign in this probability to show the effect of infection reflected in players' utilities.
3. If the two players choose the action, get vaccinated V_I, then the strategy (V_I, V_I) appears, and the payoff for both of them is I'.
4. If one player chooses the action, not get vaccinated V_I', then there is a chance to get infected and the strategy (V_I, V_I') or (V_I', V_I) appear, so the payoff is either $(I', -I)$ or $(-I, I')$.
5. If the two players choose the action, not get vaccinated V_I', then the strategy (V_I', V_I') appears, and the payoff for both of them is $-I$.

According to the above procedures, the basic payoff matrix of the first round of vaccination and getting infected VI can be created as:

$$Payoff\ matrix\ VI = \begin{bmatrix} & V_I & V_I' \\ V_I & (I', I') & (I', -I) \\ V_I' & (-I, I') & (-I, -I) \end{bmatrix}$$

Where,

V_I: Get vaccinated

V_I': Not get vaccinated and,

I': The probability of a person to be not infected

I: The probability of a person being infected

Thus, the basic payoff matrix of vaccinated and getting infected or not infected VI is

$$Payoff\ matrix\ VI = \begin{bmatrix} & V_I & V_I' \\ V_I & (\overline{0.94}, \overline{0.94}) & (\overline{0.94}, -0.06) \\ V_I' & (-0.06, \overline{0.94}) & (-0.06, -0.06) \end{bmatrix}$$

To see the gain or loss in this game, as mentioned in the method, we marked the highest value in every column of the first entry of the binary and marked the highest value in every row in the second entry of the binary. Thus, the Nash equilibrium point of this game will be (0.94,0.94), which is (I', I') if the two players decide to get vaccinated, the chance to get infected is meagre. The worst choice is $(-0.06, -0.06)$ when the two players choose not to get vaccinated; there is a chance to get infected. It's clear that the strategy $(-I, -I)$ is not worth playing and (I', I') is the best to play. Here we generate the tree diagrams of this game's repeated round if the two players decide to take the booster shots of the vaccine to see the best and worst choice using the game tree shown in Figure 1.

From the tree diagram of Figure 1, the payoff biometric of the game of the two rounds is given as follows:

$$\begin{bmatrix} & V_IV_I & V_IV_I' & V_I'V_I & V_I'V_I' \\ V_IV_I & (\overline{1.88}, \overline{1.88}) & (\overline{1.88}, 0.88) & (0.88, \overline{1.88}) & (0.88, 0.88) \\ V_IV_I' & (\overline{1.88}, 0.88) & (\overline{1.88}, -0.12) & (0.88, 0.88) & (0.88, -0.12) \\ V_I'V_I & (0.88, \overline{1.88}) & (0.88, 0.88) & (-0.12, \overline{1.88}) & (-0.12, 0.88) \\ V_I'V_I' & (0.88, 0.88) & (0.88, -0.12) & (-0.12, 0.88) & (-0.12, -0.12) \end{bmatrix}$$

To find the Nash equilibrium point from the generated payoff biometric, we will follow the same method used in the first round and marked the highest value in every column of the first entry and every row in the second entry of the binary. Thus, the Nash equilibrium point will be (1.88,1.88). Therefore, the best choice of players is to get the booster dose

to reduce the chance of infection. On the other hand, the worst strategy at $(-0.12, -0.12)$ means that the players can be infected when they decide not to get vaccinated.

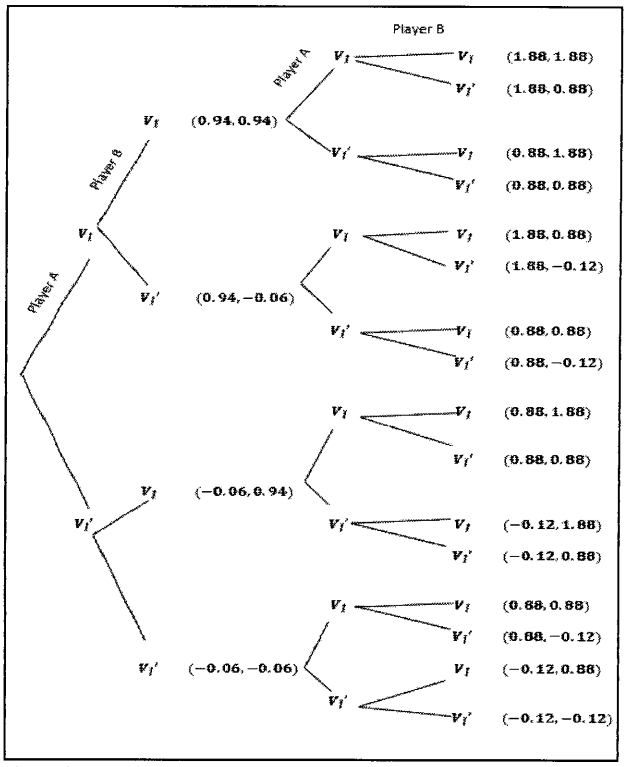


Fig. 1 The tree diagram of the two rounds of the game (part 1)

We will follow the same procedure in parts 2 and 3 to see the impact of vaccination and booster shots in reducing the chances of vaccinated individuals to end up in critical cases and dying due to COVID-19.

Part 2: Procedure to create a payoff matrix for vaccination and being in the critical case due to COVID-19 (VC)

1. The player strategy set consists of two elements as $S = (V_C, V_C')$, where
   V_C: Get vaccinated
   V_C': Not get vaccinated
2. According to the MOH statistics in Bahrain, on June 1St 2021, the total number of critical cases reached 326, of which 270 cases didn't receive the vaccine against COVID-19 [22]. Thus, in this game, we can create the payoff matrix by using the quantities obtained by the MOH statistics. we can choose the payoff for the strategy V_C to be $C'$and the payoff of the strategy V_C' to be C, where
   C': The probability of a person not being in the critical case from COVID-19 is 0.83
   C: The probability of a person being in the critical case from COVID-19 is 0.17. We will use a negative sign in this probability to show the effect of infection reflected in players' utilities.
3. If the two players choose the action, get vaccinated V_C, then the strategy (V_C, V_C) appears, and the payoff for both of them is C'.
4. If one player chooses the action, not get vaccinated V_C', then there is a chance to be in a critical case from COVID-19 and the strategy (V_C, V_C') or (V_C', V_C) appear, so the payoff is either$(C', -C)$ or $(-C, C')$.
5. If the two players choose the action, not get vaccinated V_C', then the strategy (V_C', V_C') appears, and the payoff for both of them is $-C$.

According to the above procedures, the basic payoff matrix of the first round of vaccination and being in the critical case from COVID-19 VC can be created as:

$$Payoff\ matrix\ VC = \begin{bmatrix} & V_C & V_C' \\ V_C & (C', C') & (C', -C) \\ V_C' & (-C, C') & (-C, -C) \end{bmatrix}$$

Where,

V_C: Get vaccinated

V_C': and Not get vaccinated ,

C': Probability of a person not being in the critical case from COVID-19

C: Probability of a person being in the critical case from COVID-19.

Thus, the basic payoff matrix of vaccinated and being in the critical case from COVID-19 VC is

$$Payoff\ matrix\ VC = \begin{bmatrix} & V_C & V_C' \\ V_C & (\overline{0.83}, \overline{0.83}) & (\overline{0.83}, -0.17) \\ V_C' & (-0.17, \overline{0.83}) & (-0.17, -0.17) \end{bmatrix}$$

To see the gain or loss in this game, the Nash equilibrium point of this game is (0.83,0.83), which is (C', C') if the two players decide to get vaccinated, the chance to be in the critical case from COVID-19 is low. The worst choice is $(-0.17, -0.17)$ when the two players choose not to get vaccinated; there is a chance to be in the critical case from COVID-19. It's clear that the strategy $(-C, -C)$ is not worth playing and (C', C') is the best to play.

From the tree diagram of Figure 2, the payoff biometric of the game of the two rounds is given as follows:

$$\begin{bmatrix} & V_CV_C & V_CV_C' & V_C'V_C & V_C'V_C' \\ V_CV_C & (\overline{1.66}, \overline{1.66}) & (\overline{1.66}, 0.66) & (0.66, \overline{1.66}) & (0.66, 0.66) \\ V_CV_C' & (\overline{1.66}, 0.66) & (\overline{1.66}, -0.34) & (0.66, 0.66) & (0.66, -0.34) \\ V_C'V_C & (0.66, \overline{1.66}) & (0.66, 0.66) & (-0.34, \overline{1.66}) & (-0.34, 0.66) \\ V_C'V_C' & (0.66, 0.66) & (0.66, -0.34) & (-0.34, 0.66) & (-0.34, -0.34) \end{bmatrix}$$

From the repeated round generated biometrics, the Nash equilibrium point is (1.66,1.66). So, the best choice is if players decide to get vaccinated by the booster doses to reduce the risk of being in the critical case because of COVID-19. On the other hand, the worst strategy at $(-0.34, -0.34)$ means that the chance of being in critical cases will double when they didn't decide to take the vaccine.

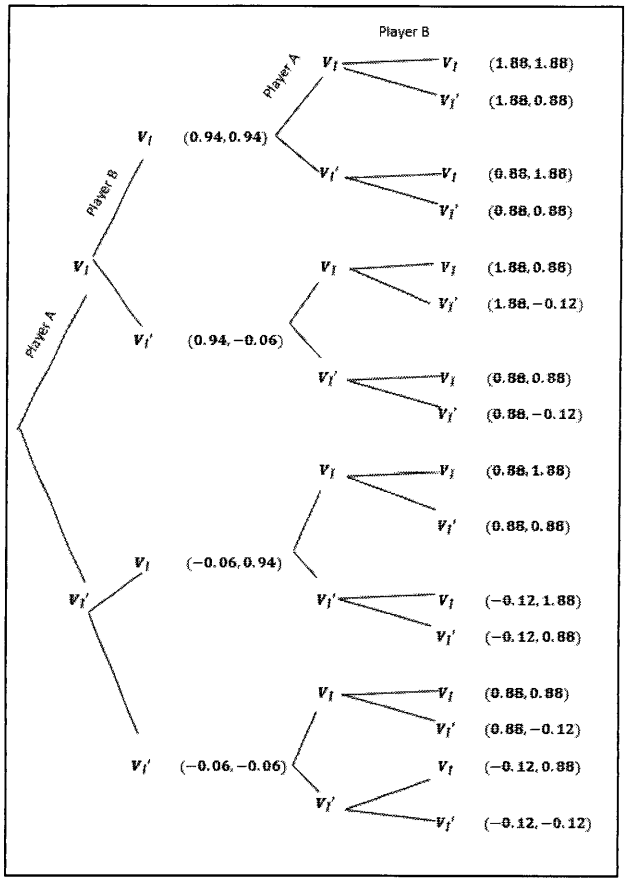


Fig. 2 The tree diagram of the two rounds of the game (part 2)

Part 3: Procedure to creating a payoff matrix for vaccination and exposure to death due to COVID-19 (VD)

1. The player strategy set consists of two elements as $S = (V_D, V_D')$, where
   V_D: Get vaccinated
   V_D': Not get vaccinated

2. According to the MOH statistics in Bahrain, from January 2021 to May 2021, the total number of death cases was 665. However, only 10% got vaccinated, and 90% didn't get vaccinated. Of the 10% who have already been vaccinated, 94% have a chronic disease [23].

   Thus, in this game, we can create the payoff matrix by using the quantities obtained by the MOH statistics. We can choose the payoff for the strategy V_D to be D' and the payoff of the strategy V_D' to be D, where
   D': Probability of a person not dying from COVID-19 is 0.9
   D: The probability of a person dying from COVID-19 is 0.1. We will use a negative sign in this probability to show the effect of infection reflected in players' utilities
3. If the two players choose the action, get vaccinated V_D, then the strategy (V_D, V_D) appears, and the payoff for both of them is D'.
4. If one player chooses the action, not get vaccinated V_D', then there is a chance to die from COVID-19 and the strategy (V_D, V_D') or (V_D', V_D) appear, so the payoff is either $(D', -D)$ or $(-D, D')$.
5. If the two players choose the action, not get vaccinated V_D', then the strategy (V_D', V_D') appears, and the payoff for both of them is $-D$.

According to the above procedures, the basic payoff matrix of the first round of vaccination and died from COVID-19 VD can be created as:

$$Payoff\ matrix\ VD = \begin{bmatrix} & V_D & V_D' \\ V_D & (D', D') & (D', -D) \\ V_D' & (-D, D') & (-D, -D) \end{bmatrix}$$

Where,

V_D: Get vaccinated

V_D': Not get vaccinated and,

D': Probability of a person not dying from COVID-19.

D: Probability of a person dying from COVID-19.

Thus, the basic payoff matrix of vaccinated and dying from COVID-19 VD is

$$Payoff\ matrix\ VD = \begin{bmatrix} & V_D & V_D' \\ V_D & (\overline{0.9}, \overline{0.9}) & (\overline{0.9}, -0.1) \\ V_D' & (-0.1, \overline{0.9}) & (-0.1, -0.1) \end{bmatrix}$$

To see the gain or loss in this game, the Nash equilibrium point of this game is (0.9,0.9), which is (D', D') if the two players decide to get vaccinated. The worst choice is $(-0.1, -0.1)$ when the two players choose not to get vaccinated; there is a chance to die from COVID-19. It's clear that the strategy $(-D, -D)$ is not worth playing and (D', D') is the best to play.

From the tree diagram of Figure 3, the payoff biometric of the game of the two rounds is given as follows:

$$\begin{bmatrix} & V_DV_D & V_DV_D' & V_D'V_D & V_D'V_D' \\ V_DV_D & (\overline{1.8}, \overline{1.8}) & (\overline{1.8}, 0.8) & (0.8, \overline{1.8}) & (0.8, 0.8) \\ V_DV_D' & (\overline{1.8}, 0.8) & (\overline{1.8}, -0.2) & (0.8, 0.8) & (0.8, -0.2) \\ V_D'V_D & (0.8, \overline{1.8}) & (0.8, 0.8) & (-0.2, \overline{1.8}) & (-0.2, 0.8) \\ V_D'V_D' & (0.8, 0.8) & (0.8, -0.2) & (-0.2.0.8) & (-0.2, -0.2) \end{bmatrix}$$

From the repeated round generated biometric, the Nash equilibrium point is (1.8,1.8). Thus, the best option is if players decide to get the vaccine's booster doses. The booster doses will reduce twice the chance of dying from COVID-19. On the other hand, the worst strategy at $(-0.2, -0.2)$ means that the players die from COVID-19 when they decide not to be vaccinated, and this chance is doubled in the next round.

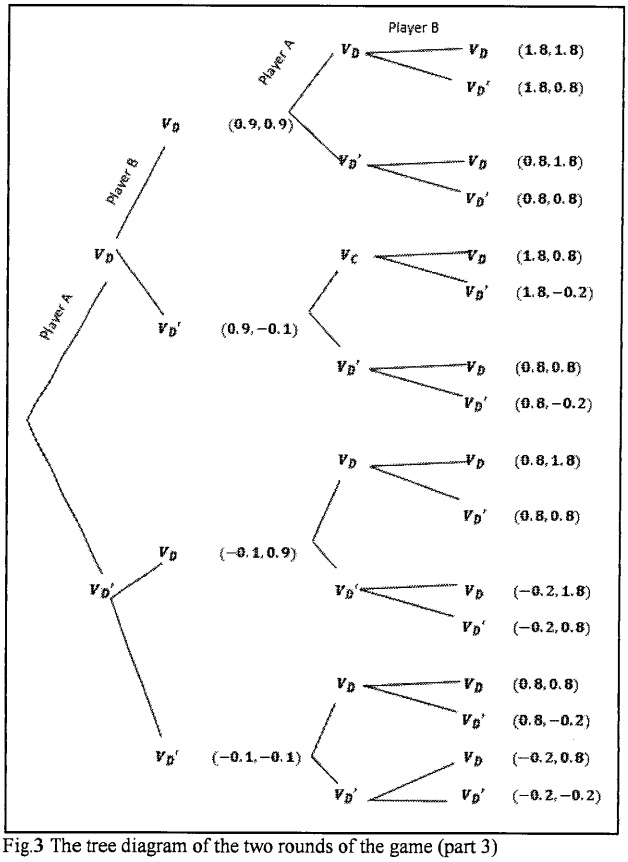


Fig.3 The tree diagram of the two rounds of the game (part 3)

III. Results and Discussion

Vaccinations significantly impact protecting everyone, reducing infection, and reducing the severity of symptoms that can lead to death (MOH, 2021b). This study used the MOH of Bahrain statistics from January to June 2021 published on its official website in using game theory in the three studied parts. The results of the three parts of game theory were as follows:

The Nash equilibrium points are (0.94,0.94) and (1.88,1.88) of the basic and repeated rounds, respectively, when players choose to take the vaccine against COVID-19 with the booster doses. That means taking the vaccine and the booster doses help reduce the probability of individuals being infected by COVID-19. On the other hand, the probability of nonvaccinated individuals' infection is 0.06 in the basic round, and this probability doubles when they choose not to take the vaccine. At the same time, other individuals decide to take the booster dose.

For the second part, the Nash equilibrium points for the two rounds are (0.83,0.83) and (1.66,1.66), when players choose to take the vaccine against COVID-19 with the booster doses. This means that taking the vaccine in booster doses helps reduce infected individuals to reach the critical cases. And the probability will double with the booster doses. On the other hand, the probability of nonvaccinated individuals' being in the critical cases is 0.17 in the basic round. This probability double when they choose not to take the vaccine and, at the same time, other individuals decide to take the booster dose.

As for the last part, the Nash equilibrium points for the two rounds are (0.9,0.9) and (1.8,1.8) when players choose to take the vaccine against COVID-19 with the booster doses. This means that taking the vaccine booster doses helps protect vaccinated individuals from the risk of death. This probability double with the booster doses. However, the risk of death for unvaccinated individuals is 0.1. Thus, the chance of death double when vaccinated individuals take the booster doses, while others do not take the vaccine.

This study proves what the MOH announced regarding the vaccination and the booster doses. For example, by July 27 2021, the MOH in Bahrain announced that 131,192 people had received booster shots. After two weeks, only 71 of those individuals (0.05%) became infected with COVID-19 and none of the people infected with the virus required hospitalization. This indicates that the booster shots effectively reduce symptoms' severity, which can sometimes result in death. Furthermore, booster shots were linked to a significant reduction in mortality, with no hospital admissions recorded in Bahrain for individuals two weeks after receiving it [24].

IV. Conclusion

This study can be considered an evidence of the efforts and studies carried out by the Ministry of Health in Bahrain, which prove the effectiveness of vaccination and booster doses in reducing the number of infected people and reducing critical cases and deaths due to COVID-19. We used game theory in this study to show that full doses of vaccination and a booster dose can reduce the risk of developing COVID-19 disease. It also reduces the possibility of patients reaching critical cases and reduces the risk of death in contrast to others not taking the vaccine.

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Appendix (C)

Application of the game theory, on a suggested model of COVID-19 coronavirus transmission in the Kingdom of Bahrain

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Abstract

To control the spread of a pandemic, preventive measures such as vaccination and social distancing are used. In this contribution, a model based on game theory is presented where a payoff function is constructed in terms of the players' available strategies. The relative cost of each strategy determines the most appropriate strategy for people to implement. In the absence of social distancing, vaccination is chosen when its cost equals the risk of infection for non-vaccinated individuals compared to vaccinated individuals. In the absence of vaccination, social distancing is chosen when its cost equals the risk of infection when social distancing is not applied. In the case of both vaccination and social distancing, the optimal strategy is to apply vaccination regardless of its cost. Using the data for Bahrain, we found that through vaccination, it is possible to create a societal immunity capable of confronting the spread of COVID-19 throughout Bahrain.

Keywords: COVID-19, game theory, Epidemic model, Reproduction number.

1 Introduction

Waves of SARS-CoV-2 infections have transfered through populations worldwide, and thus, the COVID-19 pandemic has imposed a substantial international health burden [1]. As human behaviour plays a significant role in the spread of infectious diseases, understanding this behaviour will help control and contain the spread of infectious diseases. [2]. Social distancing is a human behaviour that is known to reduce the spread of diseases. Factor-based flu simulation studies [3] suggest that small changes in behaviour can significantly affect transmission patterns during an epidemic and have stimulated theoretical work around social distancing. In addition, many studies [4] have shown that social distancing can stop epidemics if it starts quickly and lasts for a relatively long time. Also, vaccination has an important role in reducing the spread of infectious diseases [5, 6].

Mathematical modelling has become a vital tool in analyzing and monitoring the spread of infectious diseases, given the main factors that control disease progression-such as transmission and recovery rates-that help predict how the disease will spread over the years [7]. In 1927, Kermack

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and McKendrick [8] presented the first model to predict the progression of an epidemic called the SIR model, which consists of three differential equations describing disease dynamics and individual transmissions in a population between three compartments. Later many studies modified the SIR model, as describe in Ref [9–12]. Moreover, Hellewell et al. [13] developed a random transmission model to understand the effects of isolation and contact tracing for COVID-19 cases.

Game theory has also emerged as an approach to the quantitative description of the decision-making process of individuals with different intervention options. A number of studies [14–16] have used game theory models that focus on preventive measures, particularly vaccination and behavioural changes. Choi and Shim [17] created a game-theoretical epidemiological model considering vaccination and social distancing and based on the assumption that people seek to maximize their profits and individually select the optimal strategy based on Nash's strategy. They also identified the relative costs of control strategies in which individuals prefer vaccination over social distancing (or vice versa). Kabir and Tanimoto [18] combined partial epidemiological models with models of behavioural dynamics as described by evolutionary game theory. They developed simulations of how compliance with a shutdown might decline as people weigh the risk of infection against the certainty of the economic cost of staying home. Dashtbali and Mirzaie [19] added the vaccination term in their game theory model to predict how vaccination would control the epidemic. In addition, Reluga [15] reported in his game theory approach that when the basic reproduction number (R_0) was around 2, then the social distancing was more beneficial to individuals. In addition, he found that optimum social distancing never recoups more than 30% of the cost of infection in the absence of vaccination. Wei et al. [20] aggregate interaction strategies and evolutionary game analysis of government and public actions in the epidemic's early days. They concluded that the government's emergency response strategy dominated the outbreak.

In this manuscript, we study the impact of applying vaccination and social distancing strategies to reduce the spread of COVID-19 in Bahrain by using a game theoretical approach. In the suggested model we include the vaccination, exposed and hospitalization compartments with extra parameters such as the efficacy, vaccination, hospitalization, duration of the latent period and social distance rates. The manuscript is organized as follows: Sec.(2) introduces the dynamical system which governs this model. In Sec.(3), we investigate the vaccination strategy and game theory is used to study the stability of this strategy. The strategy of the social distance and the corresponding game theory is considered in Sec.(4). Sec.(5) is devoted to discuss the combined strategies; vaccination and social distance. Finally, we discussed and concluded our results in Sec.(6).

2 The suggested Model

The diagram of the suggested model is given in Fig.(1) where, susceptible individuals $S = S(t)$, number of individuals susceptible to disease at time t, vaccinated individuals $V = V(t)$, number of individuals vaccinated at time t, exposed individuals $E = E(t)$, individuals in the incubation period after being infected with the virus without apparent symptoms at time t, un-quarantined infected $U = U(t)$, infected asymptomatic individuals at time t, quarantined infected $Q = Q(t)$, infected individuals with or without symptoms at time t, confirmed infected $C = C(t)$, confirmed infected individuals at time t, hospitalized infected $H = H(t)$, infected individuals who require hospitalization at time$t$, recovered $R = R(t)$, survived individuals at time t. Where total population

$$N = N(t) = S(t) + V(t) + E(t) + U(t) + Q(t) + C(t) + H(t) + R(t)$$

is nonnegative for all $t \geq 0$. Let us assume that the total number of population is constant, namely $N(t) =$ constant. In this case, one can express the rate of the individual susceptible set $S(t)$ as,

$$\frac{dS(t)}{dt} = -\frac{dV(t)}{dt} - \frac{dE(t)}{dt} - \frac{dU(t)}{dt} - \frac{dQ(t)}{dt} - \frac{dC(t)}{dt} - \frac{dH(t)}{dt} - \frac{dR(t)}{dt}, \tag{1}$$

By using theses assumptions, the $VEUQCHR$ model may be described by the following Equations,

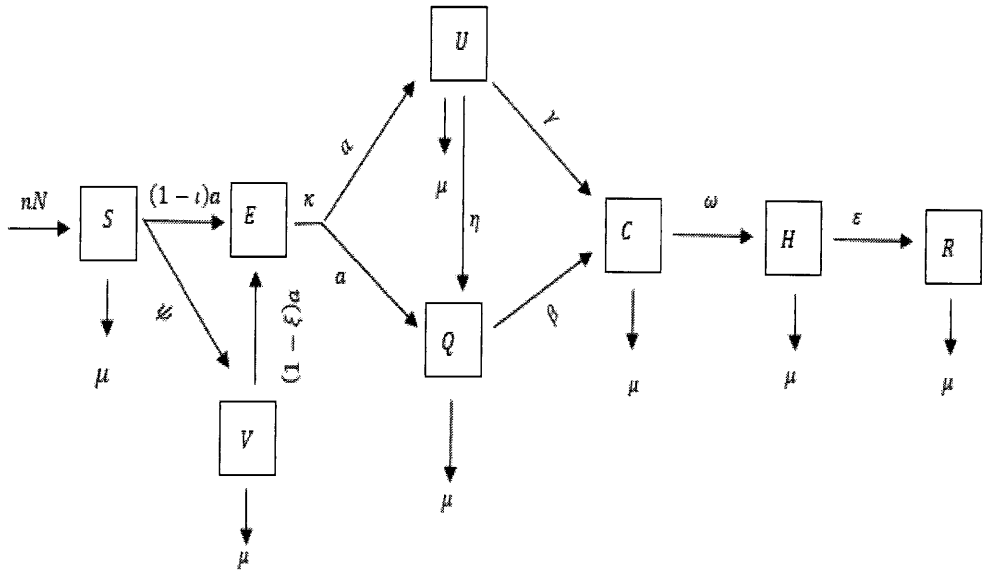


Figure 1: The diagram of the suggested model.

$$
\begin{aligned}
\frac{dV(t)}{dt} &= \psi S(t) - (1-\xi)a\frac{V(t)}{N(t)}E(t) - \mu V(t),\\
\frac{dE(t)}{dt} &= (1-\iota)a\frac{S(t)}{N(t)}E(t) + (1-\xi)a\frac{V(t)}{N(t)}E(t) - \mu E(t) - \kappa a\frac{E(t)}{N(t)}U(t) - \kappa a\frac{E(t)}{N(t)}Q(t),\\
\frac{dU(t)}{dt} &= \kappa a\frac{E(t)}{N(t)}U(t) - (\mu+\gamma+\eta)U(t),\\
\frac{dQ(t)}{dt} &= \kappa a\frac{E(t)}{N(t)}Q(t) - (\mu+\beta)Q(t) + \eta U(t),\\
\frac{dC(t)}{dt} &= \gamma U(t) + \beta Q(t) - (\mu+\omega)C(t),\\
\frac{dH(t)}{dt} &= \omega C(t) - (\mu+\varepsilon)H(t),\\
\frac{dr(t)}{dt} &= \varepsilon H(t) - \mu R(t).
\end{aligned}
\tag{2}
$$

The parameters and variables used in system (2) are defined in Table (1).

Parameter	Description	Units
a	Infection rate	Day $^{-1}$
N	Total population number	Day
n	Birth rate	Per 1000 people
μ	Death rate	Per 1000 people
γ	Non-isolated infected people rate	Day^{-1}
β	Exit tests rate	Day^{-1}
ε	Recovery rate	Day^{-1}
η	Contact tracing rate	Day^{-1}
ι	Fraction by which individuals reduces their contact rate	Day^{-1}
ψ	Vaccination rate	Day^{-1}
ω	Hospitalization rate	Day^{-1}
ξ	Vaccination efficacy	Per infected people
$\frac{1}{\kappa}$	Duration of the latent period	Day

Table 1: Definition of parameters in the $SVEUQCHR$ model

Normalizing the parameters give,

$$v(t)=\frac{v(t)}{N}, e(t)=\frac{E(t)}{N}, u(t)=\frac{U(t)}{N(t)}, q(t)=\frac{Q(t)}{N(t)}, c(t)=\frac{C(t)}{N(t)}, h(t)=\frac{H(t)}{N(t)}, r(t)=\frac{R(t)}{N(t)},$$

and the system of equation (2) reduces to

$$\begin{aligned}
\frac{dv(t)}{dt} &= \psi(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))-(1-\xi)av(t)e(t)-\mu v(t),\\
\frac{de(t)}{dt} &= (1-\iota)a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t)\\
&+ (1-\xi)av(t)e(t)-\mu e(t)-\kappa av(t)e(t)u(t)-\kappa av(t)e(t)q(t),\\
\frac{du(t)}{dt} &= \kappa av(t)e(t)u(t)-(\mu+\gamma+\eta)u(t),\\
\frac{dq(t)}{dt} &= \kappa av(t)e(t)q(t)-(\mu+\beta)q(t)+\eta u(t),\\
\frac{dc(t)}{dt} &= \gamma u(t)+\beta q(t)-(\mu+\omega)c(t),\\
\frac{dh(t)}{dt} &= \omega c(t)-(\mu+\varepsilon)h(t),\\
\frac{dr(t)}{dt} &= \varepsilon h(t)-\mu r(t). \qquad (3)
\end{aligned}$$

We consider the following initial conditions,

$$v(0)=v_0\geq 0;\ e(0)=e_0\geq 0;\ u(0)=u_0\geq 0;\ q(0)=q_0\geq 0;\ c(0)=c_0\geq 0,\ h(0)=h_0\geq 0;\ r(0)=r_0\geq 0. \qquad (4)$$

The system is then solved numerically and we investigate the dynamical stability of the suggested model, by calculating the equilibrium point of the system, and the control reproduction number R_c.

2.1 The Equilibrium point

We evaluate the equilibrium point of the system (3) under the conditions (4 and call it the disease-free equilibrium point, meaning that there is no disease exists in the population. Based on the

next-generation matrix method [21], we determine the initial reproduction number R_0 at the free equilibrium point P_0, which determines whether or not there is a single non-negative equilibrium and stability [21]. Moreover, the number R_0 may be used to check the stability of the system (3). However, if the initial reproduction number $R_0 > 1$, then the system (3) is unstable, resulting in continued pandemic spread whereas if $R_0 < 1$, the system (3) is asymptotically stable and the disease disappears [22]. The disease-free equilibrium point of the system (3) is given by

$$(\frac{\psi}{\psi+\mu}, 0, 0, 0, 0, 0, 0). \tag{5}$$

2.2 The control and basic reproduction number by the next-generation approach

We now evaluate the basic (initial) reproduction number R_0 by using the next-generation matrix approach [21]. For this aim, let us define the vectors X and Y as the infected and uninfected classes, respectively and the vector of new infection rates that flows from Y to X be $f(X;Y)$. Finally, we assume that $v(X;Y)$ be the vector of all other rates which are not new infection, where the inflow in compartment V is negative and the outflow in compartment V is positive.

The next-generation matrix G reflects the number of recently infected individuals in different generations over time, and is the number of recently infected individuals within different groups [23] as $G = FV^{-1}$ where $F = \left[\frac{df}{dx}\right]\Big|_{P_0}$ and $V = \left[\frac{dv}{dx}\right]|_{P_0}$. The reproduction number R_0 represents the average number of new cases that are created by a single case and is the dominant eigenvalue of a generation matrix $G = FV^{-1}$ [21], namely $R_0 = \rho(FV^{-1})$ where $\rho(G)$ is the spectra radius of the matrix G. Here we will calculate the control reproduction number R_c and basic reproduction number R_0 for the $SVEUQCHR$ model using the next generation matrix approach [21]. The vectors $\vec{f}$ and $\vec{v}$ are given by

$$\begin{aligned} f &= ((1-\iota)a(1-v(t)-e(t)-u(t)-q(t)-c(t)-h(t)-r(t))e(t)+(1-\xi)av(t)e(t)), \\ v &= \begin{pmatrix} \mu e(t)+\kappa ae(t)u(t)+\kappa ae(t)q(t) \\ -\kappa ae(t)u(t)+(\mu+\gamma+\eta)u(t) \\ -\kappa ae(t)q(t)+(\mu+\beta)q(t)-\eta u(t) \\ -\gamma u(t)-\beta q(t)+(\mu+\omega)c(t) \end{pmatrix}, \end{aligned} \tag{6}$$

At $(\frac{\psi}{\psi+\mu}, 0, 0, 0, 0, 0, 0)$, the matrices F and V may be written as,

$$F = \begin{bmatrix} (1-\iota)a(1-\frac{\psi}{\psi+\mu})+(1-\xi)a\frac{\psi}{\psi+\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \mu & 0 & 0 & 0 \\ 0 & \mu+\gamma+\eta & 0 & 0 \\ 0 & -\eta & \mu+\beta & 0 \\ 0 & -\gamma & -\beta & \mu+\omega \end{bmatrix}, \tag{7}$$

and consequently the next generation matrix $G = FV^{-1}$ may be written explicitly as,

$$G = \begin{bmatrix} \frac{1}{\mu}(1-\iota)a(1-\frac{\psi}{\psi+\mu})+(1-\xi)a\frac{\psi}{\mu(\psi+\mu)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

Now, by evaluating the spectra radius of the matrix G, the control reproduction number $R_c(\psi,\iota)$ of system (3) will be

$$R_c(\psi,\iota) = \frac{(1-\iota)a}{\mu} + \frac{(\iota-\xi)a\psi}{\mu(\psi+\mu)}. \tag{9}$$

By setting $\psi = 0$ and $\iota = 0$, namely there are no control measures, the control reproduction number $R_c(\psi, \iota)$ reduces to the basic reproduction number $R_0 = \frac{a}{\mu}$. The change rate of $R_c(\psi, \iota)$ with respected to vaccination $\frac{dR_c(\psi,\iota)}{d\psi} = \frac{a(\iota-\xi)}{(\psi+\mu)^2} < 0$, if $\iota < \xi$. This means that, the control reproduction number decreases if the vaccination efficacy is more than the social distancing rate ($\iota < \xi$). However, if the vaccine efficacy is not high than the social distancing, then R_c increases, and the epidemic will not be controlled.

In the following sections, we shall consider three different types of strategies, vaccination only, social distance only and the third is combining both the vaccination and the social distance. Moreover, we use the game theory as an application to discuss the three strategies

3 Vaccination strategy

We will consider vaccination as the only personal protection against COVID-19, namely it is assumed that, the vaccination is the sole personal protection against COVID-19. Therefore, if we set $\iota = 0$ in system (3), one can discuss the vaccination strategy only (VS). By following the same procedure that described in Sec.(3), the vaccination control reproduction number may be written as,

$$R_c(\psi) = R_0\Big(1 - \frac{\xi\psi}{\psi + \mu}\Big). \tag{10}$$

Moreover for the VS, the threshold value of the vaccination ψ_{thr} is given by,

$$\psi_{thr} = \frac{\mu(R_0 - 1)}{R_0(1 - \xi) - 1}, \tag{11}$$

where we solved $R_c(\psi) = 1$. In our numerical treatment we assume that the vaccination efficacy $\xi = 0.73$, and $1 < R_0 < \frac{1}{1-\xi}$. However, if $R_0 > \frac{1}{1-\xi}$ then the diseases cannot be controlled by vaccination alone. If the vaccination rate is less than the vaccination threshold, namely $\psi < \psi_{thr}$ then the disease remains endemic. Otherwise, the disease can be eradicated. The endemic point of the VS is given by

$$P^*_\psi = (v^*_\psi, e^*_\psi, u^*_\psi, q^*_\psi, c^*_\psi, h^*_\psi, r^*_\psi), \tag{12}$$

where

$$\begin{aligned}
e^*_\psi &= \frac{\mu+\gamma+\eta}{\kappa a},\ u^*_\psi = \frac{(\mu+\omega)(\gamma+\eta-\beta)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c^*_\psi,\ q^*_\psi = \frac{-\eta(\mu+\omega)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c^*_\psi,\\
h^*_\psi &= \frac{\omega}{\mu+\varepsilon}c^*_\psi,\ r^*_\psi = \frac{\varepsilon\omega}{\mu(\mu+\varepsilon)}c^*_\psi,\\
v^*_\psi &= \frac{(R_0-1)\kappa a-(\mu+\gamma+\eta)}{\xi R_0\kappa a} - \frac{(\kappa+1)((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)-(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))}{\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)}c^*_\psi,\\
c^*_\psi &= \frac{Y}{Z},
\end{aligned}$$

and,

$$\begin{aligned}
Y &= (\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta))(\xi R_0\kappa\psi(\kappa a-\mu+\gamma+\eta)\\
&\quad -((R_0-1)\kappa a-(\mu+\gamma+\eta))(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)),\\
Z &= \xi R_0\kappa^2 a(\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)((\kappa+1)((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)\\
&\quad -(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa))\\
&\quad +\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)(\psi((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)+(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))),
\end{aligned}$$

The change rate of the confirmed infected end point $c*$ with respect to the vaccination rate is given by,

$$\frac{dc_\psi^*}{d\psi} = \frac{k_1}{k_2} - \frac{L_1}{L_2}, \tag{13}$$

$$\begin{aligned}
K_1 &= \mu(\gamma(\gamma+\eta-\beta)-\beta\eta)(\mu\gamma(\gamma+\eta-\beta)-\beta\eta)(\kappa R_0\xi(\gamma-\mu+a\kappa+\eta)-\kappa(-\gamma+a\kappa(R_0-1)-\mu-\eta)),\\
K_2 &= a\kappa R_0((\kappa+1)\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)((-\omega-\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))\\
&\quad (\kappa\psi+(1-\xi)(\gamma+\mu+\eta)+\kappa\mu)+\xi\kappa(\mu\gamma(\gamma+\eta-\beta)-\beta\eta)((\omega+\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)\\
&\quad (\gamma+\eta-\beta)-\eta(\omega+\mu))\psi),
\end{aligned}$$

$$\begin{aligned}
L_1 &= ((\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)(\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)(\kappa\xi\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)\omega+\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)\\
&\quad +(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))+\kappa(\kappa+1)\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)\\
&\quad ((-\omega-\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu)))(\kappa R_0\xi(\gamma-\mu+a\kappa+\eta)\psi\\
&\quad -(-\gamma+a\kappa(R_0-1)-\mu-\eta)(\kappa\psi+(1-\xi)(\gamma+\mu+\eta)+\kappa\mu),\\
L_2 &= a\kappa R_0((\kappa+1)\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)((-\omega-\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)\\
&\quad +(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))(\kappa\psi+(1-\xi)(\gamma+\mu+\eta)+\kappa\mu)+\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\beta\eta)\\
&\quad ((\omega+\mu)(\gamma(\gamma+\eta-\beta)-\beta\eta)+(\omega+\mu)(\gamma+\eta-\beta)-\eta(\omega+\mu))\psi)^2,
\end{aligned} \tag{14}$$

and by using (14), one can show that $\frac{dc_\psi^*}{d\psi} < 0$ if $\psi < \frac{\eta-\beta+\gamma}{\eta}$.

3.1 Applying of game theory

In this section, we apply game theory to discuss the vaccination strategy, SV. For this aim, consider that we have two groups: the first group V which represents the individuals who choose to be vaccinated and the second group V' for the individuals who refuse to take the vaccine. Let π_v be the probability of infection among vaccination individuals and $\pi_{V'}$ be the probability of infection among non-vaccination individuals. These probabilities can be expressed as the proportion of susceptible individuals becoming infected over the number of individuals dying during any unit of time [14]. Thus

$$\pi_V = \frac{(1-\xi)a(\gamma u_\psi^* + \beta q_\psi^*)}{(1-\xi)a(\gamma u_\psi^* + \beta q_\psi^*)+\mu}, \quad \pi_{V'} = \frac{a(\gamma u_\psi^* + \beta q_\psi^*)}{a(\gamma u_\psi^* + \beta q_\psi^*)+\mu}. \tag{15}$$

Then by using the endemic point $P_\psi^* = (v_\psi^*, e_\psi^*, u_\psi^*, q_\psi^*, c_\psi^*, h_\psi^*, r_\psi^*)$ calculated in (12), the probabilities π_V and π_V' may be written as,

$$\begin{aligned}
\pi_V &= \frac{(1-\xi)ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))}{(1-\xi)ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)},\\
\pi_{V'} &= \frac{ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))}{ac_\psi^*((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)}.
\end{aligned} \tag{16}$$

Let $\pi_{VV'}$ be the increased risk of infection for non-vaccinated individuals compared to vaccinated individuals

$$\pi_{VV'}(c_\psi^*) = \pi_{V'} - \pi_V, \tag{17}$$

By using the general game-theoretical framework [14], the expected payoff associated with each strategy can be defined as:

$$E_V = -C^{\psi} - \pi_V, \quad E_{V'} = -\pi_{V'}, \tag{18}$$

where $C^{\psi} = \frac{C_V^{\psi}}{C_i^{\psi}}$, C_V^{ψ} is the cost of vaccination, and C_i^{ψ} is the cost of infection.

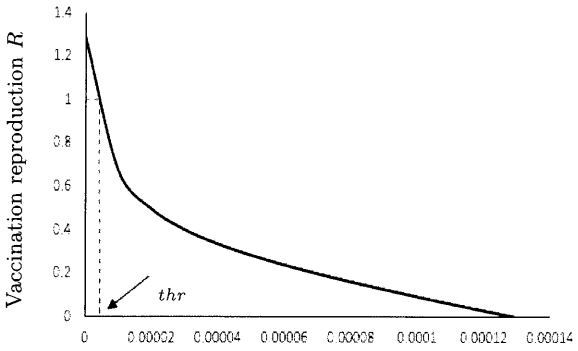


Figure 2: The vaccination control reproduction number as a function of the vaccination rate.

To analyze the expected payoffs for individuals, we assume that, ψ_{pop} be the vaccination rate of the population. However, if $\psi_{pop} > \psi_{thr}$ then $R_c(\psi) < 1$ and the population reach a disease-free equilibrium which means that the probability of infection among vaccinated individuals and non-vaccinated individuals are null. ($\pi_{V'} = \pi_V = 0$) and thus, there is no incentive for individuals to take the vaccine. If $\psi_{pop} < \psi_{thr}$ then $R_c(\psi) > 1$ and the epidemic reach endemic equilibrium P_{ψ}^*. The expected payoff of changing the strategies from non vaccinated to vaccinated is define as

$$\Delta E_{\psi} = -C^{\psi} + \pi_{VV'}(c_{\psi}^*), \tag{19}$$

Now individuals will choose the vaccine strategy if $\Delta E_{\psi} > 0$, which means that $\pi_{VV'}(c_{\psi}^*) > C^{\psi}$. They will choose the non-vaccination strategy when $\Delta E_{\psi} < 0$, which means $\pi_{VV'}(c_{\psi}^*) < C^{\psi}$. The Nash equilibrium for vaccination ψ_{Nash} is defined when the risk of infection is equal to the cost of infection $\Delta E_{\psi} = 0$, i.e. when $\pi_{VV'}(c_{\psi}^*) = C^{\psi}$. By substituting the endemic level (c_{ψ}^*) in to the equation $\pi_{VV'}(c_{\psi}^*) = C^{\psi}$, we can find equation (20) which the Nash equilibrium ψ_{Nash} must satisfied

$$\begin{aligned} J(\psi) &= C^{\psi}((a((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)) \\ &\quad ((1-\xi)((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)) \\ &\quad (\kappa\xi\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)\mu(\gamma(\gamma+\eta-\beta\beta)-\eta\beta)) \\ &\quad (\xi R_0\kappa\psi(\kappa a-\mu+\gamma+\eta)-((R_0-1)\kappa a-(\mu+\gamma+\eta))(\kappa\psi \\ &\quad +(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)))-(\xi R_0\kappa^2 a(-\mu\xi(\eta+\gamma\gamma) \\ &\quad (\mu+\omega)(\beta-\gamma)(\gamma^2+\eta\gamma-\beta\gamma-a\eta\beta))(\mu(\gamma(\gamma+\eta-\beta)-\beta\eta)((\kappa+1) \\ &\quad (\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)-(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\beta\eta))(\kappa\psi \\ &\quad +(1-\xi)(\mu+\gamma+\eta)+\mu\kappa))+\kappa\xi(\mu\gamma(\gamma+\eta-\beta)-\eta\beta)(\psi((\mu+\omega) \\ &\quad (\gamma(\gamma+\eta-\beta)-\eta(\mu+\omega)+(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))))) \\ &= 0 \end{aligned} \tag{20}$$

To analyze the existence of the vaccination Nash equilibrium, we find the roots of equation (20) defined as ψ_{root} .We find that there is one real root of $J(\psi) = 0$ using Mathematica Wolfram 12.1,

$\psi_{root} \to 0$. The vaccine efficacy of the Sinopharm vaccine at preventing infection of is 73% [24]. Daily recovery rate (99.1% of total cases), death cases (0.5% of total cases) and hospitalization (0.4% of total cases) [25] , birth rate 13.163 [26]. Estimate reproduction number in Bahrain 0.99 (0.55-1.3) [27]. The duration of the latent period ($\frac{1}{\kappa}$). The period of possible latent infection was found to range from 1-5 days before its appearance, with an average time of 2 days [28], so $\kappa = 0.5$.

Fig. (2) shows the behavior of the vaccination control reproduction number, when we apply only the vaccination strategy (SV). It is clear that, $R(\psi, t)$ decreases suddenly as the vaccination process is switched on. However, a linear behavior is predicted as the vaccination rate increases. Moreover at further values of vaccination the reproduction number vanishes completely.

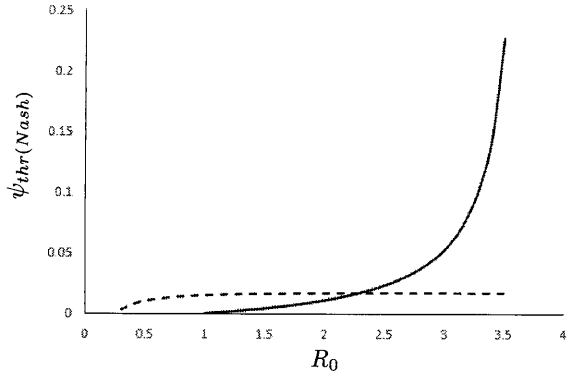


Figure 3: The vaccination rate, where the solid and dot lines represents $\psi_{thr}(R_0)$ and $\psi_{Nash}(R_0)$, respectively.

The behavior of equilibrium vaccination ψ_{thr} and that predicted from the game theory ψ_{Nash} are displayed in Fig. (3). The behavior shows that, ψ_{Nash} increases slightly and gradually at small value of the reproduction number. However, a liner behavior is depicted as $R(\psi, t)$ is increased. On the other hand the equilibrium vaccination ψ_{thr} is shown at large values of vaccination control reproduction number and increases gradually at $R(\psi, t) < 3$, while it increases suddenly as one increases the vaccination control reproduction number. From these behaviors, one can conclude that, $\psi_{thr} < \psi_{Nash}$ for all $0.2 < R_0 < 2.3$ and $\psi_{thr} > \psi_{Nash}$ for all $R_0 > 2.3$. It means that, if the basic reproduction number is less than 2.3, Bahrain can control the epidemic by vaccination only with a vaccination efficacy rate of at most 0.565, otherwise, the epidemic needs more vaccination efficacy.

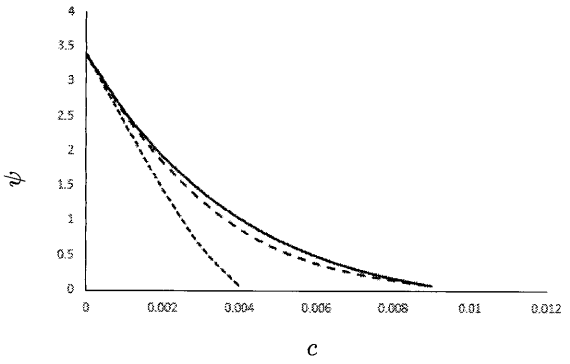


Figure 4: The vaccination rate ψ as a function of the relative vaccination cost infection c. The solid, dot and dash curves are evaluated at $R_0 = 2.9, 1.3$, and 0.5, respectively.

Fig. (4) shows the best population vaccination rate as a function of the relative vaccination cost of infection at different values of the reproduction numbers, where we set R_0 as $0.9, 1.3$ and 2.9. In general the vaccination rate decreases as the relative vaccination cost of infection increases. However, the decreasing rate depends on the initial of the reproduction number, where it decreases gradually at large values of R_0, and consequently vanishes at large values of the relative vaccination cost.

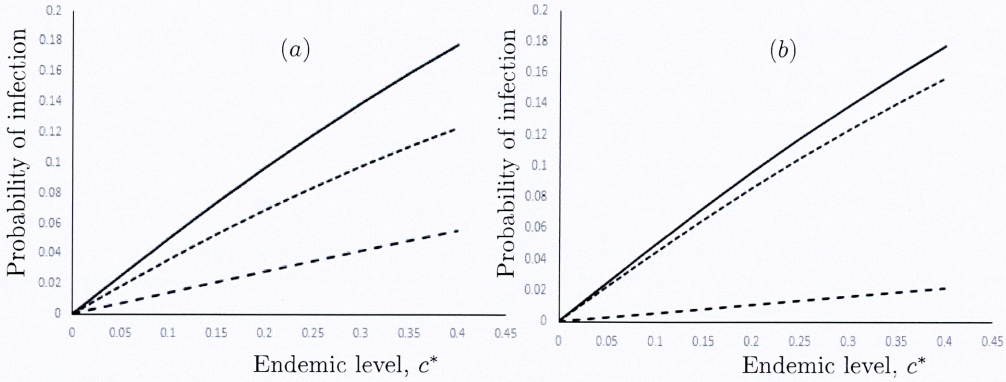


Figure 5: The probability of infection individuals who accept/reject vaccination. The solid, dash-dot and dots curves represent $\pi_{v'}, \pi_{vv'}$ and π_v, respectively.

Fig. (5) displays the effect of the efficacy, ξ on the infection probability $\pi_V(c*)$ among the individuals who accept the vaccination, where we consider different values of the vaccination efficacy. It is clear that, the probability of infection increases as the endemic level c^* increases linearly. However, the increasing rate depends on the degree of the vaccination efficacy, where $\pi_v(c*)$ decreases as one increases ξ. These results are confirmed in Fig. (5), where we investigate the behavior of the infection risk, $\pi_{VV'}$ as a function on the endemic point c^* at different values of ξ. The behavior shows that as one increases ξ the infection risk increases, which means it decreases for vaccinated individuals.

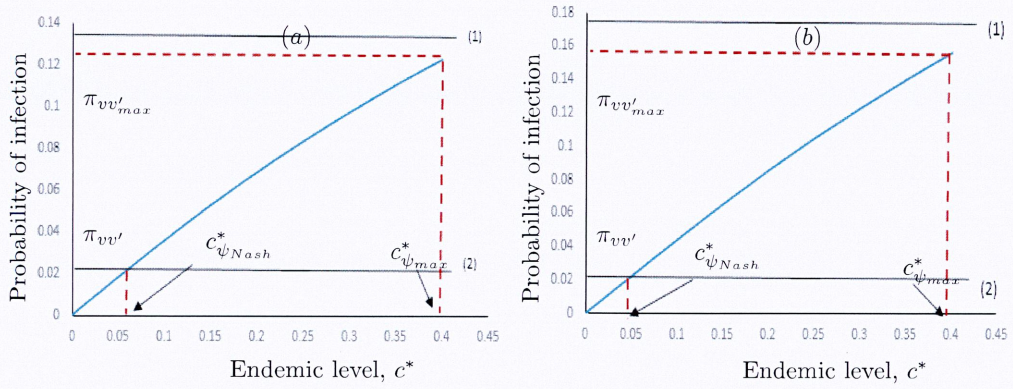


Figure 6: The probability of the increases risk of the infection for vaccinated/non-vaccinated individual where the vaccination efficacy is (a) 0.73 and (b) 0.9

The expected payoff of changing of the vaccinated and non vaccinated individual, ΔE is displayed in Fig. (6) at different values of the vaccination efficacy. We consider two cases either $C^{\psi} > \pi_{VV'}$ or $C^{\psi} = \pi_{VV'}$. For the first case, the expect payoff $\Delta E < 0$, namely the numbers of individuals who accept to be vaccinated are smaller than those reject the vaccination strategy, and consequently

there is no Nash vaccination strategy. For the second case, it is assumed that $\Delta E = 0$, namely the numbers of accepted/rejected to be vaccinated are equal, where a unique Nash vaccination strategy is displayed. As it is expected, the upper bounds of $\pi_{VV'}$ increases as the vaccination increases, namely the probability risk of infection increases. Moreover, $\pi_{VV'}(c_\psi^*)$ increases when $0 < c_\psi^* < c_{\psi_{\max}}^*$ and attends its maximum when $c_\psi^* = c_{\psi_{\max}}^*$. From equation (20) we can write as a function of the relative cost C^ψ as

$$= \frac{FE - AG}{a\kappa\mu - B + FD}, \tag{21}$$

where,

$$\begin{aligned}
A &= aR_0(\beta-\gamma)(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))\kappa\xi(\mu+\omega)^2(-\gamma-\eta)(-\gamma^2 \\
&\quad +\beta(-1+\gamma+\eta-\kappa)+\eta\kappa+\gamma(1-\eta+\kappa)), \\
B &= aR_0(\beta-\gamma)(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))\kappa\xi(\mu+\omega)^2\xi(1+\gamma+\eta)\kappa(\gamma(\gamma+\eta)\mu-\beta(\eta+\gamma\mu)) \\
F &= -C^\psi(\gamma(\gamma+\eta)\mu-\beta(\eta+\gamma\mu))(-(\beta-\gamma)(\gamma+\eta)\mu+a(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))(\mu+\omega)), \\
&\quad (-(\beta-\gamma)(\gamma+\eta)\mu+(\gamma(\gamma+\eta)-\beta(\gamma+a\eta))(1-\xi)(\mu+\omega)), \\
D &= (-(-\gamma-\eta+a(-1+R_0)\kappa-\mu)\kappa+R_0\kappa\xi[\gamma+\eta+a\kappa-\mu]), \\
G &= (\kappa\mu^2-\mu(\gamma+\eta+\mu)(-1+\xi)), \\
E &= -(-\gamma-\eta+a(-1+R_0)\kappa-\mu)(\kappa\mu-(\gamma+\eta+\mu)(-1+\xi))
\end{aligned} \tag{22}$$

4 Social distancing strategy

This strategy is one of the most applicable procedures to reduces the epidemic. Therefore, we examine the case in which individuals consider social distancing as the only infectious disease prevention strategy. In the absence of vaccination, the most effective way to reduce the transmission of the epidemic between people is to apply social distancing. Although social distancing does not eliminate the epidemic, it does play an important role in flattening the curve [14].

The dynamical system which is used to describe this strategy may be obtained be setting $\psi = 0$, system (3), namely there is no vaccination. For the social distance strategy (SD), the social control reproduction number is given by,

$$R_c(\iota) = (1-\iota)R_0, \tag{23}$$

By solving $R_c(\iota) = 1$ for ι, one gets the threshold value of the social distancing denoted as: $\iota_{thr} = 1 - \frac{1}{R_0}$.

Fig. (7) shows the control reproduction number $R_c(\iota)$ when social distancing is the only personal protection strategy and the social distancing level required to achieved herd immunity ι_{thr} we can see that if the $\iota < \iota_{thr}$ then $R_c(\iota) > 1$ and the disease remain epidemic. The endemic point of the social distance strategy, is given by,

$$P_\iota^* = (v_\iota^*, e_\iota^*, u_\iota^*, q_\iota^*, c_\iota^*, h_\iota^*, r_\iota^*), \tag{24}$$

where

$$\begin{aligned}
e_\iota^* &= \frac{\mu+\gamma+\eta}{\kappa a},\ u_\iota^* = \frac{(\mu+\omega)(-\kappa a e_\iota^*+\mu-\beta)}{\gamma(-\kappa a e_\iota^*+\mu+2\beta)}c_\iota^*,\ q_\iota^* = \frac{\eta(\mu+\omega)}{\gamma(-\kappa a e_\iota^*+\mu-\beta)}c_\iota^*, \\
h_\iota^* &= \frac{\omega}{\mu+\varepsilon}c_\iota^*, r_\iota^* = \frac{\varepsilon\omega}{\mu(\mu+\varepsilon)}c_\iota^*, \\
v_\iota^* &= \frac{((1-\iota)(1-\iota)e_\iota^*-\frac{\mu}{a})}{(\xi-1)} - \frac{V_1}{V_2}c_\iota^*, \\
c_\iota^* &= \frac{Y_1}{Z_1}.
\end{aligned}$$

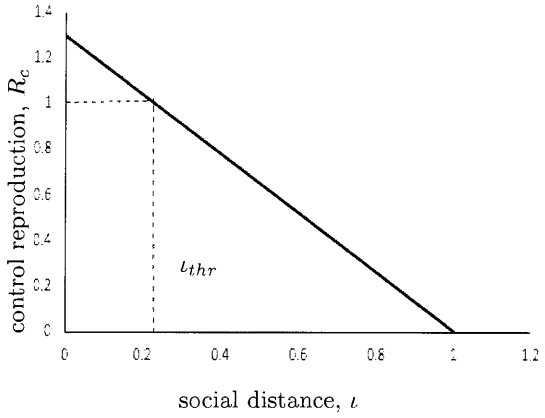


Figure 7: The social distance level required to achieve the herd immunity ι_{thr}.

with,

$$
\begin{aligned}
V_1 &= (1-\iota)(\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) \\
&\quad +(\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega)), \\
V_2 &= (\xi-1)\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta), \\
Y_1 &= ((1-\iota)(1-\iota)e_\iota^* - (1-\iota)(1-e_\iota^*))\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta), \\
Z_1 &= (1-\iota)(\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) \\
&\quad +(\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega)) + (((1-\iota)(\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) \\
&\quad +(\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega))(\kappa + (1-\iota))).
\end{aligned}
$$

4.1 Applying of game theory

Similarly, we apply game theory to analysis the stability of the system which describes this model. In this context, we assume that SD denotes individuals who choose to apply social distance and $SD^{'}$ denote individuals who refuse to apply social distance. Let π_{SD} be the probability of infection among social individuals and $\pi_{SD'}$ be the probability of infection among non social individuals. π_{SD} and $\pi_{SD'}$ can be expressed as the proportion of susceptible individuals becoming infected over the number of individuals dying during any unit of time [14]. For this strategy, we have

$$
\pi_{SD} = \frac{(1-\iota)a(\gamma u_\iota^* + \beta q_\iota^*)}{(1-\iota)a(\gamma u_\iota^* + \beta q_\iota^*) + \mu}, \quad \pi_{SD'} = \frac{a(\gamma u_\iota^* + \beta q_\iota^*)}{a(\gamma u_\iota^* + \beta q_\iota^*) + \mu}. \tag{25}
$$

and using endemic point (24) these probabilities may be written as,

$$
\begin{aligned}
\pi_{SD} &= \frac{(1-\iota)ac_\iota^*(\gamma(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \beta\eta(\mu+\omega))}{(1-\iota)ac_\iota^*(\gamma(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \beta\eta(\mu+\omega)) + \mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta)}, \\
\pi_{SD'} &= \frac{ac_\iota^*(\gamma(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \beta\eta(\mu+\omega))}{ac_\iota^*(\gamma(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \beta\eta(\mu+\omega)) + \mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta)}.
\end{aligned} \tag{26}
$$

Let us define the increased risk of infection for individuals $\pi_{SDSD'}$ as

$$\pi_{SDSD'}(c_\iota^*) = \pi_{SD'} - \pi_{SD}, \tag{27}$$

By using the general game-theoretical framework [14], the expected payoff associated with each strategy can be defined as:

$$E_{SD} = -C^\iota - \pi_{SD}, \quad E_{SD'} = -\pi_{SD'}, \tag{28}$$

where $C^\iota = \frac{C_{SD}^\iota}{C_i^\iota}$ be the relative cost of applying social distancing.

Let ι_{pop} be the social dist incing rate of the population. To analyze the expected payoffs for individuals, if $\iota_{pop} > \iota_{thr}$ then $R_c(\iota) < 1$ and the population reach a disease-free equilibrium which means that the probability of infection among vaccinated individuals and non-vaccinated individuals are null. ($\pi_{SD'} = \pi_{SD} = 0$) and thus, there is no incentive for individuals to apply social distancing. If $\iota_{pop} < \iota_{thr}$ then $R_c(\iota) > 1$ and the epidemic reach endemic equilibrium P_ι^*. The expected payoff of changing the strategies from not applying the social distancing to applying the social distancing is defining as

$$\Delta E_\iota = -C^\iota + \pi_{SDSD'}(c_\iota^*). \tag{29}$$

Now individuals will choose the social distancing if $\Delta E_\iota > 0$, which means that $\pi_{SDSD'}(c_\iota^*) > C^\iota$. They will choose the non-social distancing strategy when $\Delta E_\iota < 0$, which means $\pi_{SDSD'}(c_\iota^*) < C^\iota$. The Nash equilibrium for social distancing ι_{Nash} is defined when the risk of infection is equal to the cost of infection $\Delta E_\iota = 0$, i.e. when $\pi_{SDSD'}(c_\iota^*) = C^\iota$. By substituting the endemic level (c_ι^*)in to the equation $\pi_{SDSD'}(c_\iota^*) = C^\iota$, we can find equation (30) which the Nash equilibrium ι_{Nash} must satisfied

$$J(\iota) = C^\iota\{[A_\iota + B_\iota][(1-\iota)A_\iota + B_\iota]\} - \iota A_\iota B_\iota, \tag{30}$$

where

$$A_\iota = \frac{A_{\iota 1}}{A_{\iota 2}},$$

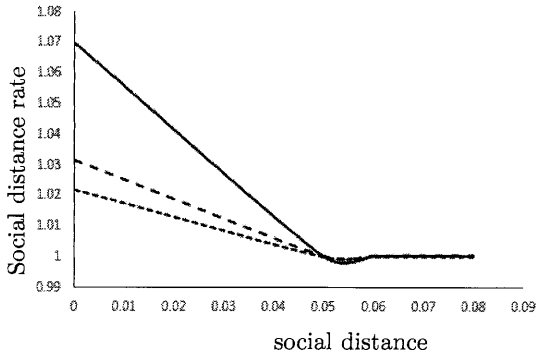


Figure 8: Population social distance rate as a function of the relative social cost infection with different reproduction number. The solid, dash and dot curves are evaluated at $R_0 = 2.9, 1.3, 0.9$, respectively.

and

$$
\begin{aligned}
A_{\iota 1} &= ((1-\iota)(1-\iota)e_\iota^* - (1-\iota)(1-e_\iota^*))\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta)a((\beta - a e_\iota^*\kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega)),\\
A_{\iota 2} &= (1-\iota)(\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta))\\
&\quad +(\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega))\\
&\quad +(((1-\iota)(\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega))(\kappa + (1-\iota))),\\
B_\iota &= \mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta).
\end{aligned}
$$

The root of $J(\iota) = 0$ using Wolfram Mathematica 12.1 is $\iota \to 0$.

Fig. (8) shows the behavior of the population social distancing rate as a function of the relative social cost of infection, where different reproduction numbers are considered. In general this rate decreases linearly as the relative cost of infection increases. It is clear that, as the initial values of R_0 increases, the upper bounds of the social distance rate is large. However, the equilibrium behavior of the distance rate is depicted at larger values of the relative cost distance.

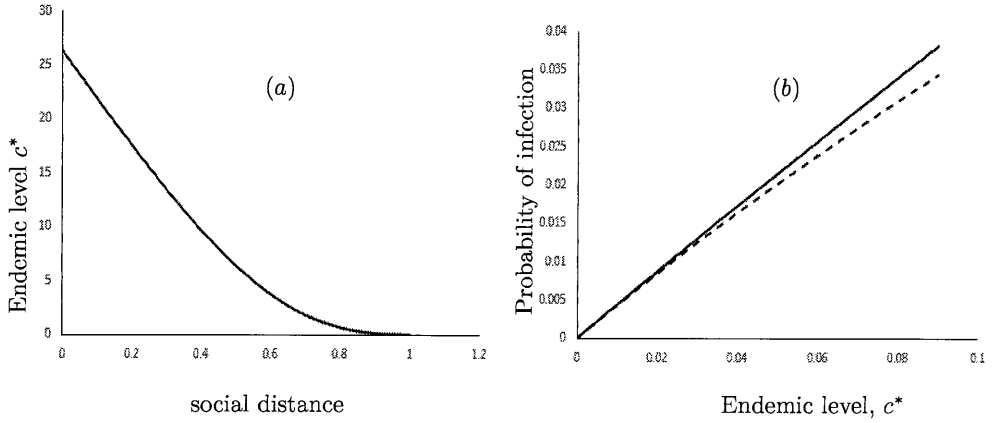


Figure 9: (a) The behavior of the endemic level $c_\iota^*(\iota)$ and (b) The dash and solid curves represent $\pi_{SD}(c^*)$ and $\pi_{SD'}(c^*)$, respectively.

The maximum point of the endemic level is obtained when $\frac{dc_\iota^*}{d\iota} < 0$. However, the maximum point is given by,

$$c_{\iota\,\max}^* = \frac{Y_{1\,\max}}{Z_{1\,\max}}, \tag{31}$$

with

$$
\begin{aligned}
Y_{1\,\max} &= (e_\iota^* - (1-e_\iota^*))\mu\gamma(-\kappa a e_\iota^* + \mu + 2\beta),\\
Z_{1\,\max} &= (\mu(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega) + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta))\\
&\quad +(\mu\kappa(\mu+\omega)(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega)) +\\
&\quad (((\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + 2\beta)) + (\mu+\omega)\gamma(-\kappa a e_\iota^* + \mu + \beta) + \eta(\mu+\omega))(\kappa+1)).
\end{aligned}
$$

Fig. (9a), displays the behavior of the endemic level as a function of the social distance ι, where it decreases gradually as one increases the social distance. Moreover, the maximum value of the endemic level is predicted at $\iota = 0$. The effect of the social distance on the probabilities π_{SD} and $\pi_{SD'}$ are displayed in Fig. (9b) . The behavior shows that both probabilities increases linearly as the endemic level c^* increases. The increasing that predicted for who do not apply the social distance is larger than that displayed for whom applied the social distance.

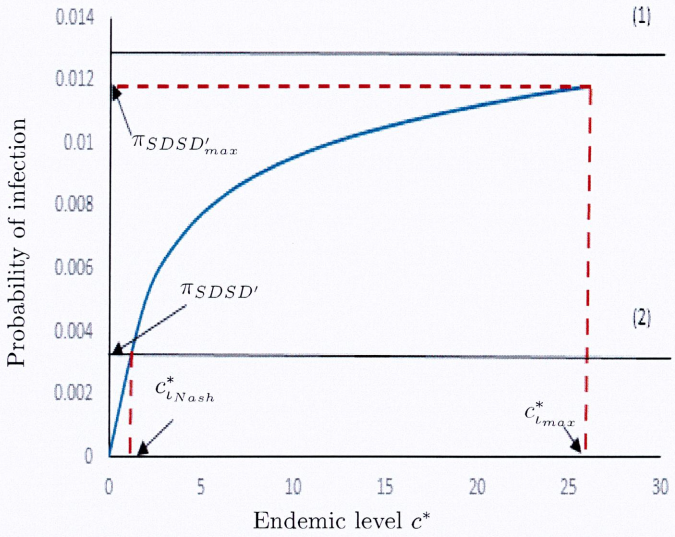


Figure 10: The probability of increased risk of infection for individuals who apply/do not apply the social distance.

In Fig. (10) we discuss the risk probability for individuals who apply/do not apply the social distance, where line (1) represents $C^\iota > \pi_{SDSD'}$, namely $\Delta E_\iota < 0$, individuals who not to apply social distancing strategy, without Nash social strategy. The second line (2) when we set $C^\iota = \pi_{SDSD'}$, i.e. Then $\Delta E_\iota = 0$ and within a unique Nash social strategy. In this case, We can write $\iota(C^\iota)$

$$\iota \to 1 - \frac{1}{a}(\frac{\iota_1}{\iota_2}), \tag{32}$$

where

$$\begin{aligned}
\iota_1 &= -\mu a c^*_\iota((\beta - a e^*_\iota \kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega))\gamma(2\beta - a e^*_\iota\kappa + \mu) - \mu^2\gamma(2\beta - a e^*_\iota\kappa + \mu)^2 \\
&\quad + C^{\iota^{-1}}(\mu\iota(a c^*_\iota((\beta - a e^*_\iota\kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega)))\gamma(2\beta - a e^*_\iota\kappa + \mu)), \\
\iota_2 &= c^*_\iota((\beta - a e^*_\iota\kappa + \mu)\gamma(\mu+\omega) + \beta\eta(\mu+\omega))(a c^*_\iota((\beta - a e^*_\iota\kappa + \mu)\gamma(\mu+\omega) \\
&\quad + \beta\eta(\mu+\omega)) + \mu\gamma(2\beta - a e^*_\iota\kappa + \mu)).
\end{aligned}$$

The behavior of the probability risk increases suddenly for any values $c^* \in [0, c^*_{\ell Nash}]$ of the endemic level c^* increases, while it increases gradually as one increases the endemic level, namely $c^* \in [c^*_{\ell Nash},\ c^*_{\ell max}]$.

5 Combining the social distancing and vaccination strategies

Now, we will find the best strategies when both vaccination and social distancing are available for individuals. The control reproduction number $R_c(\psi, \iota)$ of system (3) will be

$$R_c(\psi, \iota) = \frac{(1-\iota)a}{\mu} + \frac{a\psi(\iota - \xi)}{\mu(\psi + \mu)}.$$

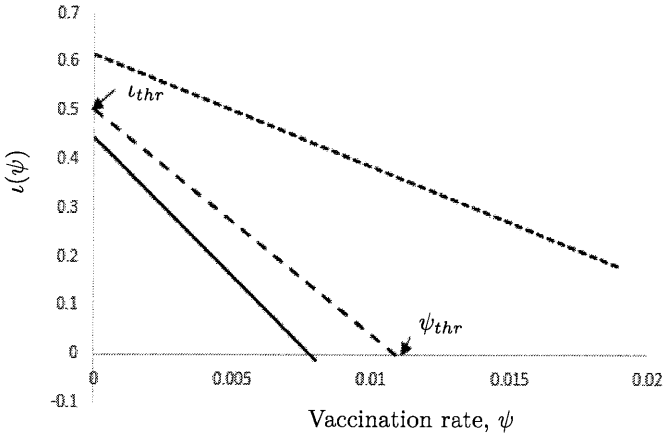


Figure 11: The social distance rate $\iota(\psi)$. The solid, dash, dot curves represent the social distance rate $\iota(\psi)$ at $R_0 = 0.9, 1, 1.3$, respectively.

Fig. (11) shows the behavior of $\iota(\xi)$ at different values of the initial reproduction number. It is clear that, small vaccination and distance rates is displayed at small values of R_0,. However, as one increases R_0 one has to increase ψ and ι.

To investigate the stability of the system which describes this strategy, we need to discuss the endemic point of system (3), which is given by,

$$P^*_{\psi,\iota} = (v^*, e^*, u^*, q^*, c^*, h^*, r^*), \tag{33}$$

where

$$\begin{aligned}
e^* &= \frac{\mu+\gamma+\eta}{\kappa a},\ u^* = \frac{(\mu+\omega)(\gamma+\eta-\beta)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c^*,\ q^* = \frac{-\eta(\mu+\omega)}{\gamma(\gamma+\eta-\beta)-\eta\beta}c^*,\\
h^* &= \frac{\omega}{\mu+\varepsilon}c^*,\ r^* = \frac{\varepsilon\omega}{\mu(\mu+\varepsilon)}c^*,\\
v^* &= \frac{(1-\iota)(1-e^*)}{(\xi-\iota)} - \frac{1}{(\xi-\iota)R_0} - \frac{V_3}{V_4}c^*,\\
c^* &= \frac{Y_2}{Z_2},
\end{aligned}$$

with

$$\begin{aligned}
V_3 &= (1-\iota)(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)+\mu(\kappa+1+\iota)((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega))),\\
V_4 &= (\xi-\iota)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta),\\
Y_2 &= (\xi-\iota)\mu(-\eta\beta+\gamma(\gamma+\eta-\beta))\kappa(\frac{\psi(\kappa a-\mu+\gamma+\eta)}{\kappa a} -\\
&\quad \frac{(1-\iota)(1-e^*)(\kappa\mu+(1-\xi)(\mu+\gamma+\eta)+\kappa\psi)}{(\xi-1)\kappa} - \frac{\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa}{(\xi-1)R_0\kappa}),\\
Z_2 &= (1-\iota)(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)+\mu(\kappa+1+\iota)((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)))\\
&\quad (\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)+(\xi-\iota)\kappa(\psi((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)+\\
&\quad (\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)).
\end{aligned}$$

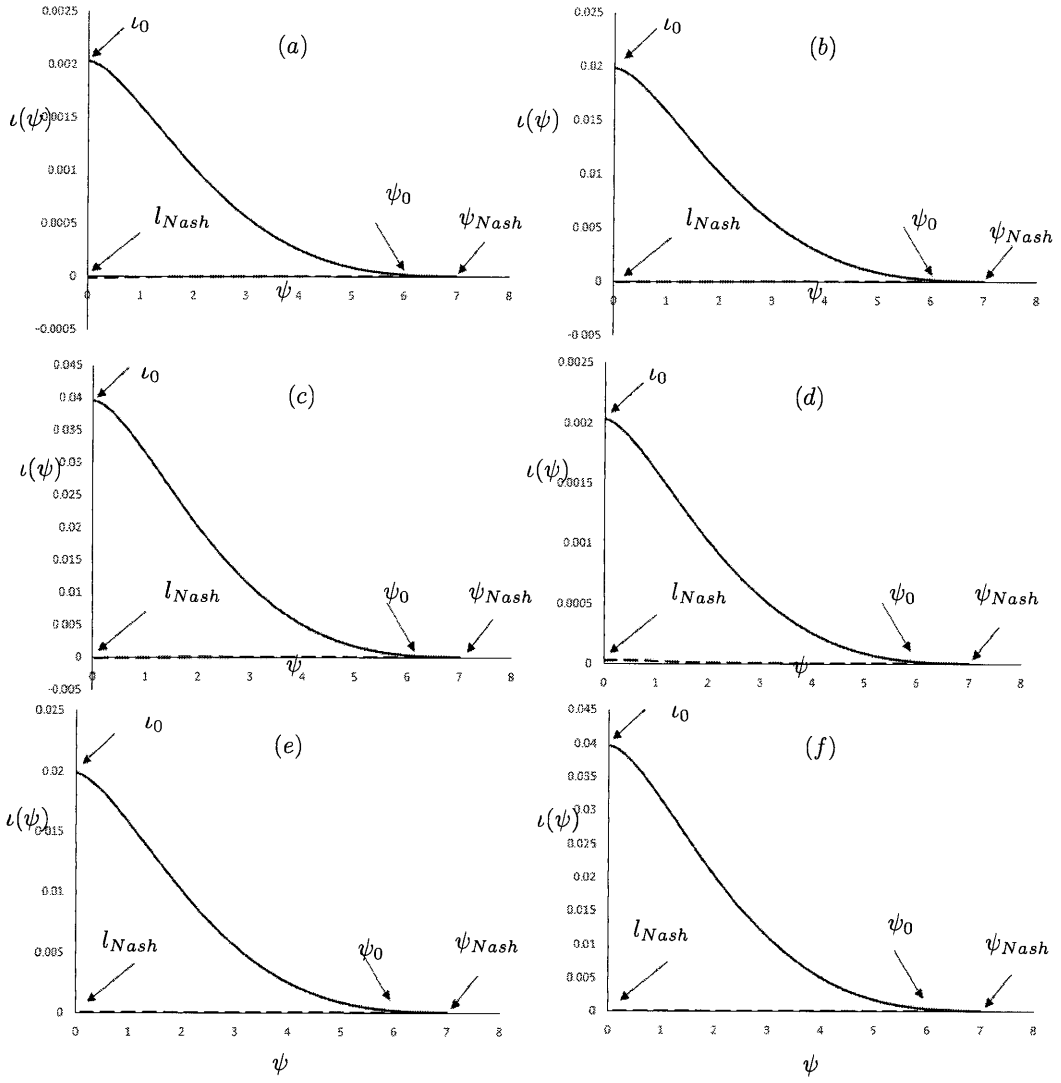


Figure 12: The social distance rate $\iota(\psi)$ plotted against ψ

When both strategies, the vaccination and social distancing, are available to individuals, then the Nash equilibrium of vaccination will be a social function $\psi_{Nash}(\iota)$, and the Nash equilibrium of social will be a function of vaccination $\iota_{Nash}(\psi)$. We substitute c^* from equation (33) in equation (17)

$$\pi_{VV'}(c^*) = C^{\psi}$$

We can obtain the equation that the Nash equilibrium of vaccination strategy $\psi_{Nash}(\iota)$ satisfies as

follow:

$$\begin{aligned}\psi_{Nash}(\iota) &= C^{\psi}((((1-\iota)(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta)+\mu(\kappa+1+\iota)\\ &((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)))(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)\\ &+(\xi-\iota)\kappa(\psi((\mu+\omega)(\gamma+\eta-\beta)-\eta(\mu+\omega)+(\mu+\omega)(\gamma(\gamma+\eta-\beta)-\eta\beta))))\\ &(a((\mu+\omega)(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta))((1-\xi)((\mu+\omega)\\ &(\gamma(\gamma+\eta-\beta)-a\beta\eta))+\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)))-\\ &(((\xi-\iota)\mu(\gamma(\gamma+\eta-\beta)-\eta\beta)\kappa(\frac{\psi(\kappa a-\mu+\gamma+\eta)}{\kappa a}-\\ &\frac{(1-\iota)(1-e^{*})(\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa)}{(\xi-\iota)\kappa}-\frac{\kappa\psi+(1-\xi)(\mu+\gamma+\eta)+\mu\kappa}{(\xi-\iota)R_0\kappa}))\\ &[-\mu\xi(\eta+\gamma)(\mu+\omega)(\beta-\gamma)(\gamma^2+\eta\gamma-\beta\gamma-a\eta\beta)]])\end{aligned} \tag{34}$$

By substituting $\psi = 0$ in equation (34) and solve for ι we get $\iota \longrightarrow 0$. Also, when substitute c^* from equation (33) in equation (27)

$$\pi_{SDSD'}(c^*) = C^{\iota}.$$

For this strategy, the Nash equilibrium of social distancing strategy $\iota_{Nash}(\psi)$ is given by,

$$\iota_{Nash}(\psi) = C^{\iota}((A+B)((1-\iota)A+B)) - \iota AB, \tag{35}$$

where

$$A = ac^*(\gamma(\mu+\omega)(-\kappa ae^*+\mu+\beta)+\beta\eta(\mu+\omega)), \quad B = \mu\gamma(-\kappa ae^*+\mu+2\beta).$$

By substituting $\iota = 0$ in equation (35) and solve for ψ we get $\psi_0 = 0$. Vaccination is the best strategy when $\iota_0 > \iota_{Nash}$.

Fig. (12) shows the relative cost function for the combined two strategies to investigate the best applicable ones in Bahrain, where we consider two cases the cases $C^{\iota} > C^{\psi}$, and $C^{\iota} = C^{\psi}$ $C^{\iota} < C^{\psi}$. It is clear that, the upper bounds of the social distance increases when the vaccination cost is larger than the distance cost.

6 Discussion

In this contribution, we introduce a model that included vaccination and social distancing as control measures to determine the best individual strategies for infectious disease precaution. We construct a payoff function for each available strategy and calculate the return from it. The relative cost of each strategy determines which strategy is most applicable to individuals. We consider three different strategies; either the individuals protect themselves by vaccination only, applying the social distance only, or combining both strategies. Our calculation found that both vaccination and social distancing play vital roles in controlling the spread of COVID-19 in Bahrain. The control reproduction number mainly decreases with increased social distancing rate, but it reduces with vaccination only if vaccination efficacy is greater than the social distancing rate.

For the first strategy, Bahrain can adopt vaccination to control the epidemic if the main reproduction number is less than 2.3, with a vaccination efficiency of 0.565 at most. Otherwise, the epidemic needs more vaccination efficacy (Fig. (3)). The vaccine's effectiveness also has an essential role in reducing the possibility of infection in individuals. Also, it found that with increased vaccination efficacy, the chance of non-vaccinated individuals being more infected than the vaccinated individuals as seen in Fig. (5). However, as the relative cost of infection increases, the population vaccination rate decreases because of fears of an increase in cases of infection due to vaccination (Fig. (4)). Also, it is found that individuals will choose a non-vaccination strategy when this cost exceeds the probability of increased risk of infection for non-vaccinated individuals compared to vaccinated individuals (Fig.

(6)). However, the best strategy for this scenario is when the relative cost of infection and the probability of increased risk of infection for non-vaccinated individuals compared to vaccinated individuals are equal. Similar results were obtained in the Choi and Shim study [17].

For the second scenario, social distancing plays a significant role in reducing contact between individuals in the absence of vaccination. Thus, to control the epidemic using social distancing only, the level of social distancing should be applied at a rate greater than achieving herd immunity (Fig. (7)). The severe social distancing between individuals can protect them effectively by reducing infection, although this distancing can lead to economic damage at the individual level and society as a whole. Moreover, the herd immunity threshold for social distancing cannot be achieved through individuals' participation, but everyone must participate in it, which results in economic losses. It has also been observed that the rate of social distancing is affected by the increase in the relative cost of applying social distancing. Also, the basic reproduction number has a role in influencing this rate. The rate of social distancing decreases according to the decrease in the basic reproduction number, as seen in Fig. (8).

The endemic level decreases with the increase in the rate of social distancing. However, with its growth, the probability of infection increases between individuals who apply social distancing and those who do not, as seen in Fig. (9a) and Fig. (9b). Therefore, the best strategy to be applied in the second scenario is when the relative social cost of infection equals the increased risk of infection for individuals who don't apply social distancing compared to individuals who apply social distancing, as shown in Fig. (10).

For the third strategy, it is found that the rate of social distancing decreases as one increases the rate of vaccination rat, as represented in Fig. (11). Moreover, best strategy that can be applied is the vaccination strategy, even if vaccination and social distancing strategies have the same cost or different costs, as shown in Fig. (12). In contrast, Choi and Shim [17] study showed that, if vaccination and social distancing are available, vaccination will be the best strategy for individuals when the relative social cost is more than vaccination. However, social distancing will be the best strategy when the relative cost of vaccination is equal to the relative cost of social distancing.

One limitation of this model is that the analysis is restricted to the assumption that the system has reached an endemic state and not taking into account the age of individuals and their susceptibility to infection. Also, the model is limited to personal costs to individuals only. However, in real life, the prevalence, severity of the disease, and the individual's age all affect susceptibility to contracting it. For example, the optimal vaccination coverage varies between age groups, and the optimal vaccination rates for all age groups are highest at the beginning of a seasonal influenza epidemic [29]. In addition, with the increase in the severity of the disease and its spread, the country needs more government interventions, such as closing schools, providing work from home and partial closure of shops. Individuals choose the most appropriate method to prevent exposure to infection, such as social distancing. Therefore, both vaccination and social distancing should be identified as case variables.

In *conclusion*, a three different strategies are considered in order to overcome/reduces the endemic of COVID-19 in Bahrain, where it is assumed that the individuals either agree/reject vaccination, apply only the social distance, or accept to be vaccinated and obeying the social distance procedure. Moreover, game theory is used as an application to investigate the stability of the dynamical systems which describes these three seniors. In the three scenarios, it is assumed that vaccination as an optimal strategy in the absence of social distancing and social distancing as an optimal strategy in the absence of vaccination. However, for the combined strategy, one applies the optimal strategy when vaccination and social distancing are present.

The obtained results show that, in the absence of social distancing, the best strategy is to apply vaccination when its cost is equal to the risk of infection for non-vaccinated individuals compared to vaccinated individuals. However, in the absence of vaccination, the optimal strategy for individuals would be to apply social distancing when the cost is equal to the increased risk of infection for individuals who don't apply social distancing compared to individuals who apply social distancing. In the presence of vaccination and social distancing, choosing vaccination is the best way, regardless

of their cost. *Therefore*, vaccination's importance must be considered a precautionary and preventive measure to control the spread of the disease throughout the country.

Author Declarations

- **Author contributions:**
  All authors contributed to study the suggested model. The numerical calculations, data collection were performed by Al-Sayegh, M., while the analysis of the results and discussions were done by Metwally, N, and Igbal, A. The manuscript was written by Al-Sayegh, M., and all authors read and approved the final manuscript.
- **Consent to Participate:**
  Not applicable.
- **Conflicts of interest/Competing interests:**
  The authors declare they have no competing interests.
- **Funding:**
  No funding was received for this study.
- **Ethics Declaration statement:**
  Not applicable.
- **Consent for publication:**
  Not applicable.
- **Availability of data and material/ Data availability:**
  Not applicable.

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Appendix (D)

Mathematical Model to suppress the spreading of COVID-19 Coronavirus in the Kingdom of Bahrain

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Abstract

This article develops a mathematical model considering time-dependent Susceptible, Unquarantined infected, Quarantined-infected, Confirmed-infected and Recovered transmission model *(SUQCR)* for the disease progression in Bahrain. The stability and the existence of an equilibrium point of the suggested model are discussed. The reproduction number R_0 by the next-generation approach is calculated. The model parameters are used as controllers to minimis the infected individuals and maximize the recovered numbers. The basic reproduction number for Bahrain, according to the $SUQCR$ model, is $R_0 < 1$, where all the infected individuals decreases, and consequently it is taken as measure of increasing or decreasing the precautionary procedures.

Keywords:
COVID-19, Mathematical modeling of COVID-19, epidemic model, Reproduction number

1 Introduction

The World Health Organization (WHO) informed of a new virus responsible for respiratory disease in Wuhan, China, on December 31, 2019 [1], resulting in great concern inside China and universally. The epidemic began with patients entering hospitals in late December 2019 with preliminary diagnosis of pneumonia. They were reported to be linked to the seafood and wet animal market in Wuhan, Hubei Province, China [2]. COVID-19 has an incubation period of 3-7 days and may last up to 14 days and this period differs from patient to patient [3]. It is believed that the new Coronavirus is contagious during the incubation period when patients do not have symptoms. It can be transmitted from individual to another by direct contact and drops of sneezing, coughing or speaking [4]. The COVID-19 virus is believed to have entered Bahrain on February 24, 2020 [4]. As of December 9, a total of 68697077 confirmed cases of COVID-19 were reported worldwide, with 1555311 deaths. Bahrain's share in these statistics was 88293 confirmed cases and 343 deaths [5].

COVID-19 has sparked intense interest internationally about its spread and improvement pattern [6]. Concern about the spread and direction of the epidemic has led to mathematical research focused on modeling its spread and evolution [1, 7, 8]. However, the mathematical models are useful for understanding the behaviour of an infection as it reaches the population and studying the conditions under which it would be wiped out or would continue. Currently, COVID-19 is of great concern to researchers, governments, and citizens, due to the high rate of spread of infection and the high number

of deaths that have occurred. Mathematical models are used as methods to gain insight into the nature of infectious diseases [9–14]. Several mathematical models were used to study the spread of COVID-19, such as Susceptible, Infected, Recovered (SIR), the Susceptible, Exposed, Infected, Recovered ($SEIR$, and the Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected ($SUQC$) models [6, 9, 15]. These models represent individuals' movement between 'compartments' in a given community depending on the population and individuals' infectious status [16].

The most popular of the compartmental models is the (SIR) model [9], where Kernack and McKendrick have developed it to describe the influenza epidemic [17] and established the famous threshold theory in 1927. It is shown that, the threshold represents the number of new cases produced by an infectious individual in a susceptible population and is determined by the basic reproduction number R_0. The severity and rate of progression of an epidemic depend on the value of R_0. However, if $R_0 > 1$, the epidemic will continue, but if $R_0 < 1$, the epidemic will disappear [18]. Also, R_0 aims to ensure equilibrium stability, fundamentally, disease-free equilibrium (DFE): if $R_0 < 1$ the DFE is stable while it will be unstable if $R_0 > 1$ [9, 19–22]. The SIR model was also used to describe the transmission and frequency of a measles outbreak in New York between 1930 and 1962. The modified SIR has been used in the context of COVID-19 [6, 23, 24]. These models help us to know how COVID-19 spreads, predict regional peaks of the epidemic and understanding the effect of various quarantine measures [25].

Wu et al. [26] used the Susceptible, Exposed, Infected, Recovered ($SEIR$) model to perform the transmission dynamics and predict the spread of COVID-19; they estimated R_0 to be 2.68. In contrast, Tang et al. [27] obtained R_0 to be 6.47 when the clinical progression of the disease and intervention measures was combined into the $SEIR$ model. The Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected ($SUQC$) model suggested by Zhao & Chen [28] is applied to data from China to analyze the dynamics of the COVID-19 outbreak and it shows accurate prediction of trends. It can be used as a useful tool for quantifying parameters and variables related to quarantine effects or methods of confirming an epidemic and providing further guidance on controlling outbreaks.

Therefore we are motivated to investigate the dynamics of the COVID-19 in Kingdom of Bahrain by using the SUQCR model. The context of the manuscript is arranged as follows. In Sec.(2), we review the SUQCR model and the differentiation equations system which describe it as well as its stability. In Sec.(3), we investigate the production number R_0 of the SUQCR model. The results are discussed in Sec. (4). Finally, we summarize and conclude

2 The SUQCR Model

In present study, a time-dependent Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected, and Recovered transmission model ($SUQCR$) is proposed for the disease progression in Bahrain, which is a modified version of the $SUQC$ model [28]. The reported cases of COVID-19 were collected from the Ministry of Health in Bahrain and also from the website [29].

The $SUQCR$ model includes the number of recovered individuals, referred as R, due to its importance of knowing the role of immunization used against COVID-19 and the role of natural immunity. The effect of individuals' disease history λ and early detection of COVID-19 by the random tests δ on the recovered numbers are explored in our model. The parameter μ representing the normal death rate at any stage of individual transmission is included in all compartments. Also, the $SUQC$ model showed the effect of quarantine measures on the spread of the disease directly. However, the $SUQCR$ model will show the impact of un-quarantine and quarantine measures on the progression of disease in Bahrain. In more detail, it will show the effect of the random tests and trace the contacts of COVID-19 patients on discovering the infected cases.

This model is described by five variables; Susceptible $S = S(t)$, individuals exposed to disease without disease resistance at time t, the Un-quarantined infected ($U = U(t)$), infected asymptomatic individuals at time t, Quarantined infected $Q = Q(t)$, people in contact with Corona patients, whether with symptoms or without symptoms at time t, Confirmed infected ($C = C(t)$), the number of

confirmed infected cases at time t, and Recovered $R = R(t)$, the number of surviving individuals contagious and had developed a natural immunity to the virus at time t [30], where the total population $N = N(t) = S(t) + U(t) + Q(t) + C(t) + R(t)$ is non-negative for all $t \geq 0$. Fig.(1), draws the diagram of the proposed $SUQCR$ model that tracking the infected cases in Bahrain

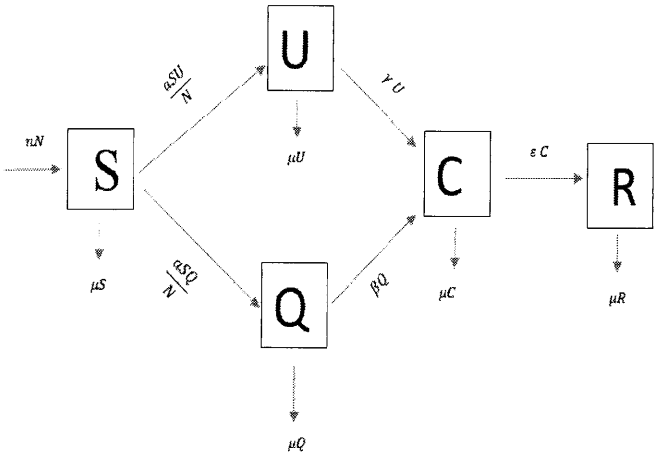


Figure 1: The diagram of the $SUQCR$ Model

Mathematically, the $SUQCR$ model may be described by the following system of differential equations, (1),

$$\begin{aligned}
\frac{dS(t)}{dt} &= nN(t) - \mu S(t) - a\frac{S(t))}{N(t)}U(t) - a\frac{S(t))}{N(t)}Q(t), \\
\frac{dU(t)}{dt} &= a\frac{S(t))}{N(t)}U(t) - (\mu + \delta + \sigma + \eta)U(t), \\
\frac{dQ(t)}{dt} &= a\frac{S(t))}{N(t)}Q(t) - (\mu + \beta)Q(t) + \eta U(t), \\
\frac{dC(t)}{dt} &= (\delta + \sigma)U(t) + \beta Q(t) - (\mu + \tau - \lambda)C(t), \\
\frac{dR(t)}{dt} &= (\tau - \lambda)C(t) - \mu R(t), \qquad (1)
\end{aligned}$$

where $\frac{S(t))}{N(t)}$ represents the probability that contact is made with a susceptible individual and the parameter a is the average number of effective contacts produced by each infectious individual [31]. However, all the parameters that displayed in Eq.(1) are described in Table (1). To simplify the model (1), we set $\gamma = \delta + \sigma + \eta$ and $\varepsilon = \tau - \lambda$, and consequently the system (1) reduces to be,

$$
\begin{aligned}
\frac{dS(t)}{dt} &= nN(t) - \mu S(t) - a\frac{S(t))}{N(t)}U(t) - a\frac{S(t))}{N(t)}Q(t), \\
\frac{dU(t)}{dt} &= a\frac{S(t))}{N(t)}U(t) - (\mu+\gamma)U(t), \\
\frac{dQ(t)}{dt} &= a\frac{S(t))}{N(t)}Q(t) - (\mu+\beta)Q(t) + \eta U(t), \\
\frac{dC(t)}{dt} &= (\gamma-\eta)U(t) + \beta Q(t) - (\mu+\varepsilon)C(t), \\
\frac{dR(t)}{dt} &= \varepsilon C(t) - \mu R(t). \qquad (2)
\end{aligned}
$$

In this context, if we assume that population $N(t)$ is a constant, then the birth rate becomes equivalent to the death rate, namely $n = \mu$. In this study the restriction will be to the positive values for S, U, Q, C, and R.

Parameter	Description	Units
a	Infection rate	Day-1
N	Total population number	Day
n	Birth rate	Per 1000 people
μ	Death rate	Per 1000 people
γ	Non-isolated infected people rate	Day-1
δ	Random tests rate	Day-1
σ	Symptoms appearing rate	Day-1
β	Exit tests rate	Day-1
ε	Recovery rate	Day-1
η	Contact tracing rate	Day-1
τ	Immunity rate (Natural immunity and immunity through vaccination)	Day-1
λ	Medical history rate	Day-1

Table1: Definition of parameters in the $SUQCR$ model.

For system (2), let $s(t) = \frac{S(t))}{N(t)}$, $u(t) = \frac{U(t))}{N(t)}$, $q(t) = \frac{Q(t))}{N(t)}$, $c(t) = \frac{C(t))}{N(t)}$, $r(t) = \frac{R(t))}{N(t)}$, and by re-scaling the system one obtains the normalized system of differential equations that describe the model as,

$$
\begin{aligned}
\frac{ds(t)}{dt} &= \mu - \mu s(t) - as(t)u(t) - as(t)q(t), \\
\frac{du(t)}{dt} &= as(t)u(t) - (\mu+\gamma)u(t), \\
\frac{dq(t)}{dt} &= as(t)q(t) - (\mu+\beta)q(t) + \eta u(t), \\
\frac{dc(t)}{dt} &= (\gamma-\eta)u(t) + \beta q(t) - (\mu+\varepsilon)c(t), \\
\frac{dr(t)}{dt} &= \varepsilon c(t) - \mu r(t), \qquad (3)
\end{aligned}
$$

This system will be solved by considering the following initial conditions,

$$
s(0) = s_0 \geq 0;\ u(0) = u_0 \geq 0;\ q(0) = q_0 \geq 0;\ c(0) = c_0 \geq 0,\ r(0) = r_0 \geq 0. \qquad (4)
$$

In this context, it is important to investigate stability and the equilibrium point of the SUQCR. It is well known that, the existence of a single nonnegative equilibrium and stability of the system (3) depends on the basic reproduction number R_0 on a free equilibrium point P_0 which is determined with the next-generation matrix method [21]. R_0 aims to check the stability of the system (3), if $R_0 > 1$, the system is unstable and continued pandemic spread occurs; however, if $R_0 < 1$, the system (3) is asymptotically stable, and the disease vanishes [32]. Based on Eq.(3), the disease-free equilibrium point, which is the point at which no disease exists in the population is given by

$$\left(\frac{n}{\mu}, 0, 0, 0, 0\right), \tag{5}$$

For all the equilibrium points, we have $U = Q = C = 0$. The positive disease-free equilibrium point for the system (3) is $S(t) = 1$. Thus, the disease-free equilibrium point will be $P_0 = (1, 0, 0, 0, 0)$.

3 The Reproduction number R_0 by the next-generation approach

To develop a meaningful definition of the basic reproduction number R_0 and the next-generation matrix approach, let X be the vector of infected classes, and let Y be the vector of uninfected classes. Let $f(X;Y)$ be the vector of new infection rates (flows from Y to X) and $v(X;Y)$ be the vector of all other rates (not new infection). For each compartment, in-flow in V is negative and out-flow in V is positive. Consider the next-generation matrix G, which is a matrix that links the number of recently infected individuals in different groups in subsequent generations [34] as $G = FV^{-1}$ where $F = \left[\frac{df}{dx}\right]\Big|_{P_0}$ and $V = \left[\frac{dv}{dx}\right]|_{P_0}$. The reproduction number R_0 which is the average number of new cases performed by a single case, is the dominant eigenvalue of the generation matrix $G = FV^{-1}$ [21]. i.e , $R_0 = \rho(FV^{-1})$ where $\rho(G)$ is the spectra radius of the matrix G. Here we will calculate the reproduction number R_0 for *SUQCR* model. The vectors f and v are given by

$$f = (as(t)u(t) + as(t)q(t)), \quad v = \begin{pmatrix} \mu u(t) + \gamma u(t) \\ \mu q(t) + \beta q(t) + \eta u(t) \\ -(\gamma - \eta)u(t) + \mu c(t) - \beta q(t) + \varepsilon c(t) \end{pmatrix}, \tag{6}$$

At $(1, 0, 0, 0, 0)$ we obtain

$$F = \begin{bmatrix} a & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \mu + \gamma & 0 & 0 \\ \eta & \mu + \beta & 0 \\ -(\gamma - \eta) & -\beta & \mu + \varepsilon \end{bmatrix}. \tag{7}$$

$$V^{-1} = \begin{bmatrix} \frac{1}{\mu+\gamma} & 0 & 0 \\ \frac{-\eta}{(\mu+\gamma)(\mu+\beta)} & \frac{1}{\mu+\beta} & 0 \\ \frac{-\beta\eta}{(\mu+\gamma)(\mu+\beta)(\mu+\varepsilon)} + \frac{\gamma-\eta}{(\mu+\gamma)(\mu+\varepsilon)} & \frac{\beta}{(\mu+\beta)(\mu+\varepsilon)} & \frac{1}{\mu+\varepsilon} \end{bmatrix}, \tag{8}$$

and consequently the matrix G takes the following form,

$$G = \begin{bmatrix} \frac{a}{\mu+\gamma} + \frac{-\eta a}{(\mu+\gamma)(\mu+\beta)} & \frac{a}{\mu+\beta} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{9}$$

Now the reproduction number of system (3) is defined as $R_0 = \rho(G)$. Explicitly it can be written as,

$$R_0 = \frac{a(\mu + \beta - \eta)}{(\mu + \delta + \sigma + \eta)(\mu + \beta)}. \tag{10}$$

For the parameters used in our *SUQCR* model, and affording to Table 2, the basic reproduction number of Bahrain according to this mode is $R_0 < 1$. However, R_0 will gradually increase as the contact tracing rate, randomized test and exit test rates all fall from 0.09, which is the rate currently calculated for Bahrain in the period of this study [29,33]. R_0 in Bahrain depends on six parameters, the infection rate, death rate, the symptoms appearing rate, the random and exit test rates, in addition to the contact tracing rate.

a	μ	δ	σ	β	η	R_0
0.2425	0.00000671	0.09	0.09	0.09	0.09	0.0000670
0.2425	0.00000671	0.0001	0.09	0.0001	0.0001	0.1690
0.2425	0.00000671	0.00004	0.09	0.00004	0.00004	0.3687
0.2425	0.00000671	0.00003	0.09	0.00003	0.00003	0.4921
0.2425	0.00000671	0.00002	0.09	0.00002	0.00002	0.6765
0.2425	0.00000671	0.00001	0.09	0.00001	0.00001	1.0816
0.2425	0.00000671	0	0.09	0	0	2.6942

Table2: Values of the reproduction numbers with different parameters

Theorem 1 *System (3) is locally stable related to the disease-free equilibrium point P_0 if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. The Jacobian matrix J of the system (3) at the J at equilibrium point P_0 will be

$$J = \begin{bmatrix} -\mu & -a & -a & 0 & 0 \\ 0 & a-(\mu+\gamma) & 0 & 0 & 0 \\ 0 & \eta & a-(\mu+\beta) & 0 & 0 \\ 0 & \gamma-\eta & \beta & -(\mu+\varepsilon) & 0 \\ 0 & 0 & 0 & \varepsilon & -\mu \end{bmatrix}. \tag{11}$$

The eigenvalues of J are given by,

$$\begin{aligned} \lambda_1 &= \lambda_5 = -\mu, \quad \lambda_2 = \frac{(\mu+\gamma)\left[(R_0-1)(\mu+\beta)+\eta\right]}{\mu+\beta-\eta}, \\ \lambda_3 &= \frac{(\mu+\beta)\left[(\mu+\gamma)(R_0-1)+(\gamma-\beta+\eta)\right]}{\mu+\beta-\eta}, \quad \lambda_4 = -(\mu+\varepsilon), \end{aligned} \tag{12}$$

From (12), it is clear that λ_1, λ_4 and λ_5 are always negative. However, at $R_0 < 1$ the eigenvalues λ_2 and λ_3 are negative.

Thus, the disease-free equilibrium point P_0 is asymptotically stable localized since all roots of Eq.(12) are negative. Moreover, P_0 is asymptotically unstable if $R_0 > 1$, because it has two roots with positive real parts. Thus, system (3) is locally stable at P_0 if $R_0 < 1$ and unstable if $R_0 > 1$. ■

4 Result

In this section we shall investigate the behavior of the individual numbers, susceptible, un-quarantined infected, quarantined infected, confirmed infected and recovers, where solved the system of (1) numerically by using using the code "NDSolve" in the Wolfram Mathematica 12.1 with the initial conditions $S(0) = 1$, $U(0) = 0.5$, $Q(0) = 0.3$, $C(0) = 0.1$ and $R(0) = 0.1$. The behavior of these individual numbers are displayed in Figs.(2-6), where we set the death rate $\mu = 2.450$ per 1000 people, which means $\mu = 0.00000671$ per day [35] and the infection rate $a = 0.2425$ [36]. Moreover, calculated parameters from Bahrain COVID -19 data [29,33] from September 24, 2020, to December 9, 2020, such as the,

recovery rate $\varepsilon = 0.978$, symptoms appearing rate $\sigma = 0.09$, contact tracing rate $\eta = 0.09$, random tests $\delta = 0.09$ and exit tests rate $\beta = 0.09$.

Fig.(2) shows the behavior of the individual numbers at different values of the parameters which describe the system. In Fig.(2), we investigate the effect of the random test number, where we set $\delta = 0.05, 1.55$ where all the other parameters are fixed. The general behavior shows that the recover individual increases gradually as the time increases. However, it behaves constantly at further time. However, the constantly behavior is depicted faster at smaller values of δ, where this observation can be seen by comparing Figs.(2a) and (2b). The quarantined individual numbers increases gradually to reach its maximum values at short time, where the maximum number is predicted at small values of δ. However, at further time the U numbers decrease gradually to completely vanishes, where the vanishing time depends on the random test number δ. The behavior of number of quarantined Q decreases gradually as time increases, where it decreases suddenly at large values of δ. A similar behavior is predicted for the confirmed infected numbers and the susceptible numbers, where they decreases suddenly/gradually at small/large values of δ.

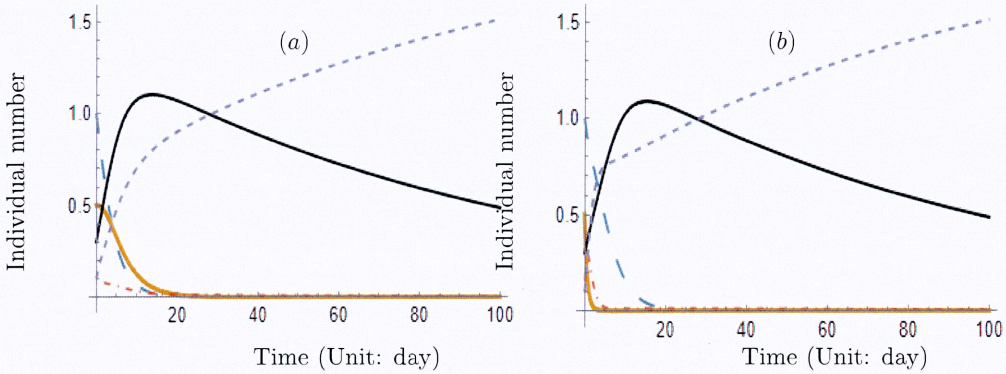


Figure 2: The susceptible (dash), un- quarantined infected (thick orange), quarantined infected (black), confirmed infected (dot dash) and recovers (tiny dash) when $a = 0.2425$, $\mu = 0.00000671, \varepsilon = 0.978, \sigma = 0.09, \beta = 0.01, \eta = 0.09$ and (a) $\delta = 0.05, (b)\delta = 1.55$

In Fig. (3), we increases the exit test rate on the behavior of all individuals number. The depicted behaviors are almost similar to those displayed in Fig.(2), namely all the individual numbers decreases except the recovers number increases. Also, the susceptible individual numbers decrease at a short time, then becomes a constant at further time. The gradually decay of the U, Q, C, S is displayed at small rate of random test, while their sudden decay is displayed at larger vales of δ

The effect of the contact tracing rate η on the behavior of all the individual number is displayed in Fig. (4). In general all the individual numbers predicted a behavior similar to that displayed in Fig.(2), (3), but only the quarantined infected number increases in a short period of time. However, as one increases the time further, it decays gradually to reach its non-vanishing minimum value. However, as one increases η (see Fig.(4b), the upper bounds of the quarantined infected numbers is larger than that displayed in (4a) It is clear that, the recovers number increases on the expanses of the other decrease individual numbers, where it increases gradually at small time due to the increasing of the individual number Q.

In Fig.(5), we discuss the effect of a large values of the exist test rate, β. The behavior shows that the recovers individual number increase on the expenses of the decreasing all the remaining individual numbers. As it is shown from Fig.(5b), the upper bounds of recover number $R(t)$ that displayed at $\beta = 0.3$ are larger than those displayed at $\beta = 0.01$ as it shown in Fig.(5a). However, the susceptible individual numbers $S(t)$, behaves as a constant at large values, while it vanishes faster at small values of β.

From Figs.(2-5) one ca say that by controlling the random test parameter δ, the contact tracing

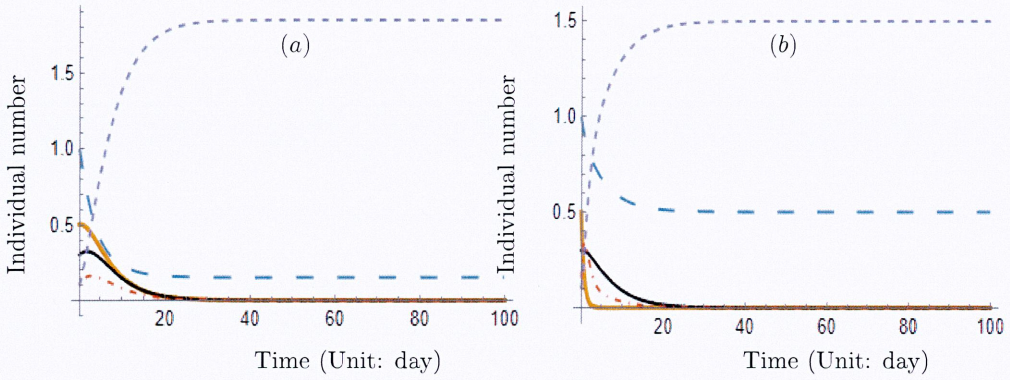


Figure 3: The same as Fig.(2)nut $\beta = 0.3$

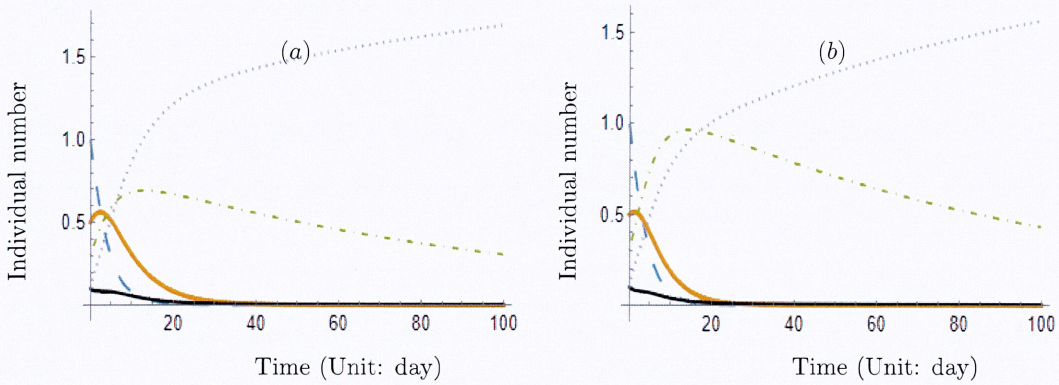


Figure 4: The susceptible (dash), un- quarantined infected (thick orange), quarantined infected (black), confirmed infected (dot dash) and recovers (tiny dash) when $a = 0.2425, \mu = 0.00000671, \varepsilon = 0.978, \sigma = 0.09, \beta = 0.01, \delta = 0.05$ and $(a)\eta = 0, (b)\eta = 0.05$

rate η, and the exist test rate, one can increase the recovers individual numbers and at meanwhile, all the other individual number could be decreases.

In Fig(6), we discuss the possibility of increasing the recovered numbers $R(t)$ in the light of the assumed constant date of the Ministry of health at Bahrain. The effect of the medical history rate, λ on the behavior of the individual recovered numbers $R(t)$ is displayed in Fig.(6a). The behavior of $R(t)$ shows that, the recovered rate number increases gradually as the one increases the time. The increasing rate depends on the initial value of λ, where if the patient has no medical history rate, namely $\lambda = 0$, the possibility of increasing the recovered numbers is larger than that displayed for those who have a medical history, namely $\lambda > 0$.

In this context, it is important to investigate the effect of the initial immunity of the patient on the possibility of incrossing the number of the recovered patient. From Fig.(6b), its clear that, as one increases the patient's immunity the recovering rate increases. However, our results show that at small values of the immunity, namely $\tau = 0.2$, there is no possibility of increasing the recovered numbers $R(t)$. Moreover, the large values of the immunity has a slightly effect on increasing $R(t)$.

Fig.(6c) describes the effect of the contact tracing rate η on the behavior of the individual recovered numbers. Although the general behavior of the recovered numbers shows that $R(t)$ increases, but the predicted increasing rate at small values of η is larger than that displayed at large value of η. This result is logic, because the more of η, the more of discovering infected people and consequently the

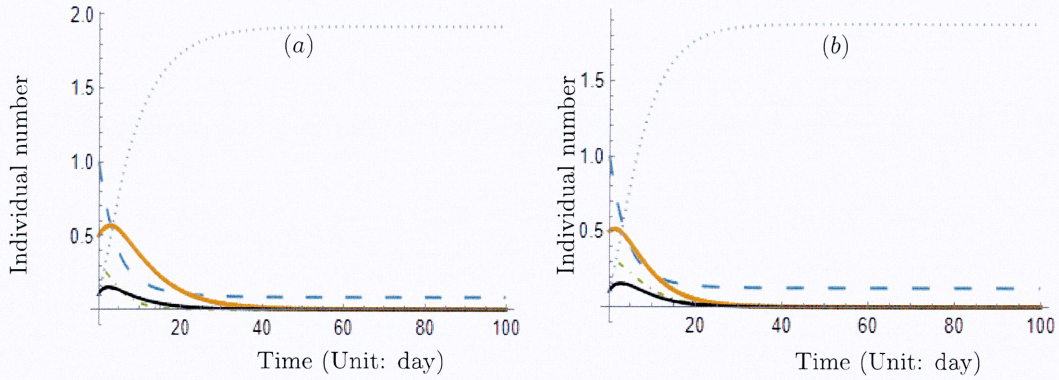


Figure 5: The same as Fig.(4), but we set the exist test rate $\beta = 0.3$.

infected sample increases.

One of the most important procedure that is taken by the Ministry of health is the exact test. Therefor it is deserve to discuss the effect of this factor β on the suggested model. Fig.(6d), shows the impact of β on the behavior of the recovered numbers $R(t)$. It is shown that, $R(t)$ increases as one increase β. However, the sudden increasing of $R(t)$ is displayed at large values of β, while it increases gradually at small values of the exact testing parameter. Moreover, at large time, the constantly behavior of $R(t)$is displayed, where the large constant values are shown at large values of β.

Fig.(7) shows the number of infected individuals in Bahrain at different values of reproduction number, namely the behavior of the total sample of $U(t), Q(t)$ and $C(t)$. The general behavior shows that the infected sample decreases gradually as time increases. The decreasing rate depends on the value of the reproduction number, where as one decreases R_0, the infected sample decreases fast and vanishes completely at short period of time. On the other hand, at large values of the reproduction number, the infected sample vanishes at longer period of time. Therefore, the precautionary procedures that have been taken by the ministry of health at Bahrain are agree with the increasing/decreasing the reproduction number.

5 Conclusion

In this contribution, we develop the $SUQCR$ model from Bahrain's daily data to investigate the dynamics of the COVID-19 outbreak and determine the best preventive measures to limit its spread. $SUQCR$ model uses five variables and few parameters shown in Table 1. The model is described mathematically by as system of differential equations which depends on some parameters, which has a clear effect on the epidemic dynamic of COVID-19. The local stability and existence equilibrium point of the mathematical model are obtained. Moreover, the reproduction number R_0 by the next-generation approach was calculated. It is shown that, the $SUQCR$ model can act as a useful tool for measuring quarantine-related parameters and variables, detecting confirmed cases by conducting the random and exit tests in addition to the contact tracing.

From the behavior of the individual numbers, it seems clear that some parameters have a high impact on controlling the spread of COVID-19 disease in Bahrain when other parameters are fixed. These results refer to COVID-19 data in Bahrain from September 24 2020, to December 9, 2020. The most critical issue in this model is decreasing the un-quarantine infected individuals and reducing the reproduction number R_0 to control the spread of COVID-19 and increasing the number of recovery individuals. The most vital parameters are the random tests δ, the exit tests β, the contact tracing rate η, the immunity rate, including the natural immunity and the immunity through vaccination τ and the medical history rate λ.

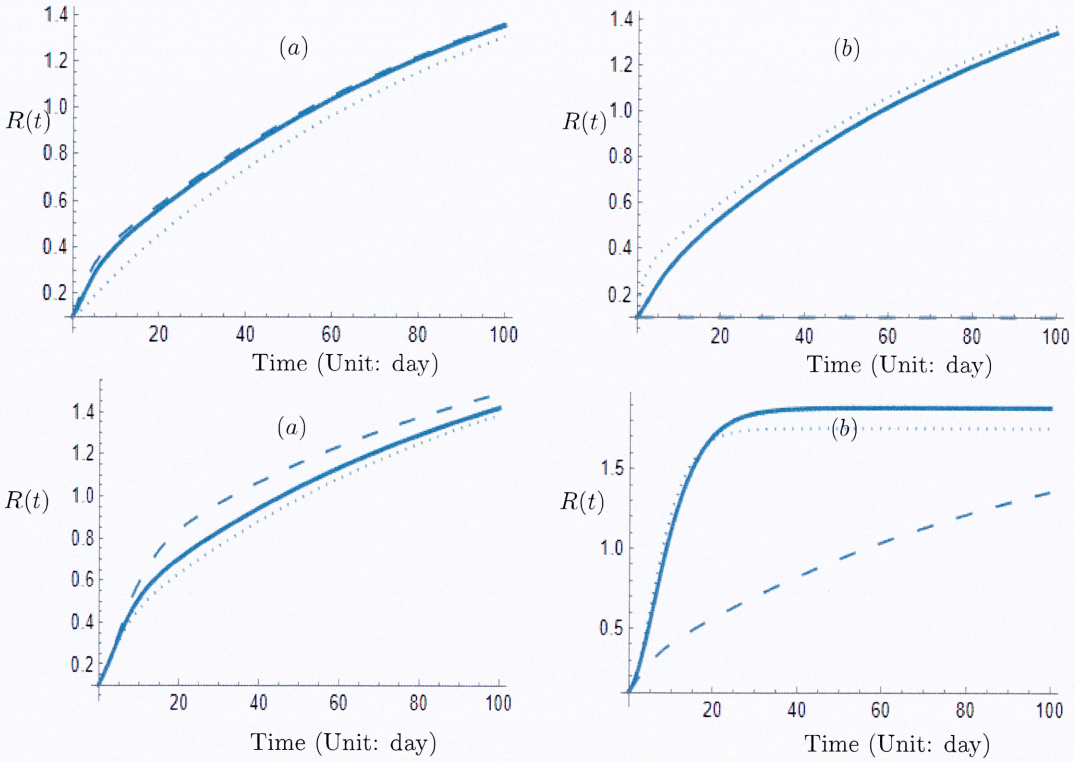


Figure 6: The recovered individuals when $a = 0.2425, \mu = 0.00000671, \sigma = 0.09$ a)Describe the effect of medical history(λ) with , $\beta = 0.01$, $\tau = 0.5$, $\eta = 0.5$, where the dash, solid and dot curves are evaluated at $\lambda = 0, 0.2$ and 0.4, respectively, (b)the effect of the immunity τ with $\beta = 0.01$, $\eta = 0.5$, $\lambda = 0.2$, where the dash, solid and dot curves are evaluated at $\tau = 0.2, 0.4, 2$ respectively, (c) describes the impact of the contact tracing rate η with $\beta = 0.01, \lambda = 0.2$ and $\tau = 0.5$, where the dash, solid and dot curves are evaluated at $\eta = 0.1, 0.2$ and 0.3 respectively. (d) Shows the effect of exit test rate β, with $\eta = 0.2, \tau = 0.5, \lambda = 0.2$, where the dash, solid and dot curves are evaluated at $\beta = 0.01, 0.2$ and 0.3 respectively.

Bahrain played a tremendous role in tracing contacts and conducting random checks. Also, it followed a targeted and strict random tests system: daily random units organize random visits to seven or eight locations, including health centers, sports clubs and various local markets. The number of tests at each site ranges between 300-400, while 2000-2300 daily random checks are performed [39]. The obtained results show that, all the individual numbers decrease as the random test parameter increases at large time. However, only the quarantined infections increases at small period of time by setting a small values of exit test rate. Moreover, the un-quarantined infected individuals decreased as one increases the random tests rate. These results are consistent with Worker et al. [37], who found that contact tracing can be used in the event of an epidemic to help along with other control methods to reduce the number of cases.

It is shown that, the immunity parameter may be considered as a control parameter to increases the recovered numbers. This immunity includes the natural immunity of individuals or vaccine-mediated immunity. Also, individuals' disease history is an essential factor in the healing process, so we notice that the number of people recovering decreases whenever there is a history of the individual's disease.

Reproduction number is a valuable tool for disease control, where the disease spread rapidly, and the number of infections increased as one increases the reproduction number. Therefore, it is

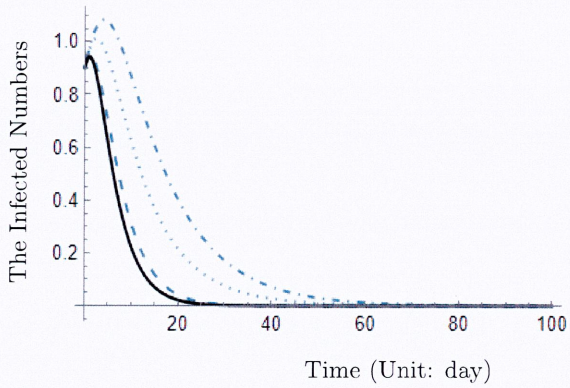


Figure 7: Infected individuals for different reproduction numbers.The dash-dot, dot, and the solid curves are evaluated at $R_0 = 1.7, 1, 0.6, 0.3$, respectively

important to reduces it. This aim can be achieved by increasing the number of random, exit tests and the contact tracing. The reproduction number of the $SUQCR$ model is calculated from Table 2 is 0.00007 when $a = 0.2425, \mu = 0.0000671, \sigma = 0.09, \delta = 0.09$, and $\eta = 0.09$, which means that the disease is not spreading rapidly and will disappear in Bahrain. According to our model, the reproduction number will gradually increase when the contact tracing, random and exit tests are reduced from their current numbers. The estimated reproduction number in March 2020 was 3.85 [44] and from September to November 2020, it was between 0.86 to 0.97 [29]. According to our model $R_0 < 1$, Bahrain's government imposes effective restrictions to reduce the spread of the disease among people. Therefore, citizens need to follow instructions to minimize COVID-19 spread between people.

Therefore, the precautionary measures followed by Bahrain's Government, such as increasing random tests to detect un-quarantined infected individuals and increasing the contact tracing and individual's immunity, have an effective impact on controlling the spread of COVID-19 in Bahrain. The Government of Bahrain imposes many restrictions and preventive measures that would reduce disease spread among people, so that must be followed and adhered to by the citizens and residents.

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ملخص الدراسة

انتشر وباء كوفيد-19 على مستوى العالم، واتخذت جميع البلدان تدابير احترازية للحد من انتشار هذا الوباء. ومن هذا المنطلق تشجعنا لنساهم في دراسة هذا الوباء من خلال تقديم بعض الحلول التي قد تحد من انتشار هذا الوباء. وبناءاً علية تم اقتراح نموذج رياضي للتحكم في انتشار كوفيد-19 في البحرين.

لتحقيق هدف الرسالة، دُرس أثر الإجراءات الاحترازية التي اتخذتها مملكة البحرين للحد من انتشار فيروس كورونا خلال عامي 2020 و2021. حيث تم الحصول على البيانات الإحصائية والإجراءات الاحترازية من موقع وزارة الصحة في مملكة البحرين. وبدراسة هذه البيانات خلال كلا العامين، كان عدد الحالات المصابة المكتشفة أقل من المتوقع، باستثناء بعض الفترات التي زاد فيها عدد الإصابات. كما كان أغلاق المؤسسات وادارتها عن بعد عبر المنصات المختلفة وأيضا بتطبيق نظام الإشارة الضوئية أثر واضح في تقليل عدد الإصابات المكتشفة. ومن ناحية أخرى ، فإن إلغاء اختبارات PCR للقادمين للبحرين والمخالطين للحالات المصابة وتقليل عدد أيام الحجر الصحي لهم، زاد من فرص الإصابة بالمرض.

كذلك، تم اقتراح نموذج $SUQCR$ لتطور انتقال المرض في مملكة البحرين. وحُددت حالة التوازن المحلية وشروط وجودها. كما تم إيجاد رقم التكاثر الأساسي R_0 بواسطة نهج الجيل التالي، حيث تم استخدام بارامترات (معاملات) النموذج الرياضي المقترح كوحدات تحكم لتقليل الأفراد المصابين وزيادة عدد الأفراد الذين تم شفاؤهم. ووفقًا لنموذج $SUQCR$ ، فإن رقم التكاثر الأساسي لمملكة البحرين هو $R_0 < 1$ حيث ينخفض عدد الأفراد المصابين، وبالتالي تم اتخاذ هذا المعمل كمؤشر يوضح إمكانية زيادة أو تقليل الإجراءات الاحترازية.

إضافة إلى ذلك، تم تقديم نموذج $SVEUQCHR$ القائم على نهج نظرية الألعاب فأُنشأت دالة المكافأة وأُوجد توازن ناش (Nash) من حيث الاستراتيجيات المتاحة للأفراد حيث تم تحديد التكلفة النسبية للاستراتيجية الخيار الأنسب للفرد. بَيّنت النتائج أن الخيار الأمثل للأفراد هو اختيار التطعيم في غياب التباعد الاجتماعي عندما تتساوى تكلفته مع خطر العدوى للأفراد غير الملقحين. أما إذا لم يكن التطعيم مُتاحًا، فيختار الأفراد تطبيق التباعد الاجتماعي عندما تتساوى تكلفته مع خطر العدوى بدون تطعيم. أما في وجود كلا من التطعيم التباعد الاجتماعي، فيُعد التطعيم استراتيجية أكثر فعالية بغض النظر عن التكلفة.

وأخيرا، أُستخدم نهج نظرية الألعاب أيضا لتحديد ما إذا كانت اللقاحات الأولية والمنشطة فعّالة في احتواء كوفيد-19 في مملكة البحرين، حيث تم إنشاء مصفوفة المُكافآت وتحديد نقاط توازن ناش من خلال تحديد القيمة القصوى في المدخل الأول لكل عمود والقيمة القصوى للمدخل الثاني لكل صف. وأستنتج أن التطعيم بجرعاته الأولية والمنشطة يقلّل عدد الأفراد المصابين والحالات الحرجة وحالات الوفاة بسبب وباء كوفيد-19.

وبالتالي، فإن الاستمرار في تطبيق الإجراءات الاحترازية وليس الحد منها أو تقليلها، وأخذ التطعيم يساهم بشكل كبير في احتواء كوفيد-19 في مملكة البحرين.

جامعة البحرين

كلية العلوم

برنامج البيئة والتنمية المستدامة

النمذجة الرياضية لانتشار فيروس كورونا الجديد والسيطرة عليه في مملكة البحرين

أطروحة مقدمة كجزء من متطلبات الحصول على درجة الدكتوراه في
برنامج البيئة والتنمية المستدامة

إعـداد

منى عبد الكريم الصائغ

الرقم الجامعي: ١٩٩٦٢٦٩٢

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فبراير ٢٠٢٣م