

THERMAL AND MHD EFFECTS ON THE STABILITY OF COUETTE FLOW
BETWEEN TWO ROTATING CYLINDERS

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DECLARATION

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other University or other Insititute of Learning.

M.A. Ali

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To My Family

ABSTRACT

The effects of heating and an applied axial magnetic field on the stability of viscous incompressible flow between two concentric rotating cylinders (Couette flow) are examined. The equations governing the marginal stability state of the flow are solved numerically and the results are presented in both tabular and graphical forms. Our predictions for some special cases of the problems considered exhibit a good agreement with those previously reported by various authors.

In chapter one, a general introduction on the topic of hydrodynamic stability is given. The introduction describes:

- (i) The first systematic experiments in this field.
- (ii) The assumptions employed in order to simplify the mathematical treatment of a stability problem.
- (iii) The patterns of flow observed at the onset of instability.
- (iv) The numerical methods used in solving different types of stability equations.
- (v) The practical importance of the studies in the field of hydrodynamic stability.
- (vi) The problems that are investigated in this thesis.

In chapter two, the stability of Couette flow in the presence of a radial temperature gradient due to constant heat flux at the inner cylinders is studied. In formulating the stability equations we assume that the gap between the two cylinders is of small and arbitrary sizes. All three types of rotation of the two cylinders are considered. The two cylinders may rotate in the same or in the opposite directions or only the inner cylinder is rotating.

In chapter three, we examine the combined effect of a radial

temperature gradient and an axial magnetic field on the stability of an electrically conducting Couette flow. The narrow gap annulus is assumed and the critical angular velocities are calculated for all types of rotation of the two cylinders. It is found that while the magnetic field always stabilizes the flow, the temperature gradient, depending on the direction of heat flow, may stabilize or destabilize the flow.

In chapter four, we explore, in the presence of various gap sizes between the cylinders, the stability effect of (i) a radial temperature gradient, (ii) axial magnetic field and (iii) the type of rotation of the two cylinders. The important role that different gap sizes play in the stability of Couette flow is manifested through the strong dependence of the critical velocity on their values.

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The topic of hydrodynamic stability is related to the concept of transition of a fluid from one hydrodynamic state to another. Osborne Reynolds (1833) was the first scientist/engineer who showed, by systematic experimental investigations, the essential features of this problem. In his experiment, Reynolds used a glass tube through which water was drawn from a large glass tank. The flow pattern was observed by injecting a thin filament of dye into the main stream. It was noticed that as long as the velocity of the water remains sufficiently low, the dye streak remains straight and parallel to the walls of the tube. As the velocity is increased beyond a certain value, the dye streak begins to oscillate and it mixes up with the surrounding water.

The first case is known as the laminar flow and the second case, which is characterized by rapid and continuous mixing of the fluid, is known as the turbulent flow. Reynolds discovered that a fluid undergoes a transition from a laminar to a turbulent flow when the non-dimensional number R , (Va/ν) called the Reynolds number exceeds some critical value (R_c) where V denotes the maximum velocity of the water, a the radius of the tube, and ν the kinematic viscosity of the water.

For values of R below R_c , all disturbances in the fluid decay, whereas for $R > R_c$ at least some disturbances are amplified. The value of R_c itself is found to be very sensitive to disturbances in the water entering the tube. It has been demonstrated experimentally that for fluid entering the tube with disturbances of some finite magnitude, R_c has a value of about 2300. But for a very smooth flow, R_c may exceed a value of 40,000. Thus, by keeping the disturbances very small, it is possible to maintain a laminar flow up to a very high Reynolds number, i.e., a fluid which undergoes a transition process for disturbances of some magnitude, will

remain in its initial state for smaller disturbances.

The following question arises from the above discussion; if a laminar flow is assumed to be subjected to infinitesimally small disturbances, will the disturbances gradually die out or will they grow in amplitude in such a way that the fluid progressively departs from its initial state, thus breaking down into turbulent flow or some other forms of laminar flow? this is the basic problem of hydrodynamic stability. The fluid is said to be stable or unstable depending, respectively, on whether it changes its initial state or not. Determining the critical conditions which separate two classes of states is one of the prime objects of hydrodynamic stability studies.

Mathematically, the problem of hydrodynamic stability can be abbreviated as follows: Suppose a system of hydrodynamic equations has a basic steady solution $U_j(x_i)$, $P_j(x_i)$ and $T_j(x_i)$ for the components of velocity, pressure and temperature respectively. The system is then subjected to infinitesimal disturbances and the solution of the new perturbed state is obtained. If the solution approaches a steady state as the time t goes to infinity, then the fluid is said to be stable. Otherwise it is said to be unstable.

It is quite difficult to obtain a solution of a hydrodynamic stability problem which is based on an exact mathematical treatment. Therefore, it was necessary to employ various simplifying assumptions which emerge from valid theoretical concepts and experimental observations. The first of these assumptions comes from the fact that the super-imposed disturbances are very small compared with the corresponding quantities of the main flow. Thus, in obtaining the equations governing the perturbed state, the products and powers greater than one of the disturbances are neglected. The theory based on this linearization process is called the linear theory

of stability.

A substantial simplification is achieved by analysing the solution of the perturbation equations in terms of a complete set of functions of a particular form. For example in the stability problem of a fluid between two coaxial rotating cylinders (known as Couette flow) a disturbance U can be expressed as:

$$U = U(r) \exp [st + i (n \theta + kz)] \quad (1)$$

where $U(r)$ denotes the amplitude of the disturbances in the radial direction r , k the wave number of the disturbance in the axial direction z , t the time, θ the azimuthal angle, n is an integer and s is a complex constant. The disturbances are damped and the laminar flow is stable when $\text{real } s < 0$, whereas the flow is unstable when $\text{real } s > 0$. The imaginary part of s represents the angular frequency of the oscillation of the disturbance.

The state where $\text{real } s = 0$ is known as the state of marginal stability. It separates the stable flow from the unstable one. Depending on how the amplitudes of the disturbances grow at the onset of instability, the marginal state can be one of two kinds. If they grow periodically then the transition from stability to instability takes place via marginal state by exhibiting a stationary pattern of motion. In this case, the principle of exchange of instabilities is said to be valid. The instability sets in the fluid as a stationary secondary flow. If the disturbances grow by oscillations of increasing amplitudes then the transition takes place via a marginal state by exhibiting oscillatory motion with a certain characteristic frequency given by the imaginary part of s . This case is known as overstability.

Determining which of the two kinds of marginal stability states occurs is usually a matter of experimental observations and it is different for different problems. For example, in the stability problem of Couette flow, Taylor (1923) showed experimentally that, for a narrow gap between the cylinders and for velocities ratio $\mu = \Omega_2/\Omega_1$ (where Ω_2 and Ω_1 are the angular velocities of the outer and inner cylinders respectively) positive or slightly negative, the principle of exchange of instabilities is valid and thus the marginal state is stationary. In his experiment, Taylor used a single outer cylinder made of glass and various inner cylinders in different runs of his experiment. The ratio μ was fixed in each run. The angular velocities were then gradually increased until the onset of instability. To detect instability, Taylor injected a dye into the fluid through six small holes near the middle of the inner cylinder. The motion pattern could then be viewed through the glass walls of the outer cylinder. At the onset of instability a steady secondary motion in the form of cellular toroidal vortices (now known as Taylor vortices) spaced regularly along the cylinder's axis was observed.

Taylor confirmed his measurement of the critical angular velocity at which the flow becomes unstable by a theoretical prediction. The agreement between his experimental and theoretical results was remarkable. His work was the basis for various researches in hydrodynamic stability. A study by Kruger, Gross and DiPrima (1964) emphasized the validity of the principle of exchange of instabilities for all values of μ greater than about -0.78 for both narrow and finite gap between the cylinders. These predictions were then supported experimentally by Snyder (1968). The principle, as pointed out by Stuart (1985), is now widely believed to be true in the specified range of μ .

To determine the critical conditions at the onset of instability of

Couette flow, an eigenvalue problem, which is defined by the stability equations and two boundary equations, has to be solved. It comes out that it is quite difficult to have an analytical solution of such a problem. However, the rapid development of high-speed digital computers in recent years made it possible to develop various numerical methods which overcome many difficulties and obtain quite accurate solution of the stability problem. Out of these methods, three have been widely used and these proved to be effective in dealing with different types of stability equations. These methods are:

- (i) the Galerkin method.
- (ii) the finite difference method.
- (iii) the Harris and Reid method.

In the Galerkin method, the disturbances in the governing equations are expanded in complete sets of functions which satisfy the boundary conditions. The coefficients in the series are determined by the requirement that the governing equations be orthogonal to the expansion functions of the disturbances. This leads to infinitely many linear, homogeneous equations for the various coefficients in the series. For non-trivial solution it is necessary that the determinant of this system of the equations equals zero. This condition gives a determinant equation for the critical conditions. In practice only a finite number of terms are used in the series of the disturbances.

Chandrasekhar (1954) was the first to develop a Galerkin-type method for solving the stability equations. Since 1954, this method has been used for a wide variety of problems in hydrodynamic and hydromagnetic stability. For example, Walowit, Tsao and DiPrima (1964) employed this method to study the effect of heating on the stability of Couette flow, whereas Kurzweg (1961) and Hassard, Chang and Ludford (1972) used it to predict the

instability of an electrically conducting fluid under the influence of a constant magnetic field applied in the axial direction of the cylinders. A general study of the application of Galerkin method to stability problems can be found in a paper given by DiPrima (1955).

The finite difference procedure starts by introducing the vorticity stream functions into the unsteady Navier-Stokes equations governing the perturbed state of the problem. The derivatives are then approximated by central differences in the space coordinates and by forward differences in time. In view of the behaviour of the perturbed flow with time the critical state can be determined. The perturbations will decay or grow with successive time steps depending on whether given parameters are below or above their critical values. Thus, by a proper adjustment of these parameters, the critical values can be found. A full description of this method and its application to the instability of Couette flow is given by Astill and Chang (1976).

The method developed by Harris and Reid (1964) converts the two-point boundary value stability problem into an initial value problem. The procedure starts by rewriting the governing equations as a system of first order equations. A set of linearly independent solutions of the system which satisfies the first boundary condition is constructed by imposing the initial conditions. Any solution of the system of equations satisfying the first boundary condition can then be represented as a linear combination of the constructed solutions. A necessary condition that this linear combination also satisfies the second boundary condition (say at $x = a$) is that the determinant of the system of the boundary conditions vanishes at $x = a$. This requirement provides the characteristic equation from which the critical conditions can be found.

Different aspects of the stability of Couette flow problem were studied

by employing the Harris and Reid method of solution, for example, in examining the stability effects of radii ratio of the two cylinders and of constant temperature gradient between the cylinders by Sparrow, Munro and Jonsson (1965) and Takhar, Smith and Soundalgekar (1985) respectively. We have adopted this method in all our problems. We find it is relatively simple, gives quite accurate results and uses less computer time.

As noted above, the development of various numerical techniques has simplified the study of the stability contributions of different factors, such as radii and velocities ratios of the two cylinders, heating and the applied magnetic field. The importance of these studies emerge from their wide engineering applications, especially in the design of journal bearings. Torque measurements show that journal bearings operating in Taylor vortices flow regime generate more torque than in laminar flow conditions and this results in an increase in frictional power loss and bearing metal temperature. These factors are important in running costs and the bearing life.

The presence of Taylor vortices, which is not beneficial in journal bearings, plays a positive and useful role in some other applications such as;

1. Drying machines.
2. Rotary heat exchangers.
3. Chemical mixing machines.

Therefore, it is of great importance to examine the various factors which can alter the critical velocity of a fluid and to determine the extent of the stability changes that occur due to the action of one or more of these influencing factors.

In Chapter Two, we study the stability effect of heating due to the presence of a constant heat flux at the inner cylinder. All three possible types of rotation of the two cylinders are considered. The inner cylinder only is rotating while the outer one is kept stationary or both the cylinders are rotating, with different velocities ratios, in the same or opposite direction. In formulating the equations governing the marginal stability state, the cylinders are assumed i) to have a narrow gap and ii) to have different gap sizes. Hence, this problem explores the stability interaction between the constant heat flux and velocities ratio in the first part and the addition of the radii ratio in the second part. The results obtained by a numerical method indicate that, as compared with isothermal flow and stationary outer cylinder, the presence of a constant heat flux at the inner cylinder and co-rotating cylinders always lead to a more stable flow. For counter-rotating cylinders, the stabilization with increasing magnitudes of μ starts after the flow attains its minimum stable state. This minimum point is found to be a function of the radii ratio η and it moves slowly towards larger negative values of μ as η decreases.

In Chapter Three, the fluid is assumed to be electrically conducting and it is under the influence of a magnetic field applied in the axial direction and also a radial temperature gradient maintained by keeping the cylinders at different constant temperatures. This hydromagnetic stability problem is solved for the narrow gap case and for all the three types of rotation. The variations of the critical parameters with the different influencing factors, presented in tabular and graphical forms, reveal that the axially applied magnetic field H inhibits the onset of the instability. The critical angular velocity Ω_1 needed to bring about instability in the flow, increases steadily with larger magnitudes of H . Moreover, the inhibiting power of H is strengthened by the stabilization tendency of the

negative temperatures gradient (temperature of the outer cylinder is less than that of the inner one) and weakened by the destabilization tendency of the positive temperature gradient. It is also worth mentioning here that the predicted critical conditions exhibit a good agreement with those previously reported for the special cases (a) isothermal non-magnetic flow, (b) isothermal magnetic flow and (c) thermal non-magnetic flow.

In Chapter Four, we remove the restriction of narrow gap and the stability effect of different radii ratios is considered. We therefore, present the simultaneous effects of the axial magnetic field, radial temperature gradient, velocities ratio and radii ratio. The stability equations were written in such a way that for a given fluid and fixed outer cylinder radius R_2 , the critical parameter determines the angular velocity Ω_1 , which sets instability in the flow. This allowed us to examine explicitly the stability contribution of one or more of the influencing factors. The important role that the radii ratio η plays in this problem, is manifested through the strong quantitative dependence of the stability potentials of the other factors on its value. The critical angular velocity associated with a fixed magnetic field, temperature gradient and velocities ratio undergoes a considerable change with the alteration of the gap size between the cylinders. Our problem also includes, as a special case, the problem of the stability effect of different η on an axially magnetic isothermal flow.

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CHAPTER 2
THE EFFECT OF CONSTANT HEAT FLUX AT THE INNER
CYLINDER ON THE STABILITY OF COUETTE FLOW

2.1 Introduction

Since the early investigation of Taylor (1923), the problem of instability of viscous flow between rotating concentric cylinders has been studied by several authors. In particular, different aspects of the isothermal case were examined by Chandrasekhar (1954, 1958), Walowit, Tsao and DiPrima (1964), Sparrow, Munro and Jansson (1964) and Astill and Chung (1976). Chandrasekhar (1954, 1958) considered two cylinders with narrow gap (i.e. $\eta \sim 1$) and a finite gap that corresponds to $\eta = \frac{1}{2}$ (i.e. $R_1 = \frac{1}{2} R_2$). The critical Taylor number, T_C , at which the flow becomes unstable, was obtained for values of μ in the range of $-0.5 \leq \mu < 0.25$. Using the Galerkin method, Walowit et al (1964) predicted instability for a wider range of η ($0.1 \leq \eta \leq 1.0$). However, the T_C , which is defined as a function of η and μ , was found for values of μ only between $-\frac{1}{2}$ and η^2 . Sparrow et al (1964) defined the Taylor number as,

$$T = \frac{\Omega_1^2 R_2^4}{\nu^2} \quad (1)$$

Thus for a given fluid and fixed R_2 , the T_C determines the angular velocity Ω_1 at the onset of instability. It was, therefore, possible to examine explicitly the stability contribution of η and μ . The T_C , calculated by employing the Harris and Reid method, indicated that the greatest and lowest stable state of the fluid is achieved when $\eta \sim 1$ and $\eta = \frac{1}{2}$ respectively. The T_C decreases as η drops from 1.0 to 0.5 and then increases for $\eta > 0.5$. In relation to the velocities ratio μ , the T_C attains its minimum when μ is slightly negative. For all other values of μ , the stability curves are seen to rise monotonically with increasing values of $|\mu|$. Finally, using finite difference technique, Chang and Astill (1976) confirmed the findings of Sparrow et al (1964). Moreover,

the cell patterns at the onset of instability were presented for different gap sizes.

Turning next to the non-isothermal case, we find that the stabilizing effects of radial temperature gradient sustained by two cylinders with different constant temperatures have received great attention. The mathematical treatment of this problem was considered by Yih (1961), Becker and Kaye (1962a), Walowit, Tsao and DiPrima (1964), Bahl (1972), Soundalgekar, Takhar and Smith (1981) and Takhar, Soundalgekar and Smith (1985). Experimental results were obtained by Bjorklund and Kays (1959), Haas and Nissan (1961), Becker and Kaye (1962b) and Ho, Nardacci and Nissan (1964). In all these studies, it was observed that the positive temperature gradient (the temperature of the outer cylinder is higher than that of the inner one) destabilizes the flow, while the negative temperature gradient stabilizes it.

The analysis of Becker and Kaye (1962a) was limited to the situation where the outer cylinder is stationary and the gap between the cylinders ($R_2 - R_1$) is much smaller than the mean radius ($\frac{1}{2} (R_1 + R_2)$). Hence, they explored the stability reaction of the fluid to the presence of heating alone. Walowit et al (1964) extended this problem and included the case of finite gap between the cylinders that corresponds to radii ratio $\eta = \frac{1}{2}$. The role of velocities ratio μ in the thermal instability of the fluid was later examined by Bahl (1972) and then by Takhar, Smith and Soundalgekar (1985). Their results indicated that as the negative value of μ is increased in magnitude, the tendencies of temperature gradients to induce stability changes are reduced considerably.

In the present study, we consider the effect of heating due to a constant heat flux at the inner cylinder, on the stability of Couette flow. All different types of rotation of the two cylinders are considered and the

stability equations are formulated for both cases, i.e., narrow gap and wide gap. Using a numerical procedure similar to the one described by Harris and Reid (1964), the critical conditions characterizing the stationary marginal state are determined. Our results for the special isothermal case exhibit a good agreement with those given by Chandrasekhar (1961), Sparrow et al (1964) and Astill and Chung (1976). The presence of constant heat flux at the inner cylinder enhances the stability state of the flow and hence postpones the appearance of the Taylor vortex flow to a higher critical angular velocity Ω_1 . In the wide gap problem, we analyse the stabilizing interaction between the three influencing factors, viz. velocities ratio μ , radii ratio η and heating. The value of T_c for given values of μ and η in the isothermal case is raised owing to the action of heating.

2.2 Problem Formulation

We consider a viscous incompressible fluid between two coaxial rotating cylinders. Heat is supplied to the inner cylinder at a constant rate whereas the outer cylinder being kept at a constant temperature T_2 . In a cylindrical co-ordinate system (r, θ, z) , with z measured along the common axis of the cylinders and associated (u, v, w) velocities in these directions, the Navier Stokes equations are given by,

Momentum Equations :

$$\frac{\partial u}{\partial t} + (u.\text{grad}) u - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u - \frac{2}{r^2} \frac{\partial v}{\partial \theta} - \frac{u}{r^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + (u.\text{grad}) v + \frac{uv}{r} = -\frac{1}{r} \frac{1}{\rho} \frac{\partial p}{\partial \theta} + \nu \left(\nabla^2 v + \frac{2}{r^2} \frac{\partial u}{\partial \theta} - \frac{v}{r^2} \right) \quad (3)$$

$$\frac{\partial w}{\partial t} + (u.\text{grad}) w = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w \quad (4)$$

Continuity Equation :

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \quad (5)$$

Energy Equation :

$$\frac{\partial T}{\partial t} + (u \cdot \text{grad}) T = \frac{k}{\rho c_p} \nabla^2 T \quad (6)$$

Equation of State :

$$\rho = \rho_2 (1 - \alpha (T - T_2)) \quad (7)$$

where

$$\begin{aligned} u \cdot \text{grad} &= u \cdot \frac{\partial}{\partial r} + \frac{v}{r} \frac{\partial}{\partial \theta} + w \frac{\partial}{\partial z} \\ \nabla^2 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \end{aligned} \quad (8)$$

Here p denotes the pressure, ρ the density, ν the kinematic viscosity, T the temperature, k the thermal conductivity, and c_p the specific heat at a constant pressure of the fluid. In the above equations ν , k and c_p are assumed to be constant. The heat due to viscous dissipation is also neglected in equation (6).

In the absence of transverse pressure gradient, equations (2) - (6) allow a stationary solution of the form,

$$u = w = 0, \quad v = V(r), \quad T = T(r), \quad P = P(r) \quad (9)$$

provided that

$$\frac{dp}{dr} = \rho \frac{V^2}{r} \quad (10)$$

and

$$\nabla^2 V - \frac{V}{r^2} = 0 \quad (11)$$

In view of equation (11), the most general form of $V(r)$ is given by

$$V = Ar + \frac{B}{r} . \quad (12)$$

The constants A and B are determined by imposing the boundary conditions on V . If Ω_1 , Ω_2 , R_1 and R_2 denote the angular velocities and the radii of the inner and the outer cylinder respectively, then, we have

$$\Omega_1 R_1 = AR_1 + \frac{B}{R_1} \quad (13)$$

$$\Omega_2 R_2 = AR_2 + \frac{B}{R_2} \quad (14)$$

solving equations (13) and (14) for A and B , we obtain

$$A = \frac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_1^2} , \quad B = \frac{R_1^2 R_2^2 (\Omega_1 - \Omega_2)}{R_2^2 - R_1^2} . \quad (15)$$

The steady temperature, T , is assumed to vary along the radial direction only, and according to equation (6), it is governed by

$$\frac{d^2 \bar{T}}{dr^2} + \frac{1}{r} \frac{d\bar{T}}{dr} = 0 \quad (16)$$

The boundary conditions on $\bar{T}$ are

$$\frac{d\bar{T}}{dr} = -\frac{q}{k} \quad \text{at } r = R_1 \quad (17)$$

$$T = T_2 \quad \text{at } r = R_2$$

where q is the constant heat flux at the inner cylinder. Solving the differential equation (16) and imposing the boundary conditions (17), gives

$$\bar{T} = T_2 - \frac{qR_1}{k} \ln \frac{r}{R_2} . \quad (18)$$

2.3 Perturbation Equations

If infinitesimal disturbances u' , v' , w' , p' , ρ' and T' are

superimposed on the steady state solution defined by (9), then the perturbed flow is described by the variables,

$$u', v' + V(r), w', p' + P(r), \rho' + \rho, T' + \bar{T}(r). \quad (19)$$

The various perturbations are assumed to be axisymmetric along the common axis of the cylinders and thus independent of θ . To obtain the perturbation equations we substitute (19) into the governing equations (2) - (7), and taking into consideration the relations, (11) and (16). We obtain, upon simplification,

$$\frac{\partial u'}{\partial t} + u' \frac{\partial u'}{\partial r} + w' \frac{\partial u'}{\partial z} - \frac{(V+v')^2}{r} = \frac{1}{(\rho+\rho')} \frac{\partial(p+p')}{\partial r} + \nu (\nabla^2 u' - \frac{u'}{r^2}) \quad (20)$$

$$\frac{\partial v'}{\partial t} + u' \frac{\partial}{\partial r} (V + v') + w' \frac{\partial v'}{\partial z} + u' \frac{V + v'}{r} = \nu (\nabla^2 v' - \frac{v'}{r^2}) \quad (21)$$

$$\frac{\partial w'}{\partial t} + u' \frac{\partial w'}{\partial r} + w' \frac{\partial w'}{\partial z} = \frac{1}{\rho + \rho'} \frac{\partial p'}{\partial z} + \nu \nabla^2 w' \quad (22)$$

$$\frac{\partial u'}{\partial r} + \frac{u'}{r} + \frac{\partial w'}{\partial z} = 0 \quad (23)$$

$$\frac{\partial T'}{\partial t} + u' \frac{\partial}{\partial r} (\bar{T} + T') + w' \frac{\partial T'}{\partial z} = \frac{k}{\rho c_p} \nabla^2 T' \quad (24)$$

$$\rho + \rho' = \rho_2 [1 - \alpha (\bar{T} - T_2 + T')] \quad (25)$$

The above equations are linearized by neglecting product and powers (greater than one) of the disturbances. They are also simplified by imposing the usual Boussinesq approximation. The density can be treated as constant except when it is multiplied by the centrifugal acceleration term,

$\frac{v^2}{r}$. This approximation is valid for small value of the thermal expansion coefficient α . For fluids, which we shall be mostly concerned (such as water), α lies in the range of 10^{-3} to 10^{-4} . Therefore, equations (20) - (24) reduce to

$$\frac{\partial u'}{\partial t} + \alpha T' \frac{V^2}{r} - 2 \frac{V}{r} v' = - \frac{1}{\rho} \frac{\partial p'}{\partial r} + \nu (\nabla^2 u' - \frac{u'}{r^2}) \quad (26)$$

$$\frac{\partial v'}{\partial t} + 2A u' = \nu (\nabla^2 v' - \frac{v'}{r^2}) \quad (27)$$

$$\frac{\partial w'}{\partial t} = - \frac{1}{\rho} \frac{\partial p'}{\partial z} + \nu \nabla^2 w' \quad (28)$$

$$\frac{\partial u'}{\partial t} + \frac{u'}{r} + \frac{\partial w'}{\partial z} = 0 \quad (29)$$

$$\frac{\partial T'}{\partial t} + \frac{\partial \bar{T}}{\partial r} u' = \frac{k}{\rho c_p} \nabla^2 T' \quad (30)$$

in view of equation (12), the term $(\frac{\partial V}{\partial r} + \frac{V}{r})$ has been replaced by $2A$ in equation (27).

The dependence on pressure in (26) - (30), can be eliminated by differentiating (26) and (28) with respect to z and r respectively. This leads to

$$\nu (\nabla^2 - \frac{1}{r^2} - \frac{1}{\nu} \frac{\partial}{\partial t}) \frac{\partial u'}{\partial z} + 2 \frac{V}{r} \frac{\partial v'}{\partial z} - \alpha \frac{V^2}{r} \frac{\partial T'}{\partial z} = \frac{1}{\rho} \frac{\partial^2 p'}{\partial z \partial r} \quad (31)$$

$$\nu (\nabla^2 - \frac{1}{r^2} - \frac{1}{\nu} \frac{\partial}{\partial t}) \frac{\partial w'}{\partial r} = \frac{1}{\rho} \frac{\partial^2 p'}{\partial z \partial r} \quad (32)$$

Eliminating $\frac{\partial^2 p'}{\partial z \partial r}$, we have

$$\nu (\nabla^2 - \frac{1}{r^2} - \frac{1}{\nu} \frac{\partial}{\partial t}) (\frac{\partial w'}{\partial r} - \frac{\partial u'}{\partial z}) = 2 \frac{V}{r} \frac{\partial v'}{\partial z} - \alpha \frac{V^2}{r} \frac{\partial T'}{\partial z} \quad (33)$$

Equations (27), (29), (30) and (33) constitute the perturbation equations.

2.4 Analysis in Terms of Normal Modes

The perturbation equations are linear, independent of θ , and have coefficients that depend on r only. Also, the disturbances are periodic in

the axial direction. Thus by analysing the disturbances into normal modes, we seek solutions of the form,

$$\begin{aligned} u' &= u(r) e^{\sigma t} \cos \lambda z & w' &= w(r) e^{\sigma t} \sin \lambda z \\ v' &= v(r) e^{\sigma t} \cos \lambda z & T' &= T(r) e^{\sigma t} \cos \lambda z \end{aligned} \quad (34)$$

where λ is the wave number of the disturbance, σ a constant which may be complex and $u(r)$, $v(r)$, $w(r)$ and $T(r)$ are the amplitudes of the corresponding disturbances. Substituting (34) into (27), (29), (30) and (33) we obtain

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \lambda^2 - \frac{\sigma}{\nu}\right) \left(\frac{dw}{dr} + \lambda u\right) = -2 \frac{V}{r} \frac{\lambda}{\nu} v + \alpha \frac{V^2}{r} \frac{\lambda}{\nu} T \quad (35)$$

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \lambda^2 - \frac{\sigma}{\nu}\right) v = 2 \frac{A}{\nu} u \quad (36)$$

$$\left(\frac{d}{dr} + \frac{1}{r}\right) u = -\lambda w \quad (37)$$

$$\left(\frac{d}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{\sigma \rho^c p}{k}\right) T = \frac{\rho^c p}{k} \frac{d\bar{T}}{dr} u \quad (38)$$

In equation (35), w , can be replaced by using its value from (37) and the above equations can then be re-written in the following form,

$$(DD^* - \lambda^2 - \frac{\sigma}{\nu})(DD^* - \lambda^2) u = 2 \frac{V}{r} \frac{\lambda^2}{\nu} v - \alpha \frac{V^2}{r} \frac{\lambda^2}{\nu} T \quad (39)$$

$$(DD^* - \lambda^2 - \frac{\sigma}{\nu}) v = \frac{2A}{\nu} u \quad (40)$$

$$(D^*D - \lambda^2 - \frac{\sigma \rho^c p}{k}) T = \frac{\rho^c p}{k} \frac{d\bar{T}}{dr} u \quad (41)$$

where $D = \frac{d}{dr}$, $D^* = \frac{d}{dr} + \frac{1}{r}$,

$$DD^* = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} \quad (42)$$

$$D^*D = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}$$

To convert equations (39) - (41) into non-dimensional form, we introduce the following dimensionless variables and quantities,

$$x = \frac{r - R_1}{d} \quad , \quad a = \lambda d \quad , \quad \eta = \frac{R_1}{R_2}$$

$$\xi = \frac{r}{R_2} \quad , \quad \mu = \frac{\Omega_2}{\Omega_1} \quad , \quad g(x) = \frac{V}{r\Omega_1} \quad , \quad \text{Pr} = \frac{\nu \rho c_p}{k} \quad (43)$$

$$\bar{u} = \frac{2Ad^2}{\nu} \frac{1}{\Omega_1 R_2} u \quad , \quad \bar{v} = \frac{v}{\Omega_1 R_2} \quad , \quad \theta = -2 \frac{A}{\Omega_1} \frac{1}{\text{Pr}} \left(\frac{k}{qR_1} \right) T \quad ,$$

where $d = R_2 - R_1$ and Pr is the Prandtl number.

The following relations are direct consequences of relations (43)

$$D = \frac{1}{d} D_1 \quad , \quad D^* = \frac{1}{d} D_1^* \quad \text{and} \quad g(x) = A_1 + \frac{B_1}{\xi^2} \quad (44)$$

where $D_1 = \frac{d}{dx}$, $D_1^* = \frac{d}{dx} + \frac{1 - \eta}{\xi}$,

$$A_1 = \frac{\mu - \eta^2}{1 - \eta^2} \quad \text{and} \quad B_1 = \frac{1 - \mu}{1 - \eta^2} \quad .$$

In terms of the new variables (43) and replacing $\frac{dT}{dr}$ by its value in (18), equations (39) - (41) become

$$(D_1 D_1^* - a^2 - \frac{\sigma d^2}{\nu})(D_1 D_1^* - a^2) \bar{u} = \frac{4A\Omega_1 d^4}{\nu^2} a^2 g(x) \bar{v}$$

$$+ \text{Pr} \frac{\Omega_1^2 d^4}{\nu^2} \left(\frac{qR_1}{k} \right) \alpha \xi a^2 (g(x))^2 \theta \quad (45)$$

$$(D_1 D_1^* - a^2 - \frac{\sigma d^2}{\nu}) \bar{v} = \bar{u} \quad (46)$$

$$(D_1^* D_1 - a^2 - \frac{\sigma \rho c_p d^2}{k}) \theta = \frac{1}{\xi} \bar{u} \quad (47)$$

Defining the Taylor and Rayleigh numbers as

$$T_a = - \frac{4A\Omega_1 d^4}{\nu^2} \quad , \quad R_a = \text{Pr} \frac{\Omega_1^2 d^4 (\frac{qR_1}{k})}{\nu^2} \alpha \quad (48)$$

Equation (45) can now be written as

$$(D_1 D_1^* - a^2 + \frac{\sigma}{\nu})(D_1 D_1^* - a^2) \bar{u} = - a^2 T_a (g(x) \bar{v} - N\xi (g(x))^2 \theta) \quad (49)$$

where $N = \frac{R_a}{T_a}$

$$\text{i.e.,} \quad N = \frac{\text{Pr} \alpha (\frac{-qR_1}{k})}{4A_1} \quad (50)$$

2.5 Stationary Marginal State

According to equation (34), the disturbances will grow or decay with time depending on whether the real part of σ is positive or negative respectively. If the real part of σ ($\text{Re } \sigma$) and the imaginary part of σ ($\text{Im } \sigma$) are both equal to zero, then the transition from stability to instability takes place via the marginal state by exhibiting a stationary pattern of motion. The transition exhibits an oscillatory motion with a certain definite characteristic frequency when $\text{Re } \sigma = 0$ but $\text{Im } \sigma \neq 0$. In this work, we assume that the marginal state is stationary. Therefore equations (49), (46) and (47) reduce to, after dropping bars and the subscripts on D and D^* ,

$$(DD^* - a^2)^2 u = - a^2 T_a (g(x) v - N\xi (g(x))^2 \theta). \quad (51)$$

$$(DD^* - a^2) v = u. \quad (52)$$

$$(D^*D - a^2) \theta = \frac{1}{\xi} u. \quad (53)$$

Solution of equations (51) - (53) are to be found which satisfy the no slip condition at the walls. Thus, from (19), u , v and w equal zero on the cylindrical walls. Also, in view of equation (37) and since $u = 0$ and $w = 0$, it follows that $Du = 0$. Similarly, for temperature, we assume that the walls are better conductors of heat. Thus on the outer cylinder, $T + \theta =$

$\bar{T}$, i.e., $\theta = 0$. On the inner cylinder, $\frac{d}{dr} (\bar{T} + \theta) = \frac{-q}{k}$, i.e., $\frac{d\theta}{dr} = 0$.

Therefore, the boundary conditions are,

$$u = Du = v = D\theta = 0 \quad \text{at } x = 0 \quad (54)$$

$$u = Du = v = \theta = 0 \quad \text{at } x = 1$$

The governing equations (51) - (54) along with the boundary conditions (54) define an eigenvalue problem of the form

$$F(n, \mu, N, a, T_a) = 0 \quad (55)$$

2.6 The Narrow Gap Approximation

We shall restrict our discussion of equation (55) first to the situation when the gap, $d = R_2 - R_1$, between the cylinders is small compared to the mean radius $R_0 = \frac{1}{2} (R_1 + R_2)$. In this case, the following approximations are valid

$$\begin{aligned} \text{(i)} \quad n &= 1 & \text{(ii)} \quad \xi &= 1, \\ & & & \\ \text{(iii)} \quad D^* &= D & \text{(iv)} \quad g(x) &= 1 - (1 - \mu)x, \end{aligned} \quad (56)$$

The validity of part (i) follows from the fact that $d \ll R_0$, i.e. $R_1 \sim R_2$.

Since $\xi = \frac{r}{R_2}$, i.e., $\xi = \eta + (1 - \eta) x$, it will follow from (i) that $\xi = 1$ which is part (ii). Part (iii) follows directly from (i), (ii) and the definition of D^* . To satisfy relation (iv), we first notice in view of equation (11), that the basic velocity $V(r)$ is governed by

$$DD^* V(r) = 0 \quad (57)$$

For a narrow gap and in view of (iii), equation (57) reduces to

$$D^2 V = 0 \quad (58)$$

Since $g(x) = \frac{V}{R_2 \Omega_1}$, it follows that $D^2 g(x) = 0$. Thus

$$g(x) = Ex + F \quad (59)$$

The constants E and F are determined by imposing the following boundary conditions on $g(x)$,

$$\begin{aligned} g(x) &= 1 & \text{at } x &= 0 \\ g(x) &= \mu & \text{at } x &= 1 \end{aligned} \quad (60)$$

It can be easily shown, from (59) and (60), that $g(x)$ takes the form given in (iv).

Under the approximation (56), the governing equations (51) - (53) reduce to

$$(D^2 - a^2)^2 u = -a^2 T_a (g(x) v - N (g(x))^2 \theta) \quad (61)$$

$$(D^2 - a^2) v = u \quad (62)$$

$$(D^2 - a^2) \theta = u \quad (63)$$

Equations (61) - (63) and the boundary conditions (54) determine an eigenvalue problem of the form

$$H(\mu, N, a, T_a) = 0 \quad (64)$$

For given values of μ and N , we would determine the minimum positive value of T_a over all real positive value of a . This value of T_c is the critical value of T_a which determines the critical speed Ω_1 at which the flow goes through a transition from its initial state to its final unstable state. The corresponding critical wave number, a , determines the spacing of the vortices in the axial direction.

2.7 Method of Solution

We adopt the Harris and Reid method to solve the two-point eigenvalue problem defined by equation (61) - (64) and the boundary conditions (54).

(a) Solution Procedure

The procedure starts by rewriting the governing equations (61) - (63) as a system of first order differential equations in the form,

$$\begin{aligned} \frac{dA}{dx} &= B, \quad \frac{dB}{dx} = C \\ \frac{dC}{dx} &= D, \quad \frac{dD}{dx} = 2a^2C - a^4A - a^2T [g(x).E - N(g(x))^2G] \end{aligned} \quad (65)$$

$$\frac{dE}{dx} = F \quad , \quad \frac{dF}{dx} = a^2E + A$$

$$\frac{dG}{dx} = H \quad , \quad \frac{dH}{dx} = a^2G + A \quad (65)$$

$$\text{where} \quad A = u \quad , \quad B = \frac{du}{dx} \quad , \quad C = \frac{d^2u}{dx^2} \quad , \quad D = \frac{d^3u}{dx^3} \quad ,$$

$$E = v \quad , \quad F = \frac{dv}{dx} \quad , \quad G = \theta \quad , \quad H = \frac{d\theta}{dx} \quad .$$

The boundary conditions (54) require that

$$A = B = E = H = 0 \quad \text{at } x = 0 \quad (66)$$

$$A = B = E = G = 0 \quad \text{at } x = 1 \quad .$$

We define four linearly independent solutions, U_i , V_i and T_i , ($i = 1, 2, 3, 4$) of equations (61) - (63) by imposing the initial conditions

$$\left[U_i = \frac{dU_i}{dx} = V_i = \frac{dT_i}{dx} \right] = 0 \quad i = 1, 2, 3, 4 \quad (67)$$

and

$$\frac{d^2U_i}{dx^2} \quad , \quad \frac{d^3U_i}{dx^3} \quad , \quad \frac{dV_i}{dx} \quad , \quad T_i = \begin{cases} (1, 0, 0, 0) & \text{for } i = 1 \\ (0, 1, 0, 0) & \text{for } i = 2 \\ (0, 0, 1, 0) & \text{for } i = 3 \\ (0, 0, 0, 1) & \text{for } i = 4 \end{cases} \quad (68)$$

at $x = 0$

A solution of equations (61) - (63) which automatically satisfies the boundary condition at $x = 0$ can be written in the form,

$$u = \sum_{i=1}^4 M_i U_i \quad , \quad v = \sum_{i=1}^4 N_i V_i \quad , \quad \theta = \sum_{i=1}^4 P_i T_i \quad . \quad (69)$$

where M_i , N_i and P_i are constants. The boundary condition at $x = 1$ requires that

$$\sum_{i=1}^4 M_i U_i = 0$$

$$\sum_{i=1}^4 M_i \frac{dU_i}{dx} = 0$$

(70)

$$\sum_{i=1}^4 N_i V_i = 0$$

$$\sum_{i=1}^4 P_i T_i = 0$$

If the constants M_i , N_i and P_i are not to vanish identically then it is necessary that the determinant of U_i , $\frac{dU_i}{dx}$, V_i and T_i vanishes at $x = 1$, i.e.,

$$H(\mu, N, a, T_a) = \begin{vmatrix} U_i \\ \frac{dU_i}{dx} \\ V_i \\ T_i \end{vmatrix} = 0, \quad \text{at } x = 1 \quad (71)$$

Equation (71) is the characteristic equation from which the critical Taylor number and the corresponding wave number can be obtained.

In terms of the variables defined in (65), if we put $A_i = U_i$, $E_i = V_i$ and $G_i = T_i$ ($i = 1, 2, 3, 4$), then there are thirty two equations to be integrated numerically using the initial conditions (67) and (68). These can be written as,

$$[A_i = B_i = E_i = H_i] = 0 \quad i = 1, 2, 3, 4$$

$$C_i, D_i, F_i, G_i = \begin{cases} (1, 0, 0, 0) & \text{for } i = 1 \\ (0, 1, 0, 0) & \text{for } i = 2 \\ (0, 0, 1, 0) & \text{for } i = 3 \\ (0, 0, 0, 1) & \text{for } i = 4 \end{cases} \quad (72)$$

Equation (71) becomes

$$H(\mu, N, a, T_a) = \begin{vmatrix} A_i \\ B_i \\ E_i \\ G_i \end{vmatrix} = 0 \quad (73)$$

(b) Numerical Integration Procedure

The system of ordinary differential equations (65) is integrated by using the Merson form of the Runge-Kutta method (Hall and Watt (1976)). We divided the range of integration into twenty equal increments. The integration was advanced from $x = 0$, over successive increments. The final values of one increment provide the initial values for the next. Within each increment the integration process can be outlined as follows,

The procedure advances the solution of the differential equations

$$\frac{dy_i}{dx} = f_i(x, y_1, y_2, \dots, y_n) \quad (74)$$

from x_n to $x_n + H (= x_{n+1})$, where H is the integration increment (i.e., $H = \frac{1}{20}$), using a number of steps of the Merson form of the Runge-Kutta method. This can be described as

$$y_{n+1} = y_n + \frac{h}{6} (k_1 + 4k_4 + k_5) \quad (75)$$

where

$$k_1 = f(x_n, y_1, y_2, \dots, y_n)$$

$$k_2 = f(x_n + \frac{h}{3}, y_1 + \frac{k_1}{3}, y_2 + \frac{k_1}{3}, \dots, y_n + \frac{k_1}{3})$$

$$k_3 = f(x_n + \frac{h}{3}, y_1 + \frac{1}{6} (k_1 + k_2), y_2 + \frac{1}{6} (k_1 + k_2), \dots, y_n + \frac{1}{6} (k_1 + k_2))$$

$$k_4 = f(x_n + \frac{h}{2}, y_1 + \frac{1}{8}(k_1 + 3k_3), y_2 + \frac{1}{8}(k_1 + 3k_3), \dots, y_n + \frac{1}{8}(k_1 + 3k_3))$$

$$k_5 = f(x_n + h, y_1 + \frac{1}{2}(k_1 - 3k_3 + 4k_4), + \frac{1}{2}(k_1 - 3k_3 + 4k_4), \dots, y_n +$$

$$-(k_1 - 3k_3 + 3k_4)).$$

The procedure obtains an estimate of the local truncation error at each step and varies the step length automatically to keep this estimate below a specified error bound. If the step length becomes less than 10^{-4} times the initial step length, the procedure stops the calculation and an error message is printed. If E_i is the estimated magnitude of the truncation error at the end of a particular step, then the step length will be halved until

$$E_i < \text{TOL} \times \max \{1.0, |Y(1)|, |Y(2)|, \dots, |Y(n)|\} \quad (76)$$

where TOL, is the specified error bound. For all our calculations, the value of TOL, is set equal to 10^{-4} . The truncation error per step length is controlled by varying the step length. Over a number of steps the errors can accumulate in various ways and the total error may exceed the specified error bound. This can be controlled by reducing the error bound for a large number of steps. This was found to be unnecessary in our problem.

(c) Determination of Critical Taylor and Wave Numbers

The critical Taylor number, T_C , is the minimum positive value, which satisfies equation (73), over all positive wave numbers. We employed the bisection method to obtain the critical Taylor number, T_{C1} , for a given wave number, a_1 . The method proceeds as follows,

- i) For a given small Taylor number, T_{a1} , equations (65) are integrated, as in section a, and the value of the determinant (71), $\mathfrak{S}(T_{a1})$ is obtained.
- ii) An increment ΔT_a is added to T_{a1} and for $T_{a1} + \Delta T_a (= T_{a2})$, $\mathfrak{S}(T_{a2})$ is found.
- iii) The signs of $\mathfrak{S}(T_{a1})$ and $\mathfrak{S}(T_{a2})$ are then compared. If they have the same sign, then T_{a2} is again incremented by ΔT_a . The addition of ΔT_a continues until $\mathfrak{S}(T_{an-1})$ and $\mathfrak{S}(T_{an})$ have opposite signs (This will indicate that T_{C1} is between T_{an-1} and T_{an}). At this stage ΔT_a is halved and T_{an-1} is set to equal a new T_{a1} .
- iv) The steps (ii) and (iii) are repeated until the value of $\mathfrak{S}(T_a)$ becomes less than 10^{-5} . This value is the critical Taylor number, T_{C1} .

The wave number, a_1 and the critical Taylor number, T_{C1} define a point on the neutral curve which separates the stable and the unstable regions. The critical wave number, a_c , is the minimum point on the neutral curve. Following a procedure similar to the one described in the last paragraph, the value of a_c can be determined. For a given value of a_1 , T_{C1} is obtained as described previously. The value of a_1 is then successively incremented by a given amount, Δa_1 , until the minimum is passed. That is when T_{Cn} becomes greater than T_{Cn-1} . The increment Δa , is then halved and the process is repeated until Δa_1 becomes less than 10^{-4} . The value of a_c along with T_c define the critical conditions at which instability first sets in the fluid. We repeated the calculations for different values of μ and N .

2.8 The Wide Gap

It has been common practice to start the mathematical treatment of Couette flow stability problem under the narrow gap restriction. Even through this restriction simplifies the underlying problem, but it also cancels out the important role that different values of η can play in varying the critical conditions of a fluid. Hence, we propose, in the wide gap problem to examine the stability influence of η and its relation with other factors.

For an arbitrary value of η , the equations governing the marginal state are given by (51) - (53) and the boundary conditions by (54). In order to analyse the stability effects of η , μ and heating explicitly, we re-write equation (51) as

$$(DD^* - a^2)^2 u = - a^2 \bar{T}_a (-4 A_1 (1 - \eta)^4 g(x) v - \bar{N} \xi \eta (1 - \eta)^4 (g(x))^2 \theta) \quad (77)$$

where now the Taylor number T_a and N are defined as

$$\bar{T}_a = \Omega_1^2 R_2^4 / \nu^2 \quad \bar{N} = \text{Pr} \propto (-\frac{qR_2}{k}) \quad (78)$$

The other equations remain unchanged. Following a solution procedure similar to the one described in section 2.8 the critical Taylor number $\bar{T}_c$ and the corresponding wave number a_c can be determined for any assigned values of η , μ and $\bar{N}$. It is sufficient here to show the transformation of equations (52), (53) and (77) to a system of first order equations. We put

$$\begin{aligned}
A &= u & B &= D^*u \\
C &= (DD^* - a^2) u & E &= D^*(DD^* - a^2) u \\
F &= v & G &= D^*v \\
H &= \theta & I &= D\theta
\end{aligned} \tag{79}$$

The following relations can be now easily derived from (79),

$$\begin{aligned}
(m &= \frac{1 - \eta}{\xi}) \\
B &= DA + mA & C &= DB - a^2A \\
E &= DC + mC & (DD^* - a^2)^2 u &= DE - a^2C \\
G &= DF + mF & (DD^* - a^2)v &= DG - a^2F \\
(DD^* - a^2)\theta &= DI + mI - a^2H
\end{aligned} \tag{80}$$

In view of (80), the governing equations can be now written as a system of

first order equations in the form, ($\frac{d}{dx} = D$)

$$\begin{aligned}
\frac{dA}{dx} &= B - mA \\
\frac{dB}{dx} &= C + a^2A \\
\frac{dC}{dx} &= E - mC \\
\frac{dE}{dx} &= a^2C - a^2 \bar{T}a (-4A_1(1 - \eta)^4 g(x)F - \bar{N}\xi\eta(1 - \eta)^4 (g(x))^2H) \\
\frac{dF}{dx} &= G - mF \\
\frac{dG}{dx} &= a^2F + A \\
\frac{dH}{dx} &= I \\
\frac{dI}{dx} &= mI + a^2H + \frac{1}{\xi} A
\end{aligned} \tag{81}$$

The boundary conditions (54), then require

$$\begin{aligned} A = B = F = I = 0 & \quad \text{at } x = 0 \\ A = B = F = H = 0 & \quad \text{at } x = 1 \end{aligned} \tag{82}$$

2.9 Results and Discussion

The results which have been derived by the application of the foregoing solution procedure are listed in Tables 1 and 2-5 respectively for narrow and wide gap cases. Also, the variation of T_C with different parameters are presented graphically in Figures 1 through 17. It is obvious from the type of dependence of T_C on N , that the constant heat flux on the inner cylinder has a stabilizing effect on the flow. With $N \neq 0$, the flow stays in its initial laminar state under the transition velocity Ω_1 of the isothermal case. Thus, the supply of heat to the inner cylinder at a constant rate may prove to be one solution to a physical problem where the presence of Taylor vortex flow is not desirable and it is necessary to delay its appearance to a higher critical velocity Ω_1 .

We now examine, for the narrow gap situation and as compared with the isothermal case, the stabilizing potential of N which is a function of μ . Inspection of Table (1) and Figures (1) and (2) reveals that the critical Taylor number T_C is an increasing function of N . This indicates that for greater supply of heat at the inner cylinder, larger values of Ω_1 are required to set instability in the fluid. For the stationary outer cylinder ($\mu = 0$), T_C at $N = 0.25$ is about one and half times that for $N = 0$. It rises to three times as much when $N = 1.0$. When the two cylinders rotate in the same direction ($\mu > 0$), the rate of stabilization of the flow

with N is faster than that for $\mu = 0$. Although when $N = 0$, the T_C for any $\mu > 0$ is less than that of $\mu = 0$. But as N gradually increases the difference between the two T_C gets smaller and eventually, the T_C of $\mu > 0$ becomes greater than that of $\mu = 0$. This feature is evident from Table (1) and Figure (1). It is also clear that the higher the value of positive μ , the higher is the changes in T_C . Hence we may state that the presence of constant heat flux at the inner cylinder is most effective, in terms of flow stability, when the two cylinders rotate in the same direction with high velocity ratio μ . Finally, for counter-rotating cylinders, the stabilization with N occurs but with a rate lower than that for $\mu > 0$. As N rises from 0 to 1.0, the T_C increases to three times as much when $\mu = -0.5$, whereas it increases to more than seven times when $\mu = 0.5$.

It appears that by continuously increasing the value of N , we can keep the flow stable up to any high angular velocity Ω_1 . This situation has its limitations. Larger values of N mean greater supply of heat at the inner cylinder and this will cause a variation in the physical properties of the fluid which are assumed to be constant. Hence to be consistent with our assumptions, the supply of heat should be such that the physical properties viz. viscosity, density, thermal conductivity and specific heat at constant pressure, remain constant.

Attention may now be given to the critical wave number a_C associated with the onset of instability. As Table (1) reveals, the a_C undergoes a gradual increase with greater supply of heat at the inner cylinder. The pattern of change of a_C with different N and μ is the same as described before for T_C . A greater effect of N on a_C is predicted when $\mu > 0$ as compared with $\mu < 0$ or $\mu = 0$. It is also interesting to note, as previously reported by different authors, (Walowit et al (1967) and Astill and Chung (1976)), and also presented here, that in the absence of heating,

the value of a_c is fixed around 3.12 for positive and slightly negative values of μ .

We now turn to analyse the results obtained for the wide gap problem. For this case, we have re-defined the Taylor number in such a way that for a given fluid and fixed R_2 , the $\bar{T}_c$ determines the critical angular velocity Ω_{1c} occurring at the onset of instability. This allows us to examine explicitly the stabilizing effect of μ and η on Ω_{1c} . First, from the variations of $\bar{T}_c$ with $\bar{N}$ for different μ and η , presented in Table 2 and Figures (3) - (17), we may state the following conclusions,

1. It can readily be seen that the stabilizing effect of constant heat flux described before for the narrow gap case also appears for all different values of η .
2. The stabilizing contribution of $\bar{N}$ is greater in the presence of (a) co-rotating cylinders ($\mu > 0$) and (b) small radii ratio η . For example, as $\bar{N}$ rises from 0 to 3.0, the $\bar{T}_c$ increases by about (i) 20% when $\eta = 0.85$ and $\mu = 0$, (ii) 190% when $\eta = 0.2$ and $\mu = 0$ and (iii) 330% when $\eta = 0.2$, $\mu = 0.01$.
3. For counter-rotating cylinders ($\mu < 0$), the effect of $\bar{N}$ is very small as compared with the same positive magnitude of μ . Heating the fluid by an amount that corresponds to $\bar{N} = 2$ enhances its stability by more than four times when $\eta = 0.5$ and $\mu = 0.1$ and by less than 50% when $\eta = 0.5$ but $\mu = -0.1$. Also as negative μ increases in magnitude, the tendency of $\bar{N}$ to stabilize the flow is reduced considerably. This phenomenon has been predicted by Bahl (1972) in connection with the problem of two cylinders with different constant temperatures.

Turning next to the relation between $\bar{T}_C$ and η , we note that η plays an important role in determining the value of $\bar{T}_C$. Moreover, the stabilizing potential of $\bar{N}$ or μ is strongly related to the value of η . For any given $\bar{N}$, the maximum $\bar{T}_C$ is predicted when η is very large (narrow gap between the cylinders). But as η then gets smaller, the $\bar{T}_C$ also gets smaller at a decelerating rate. At η around 0.5 ($R_1 = \frac{1}{2} R_2$), the $\bar{T}_C$ attains its minimum. Any further decrease in η will cause $\bar{T}_C$ to increase, but now, at accelerating rate. As represented in Figures (10) - (13) the minimum $\bar{T}_C$ occurs at about $\eta = 0.5$ not only for $\bar{N} = 0$, as first predicted by Sparrow et al (1964), but also for $\bar{N} \neq 0$. However, the value of $\bar{T}_C$ at this minimum point, as well as all other points, increases with increasing values of $\bar{N}$. For negative μ , the minimum point moves slightly toward the right side of $\eta = 0.5$.

The above mentioned $\bar{T}_C - \eta$ relationship can also be deduced from Figures (3) - (9). For the specified values of μ and for $\bar{N}$ between zero and three, the range of $\bar{T}_C$ at $\eta = 0.95$ is between 15×10^6 and 21×10^6 . The $\bar{T}_C$ then decreases with smaller η and becomes below 6×10^5 at $\eta = 0.5$. For η lower than 0.5, $\bar{T}_C$ rises and we have the range of 2×10^6 to 8×10^6 when $\eta = 0.1$.

Finally, the variation of $\bar{T}_C$ with μ shown in Table (2) and Figures (14) - (17), indicates the following,

- i) For a given $\bar{N}$ and η , the value of μ could be adjusted in order to maximize or minimize the critical angular velocity Ω_1 needed to bring about instability in the fluid. To achieve the most stable flow, the value of μ should be close to η^2 , where for the lowest stable flow, μ depends on η . For large η (narrow gap between the cylinders) the minimum $\bar{T}_C$ is detected when $\mu = 0$ (stationary outer cylinder). But as η gradually decreases the minimum $\bar{T}_C$ moves slowly toward negative

values of μ . When $\eta = 0.75$, the minimum occurs at $\mu = -0.1$, and when $\eta = 0.2$ it occurs at $\mu = -0.02$.

- ii) Driving the two cylinders in the same direction ($\mu > 0$) always leads to a more stable flow, as compared with the stationary outer cylinder case ($\mu = 0$). Also the higher the positive μ , the greater the stability of the flow.
- iii) For counter-rotating cylinders ($\mu < 0$), the flow starts to stabilize with larger magnitudes of negative μ after $\bar{T}_C$ attains its minimum. However, the rate of this stabilization process is smaller than that for positive μ .
- iv) It is interesting to note that the stability curves presented in Figures (14) - (17) are of parabolic character with the vertex moving slowly towards negative μ as η decreases. The spreading of these curves is wide for large η and narrow for small η . This indicates that the change in $\bar{T}_C$ with μ is faster with small η . Also the stabilizing effect of larger $\bar{N}$ is manifested through the rising of all the curves up the $\bar{T}_C$ axis.

We now examine the effects of various factors; $\bar{N}$, μ and η on the critical wave number a_C associated with $\bar{T}_C$. Inspection of Table (2) shows that these effects are quite small and we state them as follows,

- (a) The slow rate of increase of a_C with increasing values of $\bar{N}$ is enhanced by the presence of positive μ and small η .
- (b) For isothermal flow ($\bar{N} = 0$), as compared with the stationary outer cylinder case ($\mu = 0$), the a_C decreases with the co-rotating cylinders and increases with the counter-rotating cylinders. This process is predicted for all η but with different rates.

(c) The a_c suffers a small reduction on its value as the gap size between the cylinders gets smaller. Narrowing the gap from $\eta = 0.1$ to $\eta = 0.95$ at $\bar{N} = 0$, causes a_c to lose about 6% of its original value.

It is significant at this stage to report Rayleigh's criterion for stability of inviscid flow and how the presence of viscosity and radial temperature gradient modify this criterion. By a simple physical argument Rayleigh (1920) derived a stability criterion which states that a necessary and sufficient condition for stability of axisymmetric disturbances is that the square of the circulation does not decrease anywhere, i.e. that $\Phi \geq 0$ everywhere in the field of the flow, where Φ is the Rayleigh discriminant defined by

$$\Phi(r) = \frac{1}{r^3} \frac{d}{dr} (r^2 \Omega^2) \quad (83)$$

here Ω is the angular velocity which is an arbitrary function of r , the distance from the axis of rotation. When Ω takes the particular distribution described by (12), Φ takes the form

$$\Phi = 4A \left(A + \frac{B}{r^2} \right) \quad (84)$$

with the values of A and B given in (15), we have

$$\Phi = 4 \Omega_1 \frac{(1 - \mu)(\mu - \eta^2)}{(1 - \eta^2)^2} \left(\frac{\eta^2}{r^2} + \frac{\mu - \eta^2}{1 - \mu} \right) \quad (85)$$

For two cylinders rotating in the same direction, it is easily shown that for all r in $\eta \leq r \leq 1$

$$\begin{aligned} \Phi &> 0 && \text{if } \mu > \eta^2 \\ \Phi &< 0 && \text{if } 0 < \mu < \eta^2 \end{aligned} \quad (86)$$

Therefore, Rayleigh criterion applied to distribution (12) requires that for stability, the outer cylinder must rotate with an angular speed greater than η^2 times that of the inner cylinder and in the same sense.

When the two cylinders are rotating in opposite direction (i.e., $\mu < 0$), we have

$$\begin{aligned} \Phi > 0 & \quad \text{for all } r \text{ in } \eta_0 < r < 1 \\ \Phi < 0 & \quad \text{for all } r \text{ in } \eta < r < \eta_0 \end{aligned} \tag{87}$$

where the nodal surface $r = \eta_0$ on which Ω and Φ vanish is given by

$$\eta_0 = \eta \left(\frac{1 + |\mu|}{\mu^2 + |\mu|} \right)^{1/2} . \tag{88}$$

Thus Rayleigh criterion is violated only in the layer between the inner cylinder and the nodal surface. The instability which is predicted for all negative μ is to be associated with the instability of the flow interior to η_0 . As can be seen from (88), if $\mu \rightarrow -\infty$, then $\eta_0 \rightarrow \eta$ and thus the unstable layer becomes vanishingly small.

Yih (1961) has pointed out that if the density of the fluid has a radial variation, then the arguments of Vonkárman, as presented in Lin's book (1955), can be used to show that, in the absence of viscosity, the flow in the case of a liquid is always stable if $\rho \Phi^2$ increases with the radial distance, with ρ denoting the density. However, this criterion is violated when viscosity along with radial density variation are present. That is even if $\rho \Phi^2$ increases with the radial distance, the flow can become less stable or even unstable;

(1) for increasing kinematic viscosity if Φ increases but ρ decreases outward, and

(2) for increasing thermal diffusivity if Φ decreases but ρ increases outward.

The right stability criterion which counts for this case, as shown by Yih (1961) is that both ρ and Φ increase outward with the radial distance r .

In our present problem, the constant heat flux at the inner cylinder gives rise to an outward increase of the density with r . Thus, according to Yih's criterion, the flow is stable for $\Phi > 0$ (i.e., $\mu > n^2$) and we determine the instability conditions only for values of $\mu \leq n^2$. For counter-rotating cylinders (i.e. $\mu < 0$), the increase in the magnitude of μ reduces the unstable layer between the nodal surface and the inner cylinder, thus it contributes to more stable flow. This feature is clear from the results obtained for $\mu < 0$.

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TABLE 1: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ AND N
- NARROW GAP PROBLEM -

μ	N	a_c	T_c	μ	N	a_c	T_c
0	0	3.127	3389	-0.25	0	3.144	4461
	0.25	3.251	4196		0.25	3.266	5435
	0.5	3.419	5432		0.5	3.424	6867
	0.75	3.637	7470		0.75	3.625	9096
	1.0	3.910	11106		1.0	3.868	12757
0.25	0	3.119	2725	-0.5	0	3.198	6414
	0.25	3.256	3473		0.25	3.342	7882
	0.5	3.448	4708		0.5	3.533	10082
	0.75	3.716	7000		0.75	3.783	13597
	1.0	4.073	11937		1.0	4.101	19605
0.5	0	3.119	2275	-0.75	0	3.407	10518
	0.25	3.268	3009		0.25	3.669	13501
	0.5	3.499	4348		0.5	4.018	18132
	0.75	3.848	7302		0.75	4.450	25642
	1.0	4.349	16146		1.0	4.942	38352
0.75	0	3.117	1952	-1.0	0	3.999	18663
	0.25	3.288	2694		0.25	4.382	24215
	0.5	3.564	4221		0.5	4.783	32323
	0.75	4.044	8532		0.75	5.216	44917
	1.0	4.787	33164		1.0	5.696	65865
1.0	0	3.117	1708				
	0.25	3.308	2472				
	0.5	3.652	4286				
	0.75	4.347	11673				

TABLE 2: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ AND N
- WIDE GAP PROBLEM -

η	μ	N	a_C	T_C	η	μ	N	a_C	T_C
0.95	0.0	0.0	3.127	1.517 (7)*	0.95	-0.75	0.0	3.503	2.899 (7)
		0.5	3.132	1.532 (7)			0.5	3.511	2.918 (7)
		1.0	3.139	1.547 (7)			1.0	3.520	2.937 (7)
		1.5	3.144	1.562 (7)			1.5	3.525	2.956 (7)
		2.0	3.150	1.578 (7)			2.0	3.533	2.976 (7)
		2.5	3.155	1.594 (7)			2.5	3.541	2.996 (7)
		3.0	3.162	1.610 (7)			3.0	3.547	3.016 (7)
	0.1	0.0	3.123	1.545 (7)	0.85	0.0	0.0	3.130	7.217 (5)
		0.5	3.130	1.563 (7)			0.5	3.147	7.433 (5)
		1.0	3.136	1.581 (7)			1.0	3.163	7.661 (5)
		1.5	3.144	1.60 (7)			1.5	3.181	7.902 (5)
		2.0	3.150	1.619 (7)			2.0	3.199	8.158 (5)
		2.5	3.156	1.638 (7)			2.5	3.218	8.429 (5)
		3.0	3.162	1.658 (7)			3.0	3.237	8.717 (5)
	0.5	0.0	3.118	2.231 (7)	0.1	0.0	0.0	3.125	7.474 (5)
		0.5	3.133	2.295 (7)			0.5	3.145	7.749 (5)
		1.0	3.146	2.363 (7)			1.0	3.166	8.043 (5)
		1.5	3.162	2.435 (7)			1.5	3.188	8.358 (5)
		2.0	3.176	2.511 (7)			2.0	3.210	8.697 (5)
		2.5	3.197	2.592 (7)			2.5	3.230	9.062 (5)
		3.0	3.215	2.678 (7)			3.0	3.259	9.457 (5)
	-0.1	0.0	3.134	1.523 (7)	0.5	0.0	0.0	3.120	1.457 (6)
		0.5	3.138	1.536 (7)			0.5	3.188	1.665 (6)
		1.0	3.143	1.549 (7)			1.0	3.270	1.937 (6)
		1.5	3.149	1.562 (7)			1.5	3.372	2.306 (6)
		2.0	3.152	1.576 (7)			2.0	3.501	2.826 (6)
		2.5	3.157	1.590 (7)			2.5	3.661	3.602 (6)
		3.0	3.163	1.604 (7)			3.0	3.859	4.838 (6)
	-0.5	0.0	3.216	1.962 (7)	-0.1	0.0	0.0	3.138	7.202 (5)
		0.5	3.220	1.974 (7)			0.5	3.152	7.383 (5)
		1.0	3.225	1.987 (7)			1.0	3.166	7.573 (5)
		1.5	3.229	1.999 (7)			1.5	3.179	7.771 (5)
		2.0	3.233	2.012 (7)			2.0	3.196	7.979 (5)
		2.5	3.238	2.025 (7)			2.5	3.212	8.198 (5)
		3.0	3.243	2.038 (7)			3.0	3.219	8.428 (5)

* numbers in the parenthesis denote powers of 10

TABLE 3: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ AND N
- WIDE GAP PROBLEM -

η	μ	N	a_C	T_C	η	μ	N	a_C	T_C
0.85	-0.5	0.0	3.276	1.004 (6)*	0.75	-0.1	0.0	3.145	2.069 (5)
		0.5	3.290	1.023 (6)			0.5	3.168	2.153 (5)
		1.0	3.304	1.042 (6)			1.0	3.191	2.244 (5)
		1.5	3.319	1.062 (6)			1.5	3.216	2.342 (5)
		2.0	3.334	1.083 (6)			2.0	3.240	2.448 (5)
		2.5	3.348	1.104 (6)			2.5	3.269	2.563 (5)
		3.0	3.365	1.127 (6)			3.0	3.296	2.688 (5)
0.85	-0.75	0.0	3.829	1.661 (6)	0.75	-0.25	0.0	3.175	2.226 (5)
		0.5	3.856	1.693 (6)			0.5	3.194	2.298 (5)
		1.0	3.882	1.725 (6)			1.0	3.215	2.375 (5)
		1.5	3.910	1.758 (6)			1.5	3.235	2.457 (5)
		2.0	3.936	1.792 (6)			2.0	3.257	2.544 (5)
		2.5	3.964	1.827 (6)			2.5	3.280	2.637 (5)
		3.0	3.991	1.863 (6)			3.0	3.303	2.736 (5)
0.75	0	0.0	3.136	2.093 (5)	0.75	-0.5	0.0	3.413	3.290 (5)
		0.5	3.161	2.199 (5)			0.5	3.442	3.393 (5)
		1.0	3.192	2.314 (5)			1.0	3.471	3.502 (5)
		1.5	3.218	2.442 (5)			1.5	3.503	3.616 (5)
		2.0	3.254	2.583 (5)			2.0	3.534	3.738 (5)
		2.5	3.290	2.739 (5)			2.5	3.569	3.866 (5)
		3.0	3.324	2.914 (5)			3.0	3.604	4.002 (5)
0.75	0.1	0.0	3.128	2.228 (5)	0.75	-0.75	0.0	4.318	5.968 (5)
		0.5	3.164	2.375 (5)			0.5	4.355	6.133 (5)
		1.0	3.201	2.541 (5)			1.0	4.005	6.304 (5)
		1.5	3.241	2.730 (5)			1.5	4.441	6.481 (5)
		2.0	3.287	2.945 (5)			2.0	4.479	6.666 (5)
		2.5	3.332	3.194 (5)			2.5	4.525	6.858 (5)
		3.0	3.390	3.483 (5)			3.0	4.566	7.059 (5)
0.75	0.25	0.0	3.125	2.774 (5)	0.5	0.0	0.0	3.161	7.439 (4)
		0.5	3.181	3.086 (5)			0.5	3.213	8.208 (4)
		1.0	3.244	3.470 (5)			1.0	3.270	9.133 (4)
		1.5	3.314	3.952 (5)			1.5	3.339	1.026 (5)
		2.0	3.405	4.571 (5)			2.0	3.414	1.165 (5)
		2.5	3.517	5.386 (5)			2.5	3.502	1.339 (5)
		3.0	3.636	6.487 (5)			3.0	3.599	1.559 (5)

* numbers in the parenthesis denote powers of 10

TABLE 4: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ AND N
- WIDE GAP PROBLEM -

η	μ	N	a_c	T_c	η	μ	N	a_c	T_c
0.5	0.1	0.0	3.151	9.949 (4)*	0.35	0.05	0.0	3.182	1.331 (5)
		0.5	3.246	1.198 (5)			0.5	3.302	1.682 (5)
		1.0	3.371	1.490 (5)			1.0	3.464	2.232 (5)
		1.5	3.533	1.933 (5)			1.5	3.674	3.139 (5)
		2.0	3.732	2.635 (5)			2.0	3.899	4.623 (5)
		2.5	3.946	3.769 (5)			2.5	4.071	6.852 (5)
		3.0	4.125	5.547 (5)			3.0	4.156	9.894 (5)
0.5	-0.1	0.0	3.189	7.008 (4)	0.35	-0.05	0.0	3.222	8.381 (4)
		0.5	3.225	7.492 (4)			0.5	3.264	9.137 (4)
		1.0	3.264	8.041 (4)			1.0	3.313	1.002 (5)
		1.5	3.307	8.666 (4)			1.5	3.367	1.107 (5)
		2.0	3.354	9.383 (4)			2.0	3.430	1.232 (5)
		2.5	3.405	1.021 (5)			2.5	3.50	1.383 (5)
		3.0	3.462	1.116 (5)			3.0	3.573	1.563 (5)
0.5	-0.25	0.0	3.358	8.867 (4)	0.35	-0.1	0.0	3.278	8.537 (4)
		0.5	3.397	9.372 (4)			0.5	3.316	9.159 (4)
		1.0	3.441	9.932 (5)			1.0	3.358	9.868 (4)
		1.5	3.489	1.056 (5)			1.5	3.404	1.068 (4)
		2.0	3.542	1.125 (5)			2.0	3.455	1.162 (5)
		2.5	3.599	1.203 (5)			2.5	3.511	1.271 (5)
		3.0	3.661	1.246 (5)			3.0	3.575	1.399 (5)
0.5	-0.5	0.0	4.801	2.089 (5)	0.35	-0.25	0.0	4.334	1.628 (5)
		0.5	4.867	2.185 (5)			0.5	4.420	1.731 (5)
		1.0	4.936	2.288 (5)			1.0	4.508	1.843 (5)
		1.5	5.004	2.399 (5)			1.5	4.599	1.966 (5)
		2.0	5.076	2.518 (5)			2.0	4.693	2.103 (5)
		2.5	5.150	2.646 (5)			2.5	4.790	2.255 (5)
		3.0	5.225	2.784 (5)			3.0	4.891	2.423 (5)
0.35	0.0	0.0	3.196	9.464 (4)	0.35	-0.5	0.0	6.606	4.218 (5)
		0.5	3.258	1.073 (5)			0.5	6.692	4.396 (5)
		1.0	3.332	1.233 (5)			1.0	6.778	4.585 (5)
		1.5	3.421	1.439 (5)			1.5	6.865	4.787 (5)
		2.0	3.524	1.707 (5)			2.0	6.957	5.002 (5)
		2.5	3.636	2.057 (5)			2.5	7.028	5.234 (5)
		3.0	3.755	2.512 (5)			3.0	7.150	5.481 (5)

* numbers in the parenthesis denote powers of 10

TABLE 5: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ AND N
- WIDE GAP PROBLEM -

n	μ	N	a_C	T_C	n	μ	N	a_C	T_C
0.2	0.0	0.0	3.263	3.035 (5)*	0.1	0.0	0.0	3.339	2.437 (6)
		0.5	3.326	3.506 (5)			0.5	3.391	2.799 (6)
		1.0	3.407	4.118 (5)			1.0	3.455	3.253 (6)
		1.5	3.502	4.918 (5)			1.5	3.531	3.821 (6)
		2.0	3.612	5.961 (5)			2.0	3.618	4.514 (6)
		2.5	3.725	7.284 (5)			2.5	3.707	5.325 (6)
		3.0	3.831	8.888 (5)			3.0	3.790	6.227 (6)
	0.01	0.0	3.253	3.709 (5)		0.005	0.0	3.315	4.286 (6)
		0.5	3.346	4.520 (5)			0.5	3.436	5.690 (6)
		1.0	3.466	5.677 (5)			1.0	3.604	7.850 (6)
		1.5	3.608	7.337 (5)			1.5	3.779	1.082 (7)
		2.0	3.762	9.638 (5)			2.0	3.916	1.429 (7)
		2.5	3.900	1.257 (6)			2.5	3.985	1.802 (7)
		3.0	3.992	1.600 (6)			3.0	4.039	2.207 (7)
	-0.01	0.0	3.275	2.668 (5)		-0.005	0.0	3.371	1.877 (6)
		0.5	3.326	2.992 (5)			0.5	3.407	2.064 (6)
		1.0	3.386	3.391 (5)			1.0	3.445	2.285 (6)
		1.5	3.456	3.884 (5)			1.5	3.491	2.544 (6)
		2.0	3.535	4.496 (5)			2.0	3.542	2.847 (6)
		2.5	3.623	5.251 (5)			2.5	3.600	3.198 (6)
		3.0	3.712	6.164 (5)			3.0	3.662	3.599 (6)
	-0.1	0.0	4.250	3.436 (5)		-0.01	0.0	3.423	1.657 (6)
		0.5	4.334	3.673 (5)			0.5	3.453	1.789 (6)
		1.0	4.424	3.938 (5)			1.0	3.487	1.940 (6)
		1.5	4.517	4.234 (5)			1.5	3.520	2.113 (6)
		2.0	4.616	4.568 (5)			2.0	3.563	2.311 (6)
		2.5	4.722	4.944 (5)			2.5	3.606	2.537 (6)
		3.0	4.830	5.371 (5)			3.0	3.655	2.794 (6)
	-0.25	0.0	7.238	8.893 (5)		-0.1	0.0	8.120	4.181 (6)
		0.5	7.330	9.278 (5)			0.5	8.184	4.341 (6)
		1.0	7.422	9.689 (5)			1.0	8.261	4.511 (6)
		1.5	7.512	1.013 (6)			1.5	8.339	4.694 (6)
		2.0	7.626	1.060 (6)			2.0	8.425	4.887 (6)
		2.5	7.717	1.111 (6)			2.5	8.603	5.096 (6)
		3.0	7.821	1.166 (6)			3.0	8.694	5.317 (6)

* numbers in the parenthesis denote powers of 10

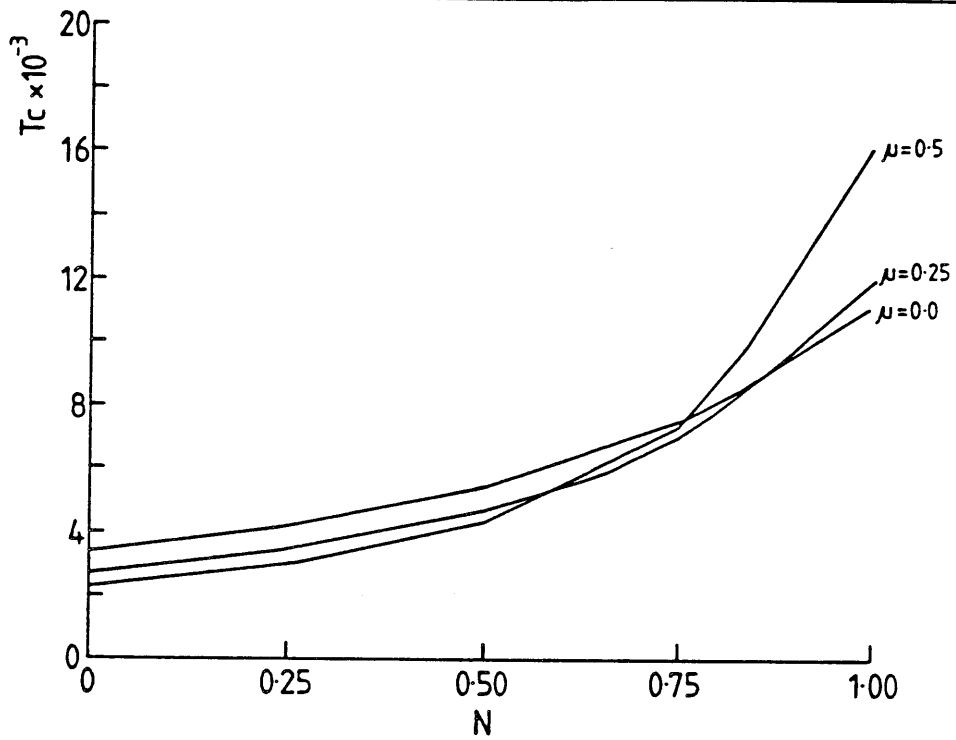


Fig.1. Variation of T_c with N for different values of μ .

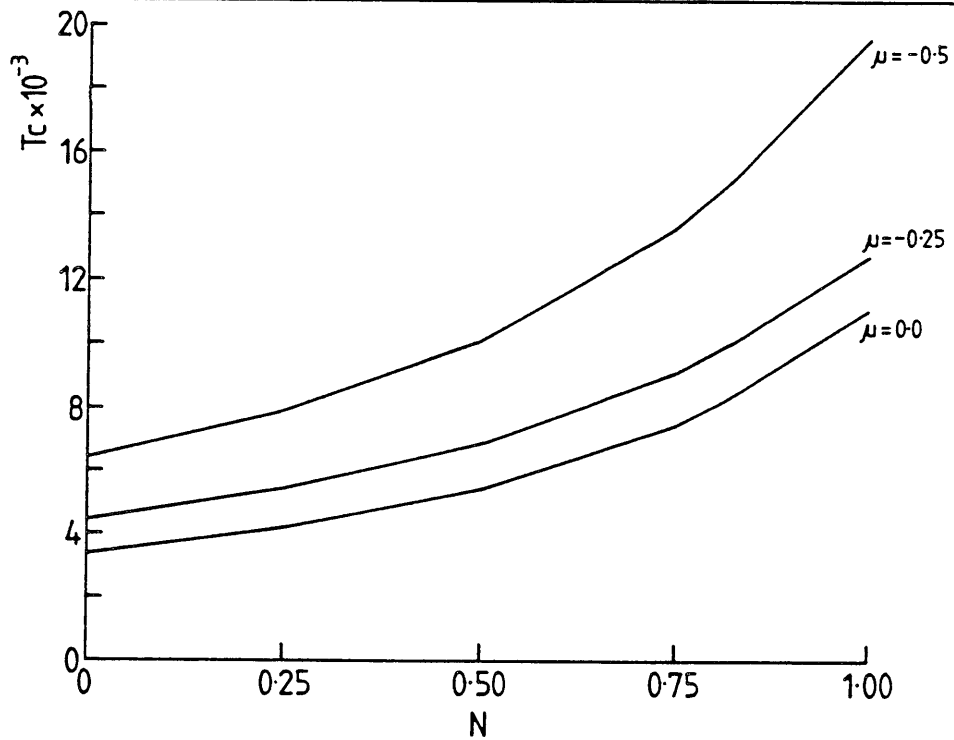
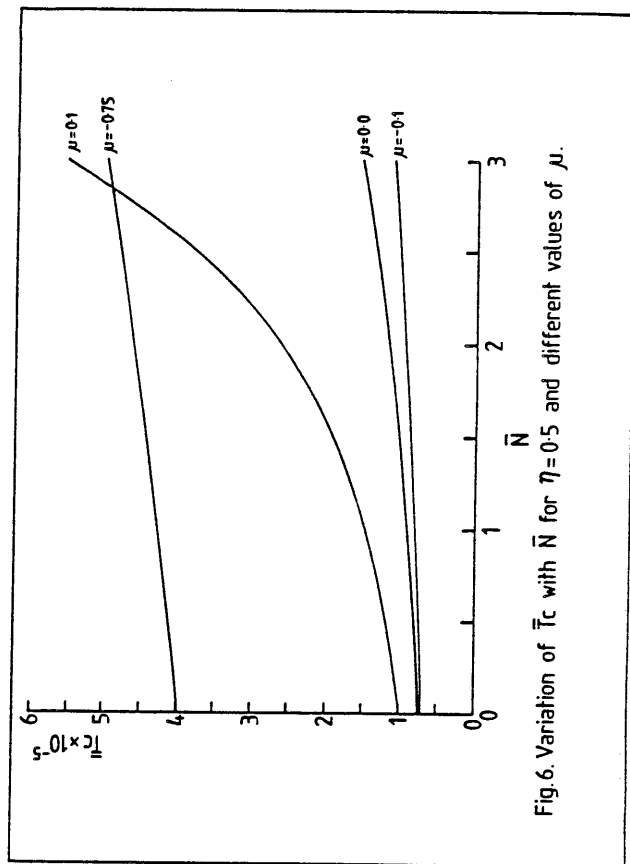
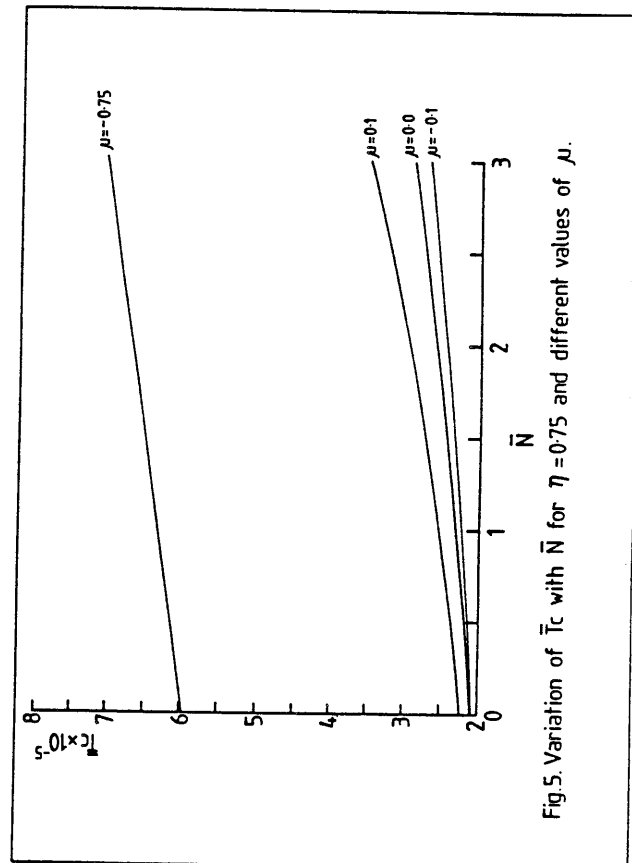
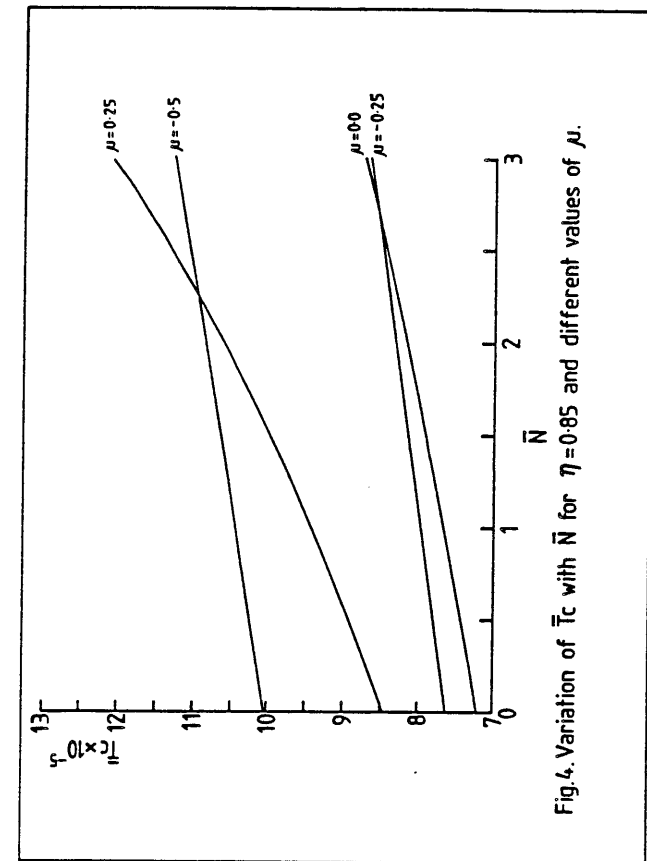
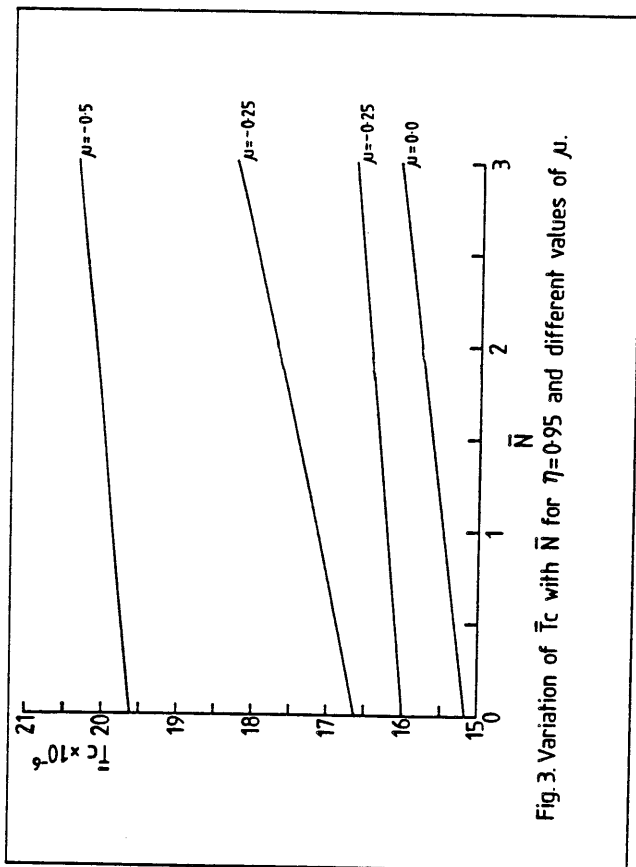
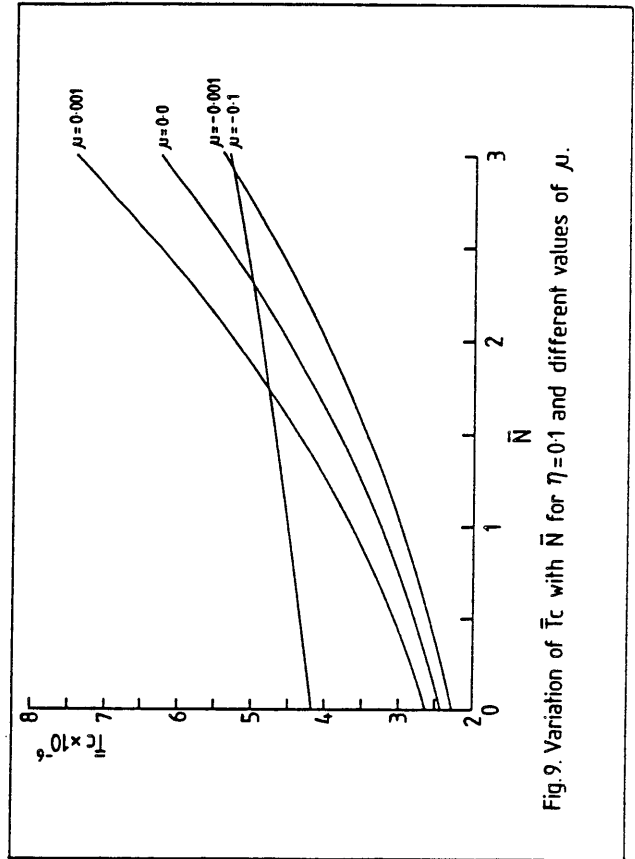
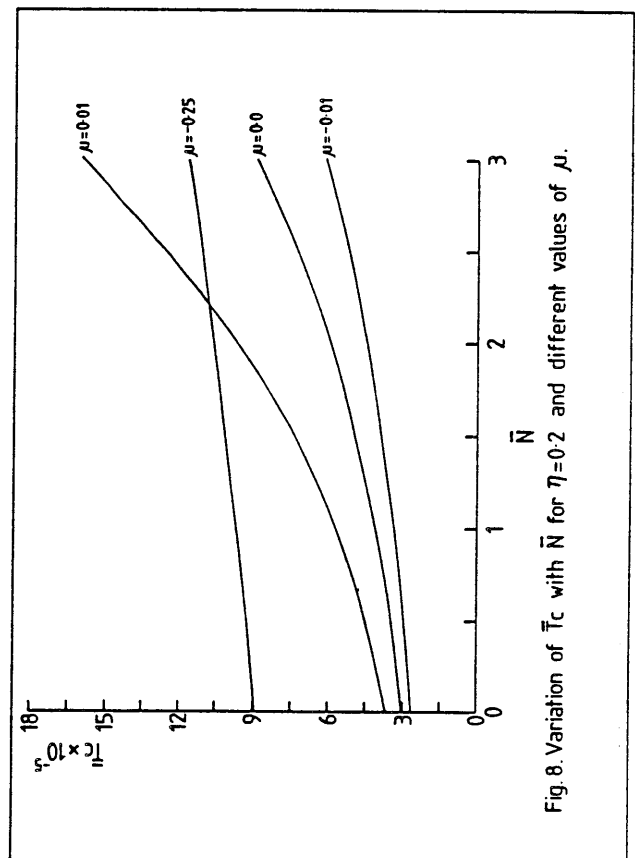
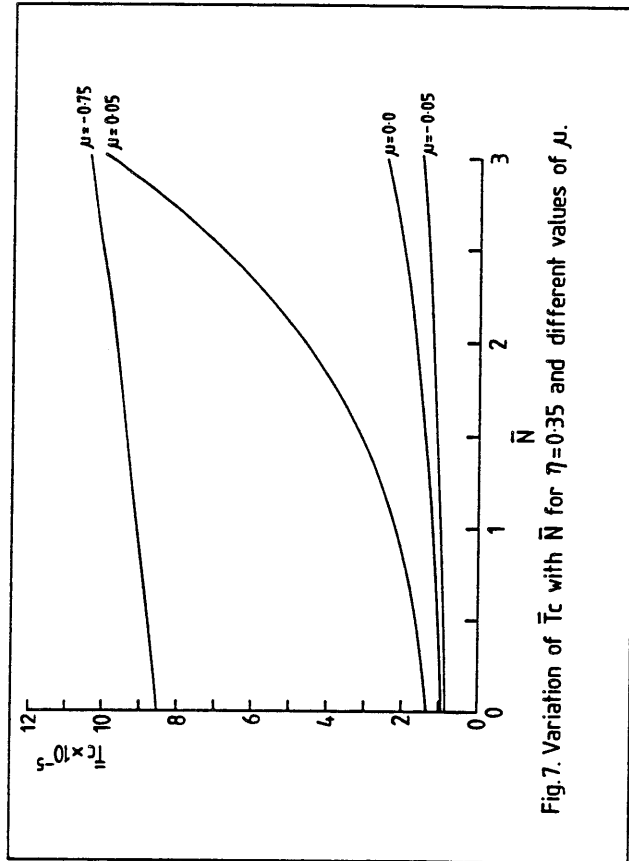
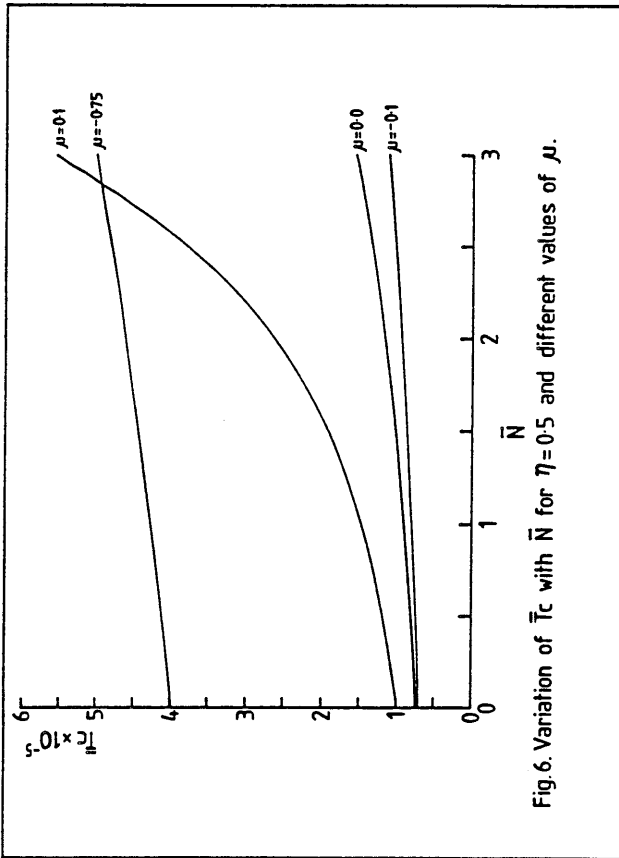
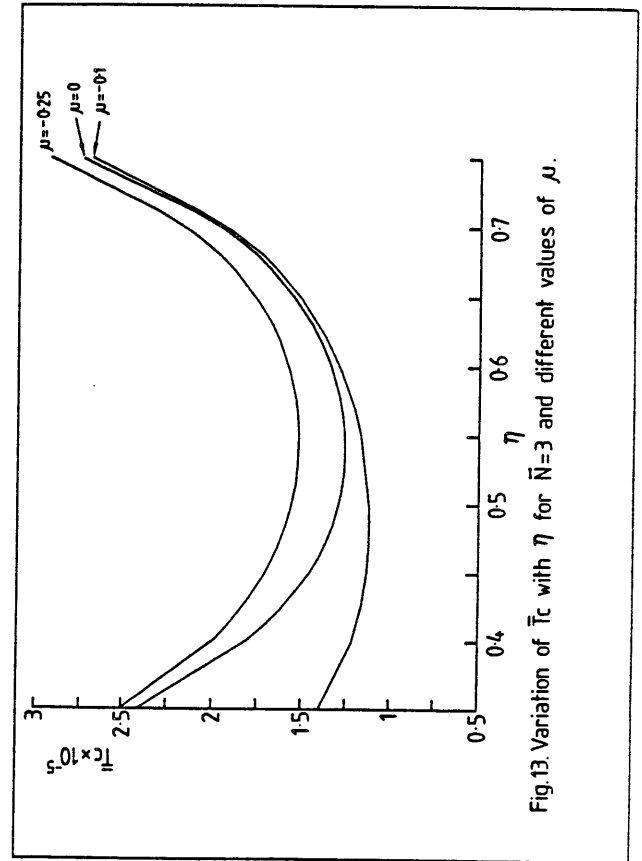
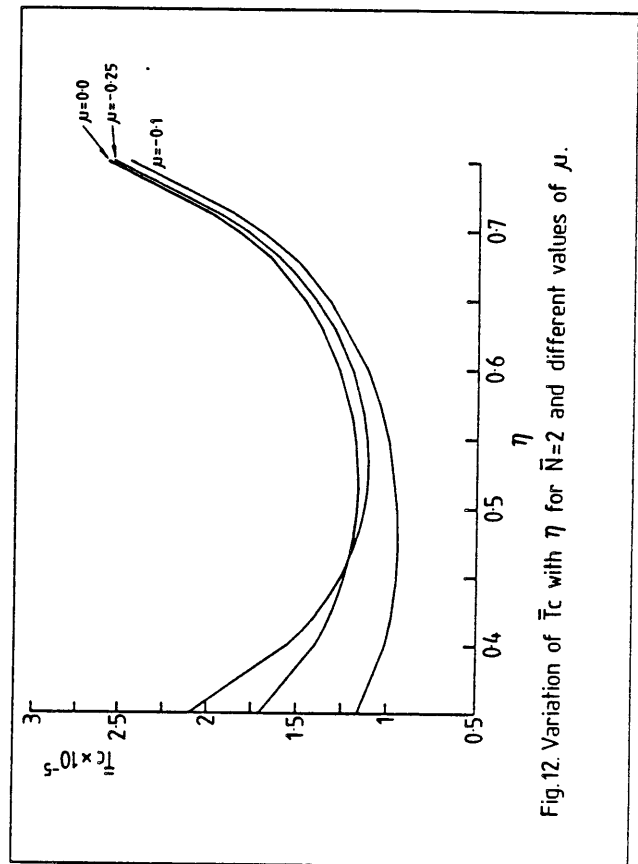
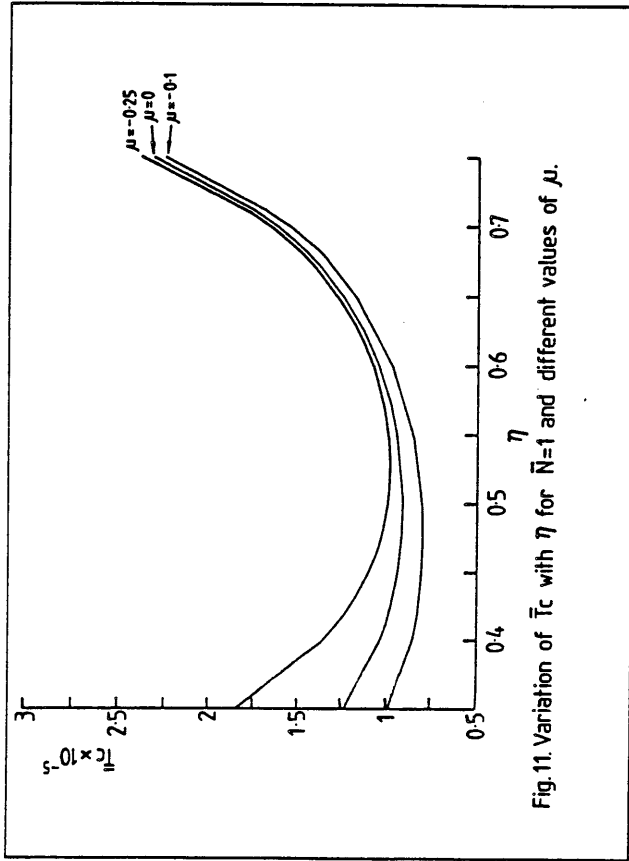
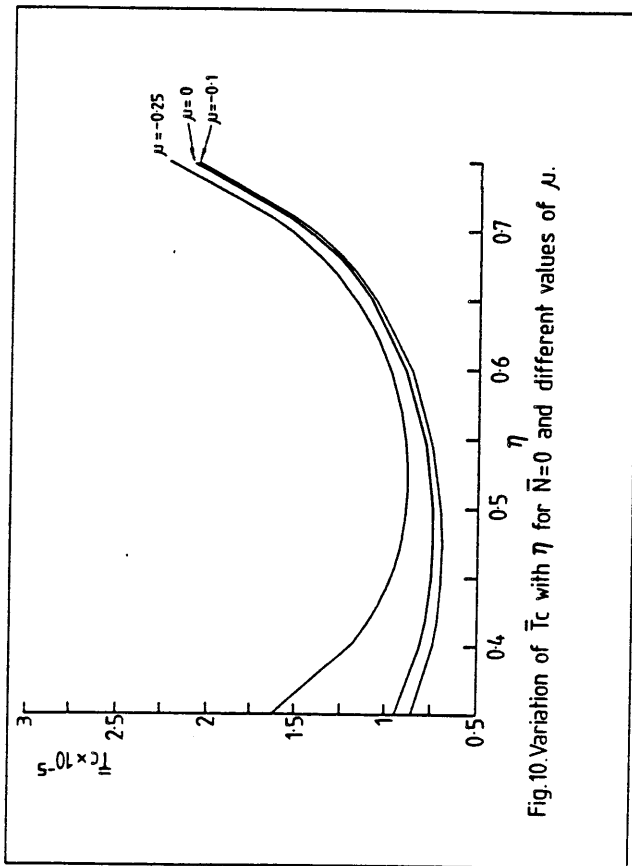


Fig.2. Variation of T_c with N for different values of μ .







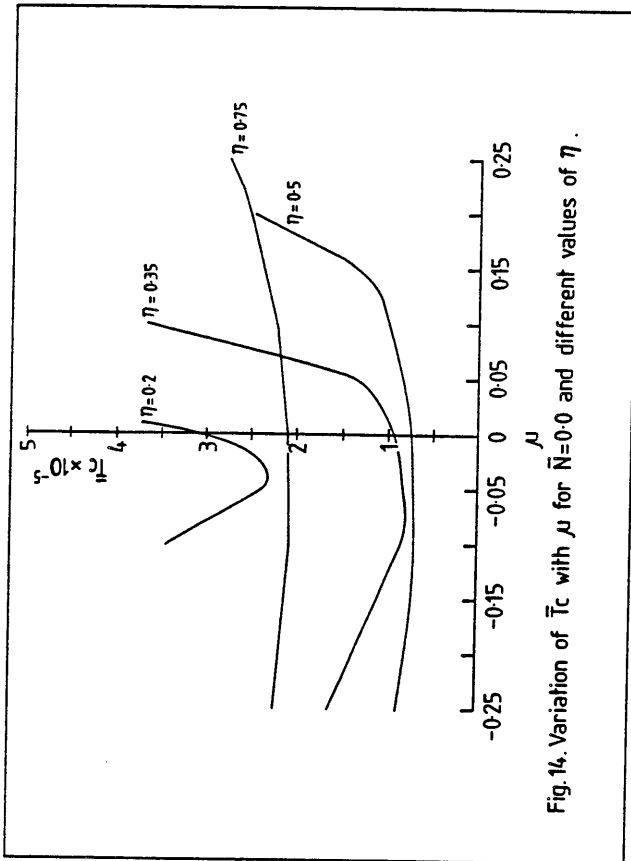


Fig. 14. Variation of $\bar{T}_c$ with μ for $\bar{N}=0$ and different values of η .

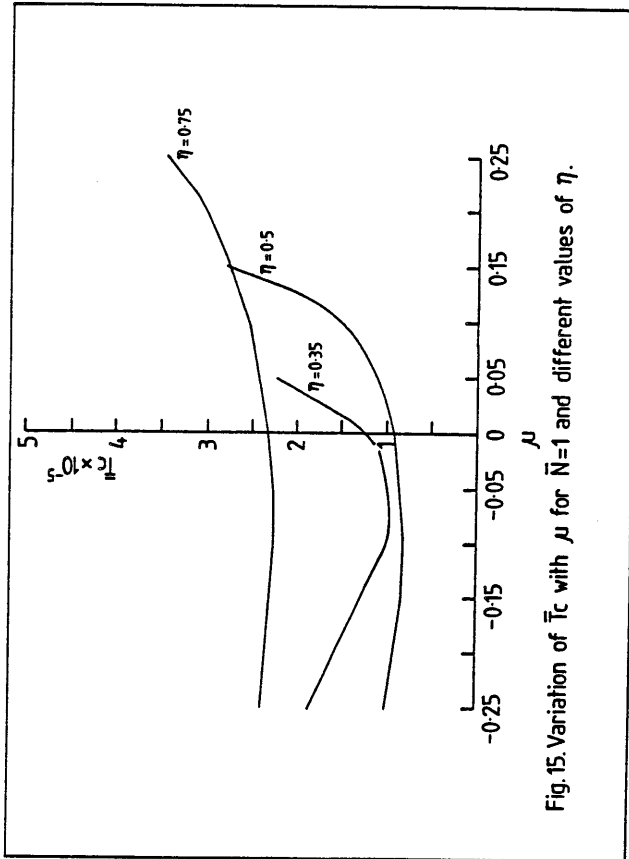


Fig. 15. Variation of $\bar{T}_c$ with μ for $\bar{N}=1$ and different values of η .

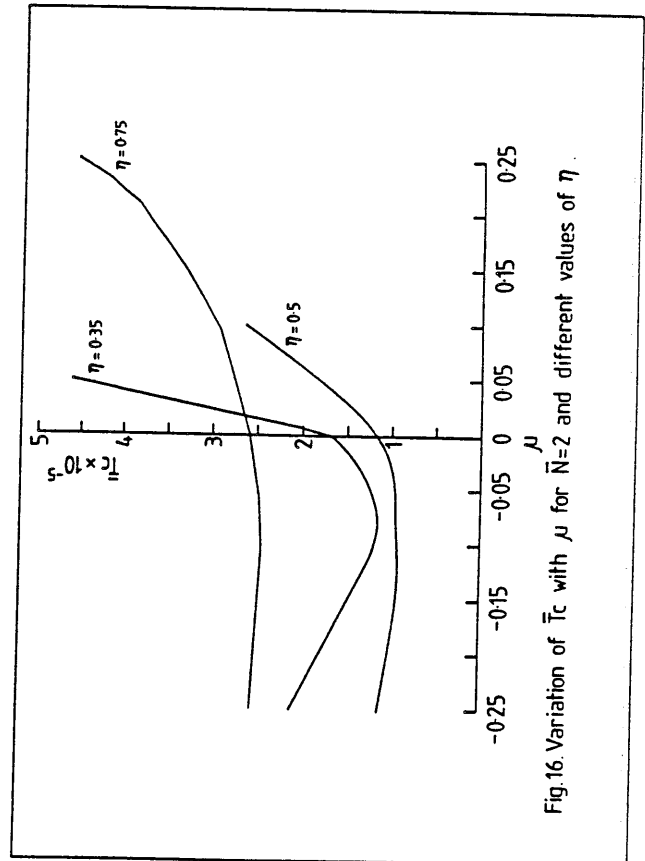


Fig. 16. Variation of $\bar{T}_c$ with μ for $\bar{N}=2$ and different values of η .

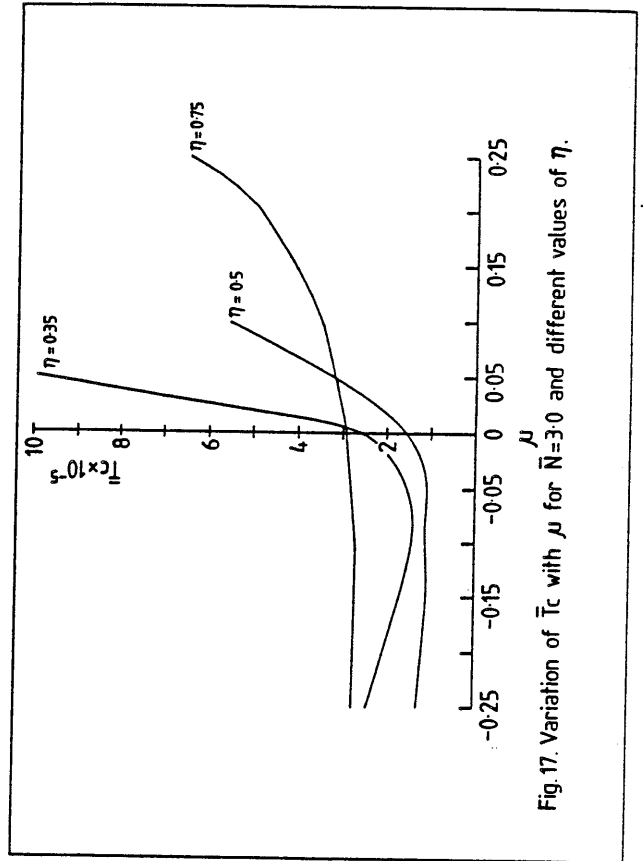


Fig. 17. Variation of $\bar{T}_c$ with μ for $\bar{N}=3.0$ and different values of η .

THE EFFECT OF RADIAL TEMPERATURE GRADIENT AND AXIAL MAGNETIC
FIELD ON THE STABILITY OF COUETTE FLOW - NARROW GAP PROBLEM.

3.1 Introduction

The theory of stability of viscous flow between two rotating coaxial cylinders was first developed by Taylor (1923). It was then extended by Chandrasekhar (1953, 1961) to the case when the fluid is an electrical conductor and a magnetic field along the common axis of the cylinders is present. Chandrasekhar formulated his problem under the assumptions of a narrow gap and a conducting fluid of very small magnetic Prandtl number

$Pm = \frac{\nu}{\nu_H}$, where ν and ν_H are the kinematic and magnetic viscosities of the

fluid respectively. For perfect electrically conducting walls and perfectly insulated walls, appropriate boundary conditions on the magnetic field were derived. The Galerkin type method was employed to solve the underlying characteristic value problem and to determine the critical Taylor number T_c . It was shown that the effect of the magnetic field is to inhibit the onset of instability. This inhibiting effect becomes more pronounced as the strength of the magnetic field H and the electrical conductivity σ are increased. The non-dimensional number

$Q = \frac{\mu_e^2 H^2 d^2 \sigma}{\rho \nu}$, where ρ is the density, μ_e is the magnetic permeability and

d is the difference in the radii of the cylinders, was introduced as a measure for the effect of the magnetic field. (here $Q = (Ha)^2$, where Ha is the Hartmann number).

The experimental investigation of Chandrasekhar's problem was carried out by Donnelly and Ozima (1960). In their experiment, they used an inner rotating cylinder viscometer and a suspended outer cylinder. Both cylinders were made of stainless steel and the electrically conducting fluid was mercury. The overall height of the viscometer was 8½ inches and could just be fitted between the pole pieces of 8½ inches magnet. A

variable-speed transmission was used to drive the inner cylinder and the speed of rotation was maintained as constant to well within 0.1 percent. It is known that so long as the flow between the cylinders is laminar and conforms to the expression (11) given in Section 2.2, the relation between torque and the angular velocity of the inner cylinder Ω_1 is linear and takes the following form:

$$\text{torque} = 4 \pi \rho \nu \frac{R_1^2 R_2^2 L}{R_2^2 - R_1^2} \Omega_1, \quad (1)$$

where L is the length of the suspended cylinder. At the onset of instability, the effective viscosity will abruptly start increasing, and this will be reflected in a sudden break in the relation (1). In conducting the experiment, the speed of the inner cylinder was slowly increased from the lowest setting until a sudden increase in the torque measurement was detected. This characterizes the beginning of transition of the fluid to the unstable state. The speed Ω_1 measured there is the critical speed. It is found that the experimental measurements confirm the broad aspects of the theoretical calculations. Moreover, for the case of non-conducting walls, the measured velocities at the onset of instability come out close to those predicted theoretically.

Chang and Sartory (1965) considered the instability of a conducting fluid but with no restriction on the gap width between the cylinders. However, they assumed the presence of perfectly conducting walls and a pure vortex flow (i.e. vortex flow with $\frac{1}{r}$ - transverse velocity profile). Their conclusions, based on results obtained by a finite difference technique, is "that an increase in the magnetic field can be either stabilizing or destabilizing, while an increase in the electrical conductivity is never stabilizing but can sometimes be destabilizing".

The instability of fluids of arbitrary magnetic Prandtl number P_m was studied by Kurzweg (1962) for the case of perfectly non-conducting walls and co-rotating cylinders with narrow gap. To solve the underlying characteristic equation, the Galerkin method was employed and the predicted critical Taylor numbers T_C confirm the findings of Chandrasekhar for fluids of very small P_m . In general, it was found that, for sufficiently large magnetic fields, T_C is directly proportional to Q when $P_m < 0.1$ and

directly proportional to the parameter $R = \frac{\mu_e}{\rho} \frac{H^2 d^2}{\nu^2}$ when $P_m > 0.1$.

The contribution of a circular, rather than an axial, magnetic field to the stability of Couette flow has been examined by Edmonds (1958). A circular magnetic field is one whose lines of force are concentric with the walls of the cylinders. This kind of field is produced by an electric current which is distributed symmetrically about the flows parallel to the common axis of rotation. The steady state solution is described by

$$u = v = 0, \quad w = V(r), \quad f = h = 0, \quad g = G(r) \quad (2)$$

where (u, v, w) and (f, g, h) are the components of the velocity and magnetic fields in the cylindrical coordinate directions (r, θ, z) respectively. The radial functions $V(r)$ and $G(r)$ are determined by substituting (2) in the general magnetohydrodynamic equations and imposing the boundary conditions appropriate to rotating and perfectly conducting cylinders. Narrow gap simplifications were assumed and the characteristic eigenvalue problem was solved by the Galerkin method for the situations of co-rotating and counter-rotating cylinders. The results reveal that the circular magnetic field inhibits the onset of instability, but this effect turns out to be quite small when it is compared with the effect of the axial magnetic field.

Lai (1962) extended the work of Edmonds by adding the effect of heating to that of the circular magnetic field. The temperature was assumed to vary along the radial direction only and the heat generated through viscous and magnetic dissipations was neglected. It was shown that a negative temperature gradient (i.e. temperature of the outer cylinder is less than that of the inner one) enhances the stabilizing effect of the magnetic field, where the positive temperature gradient destabilizes the flow.

The combined effect of the temperature gradient and the axial magnetic field has not yet been investigated. Thus we propose here to consider this problem in a manner that will include some aspects, not previously studied, of the special isothermal case. In this chapter, we will examine for the narrow gap situation and for all types of rotation of the two cylinders, the cases of perfectly conducting walls and perfectly non-conducting walls. In section 2, we derive the momentum and energy equations that describe the flow of an electrically conducting fluid moving through electric and magnetic fields. In deriving these magnetofluid-dynamic equations, knowledge of the usual fluid dynamic equations is assumed. In section 3 through 9, the mathematical analysis is presented. Starting from the known steady state solution, we obtain the perturbation equations for fluids (i) with very small magnetic Prandtl number P_m , (ii) between two cylinders with a narrow gap between them and (iii) when the marginal stability state is stationary. In section 10, the method of solution is described and in section 11, the discussion of the results is set out.

3.2 Derivation of the Magnetohydrodynamic Equations

The subject of magnetohydrodynamic (MHD) concerns with the study of flow of electrically conducting fluid across a magnetic field (H). It combines in a common framework the electromagnetic and fluid dynamic

theories to yield a description of the simultaneous effect of H on the flow and the flow on H . The motion of a fluid across H contributes to effect of two kinds: first, electric currents are generated and the associated induced magnetic field contributes to changes in the existing fields; and second, the resulting currents and the magnetic fields combine to produce an electromagnetic force on the field which acts to resist the fluid motion.

Faraday and Ritchie (1832) were the first scientists who demonstrated experimentally the principal MHD effects. In his experiment, Faraday allowed the electrically conducting fluid mercury to flow in glass tubes placed between poles of magnets. He found that a voltage is induced across the tubes which is perpendicular to the directions of the flow and the magnetic field. The generated current produces its own magnetic field which distorts the field of the magnets and slows down the motion of the fluid. Faraday, then suggested that electric power could be generated in a closed circuit by the interaction of a flowing conducting fluid and a magnetic field.

The counterpart of Faraday's conclusion was discovered by Ritchie (1832). He showed that when an electric field is applied to a conducting fluid perpendicular to a magnetic field, the fluid is pumped in a direction perpendicular to both the magnetic and the electric fields.

The relations between various electromagnetic parameters were discovered as results of experiments conducted around the beginning of the nineteenth century. These physical laws are known as Maxwell equations and when the displacement currents are neglected, they take the following forms:

$$\nabla \underline{J} = 0 \quad (3)$$

$$\nabla \underline{H} = 0 \quad (4)$$

$$\text{curl } \underline{H} = 4\pi \underline{J} \quad (5)$$

$$\text{curl } \underline{E} = - \mu_e \frac{\partial \underline{H}}{\partial t} \quad (6)$$

Here $\underline{E}$ and $\underline{H}$ are the intensities of the electric and the magnetic fields, $\underline{J}$ is the current density and μ_e is the magnetic permeability. Equations (3) and (4) are respectively the electric current and the magnetic field continuity equations. Equation (5), known as Amperé's Law, describes the magnetic field strength produced by a current. Faraday's law (6) states that for any closed conducting path, which is subjected to a changing magnetic field, the induced voltage around the path is equal to the negative, time rate of change of the total magnetic flux through the closed path.

The current density $\underline{J}$ produced by a fluid moving with velocity $\underline{U}$ across $\underline{H}$ is given by

$$\underline{J} = \sigma (\underline{E} + \mu_e \underline{U} \times \underline{H}) \quad (7)$$

where σ is the electrical conductivity of the fluid. The current and the magnetic field interact to produce a magnetic force on the fluid. This force is known as Lorentz force and is given by

$$\underline{L} = \mu_e \underline{J} \times \underline{H} \quad (8)$$

By adding the Lorentz force to the other forces acting on the fluid in the

usual hydrodynamic equation, we obtain, for viscous incompressible fluid and in vector form, the following magnetohydrodynamic equation of motion.

$$\rho \frac{D\mathbf{U}}{Dt} = - \text{grad } P + \mu \nabla^2 \mathbf{U} + \mu_e \mathbf{J} \times \mathbf{H} \quad (9)$$

We now derive the equation of motion for the magnetic field. In view of equations (6) and (7), we have

$$\frac{\partial \mathbf{H}}{\partial t} = - \frac{1}{\mu_e} \text{curl} \left(\frac{1}{c} \mathbf{J} - \mu_e \mathbf{U} \times \mathbf{H} \right) \quad (10)$$

making use of equation (5), we have

$$\frac{\partial \mathbf{H}}{\partial t} - \text{curl} (\mathbf{U} \times \mathbf{H}) = - \text{curl} (e \text{curl } \mathbf{H}), \quad (11)$$

where

$$e = \frac{1}{4\pi\mu_e} \quad (12)$$

The parameter e is called the resistivity of the fluid and has the dimension of $\text{cm}^2\text{sec}^{-1}$. If e is assumed to be a constant, then the equation of motion of the magnetic field (11) reduces to

$$\frac{\partial \mathbf{H}}{\partial t} - \text{curl} (\mathbf{U} \times \mathbf{H}) = - e \nabla^2 \mathbf{H} \quad (13)$$

To formulate the MHD energy equation, we first note that a single charge q moving with velocity $\mathbf{U}$ across a magnetic field experiences the electromagnetic force $q(\mathbf{E} + \mu_e \mathbf{U} \times \mathbf{H})$, which does work on it at a rate $q\mathbf{U} \cdot \mathbf{E} + q\mu_e \mathbf{U} \cdot (\mathbf{U} \times \mathbf{H})$. The second term vanishes because the force is perpendicular to the motion. Next, a moving charge is equivalent to a current $\mathbf{J}$. Thus the electromagnetic field does work on the charges, and via them on the fluid, at a rate of $\mathbf{E} \cdot \mathbf{J}$. According to equation (7)

$$\underline{E} \cdot \underline{J} = \frac{J^2}{\sigma} - \underline{J} \cdot (\mu_e \underline{U} \times \underline{H}) \quad (14)$$

The first term is called the joule heating and is due to the resistance of the fluid to the flow of currents. For the second term, $-\underline{J} \cdot (\mu_e \underline{U} \times \underline{H}) = \underline{U} \cdot (\mu_e \underline{J} \times \underline{H})$ and this represents the rate at which the magnetic force $(\mu_e \underline{J} \times \underline{H})$ does work on the fluid as a whole.

Including the term $\underline{E} \cdot \underline{J}$ in the hydrodynamic energy equation gives, for viscous incompressible flow, the following complete MHD energy equation

$$\rho \frac{D}{Dt} \left(c_p T + \frac{U^2}{2} \right) = \underline{E} \cdot \underline{J} - \text{div } \underline{Q} - \underline{U} \cdot \text{grad } P + \nu \nabla^2 U + \Phi \quad (15)$$

where $\underline{Q}$ is the heat flux and Φ is the viscous dissipation. If the transfer of heat by radiation is neglected then according to Fourier's law, $\underline{Q} = -k \text{grad } T$, k being the thermal conductivity. If k is constant, then $\text{div } \underline{Q} = -k \nabla^2 T$. Thus equation (15) reduces to

$$\rho \frac{D}{Dt} \left(c_p T + \frac{U^2}{2} \right) = \underline{E} \cdot \underline{J} + k \nabla^2 T - \underline{U} \cdot \text{grad } P + \nu \nabla^2 U + \Phi \quad (16)$$

The mechanical energy equation can be derived by forming the scalar product of the velocity vector $\underline{U}$ and the equation of motion

$$\rho \frac{D}{Dt} \left(\frac{U^2}{2} \right) = -\underline{U} \cdot \text{grad } P + \nu \underline{U} \cdot \nabla^2 \underline{U} + \underline{U} \cdot (\mu_e \underline{J} \times \underline{H}) \quad (17)$$

subtracting (17) from the complete energy equation (16) leaves the entropy equation

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T + \frac{J^2}{\sigma} + \Phi \quad (18)$$

The complete energy equation, the equation of motion and the entropy equation are not all independent. We have deduced the entropy equation

from the other two. Thus we need only to use two of them in any problem.

When the Joule and viscous dissipations are neglected, equation (18) reduce to

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T \quad (19)$$

Since we are dealing with a fluid between two concentric rotating cylinders, it is appropriate to rewrite the various MHD equations in the cylindrical coordinate system. Thus if (u, v, w) and (f, g, h) represents the components of the velocity and the magnetic fields in (r, θ, z) directions respectively, then with the aid of equation (5), the three components of the Lorentz force (8) take the following form:

$$L_r = \frac{\mu_e}{4\pi} \left[f \frac{\partial f}{\partial r} + \frac{g}{r} \frac{\partial f}{\partial \theta} + h \frac{\partial f}{\partial z} - \frac{g^2}{r} \right] - \frac{\mu_e}{8\pi} \frac{\partial}{\partial r} |H|^2 \quad (20)$$

$$L_\theta = \frac{\mu_e}{4\pi} \left[f \frac{\partial g}{\partial r} + \frac{g}{r} \frac{\partial g}{\partial \theta} + h \frac{\partial g}{\partial z} + \frac{fg}{r} \right] - \frac{\mu_e}{8\pi} \frac{1}{r} \frac{\partial}{\partial \theta} |H|^2 \quad (21)$$

$$L_z = \frac{\mu_e}{4\pi} \left[f \frac{\partial h}{\partial r} + \frac{g}{r} \frac{\partial h}{\partial \theta} + h \frac{\partial h}{\partial z} \right] - \frac{\mu_e}{8\pi} \frac{\partial}{\partial z} |H|^2 \quad (22)$$

Making use of equations (20) - (22), the momentum equations can be written as:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} - \frac{\mu_e}{4\pi\rho} \left(f \frac{\partial f}{\partial r} + \frac{g}{r} \frac{\partial f}{\partial \theta} + h \frac{\partial f}{\partial z} - \frac{g^2}{r} \right) \\ = - \frac{\partial s}{\partial r} + \nu \left(\nabla^2 u - \frac{2}{r^2} \frac{\partial v}{\partial \theta} - \frac{u}{r^2} \right) \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial z} + \frac{uv}{r} - \frac{\mu_e}{4\pi\rho} \left(f \frac{\partial g}{\partial r} + \frac{g}{r} \frac{\partial g}{\partial \theta} + h \frac{\partial g}{\partial z} + \frac{fg}{r} \right) \\ = - \frac{1}{r} \frac{\partial s}{\partial \theta} + \nu \left(\nabla^2 v + \frac{2}{r^2} \frac{\partial u}{\partial \theta} - \frac{v}{r^2} \right) \end{aligned} \quad (24)$$

$$\begin{aligned}
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} - \frac{\mu_e}{4\pi\rho} \left(f \frac{\partial h}{\partial r} + \frac{g}{r} \frac{\partial h}{\partial \theta} + h \frac{\partial h}{\partial z} \right) \\
= - \frac{\partial s}{\partial z} + \nu \nabla^2 w
\end{aligned} \tag{25}$$

$$\begin{aligned}
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial r} + \frac{v}{r} \frac{\partial f}{\partial \theta} + w \frac{\partial f}{\partial z} - f \frac{\partial u}{\partial r} - \frac{g}{r} \frac{\partial u}{\partial \theta} - h \frac{\partial u}{\partial z} \\
= e \left(\nabla^2 f - \frac{2}{r^2} \frac{\partial g}{\partial \theta} - \frac{f}{r^2} \right)
\end{aligned} \tag{26}$$

$$\begin{aligned}
\frac{\partial g}{\partial t} + u \frac{\partial g}{\partial r} + \frac{v}{r} \frac{\partial g}{\partial \theta} + w \frac{\partial g}{\partial z} - f \frac{\partial v}{\partial r} - \frac{g}{r} \frac{\partial v}{\partial \theta} - h \frac{\partial v}{\partial z} + \frac{1}{r} (vf - ug) \\
= e \left(\nabla^2 g + \frac{2}{r^2} \frac{\partial f}{\partial \theta} - \frac{g}{r^2} \right)
\end{aligned} \tag{27}$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + \frac{v}{r} \frac{\partial h}{\partial \theta} + w \frac{\partial h}{\partial z} - f \frac{\partial w}{\partial r} - \frac{g}{r} \frac{\partial w}{\partial \theta} - h \frac{\partial w}{\partial z} = e \nabla^2 h \tag{28}$$

where

$$s = \frac{1}{\rho} \left(p + \mu_e \frac{|H|^2}{8\pi} \right) , \tag{29}$$

and ∇^2 stands for the Laplacian operator.

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{30}$$

The continuity equations of the velocity and the magnetic fields take the form

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{31}$$

$$\frac{\partial f}{\partial r} + \frac{f}{r} + \frac{1}{r} \frac{\partial g}{\partial \theta} + \frac{\partial h}{\partial z} = 0 \tag{32}$$

Finally the energy equation (19) in cylindrical coordinates becomes

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + \frac{v}{r} \frac{\partial T}{\partial \theta} + w \frac{\partial T}{\partial z} \right) = k \nabla^2 T \tag{33}$$

and the equation of state is given by

$$\rho = \rho_1 [1 - \alpha (T - T_1)] \quad . \quad (34)$$

3.3 Steady State Solution

For an electrically conducting fluid between two coaxial cylinders with a constant magnetic field applied in the axial direction, the MHD equations (23) - (34) admit a steady state solution of the form

$$\begin{aligned} u = w = 0 \quad , \quad v = V(r) \\ f = g = 0 \quad , \quad h = H = \text{constant} \\ T = \bar{T}(r) \end{aligned} \quad (35)$$

provided that

$$\frac{\partial s}{\partial r} = \frac{V^2}{r} \quad (36)$$

and

$$\nabla^2 V - \frac{V}{r^2} = 0 \quad (37)$$

The general solution of (36) is

$$V = Ar + \frac{B}{r} \quad (38)$$

where the constants A and B are as given by (15) in Section 2.2.

The steady temperature T is governed by

$$\frac{d^2 \bar{T}}{dr^2} + \frac{1}{r} \frac{d\bar{T}}{dr} = 0 \quad (39)$$

For cylinders with different constant temperatures, the boundary conditions on $\bar{T}$ are

$$\begin{aligned} \bar{T} &= T_1 & \text{at } r &= R_1 \\ \bar{T} &= T_2 & \text{at } r &= R_2 \end{aligned} \quad (40)$$

From (39) and (40), we have

$$T = T_2 - \frac{T_2 - T_1}{\ln \frac{R_1}{R_2}} \ln \frac{r}{R_2} \quad (41)$$

3.4 Perturbation Equations

Denoting the disturbances in the velocity field by (u', v', w') , that in magnetic field by (f', g', h') and those in temperature, density and pressure by T', ρ', s' respectively, the perturbed state of the fluid is thus described by

$$u', V + v', w', f', g', H + h', T + T', \rho + \rho', s + s' \quad (42)$$

The variation in the disturbances are assumed to be axisymmetric about the z-axis and so they are independent of θ . Substituting (42) in the MHD equations (23) - (34) and neglecting all terms that involve products, and powers greater than one, of the disturbance quantities, we obtain the following linearized equations.

$$\rho \frac{\partial u'}{\partial t} - 2\rho \frac{V}{r} v' - \rho' \frac{V^2}{r} - \frac{\mu_e}{4\pi} H \frac{\partial f'}{\partial z} = - \frac{\partial s'}{\partial r} + \rho \nu (\nabla^2 u' - \frac{u'}{r^2}) \quad (43)$$

$$\rho \frac{\partial v'}{\partial t} + \rho \left(\frac{\partial V}{\partial r} + \frac{V}{r} \right) u' - \frac{\mu_e}{4\pi} H \frac{\partial g'}{\partial z} = \rho \nu (\nabla^2 v' - \frac{v'}{r^2}) \quad (44)$$

$$\rho \frac{\partial w'}{\partial t} - \frac{\mu_e}{4\pi} H \frac{\partial h'}{\partial z} = - \frac{\partial s'}{\partial z} + \rho \nu \nabla^2 w' \quad (45)$$

$$\frac{\partial f'}{\partial t} - H \frac{\partial u'}{\partial z} = e (\nabla^2 f' - \frac{f'}{r^2}) \quad (46)$$

$$\frac{\partial g'}{\partial t} - \left(\frac{\partial V}{\partial r} - \frac{V}{r} \right) f' - H \frac{\partial v'}{\partial z} = e (\nabla^2 g' - \frac{g'}{r^2}) \quad (47)$$

$$\frac{\partial h'}{\partial t} - H \frac{\partial w'}{\partial z} = e \nabla^2 h' \quad (48)$$

$$\frac{\partial u'}{\partial r} + \frac{u'}{r} + \frac{\partial w'}{\partial z} = 0 \quad (49)$$

$$\frac{\partial f'}{\partial r} + \frac{f'}{r} + \frac{\partial h'}{\partial z} = 0 \quad (50)$$

$$\frac{\partial T'}{\partial t} + \frac{\partial \bar{T}}{\partial r} u' = \frac{k}{\rho c_p} \nabla^2 T' \quad (51)$$

$$\rho + \rho' = \rho_1 [1 - \alpha (\bar{T} + T' - T_1)] \quad (52)$$

Due to the smallness of the thermal expansion coefficient α , a small temperature variation will lead to a small variation in the density. Thus, we will assume the density is a constant ρ , except when multiplied by the centrifugal acceleration term $\frac{V^2}{r}$. This is the classic Buossinesq approximation. In view of (52), the term $\rho' \frac{V^2}{r}$ in equataion (43) can be replaced by $-\rho \alpha T' \frac{V^2}{r}$.

3.5 Analysis in Terms of Normal Modes

By analysing the disturbances into normal modes, we seek solutions of the equations (43) - (52), of the form

$$\begin{aligned} u' &= u(r) e^{\sigma t} \cos \lambda z & f' &= f(r) e^{\sigma t} \sin \lambda z \\ v' &= v(r) e^{\sigma t} \cos \lambda z & g' &= g(r) e^{\sigma t} \sin \lambda z \\ w' &= w(r) e^{\sigma t} \sin \lambda z & h' &= h(r) e^{\sigma t} \cos \lambda z \\ s' &= s(r) e^{\sigma t} \cos \lambda z & T' &= T(r) e^{\sigma t} \cos \lambda z \end{aligned} \quad (53)$$

Substituting (53) into (43) - (52) and replacing $(\frac{dV}{dr} + \frac{V}{r})$ by $2A$, $(\frac{dV}{dr} - \frac{V}{r})$ by $2 \frac{B}{r^2}$ and $\frac{d\bar{T}}{dr}$ by its value from (41), we obtain

$$\nu \left(\frac{d}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{1}{r^2} - \frac{\sigma}{\nu} \right) u = \frac{1}{\rho} \frac{ds}{dr} - 2 \frac{V}{r} v + \alpha \frac{V^2}{r} T - \frac{\lambda \mu_e}{4\pi \rho} H f \quad (54)$$

$$\nu \left(\frac{d}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{1}{r^2} - \frac{\sigma}{\nu} \right) v = 2A u - \frac{\lambda \mu_e}{4\pi \rho} H g \quad (55)$$

$$\nu \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{\sigma}{\nu} \right) w = - \frac{\lambda}{\rho} s + \frac{\lambda \mu_e}{4\pi \rho} H h \quad (56)$$

$$e \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{1}{r^2} - \frac{\sigma}{e} \right) f = \lambda H u \quad (57)$$

$$e \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{1}{r^2} - \frac{\sigma}{e} \right) g = \lambda H v - 2 \frac{B}{r^2} f \quad (58)$$

$$e \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{\sigma}{e} \right) h = - \lambda H w \quad (59)$$

$$\left(\frac{d}{dr} + \frac{1}{r} \right) u = - \lambda w \quad (60)$$

$$\left(\frac{d}{dr} + \frac{1}{r} \right) f = \lambda h \quad (61)$$

$$\frac{k}{\rho c_p} \left(\frac{d}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 - \frac{\sigma \rho c_p}{k} \right) T = - \frac{(T_2 - T_1)}{r \ln \frac{R_1}{R_2}} u \quad (62)$$

The dependence of the above equations on s can be eliminated by first differentiating (56) and then combining it with (54). Moreover, we can rewrite the above equations as

$$(DD^* - \lambda^2 - \frac{\sigma}{\nu}) \left(\frac{1}{\lambda} Dw + u \right) = - \frac{2}{\nu} \frac{V}{r} v + \frac{\alpha}{\nu} \frac{V^2}{r} T - \frac{\lambda \mu_e}{4\pi \rho \nu} H \left(f - \frac{1}{\lambda} Dh \right) \quad (63)$$

$$(DD^* - \lambda^2 - \frac{\sigma}{\nu}) v = \frac{2A}{\nu} u - \frac{\lambda \mu_e}{4\pi \rho \nu} H g \quad (64)$$

$$(DD^* - \lambda^2 - \frac{\sigma}{e}) f = \frac{\lambda}{e} H u \quad (65)$$

$$(DD^* - \lambda^2 - \frac{\sigma}{e}) g = \frac{\lambda}{e} H v - 2 \frac{B}{er^2} f \quad (66)$$

$$(D^*D - \lambda^2 - \frac{\sigma}{e}) h = - \frac{\lambda}{e} H w \quad (67)$$

$$D^*u = - \lambda w \quad (68)$$

$$D^*f = \lambda h \quad (69)$$

$$(D^*D - \lambda^2 - \frac{\sigma \rho c_p}{k}) T = - \rho c_p \frac{(T_2 - T_1)}{r k \ln \frac{R_1}{R_2}} u \quad (70)$$

$$\text{where } D = \frac{d}{dr}, \quad D^* = \frac{d}{dr} + \frac{1}{r}. \quad (71)$$

$$\text{Thus } DD^* = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} \quad (72)$$

$$D^*D = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}.$$

In equations (63) - (70), we note that (65) and (67) are dependent. By applying D^* on (65) and making use of (68) and (69), we retain (67). Also, the number of equations can be further reduced by substituting for w and h in (63) by their values from (68) and (69). Equation (63) becomes

$$(DD^* - \lambda^2 - \frac{\sigma}{\nu})(DD^* - \lambda^2)u = 2 \frac{\lambda^2}{\nu} \frac{V}{r} v - \frac{\alpha \lambda^2}{\nu} \frac{V^2}{r} T - \frac{\lambda \mu_e}{4\pi\rho\nu} H (DD^* - \lambda^2) f \quad (73)$$

3.6 Stationary Marginal State

We shall investigate the situation when instability sets in as a stationary secondary flow. In this case, the equations governing the marginal state can be obtained by setting $\sigma = 0$ in equations (64), (65), (66), (70) and (73)

$$(DD^* - \lambda^2)^2 u = 2 \frac{\lambda^2}{\nu} \frac{V}{r} v - \alpha \frac{\lambda^2}{\nu} \frac{V^2}{r} T - \frac{\lambda \mu_e}{4\pi\rho\nu} H (DD^* - \lambda^2) f \quad (74)$$

$$(DD^* - \lambda^2) v = 2 \frac{A}{\nu} u - \frac{\lambda \mu_e}{4\pi\rho\nu} H g \quad (75)$$

$$(DD^* - \lambda^2) f = \frac{\lambda}{e} H u \quad (76)$$

$$(DD^* - \lambda^2) g = \frac{\lambda}{e} H v - 2 \frac{B}{er^2} f \quad (77)$$

$$(D^*D - \lambda^2) T = - \frac{\rho c_p (T_2 - T_1)}{r k \ln \frac{R_1}{R_2}} u \quad (78)$$

The non-dimensional variables are introduced as follows

$$\begin{aligned} x &= \frac{r - R_1}{d}, \quad a = \lambda d, \quad \eta = \frac{R_1}{R_2}, \quad \mu = \frac{\Omega_1}{\Omega_2}, \quad \xi = \frac{r}{R_2} \\ G(x) &= \frac{V}{r\Omega_1} = A_1 + \frac{B_1}{\xi^2}, \quad \tilde{u} = \frac{2A d^2}{\nu} u, \quad \theta = - \rho c_p \frac{2 k R_2 A \ln \eta}{\nu (T_2 - T_1)} T \\ \tilde{f} &= \frac{2 e A d}{\nu a H} f, \quad \tilde{g} = \frac{e}{aHd} g \end{aligned} \quad (79)$$

where

$$A_1 = \frac{\mu - \eta^2}{1 - \eta^2}, \quad B_1 = \eta^2 \frac{1 - \mu}{1 - \eta^2} \quad (80)$$

In terms of (79) and substituting for $(DD^* - a^2) f$ from (76) into (74), equation (74) - (78) become

$$\begin{aligned} [(DD^* - a^2)^2 + \frac{\mu_e H^2 d^2}{4\pi\rho\nu e} a^2] \tilde{u} &= \frac{4A \Omega_1 d^4}{\nu^2} a^2 G(x) \nu \\ &+ \frac{\alpha \rho c_p (T_2 - T_1) d^4 \Omega_1^2 \xi a^2 [G(x)]^2}{\nu k \ln \eta} \theta \end{aligned} \quad (81)$$

$$(DD^* - a^2) v = \tilde{u} - \frac{\mu_e H^2 d^2}{4\pi\rho\nu e} a^2 \tilde{g} \quad (82)$$

$$(DD^* - a^2) \tilde{f} = \tilde{u} \quad (83)$$

$$(DD^* - a^2) \tilde{g} = \frac{B_1}{A_1} \frac{\nu}{e} \frac{1}{\xi^2} \tilde{f} + v \quad (84)$$

$$(D^*D) - a^2) \theta = \frac{1}{\xi} \tilde{u} \quad (85)$$

where now D and D^* stands for $\frac{d}{dx}$ and $(\frac{d}{dx} + \frac{1 - \eta}{\xi})$ respectively.

Defining the Taylor number, Rayleigh number, Prandtl number, magnetic

Prandtl number, and Q as;

$$T_a = \frac{-4A \Omega_1 d^4}{\nu^2}, \quad R_a = \frac{-\alpha \rho c_p (T_2 - T_1) d^4 \Omega_1^2}{\nu k \ln \eta}, \quad (86)$$

$$Pr = \frac{\nu \rho c_p}{k}, \quad P_m = \frac{\nu}{e}, \quad Q = \frac{\mu_e H^2 d^2}{4\pi \rho \nu e}$$

We can rewrite equations (81), (82) and (84), after dropping the superscripts on μ , f and g as,

$$[(DD^* - a^2)^2 + Q a^2] u = -a^2 T_a [G(x) v + N \xi (G(x))^2 \theta] \quad (87)$$

$$(DD^* - a^2) v = U - Q a^2 g \quad (88)$$

$$(DD^* - a^2) g = \frac{B_1}{A_1 \xi^2} P_m f + v \quad (89)$$

where

$$N = \frac{R_a}{T_a} = \frac{Pr \propto (T_2 - T_1)}{4 A_1 \ln \eta} \quad (90)$$

3.7 Small Magnetic Prandtl Number

For many electrically conducting fluids, the magnetic Prandtl number is very small. As an example, for mercury $P_m = 1.45 \times 10^{-7}$. Therefore we set P_m equals zero and then substitute for v in (87) and (88) by its value from (89). The governing equations reduce to

$$[(DD^* - a^2)^2 + Q a^2] u = -a^2 T_a [G(x)(DD^* - a^2) g + N \xi (G(x))^2 \theta] \quad (91)$$

$$[(DD^* - a^2)^2 + Q a^2] g = u \quad (92)$$

$$(DD^* - a^2) \theta = \frac{1}{\xi} u \quad (93)$$

3.8 Boundary Conditions

Firstly, the boundary conditions on the velocities require that they vanish on the walls (no slip condition). According to equation (60), the condition $w = 0$ can be replaced by $D^*u = 0$ and since $u = 0$, it follows that $Du = 0$. Hence the conditions on u and v are

$$u = Du = v = 0 \quad \text{at } x = 0, 1 \quad . \quad (94)$$

Secondly, the boundary conditions on the magnetic fields depend on the nature of the walls confining the fluid. For walls of finite electrical conductivity, the magnetic perturbations will penetrate into the material of the cylinder. This will complicate the problem because the interaction of the fluid and the cylinders will not be restricted to the surfaces $r = R_1$ and $r = R_2$. Since our prime interest is to explore the manner in which the magnetic field affects the onset of instability of the flow, it is appropriate to limit the magnetic perturbation to the fluid between the cylinders. This can be achieved if we consider the following two extreme cases:

- (a) perfectly non-conducting cylinders.
- (b) perfectly conducting cylinders.

For case (a), the electrical conductivity of the walls is zero. There is no current which can cross the walls. Therefore, the current density normal to the walls must be zero, i.e., $J_r = 0$. This condition implies, according to equation (5), that

$$g = 0 \quad \text{at } x = 0, 1 \quad . \quad (95)$$

For perfectly conducting walls, the electrical field tangent to the walls must be zero so that the current density in the conductor remain finite. Since the velocities at the walls are zero, it follows from equation (7) that $J_\theta = J_z = 0$. Using the relation between the current density and the magnetic field (5), we have

$$f = D^*g = 0 \quad \text{at } x = 0, 1 \quad . \quad (96)$$

Finally, the boundary conditions on the temperature are

$$\theta = 0 \quad \text{at } x = 0, 1 \quad . \quad (97)$$

3.9 Narrow Gap Approximation

If the gap spacing $d = R_2 - R_1$ is small compared to the mean radius $R_0 = \frac{1}{2} (R_1 + R_2)$, then the narrow gap approximations given by (56) in Section 2.6 can be applied and the governing equations (91) - (93) reduce to

$$[(D^2 - a^2)^2 + Q a^2] u = - a^2 T_a [\bar{G}(x)(D^2 - a^2) g + N (\bar{G}(x))^2 \theta] \quad (98)$$

$$[(D^2 - a^2)^2 + Q a^2] g = u \quad (99)$$

$$(D^2 - a^2) \theta = u \quad . \quad (100)$$

Boundary Conditions

a) Conducting Walls

$$u = Du = Dg = (DD^* - a^2) g = \theta = 0 \quad \text{at } x = 0, 1 \quad (101)$$

b) Non-Conducting Walls

$$u = Du = g = (DD^* - a^2) g = \theta = 0 \quad \text{at } x = 0, 1 \quad (102)$$

where now

$$\bar{G}(x) = 1 - (1 - \mu) x$$

$$N = Pr \propto (T_2 - T_1)/2 (1 - \mu) \quad (103)$$

Equations (98) - (100) and the boundary condition (101) or (102) define an eigenvalue problem of the form

$$F(T_a, a, \mu, Q, N) = 0 \quad (104)$$

For fixed values of μ , Q and N we wish to determine the minimum positive T_a over all real positive wave numbers, a , for which there exists a non-trivial solution of the eigenvalue problem. The critical value of T_a determines the critical speed at which instability first sets in the fluid. The corresponding critical value of a , determines the spacing of the vortices in the z -direction.

3.10 Method of Solution

The eigenvalue problem (104) is solved by the Harris and Reid Method (1964). The method has been described in details in Section 2.7, thus it is sufficient here to demonstrate only how the method can be adapted for the present problem. We start by rewriting equations (98) - (100) as a system of first order equations. So we put

$$\begin{aligned}
A &= u & B &= Du \\
C &= (D^2 - a^2) u & E &= D (D^2 - a^2) u \\
F &= g & H &= Dg \\
I &= (D^2 - a^2) g & J &= D (D^2 - a^2) g \\
K &= \theta & L &= D\theta
\end{aligned} \tag{105}$$

Thus

$$\begin{aligned}
DA &= B & DB &= C + a^2 A \\
DC &= E & DE &= a^2 C - Q a^2 A - a^2 T_a [G(x).I + N (G(x))^2 K] \\
H &= DF & DH &= I + a^2 F \\
DI &= J & DJ &= a^2 I - Q a^2 F + A \\
DK &= L & DL &= a^2 K + A
\end{aligned} \tag{106}$$

The boundary conditions for conducting walls case (the corresponding case of non-conducting walls is referred to in brackets) require that

$$A = B = H(F) = I = K = 0 \quad \text{at } x = 0, 1 \tag{107}$$

We define five linearly independent solutions A_i , F_i and K_i , ($i = 1, 2, 3, 4, 5$) of equation (98) - (100) by imposing the initial conditions

$$\left[A_i = B_i = H_i(F_i) = I_i = K_i \right] = 0, \quad i = 1, 2, 3, 4, 5 \quad (108)$$

and

$$C_i = E_i = F_i(H_i) = J_i = L_i = \begin{cases} (1, 0, 0, 0, 0) & i=1 \\ (0, 1, 0, 0, 0) & i=2 \\ (0, 0, 1, 0, 0) & i=3 \\ (0, 0, 0, 1, 0) & i=4 \\ (0, 0, 0, 0, 1) & i=5 \end{cases} \quad (109)$$

at $x = 0$

A solution of (98) - (100) which automatically satisfies the boundary condition at $x = 0$ can be written as:

$$u = \sum_{i=1}^5 M_i A_i, \quad g = \sum_{i=1}^5 N_i F_i, \quad \theta = \sum_{i=1}^5 P_i K_i \quad (110)$$

where M_i , N_i and P_i are constants. The boundary conditions at $x = 1$ then require that

$$\begin{aligned} \sum_{i=1}^5 M_i A_i = 0, \quad \sum_{i=1}^5 M_i B_i = 0, \quad \sum_{i=1}^5 N_i H_i(F_i) = 0 \\ \sum_{i=1}^5 N_i I_i = 0, \quad \sum_{i=1}^5 P_i K_i = 0. \end{aligned} \quad (111)$$

If the constants M_i , N_i and P_i are not to vanish identically then it is necessary to have

$$\delta(a, T, \mu, Q, N) = \begin{vmatrix} A_i \\ B_i \\ H_i(F_i) \\ I_i \\ K_i \end{vmatrix} = 0, \quad \text{at } x = 1. \quad (112)$$

This is the characteristic equation from which the critical values of a and

T_a can be obtained for any given values of μ , N and Q .

The process of numerically integrating the fifty equations defined by (106) and the process of determining the Taylor and wave numbers that satisfy equation (112) are the same as described in Section 2.7.

3.11 Discussion of the Results

At the beginning, we note that our stability analysis combines in one problem the simultaneous effects of heating, axial magnetic field, velocities ratio and radii ratio. The study includes, as special cases, the stabilizing influence of only one or two factors, previously examined by different authors. Setting $Q = 0$ and $N = 0$, the governing equations reduce to that derived first by Taylor (1923) under the narrow gap approximations and then by Chandrasekhar (1958) and Sparrow, Munro and Jonsson (1964) for the wide gap case. When $Q = 0$ but $N \neq 0$, we recover the stability equations of radial temperature gradient first obtained by Walowit, Tsao and DiPrima (1964) and later by Soundalgekar, Takhar and Smith (1981). Finally, if $N = 0$ but $Q \neq 0$, then we come to the problem first considered by Chandrasekhar (1953) for an electrically conducting Couette flow with an axial magnetic field and narrow-gap assumptions. It is worth mentioning here that the results obtained for the above special problems are in good agreement with our predictions.

For both conducting and non-conducting walls, an inspection of Tables 1-8 and Figs.1-24 allows us to write down the following general statements about the effect of Q , N and μ on the stability of the flow.

1. The presence of an axial magnetic field inhibits the onset of

instability. The greater the strength of the magnetic field, the greater is the angular velocity Ω_1 needed to bring about the instability. The reason for this inhibiting effect of Q is that the magnetic lines of force have a tendency to be dragged with the fluid. This attachment of the fluid to the lines of force gets stronger with large values of Q . Consequently, it becomes difficult for the fluid to move at right angles to the magnetic field which is required for instability.

2. When the temperature of the fluid has a radial variation then, depending on the direction of heat flow, the fluid becomes more or less stable. The transfer of heat from or towards the inner cylinder raises or lowers respectively the fluid transition velocity. Consequently, the stabilization effect of the magnetic field can be strengthened or weakened by the presence of heating.

3. The third factor that creates changes in T_c is the angular velocities ratio of the cylinders μ . For different values of μ , different quantitative changes of the same type of effects mentioned above for Q and N are predicted. The critical Taylor number which results from rotating the inner cylinder only is decreased or increased when the two cylinders rotate in the same or in the opposite direction respectively.

In view of (1), (2) and (3), we may now analyse the T_c 's calculated for different values of the axial magnetic field, the temperature gradient and the velocities ratio of the two cylinders. The case of perfectly conducting walls will be referred to by (a) and that of non-conducting walls by (b). Starting with the variation of T_c with Q for different N and μ , the tabular results and Figs.1-8 reveal the following.

1. For an isothermal flow and non-rotating outer cylinder the inhibiting power of Q is manifested through its influence on T_c . As Table 1 and Fig.1

demonstrate, for case (a) the T_c at $Q = 10$ is about $1\frac{1}{2}$ times that at $Q = 0$. It rises to a value more than forty times as great when $Q = 300$. This indicates that the flow stabilizes at an accelerating rate with stronger magnetic fields. For case (b), the capability of Q to maintain a stable flow is considerably smaller than (a). With Q reaching 300, the critical angular velocity increases by only about ten times Ω_1 at $Q = 0$.

2. A further inspection of Table 1 and Fig.1 also suggests that the contribution of heating in varying the critical velocity is more effective at high values of Q . For (a), due to the presence of a positive temperature gradient that corresponds to $N = 0.1$, the T_c of an isothermal flow suffers a reduction by 5% when $Q = 0$ and by 40% when $Q = 300$. The equivalent negative temperature gradient (i.e., $N = -0.1$) causes a larger change in T_c . For $Q = 0$ and $Q = 300$, the T_c 's rise by 6% and 97% respectively. That the same amount of heat has a greater effect when it transfers from the inner rather than the outer cylinder is predicted for all calculated values of N . The difference in case (b) is that the changes in T_c created by N are less significant as compared with (a). With $N = 0.1$ or $N = -0.1$, the T_c goes up or down by about 16% or 20% respectively.

3. The same type of interaction between Q and N described in (1) and (2) is also observed when both cylinders are rotating (i.e., $\mu \neq 0$). However, as one can deduce from Figs.2-8, the stabilizing role that μ plays is manifested through the quantitative dependence of the critical conditions, associated with given temperature gradient and magnetic field, on its value. This can be seen if we note, for example for case (a) that as the magnitude of μ changes from $|\mu| = 0.25$ to $|\mu| = 1.0$, the T_c results from the combined effect of $Q = 300$ and $N = -0.1$ loses about 40% of its original value for co-rotating cylinders and about 37% for counter-rotating cylinders.

The variations of T_c with N , shown in tabular and graphical forms, demonstrate the role that heating can play in the control of the velocity at the onset of instability for an electrically conducting flow under the influence of a fixed magnetic field and velocities ratio μ . It can be readily seen from the tabular results and Figs.9-24 that the critical Taylor number varies smoothly and slowly with N and the fastest rate of this variation occurs in the presence of a strong magnetic field. As an example, for (a) and $\mu = 0$, the gain in T_c , due to reversing the temperature gradient between the cylinders from $N = 0.1$ to $N = -0.1$, is about 12% when $Q = 0$ and it is more than 220% when $Q = 300$. This shows that a greater stability reaction is obtained when the alteration of the constant temperatures of the cylinders occurs in the presence of a strong magnetic field.

A comparison between Figs.9-24 indicates that μ has a quantitative role in the changes in T_c created by N for a given Q . Co-rotating and counter-rotating cylinders will respectively reduce and magnify the forementioned effects of N on T_c . We note, for example, for (a) and $Q = 50$, that by heating the isothermal fluid to $N = 0.1$, the T_c changes by about 14% for $\mu = 0$, 22% for $\mu = 1.0$ and by 10% for $\mu = -1.0$.

It is quite evident from our calculations that the vital contribution to the fluid stability comes from the magnetic field rather than from heating. This is due to the fact that while the magnetic field can be strengthened to a large extent the heating (restricted by our assumptions that ν , k and c_p are considered as constants) is to be of moderate magnitude. Therefore, if we are to examine the effect of great heating, then a stability study which accounts for the presence of a large temperature gradient is needed. That such an important study has not yet been reported is mainly attributed to its complexity.

We now turn to analyse the effect of Q , N and μ on the critical wave number a_c . With the aid of Tables 1-4, we may state the following for the case of perfectly conducting walls,

1. When an axial magnetic field is applied to an isothermal flow with a stationary outer cylinder, the critical wave number a_c for $Q \neq 0$ may move to a higher or lower value depending on the strength of the magnetic field. For Q less than about 100, the a_c undergoes a gradual increase with larger Q , where for $Q > 100$, the a_c suffers a continuous reduction in its value.
2. Keeping the outer cylinder at a constant temperature, greater or lower than that of the inner cylinder, causes the a_c for $N = 0$ to decrease or increase respectively. Also, as in the case of T_c , the changes of a_c with the larger magnitude of N are accelerated in the presence of a strong magnetic field. When a negative temperature gradient that corresponds to $N = -0.1$ is maintained between the cylinders, this causes a_c to increase by about 0.8% when $Q = 10$ and by more than 70% when $Q = 200$.
3. The type of variation of a_c with μ is a function of the temperature gradient that prevails in the fluid. Driving the two cylinders in the same direction with increasing values of μ leads to a lower a_c when the fluid is under the influence of $N \geq 0$ or small negative N accompanied by small μ , whereas for large negative N , the a_c attains higher values with μ . On the other hand, for counter-rotating cylinders, the relation between a_c and μ is simpler. The a_c decreases or increases according as $N \geq 0$ or $N < 0$.

When the cylinders are made of non-conducting materials, the a_c exhibits a smooth pattern of variation with Q , N and μ . For any fixed N and μ , the a_c shows a gradual decrease with larger values of Q . For rotating cylinders with $\mu \geq -0.5$ and also for $\mu < 0.5$ accompanied by a strong magnetic field ($Q > 100$), the a_c smoothly increases or decreases with larger magnitudes of positive or negative N respectively. It is also

worth mentioning here that, as shown before for T_C , the a_c associated with any values of μ , N and $Q \neq 0$ is always smaller for the case of non-conducting walls as compared with that of the conducting walls.

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TABLE 1: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- CONDUCTING WALLS -

μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
0.0	0.0	0.1	3.127	3210	0.25	0.0	0.1	3.121	2558
		0.05	3.127	3297			0.05	3.120	2638
		0.0	3.127	3390			0.0	3.120	2724
		-0.05	3.126	3487			-0.05	3.120	2816
		-0.1	3.126	3590			-0.1	3.120	2914
	10	0.1	3.305	4934		10	0.1	3.291	3929
		0.05	3.315	5119			0.05	3.303	4100
		0.0	3.327	5319			0.0	3.317	4287
		-0.05	3.339	5535			-0.05	3.331	4491
		-0.1	3.352	5769			-0.1	3.348	4715
	50	0.1	3.364	13442		50	0.1	3.317	10681
		0.05	3.425	14436			0.05	3.386	11594
		0.0	3.498	15576			0.0	3.471	12664
		-0.05	3.584	16893			-0.05	3.573	13928
		-0.1	3.687	18425			-0.1	3.699	15437
	100	0.1	3.134	26817		100	0.1	3.053	21219
		0.05	3.253	29939			0.05	3.180	24072
		0.0	3.417	33788			0.0	3.361	27703
		-0.05	3.646	38586			-0.05	3.627	32385
		-0.1	3.946	44596			-0.1	4.010	38436
	200	0.1	2.632	57763		200	0.1	2.536	45221
		0.05	2.777	68859			0.05	2.673	55155
		0.0	3.060	84806			0.0	2.949	70123
		-0.05	3.768	108414			-0.05	3.667	93535
		-0.1	5.207	140084			-0.1	5.307	126497
	300	0.1	2.292	90274		300	0.1	2.208	70071
		0.05	2.403	112843			0.05	2.306	89843
		0.0	2.686	150345			0.0	2.554	124235
		-0.05	4.750	219046			-0.05	4.829	195534
		-0.1	6.953	295764			-0.1	7.364	276369

TABLE 2: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- CONDUCTING WALLS -

μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
0.5	0.0	0.1	3.117	2114	1.0	0.0	0.1	3.116	1552
		0.05	3.117	2191			0.05	3.116	1626
		0.0	3.117	2275			0.0	3.116	1707
		-0.05	3.117	2364			-0.05	3.116	1797
		-0.1	3.282	2462			-0.1	3.116	1897
	10	0.1	3.282	3239		10	0.1	3.271	2360
		0.05	3.297	3402			0.05	3.290	2515
		0.0	3.312	3582			0.0	3.311	2690
		-0.05	3.331	3783			-0.05	3.335	2892
		-0.1	3.351	4007			-0.1	3.364	3125
	50	0.1	3.285	8729		50	0.1	3.237	6211
		0.05	3.362	9590			0.05	3.331	6997
		0.0	3.459	10622			0.0	3.454	7990
		-0.05	3.582	11875			-0.05	3.622	9271
		-0.1	3.737	13416			-0.1	3.850	10960
	100	0.1	2.999	17174		100	0.1	2.926	11917
		0.05	3.134	19825			0.05	3.075	14248
		0.0	3.338	23320			0.0	3.327	17570
		-0.05	3.653	28005			-0.05	3.770	22439
		-0.1	4.126	34284			-0.1	4.481	29505
	200	0.1	2.479	36021		200	0.1	2.414	24109
		0.05	2.613	44988			0.05	2.547	31505
		0.0	2.903	59241			0.0	2.882	44711
		-0.05	3.709	82777			-0.05	4.046	69448
		-0.1	5.476	116446			-0.1	6.059	104850
	300	0.1	2.160	55229		300	0.1	2.114	36171
		0.05	2.253	72695			0.05	2.202	49974
		0.0	2.502	104902			0.0	2.479	79150
		-0.05	4.645	178355			-0.05	3.468	137178
		-0.1	7.880	261507			-0.1	3.123	165925

TABLE 3: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- CONDUCTING WALLS -

μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
-0.25	0.0	0.1	3.145	4251	-0.5	0.0	0.1	3.199	6115
		0.05	3.145	4353			0.05	3.199	6261
		0.0	3.144	4461			0.0	3.198	6413
		-0.05	3.143	4574			-0.05	3.198	6574
		-0.1	3.142	4693			-0.1	3.197	6742
	10	0.1	3.334	6506		10	0.1	3.408	9196
		0.05	3.343	6723			0.05	3.417	9493
		0.0	3.352	6954			0.0	3.427	9808
		-0.05	3.362	7202			-0.05	3.438	10145
		-0.1	3.373	7464			-0.1	3.450	10506
	50	0.1	3.442	17509		50	0.1	3.594	23667
		0.05	3.499	18645			0.05	3.653	25074
		0.0	3.564	19929			0.0	3.721	26643
		-0.05	3.639	21386			-0.05	3.798	28403
		-0.1	3.728	23049			-0.1	3.886	30384
	100	0.1	3.267	34686		100	0.1	3.499	45693
		0.05	3.386	38200			0.05	3.633	49787
		0.0	3.544	42419			0.0	3.801	54575
		-0.05	3.754	47528			-0.05	4.012	60198
		-0.1	4.035	53740			-0.1	4.272	66813
	200	0.1	2.798	74765		200	0.1	3.091	97317
		0.05	2.971	87323			0.05	3.340	111608
		0.0	3.307	104542			0.0	3.770	129975
		-0.05	4.051	128199			-0.05	4.455	153110
		-0.1	5.200	157971			-0.1	5.302	181087
	300	0.1	2.442	117461		300	0.1	2.716	153318
		0.05	2.591	143615			0.05	2.985	183827
		0.0	3.024	185058			0.0	3.700	227577
		-0.05	4.840	248640			-0.05	5.076	284963
		-0.1	6.663	320695			-0.1	6.511	352370

TABLE 4: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- CONDUCTING WALLS -

μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
-0.75	0.0	0.1	3.394	9939	-1.0	0.0	0.1	3.954	17514
		0.05	3.400	10220			0.05	3.997	18075
		0.0	3.406	10517			0.0	3.999	18662
		-0.05	3.413	10831			-0.05	4.021	19279
		-0.1	3.420	11162			-0.1	4.044	19927
	10	0.1	3.622	14157		10	0.1	4.095	22680
		0.05	3.641	14655			0.05	4.130	23512
		0.0	3.662	15186			0.0	4.166	24390
		-0.05	3.685	15754			-0.05	4.202	25319
		-0.1	3.708	16362			-0.1	4.237	26302
	50	0.1	3.880	32869		50	0.1	4.304	45783
		0.05	3.956	34720			0.05	4.392	48192
		0.0	4.038	36755			0.0	4.481	50793
		-0.05	4.127	38998			-0.05	4.873	53616
		-0.1	4.224	41479			-0.1	4.664	56689
	100	0.1	3.872	60606		100	0.1	4.369	79969
		0.05	4.030	65438			0.05	4.531	85604
		0.0	4.210	70919			0.0	4.699	91849
		-0.05	4.412	77158			-0.05	4.873	98812
		-0.1	4.638	84283			-0.1	5.058	106618
	200	0.1	3.597	126145		200	0.1	4.293	161005
		0.05	3.993	141920			0.05	4.611	187110
		0.0	4.370	160853			0.0	4.964	197195
		-0.05	4.904	183468			-0.05	5.364	219879
		-0.1	5.516	210320			-0.1	5.820	246469
	300	0.1	3.252	198958		300	0.1	4.071	252426
		0.05	3.716	232622			0.05	4.522	287034
		0.0	4.439	275447			0.0	5.085	328983
		-0.05	5.408	328472			-0.05	5.792	379814
		-0.1	6.504	392037			-0.1	6.631	440823

TABLE 5: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- NON-CONDUCTING WALLS -

μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
0.0	0.0	0.1	3.128	3210	0.25	0.0	0.1	3.119	2558
		0.05	3.127	3297			0.05	3.119	2639
		0.0	3.127	3390			0.0	3.119	2725
		-0.05	3.126	3487			-0.05	3.118	2817
		-0.1	3.125	3590			-0.1	3.118	2915
	10	0.1	2.997	4410		10	0.1	2.986	3511
		0.05	2.995	4559			0.05	2.985	3649
		0.0	2.993	4719			0.0	2.983	3798
		-0.05	2.992	4890			-0.05	2.983	3959
		-0.1	2.990	5073			-0.1	2.980	4135
	50	0.1	2.426	10162		50	0.1	2.408	8063
		0.05	2.398	10742			0.05	2.378	8595
		0.0	2.365	11390			0.0	2.341	9199
		-0.05	2.326	12120			-0.05	2.295	9891
		-0.1	2.278	12943			-0.1	2.238	10689
	100	0.1	1.879	18364		100	0.1	1.870	14518
		0.05	1.807	19741			0.05	1.792	15769
		0.0	1.725	21296			0.0	1.701	17212
		-0.05	1.635	23053			-0.05	1.601	18872
		-0.1	1.542	25029			-0.1	1.497	20776
	200	0.1	1.340	25534		200	0.1	1.342	27989
		0.05	1.249	38666			0.05	1.243	30832
		0.0	1.162	42146			0.0	1.147	34052
		-0.05	1.082	45995			-0.05	1.061	37677
		-0.1	1.010	50238			-0.1	0.981	41744
	300	0.1	1.088	52906		300	0.1	1.094	41609
		0.05	1.002	57814			0.05	0.999	46068
		0.0	0.926	63206			0.0	0.915	51056
		-0.05	0.860	69115			-0.05	0.843	56618
		-0.1	0.803	75588			-0.1	0.781	62815

TABLE 6: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- NON-CONDUCTING WALLS -

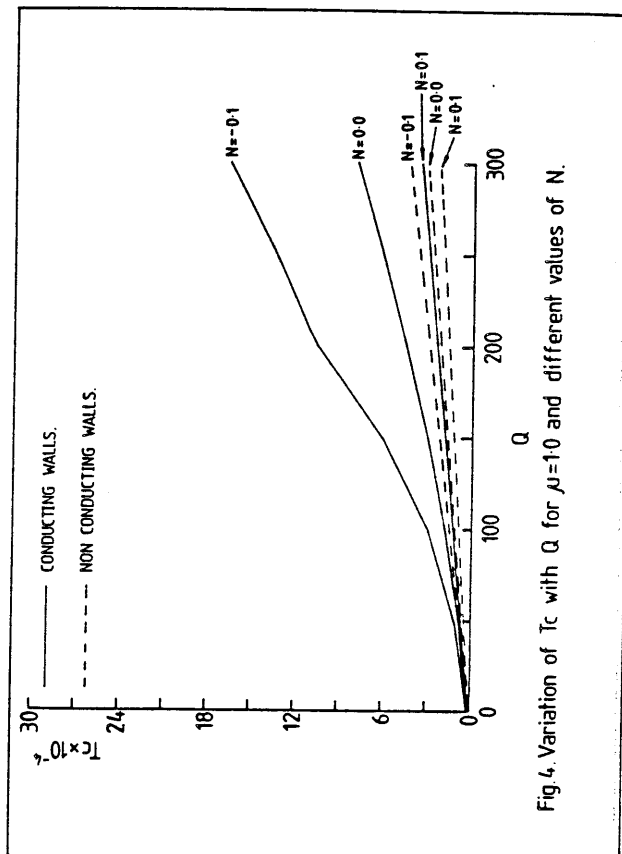
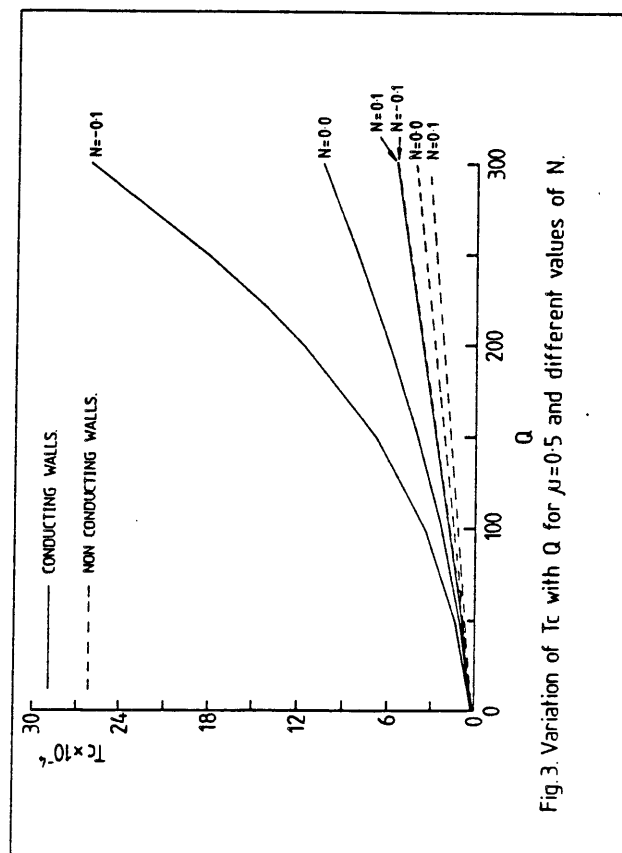
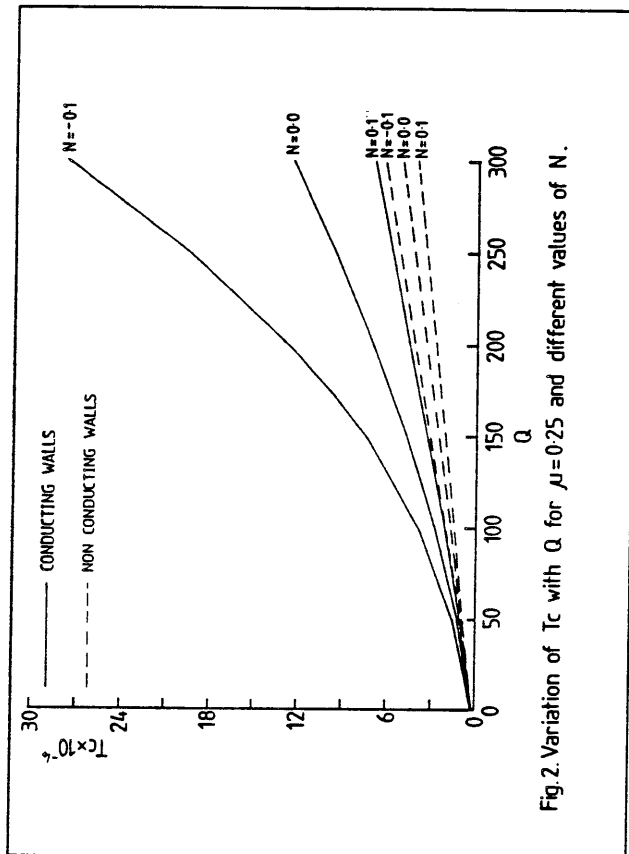
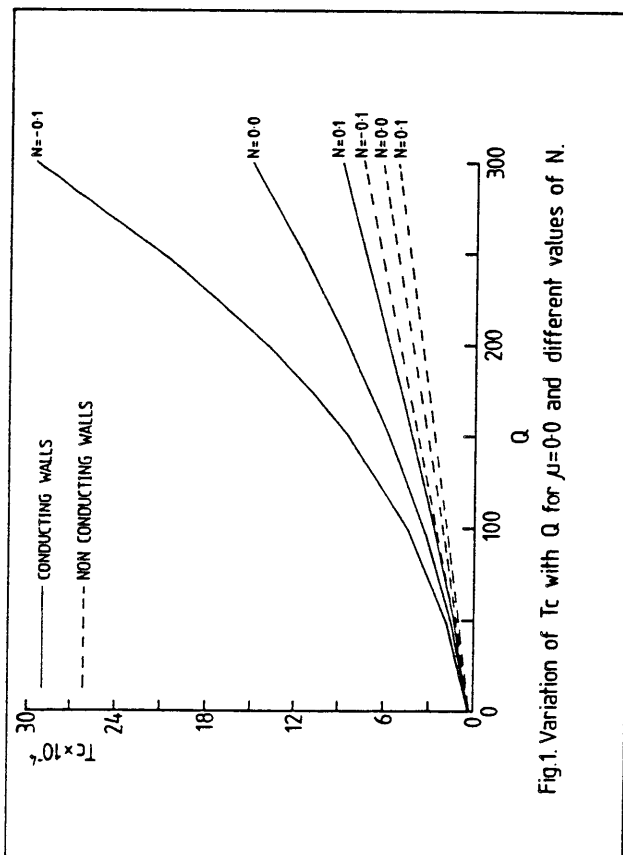
μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
0.5	0.0	0.1	3.118	2114	1.0	0.0	0.1	3.116	1588
		0.05	3.118	2192			0.05	3.118	1626
		0.0	3.119	2275			0.0	3.118	1707
		-0.05	3.118	2365			-0.05	3.118	1797
		-0.1	3.118	2462			-0.1	3.117	1897
	10	0.1	2.982	2897		10	0.1	2.981	2117
		0.05	2.981	3028			0.05	2.980	2241
		0.0	2.981	3172			0.0	2.978	2382
		-0.05	2.978	3331			-0.05	2.976	2541
		-0.1	2.976	3506			-0.1	2.974	2722
	50	0.1	2.407	6609		50	0.1	2.419	4752
		0.05	2.374	7112			0.05	2.380	5217
		0.0	2.332	7695			0.0	2.327	5780
		-0.05	2.278	8377			-0.05	2.254	6473
		-0.1	2.208	9183			-0.1	2.151	7339
	100	0.1	1.879	11837		100	0.1	1.919	8408
		0.05	1.794	13013			0.05	1.818	9484
		0.0	1.692	14401			0.0	1.688	10820
		-0.05	1.578	16035			-0.05	1.537	12476
		-0.1	1.458	17951			-0.1	1.383	14514
	200	0.1	1.142	28487		200	0.1	1.422	15959
		0.05	1.251	25390			0.05	1.281	18414
		0.0	1.142	22715			0.0	1.138	21402
		-0.05	1.042	32045			-0.05	1.012	24993
		-0.1	0.955	36117			-0.1	0.905	29282
	300	0.1	1.115	33705		300	0.1	1.178	25414
		0.05	1.007	37908			0.05	1.038	27444
		0.0	0.911	42709			0.0	0.910	32086
		-0.05	0.829	48164			-0.05	0.805	37584
		-0.1	0.760	54362			-0.1	0.721	44097

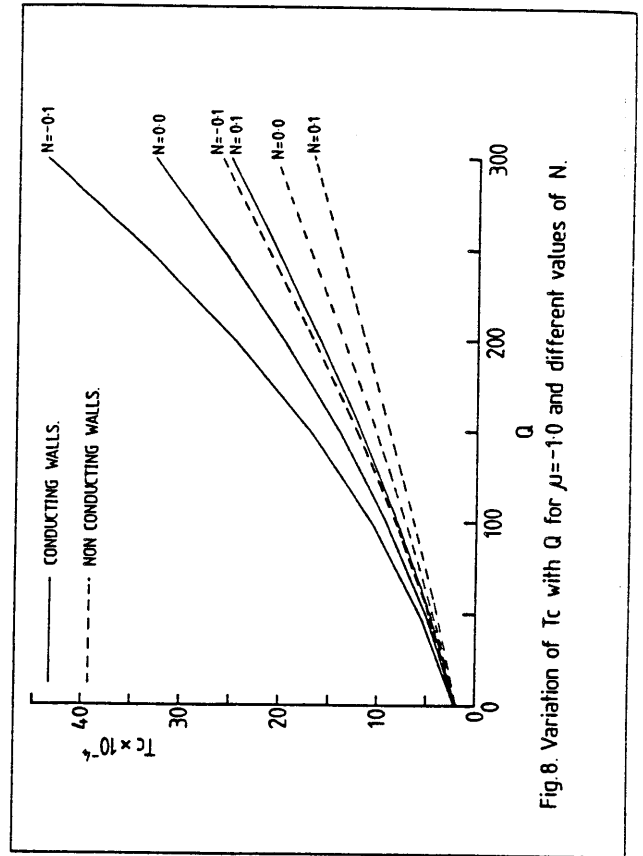
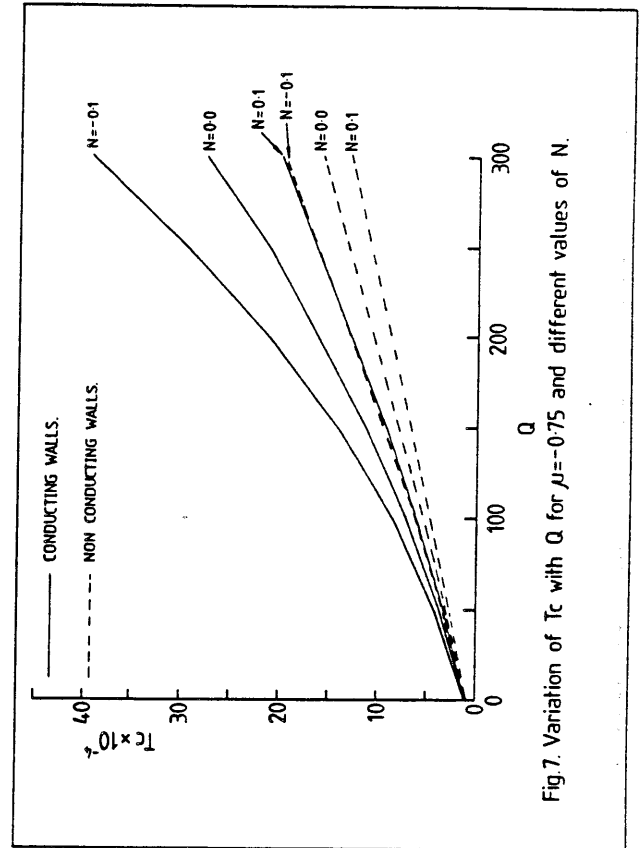
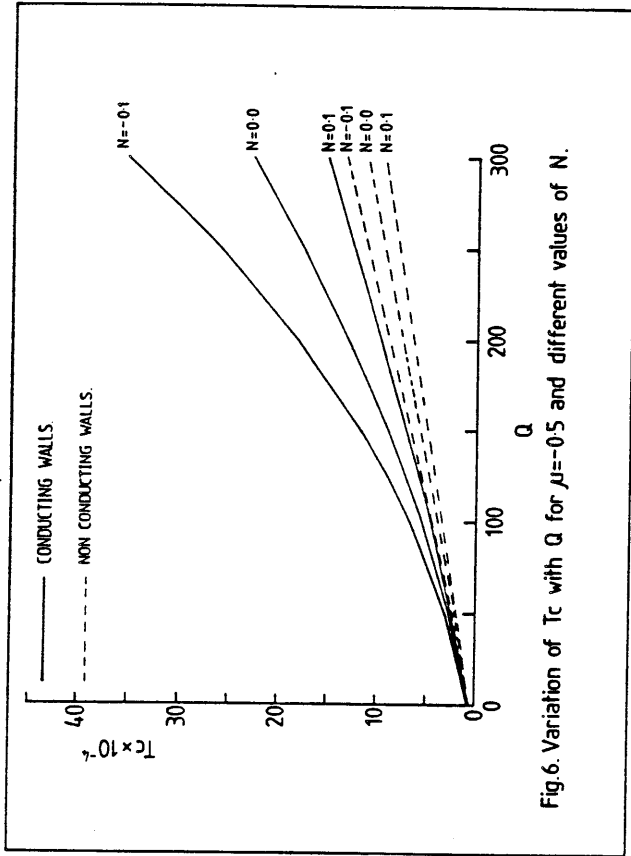
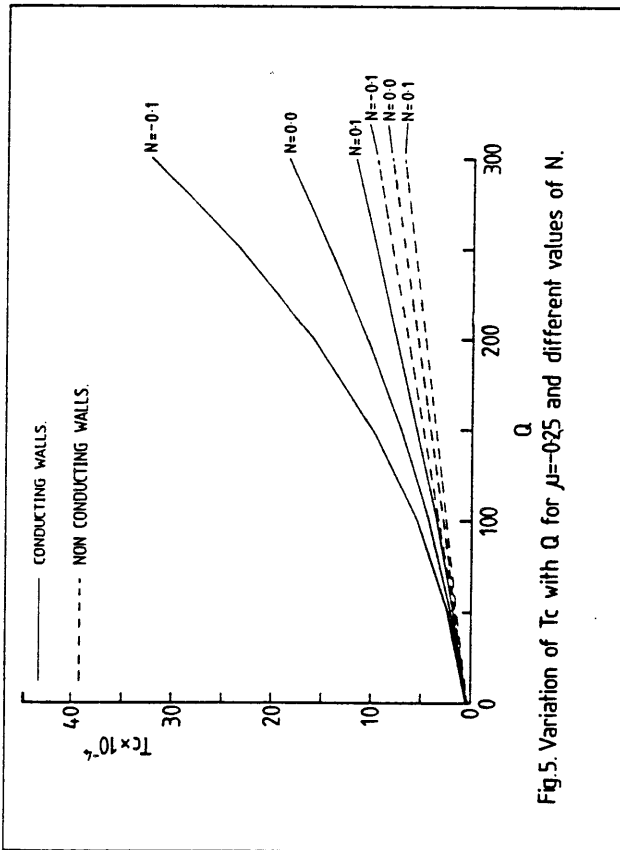
TABLE 7: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- NON-CONDUCTING WALLS -

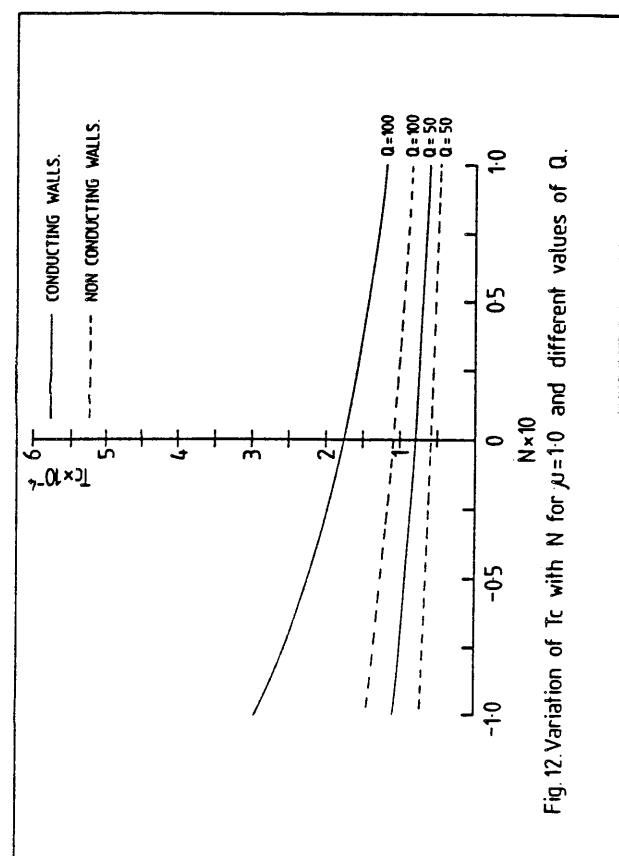
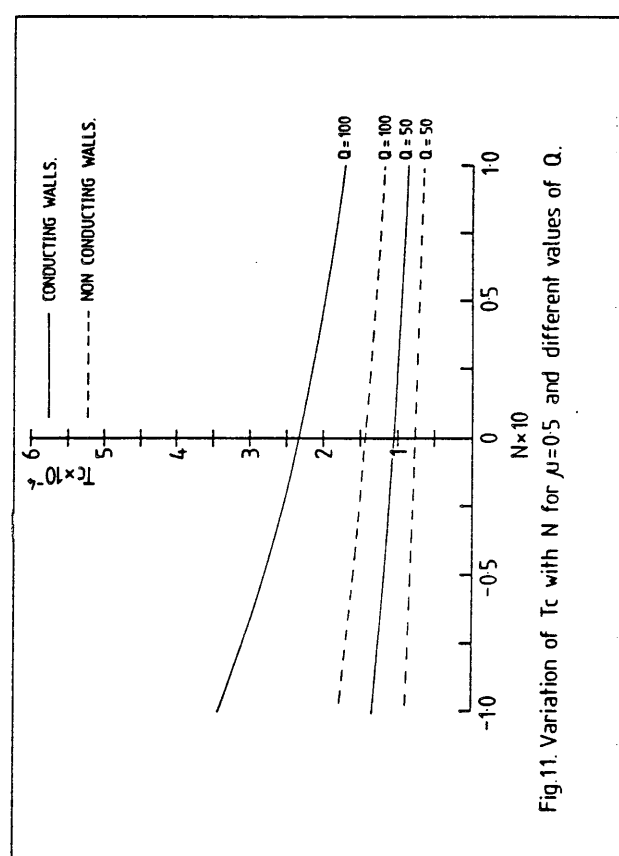
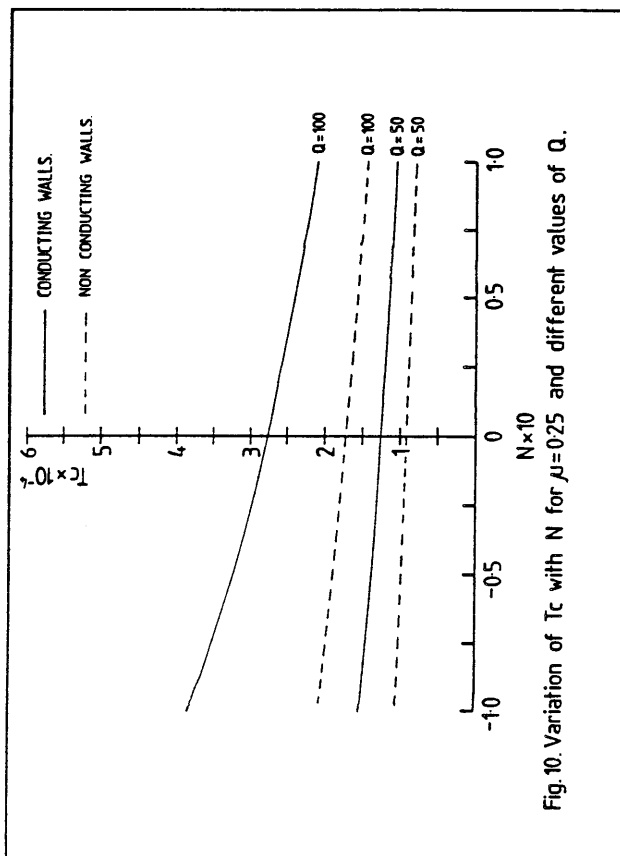
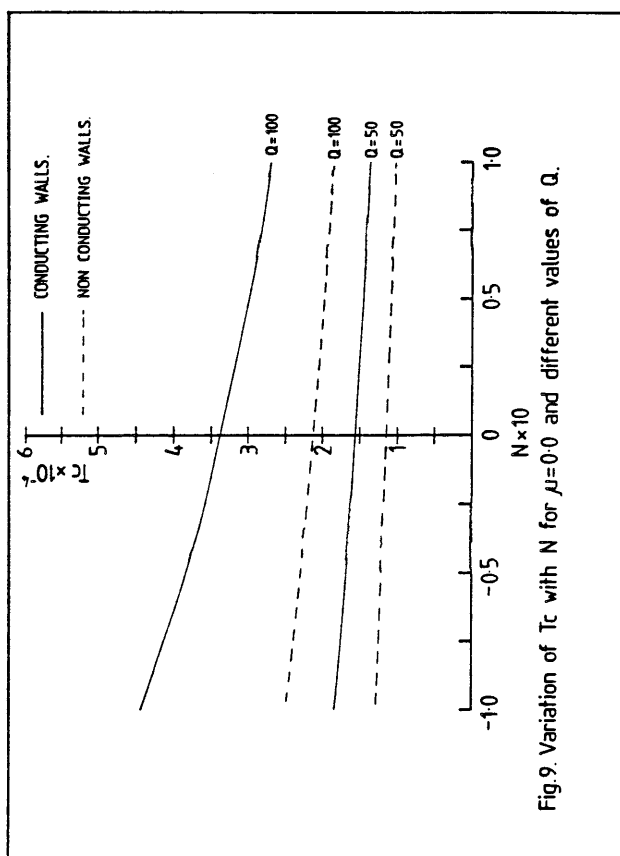
μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
-0.25	0.0	0.1	3.146	4251	-0.5	0.0	0.1	3.199	6115
		0.05	3.145	4353			0.05	3.199	6261
		0.0	3.144	4461			0.0	3.198	6413
		-0.05	3.143	4574			-0.05	3.198	6574
		-0.1	3.142	4693			-0.1	3.197	6742
	10	0.1	3.020	6829		10	0.1	3.100	8308
		0.05	3.022	6004			0.05	3.100	8551
		0.0	3.020	6190			0.0	3.100	8810
		-0.05	3.018	6387			-0.05	3.099	9084
		-0.1	3.016	6597			-0.1	3.098	9376
	50	0.1	2.500	13377		50	0.1	2.654	18594
		0.05	2.458	14057			0.05	2.639	19503
		0.0	2.429	14808			0.0	2.621	20505
		-0.05	2.395	15642			-0.05	2.599	21615
		-0.1	2.354	16570			-0.1	2.573	22850
	100	0.1	1.941	24171		100	0.1	2.159	33214
		0.05	1.872	25799			0.05	2.097	35414
		0.0	1.793	27620			0.0	2.020	37893
		-0.05	1.705	29651			-0.05	1.927	40687
		-0.1	1.614	31911			-0.1	1.820	43828
	200	0.1	1.385	46847		200	0.1	1.580	64090
		0.05	1.292	50593			0.05	1.466	69356
		0.0	1.204	54704			0.0	1.351	75919
		-0.05	1.123	59194			-0.05	1.244	81593
		-0.1	1.051	64082			-0.1	1.150	88573
	300	0.1	1.121	69820		300	0.1	1.288	95461
		0.05	1.034	75706			0.05	1.169	103886
		0.0	0.957	82088			0.0	1.064	113055
		-0.05	0.890	88991			-0.05	0.976	122967
		-0.1	0.833	96456			-0.1	0.902	133660

TABLE 8: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF μ , Q AND N
- NON-CONDUCTING WALLS -

μ	Q	N	a_c	T_c	μ	Q	N	a_c	T_c
-0.75	0.0	0.1	3.394	10078	-1.0	0.0	0.1	3.954	17514
		0.05	3.400	10221			0.05	3.976	18075
		0.0	3.407	10517			0.0	3.999	18662
		-0.05	3.413	10831			-0.05	4.021	19279
		-0.1	3.420	11163			-0.1	4.044	19927
	10	0.1	3.351	13063		10	0.1	3.918	21485
		0.05	3.362	13497			0.05	3.947	22248
		0.0	3.373	13959			0.0	3.976	23052
		-0.05	3.384	14452			-0.05	4.005	23902
		-0.1	3.397	14978			-0.1	4.034	24800
	50	0.1	2.080	26977		50	0.1	3.661	38844
		0.05	3.101	28332			0.05	3.711	40733
		0.0	3.125	29820			0.0	3.761	42769
		-0.05	3.152	31460			-0.05	3.813	44972
		-0.1	3.183	33275			-0.1	3.866	47363
	100	0.1	2.679	46307		100	0.1	3.263	63015
		0.05	2.683	49441			0.05	3.321	67038
		0.0	2.685	53003			0.0	3.380	71498
		-0.05	2.684	57082			-0.05	3.442	76472
		-0.1	2.678	61797			-0.1	3.507	82056
	200	0.1	2.070	87364		200	0.1	2.576	115275
		0.05	1.999	95282			0.05	2.586	125480
		0.0	1.900	104547			0.0	2.589	137311
		-0.05	1.770	115380			-0.05	2.554	151172
		-0.1	1.616	127935			-0.1	2.489	167581
	300	0.1	1.720	129273		300	0.1	2.137	169216
		0.05	1.609	142555			0.05	2.090	186916
		0.0	1.474	158006			0.0	2.041	207776
		-0.05	1.331	175731			-0.05	1.904	232413
		-0.1	1.198	195743			-0.1	1.767	261381







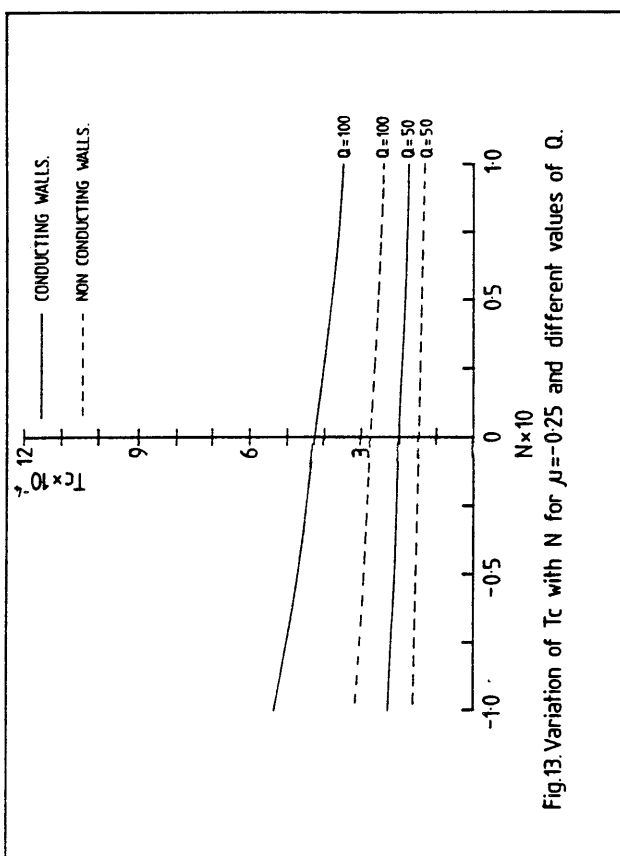


Fig.13. Variation of T_c with N for $\mu=-0.25$ and different values of Q .

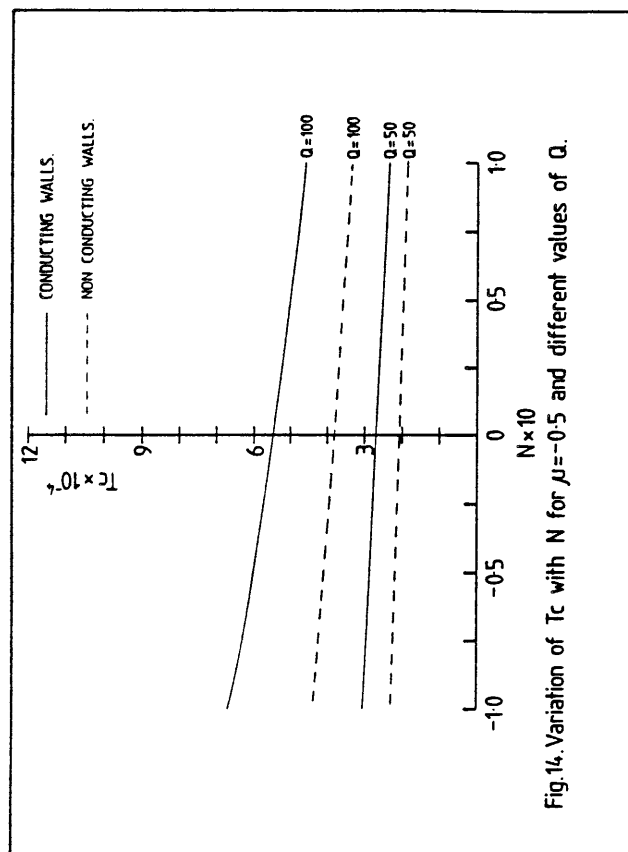


Fig.14. Variation of T_c with N for $\mu=-0.5$ and different values of Q .

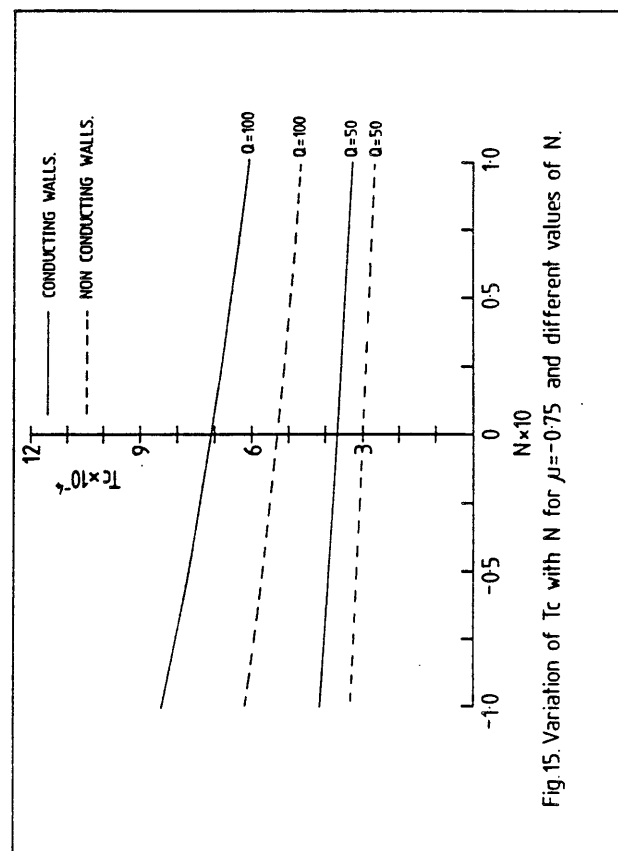


Fig.15. Variation of T_c with N for $\mu=-0.75$ and different values of N .

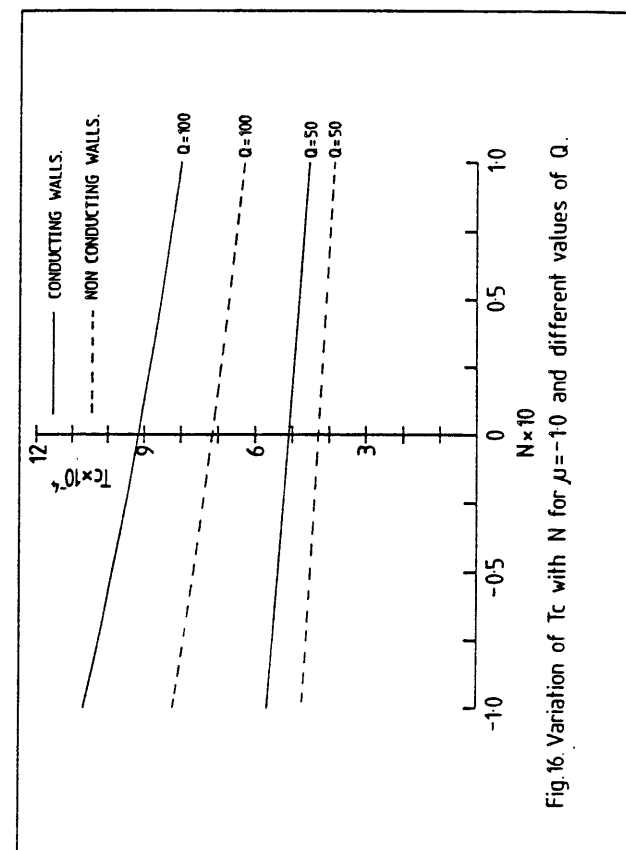
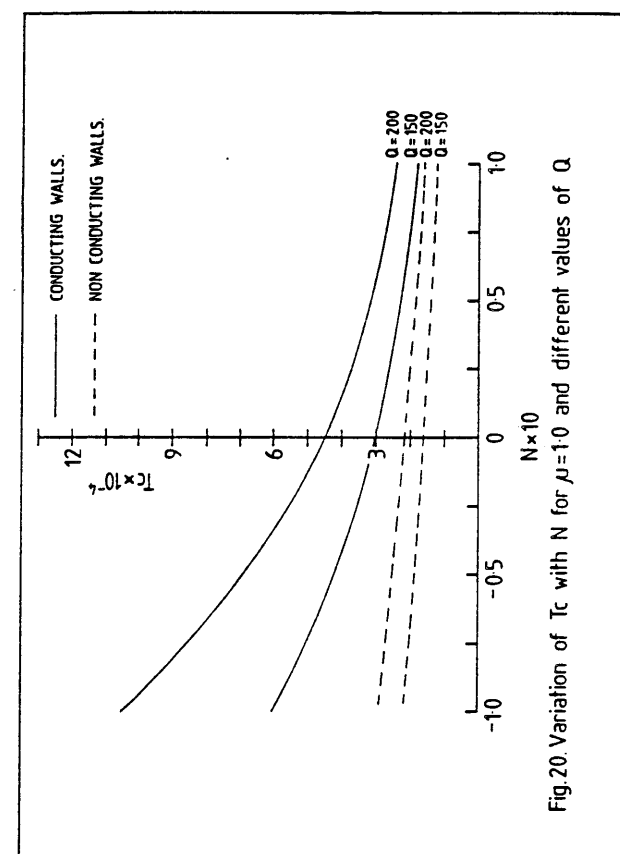
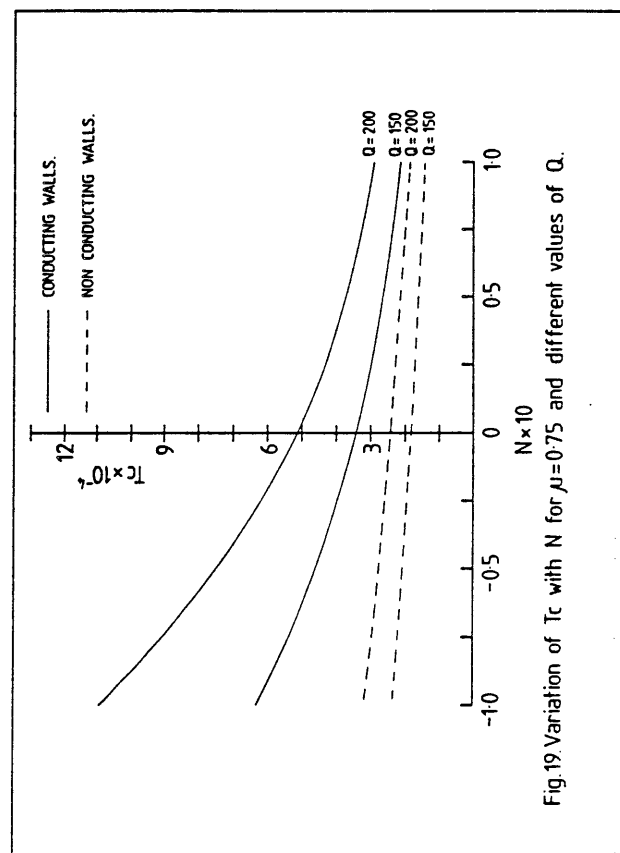
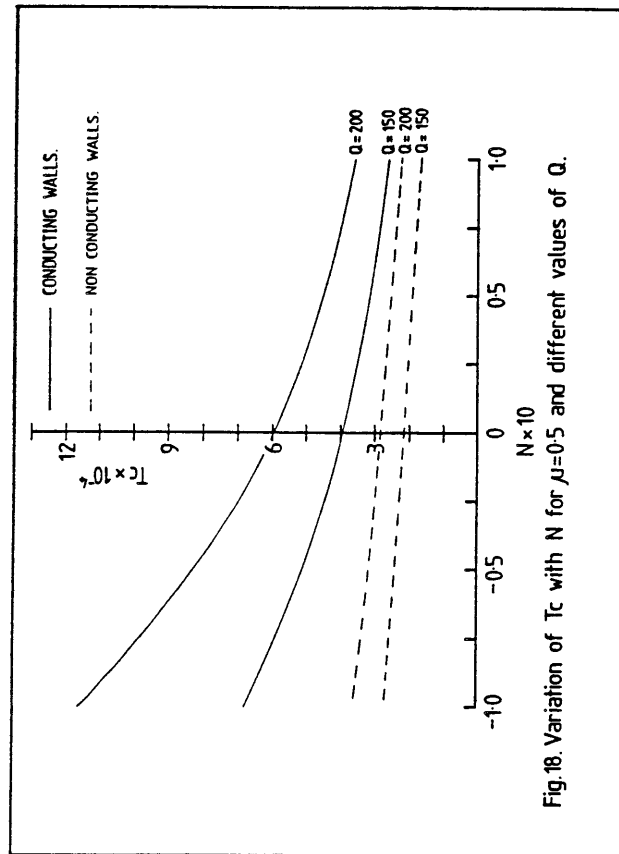
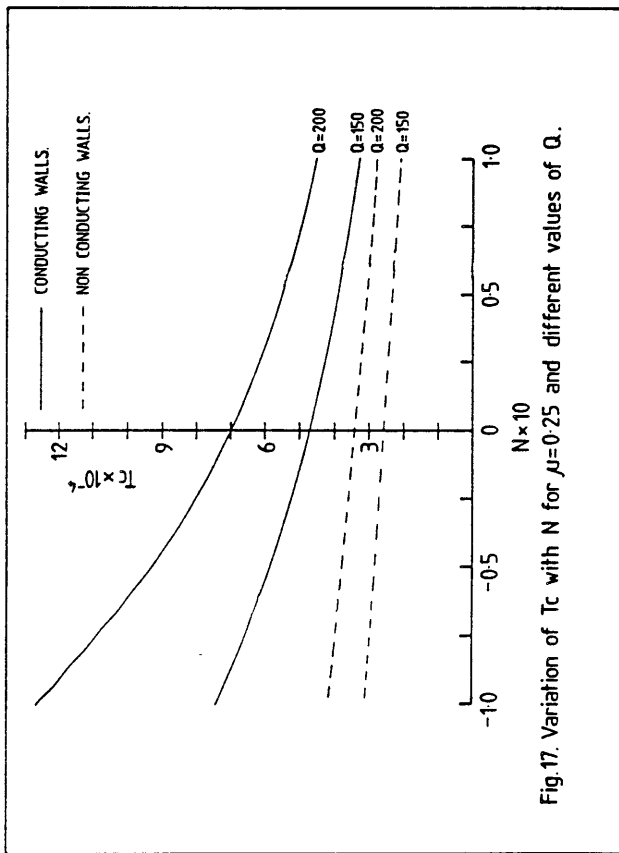
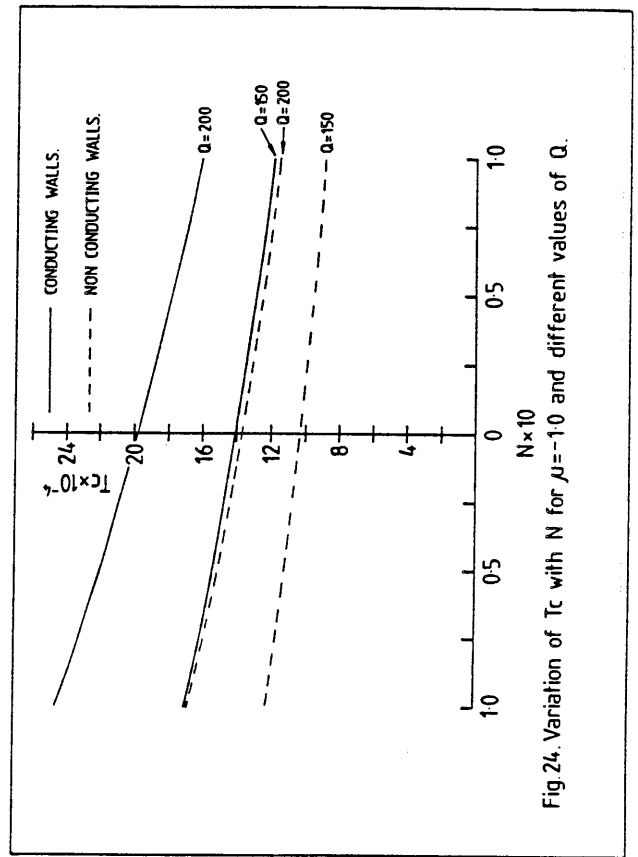
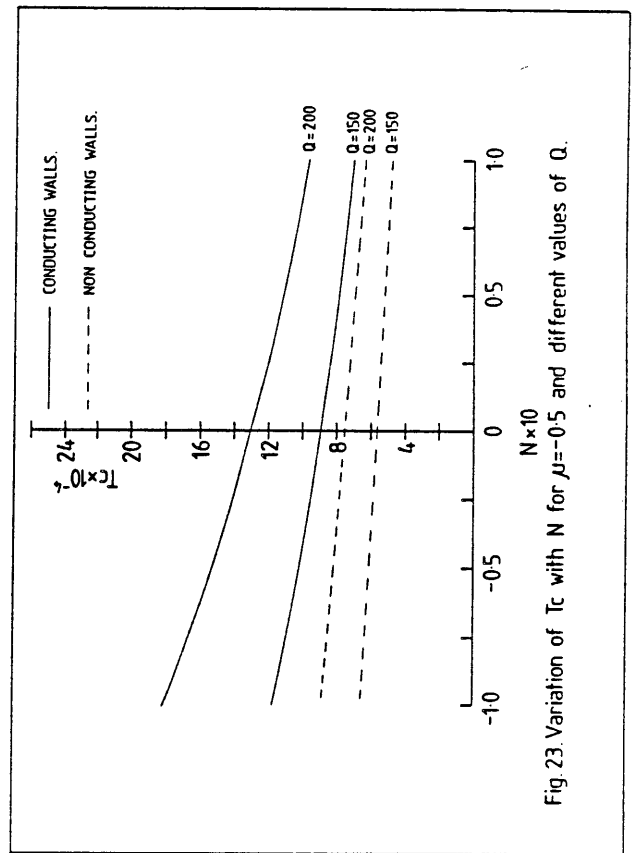
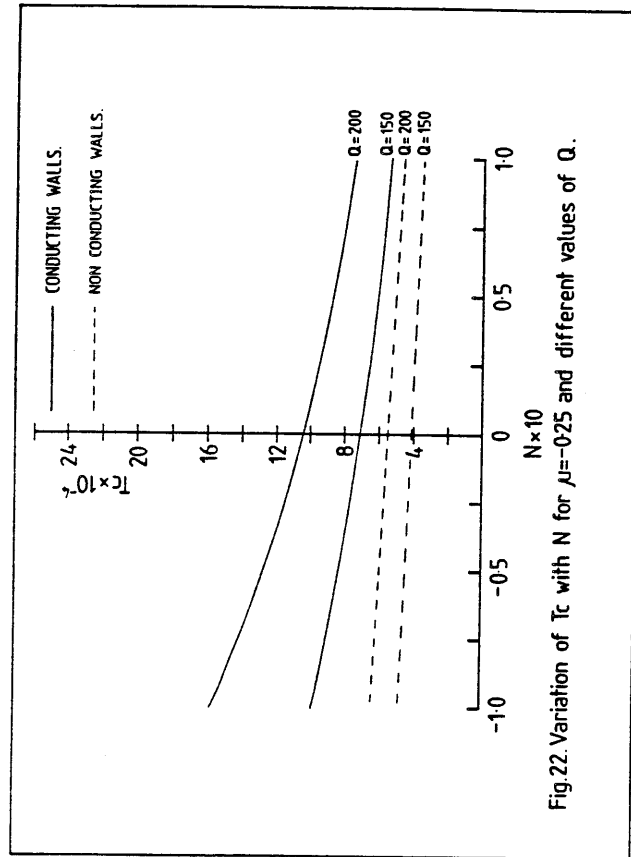
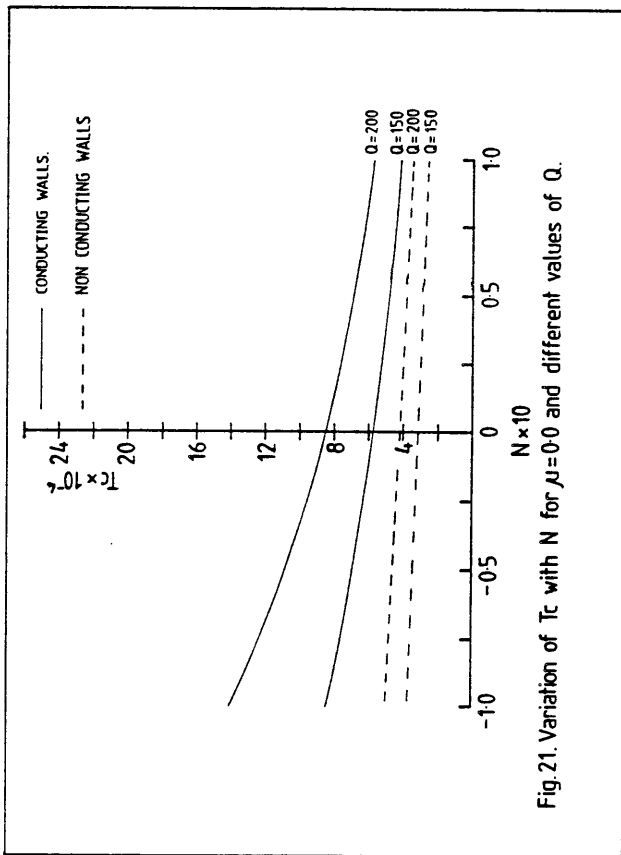


Fig.16. Variation of T_c with N for $\mu=-1.0$ and different values of Q .





THE EFFECT OF RADIAL TEMPERATURE GRADIENT AND AXIAL MAGNETIC
FIELD ON THE STABILITY OF COUETTE FLOW - WIDE GAP PROBLEM.

4.1 Introduction

Under the restriction of a narrow gap between the cylinders, the stability of a Couette flow under various influencing factors has received great attention. Out of an enormous number of studies in this field, we mention that of Taylor (1923), Chandrasekhar (1953, 1954, 1961), Edmonds (1958), Yih (1961), Becker and Kaye (1962), Lai (1962), Kurzweg (1962), Harris and Reid (1964), Walowit, Tsao and DiPrima (1964), Krueger, Gross and DiPrima (1966), Hassard, Chang and Ludford (1972), Bahl (1972), Sastry and Rama (1977), Soundalgekar, Takhar and Smith (1981) and Takhar, Smith and Soundalgekar (1985). In Chapter Three, we have presented the concurrent stability effects of an applied axial magnetic field, temperature gradient and velocities ratio.

Considerably less investigations have been carried out for the finite gap problem. The complementary part of the narrow gap case. For isothermal flow, the problem was studied by Chandrasekhar (1958), Chandrasekhar and Elbert (1962), Walowit, Tsao and DiPrima (1964), Sparrow, Munro and Jonsson (1964), Astill and Chung (1976) and by us, in Chapter Two, as a special case of the constant heat flux problem. The stability effect of heating due to the presence of a radial variation in the flow temperature, was examined by Walowit et al (1964) and by us in Chapter Two. In their study, Walowit et al (1964) assumed the situation of co-rotating cylinders with different constant temperatures and with a gap width that corresponds to $\eta = \frac{1}{2}$ (i.e, $R_1 = \frac{1}{2} R_2$) only. In Chapter Two, we have examined the stability contribution of heating due to the presence of a constant heat flux at the inner cylinder. No restrictions have been made on the type of rotation and the radii ratio of the two cylinders. The critical conditions at the onset of instability were determined for the stationary outer cylinder, for both the co-rotating and the

counter-rotating cylinders and for a range of gap sizes that correspond to values of η between 0.1 and 0.95.

When considered the stabilizing effect of an axial magnetic field in the presence of various η , we could not find any study in this field except for the one given by Chang and Sartory (1965). In their study, they employed the finite difference techniques in order to determine the critical conditions for a wide range of radii ratio η . However, their investigation was limited to the case of perfectly conducting cylinders. Moreover, they

assumed that the basic velocity V is of the form $\frac{A}{r}$, where A is a constant. This form is only a special solution of the differential equation governing the basic velocity V .

In this Chapter, we examine the stability of a viscous incompressible Couette flow in an axially applied magnetic field. The flow has also a radial temperature variation sustained by keeping the cylinders at different constant temperatures. The equations governing the marginal stability state are solved by employing a numerical procedure similar to the one described by Harris and Reid (1964) for the non-magnetic isothermal flow. We determined the critical conditions for all possible combinations of the following:

- (i) perfectly conducting walls
- (ii) perfectly non-conducting walls
- (iii) flow under the influence of an axially applied magnetic field
- (iv) non-magnetic flow
- (v) isothermal flow
- (vi) flow with a radial temperature variation
- (vii) stationary outer cylinder
- (viii) co-rotating cylinders

- (ix) counter-rotating cylinders
- (x) radii ratios ranging from 0.1 to 0.95.

We, therefore, include only as special case, the role of different η on the stabilizing effect of the axially applied magnetic field, solved by Chang and Sartory (1965). However, we solve this problem for both conducting and non-conducting walls. Also, we make no restriction on the steady state velocity V . The disturbances are super-imposed on the general form of V .

In the narrow gap problem, the parameter η was eliminated from the characteristic equation and the critical conditions were given as a function of only μ , Q and N . The importance of retaining η in this wide gap problem emerges from the following considerations.

1. In some practical situations, the gap width between the cylinders is finite and thus the narrow gap assumption is not valid.
2. When the critical velocity, at which the Taylor vortices appear is to be modified by means of adjusting the gap size between the cylinders only.
3. To exhibit the stability interaction between η and the other influencing factors viz - velocities ratio, magnetic field and heating.

The variations of the critical Taylor number, T_C with various parameters, presented in tabular and graphical forms, indicate that η plays an important role in bringing about stability changes in the flow. The type and magnitude of these changes depend on the specific value of η . In the absence of rotation of the outer cylinder, heating and magnetic field, incrementing a large η by a small amount results in a great decrease in the value of the critical Taylor number T_C . Where incrementing $\eta = 0.5$ ($R_1 = \frac{1}{2} R_2$) by the same amount produces a relatively small effect on T_C . The influence of η on the other factors is manifested through the strong dependence of the stabilizing potentials of these factors on its value.

4.2 The Governing Equations

For an arbitrary value of η , the marginal stability state is governed by equations (91) - (93) and the boundary condition (94) - (97) (Chapter 3, Section 3.7 and 3.8). These equations have been derived by first super-imposing infinitesimal disturbances on the steady state solution of the velocity field, magnetic field and temperature. Then, substituting the new quantities in the general hydromagnetic equations, we obtained the equations that describe the perturbed state of the flow. These equations are simplified by (i) neglecting products and powers greater than one of the disturbances and (ii) analysing the disturbances into normal modes. Finally, the assumptions that the principle of exchange of instabilities is valid and the electrical conducting fluid between the cylinders is of very small magnetic Prandtl number P_m , reduced the governing equations to that given by (91) - (93) (Section 3.7).

The Taylor number T_a and the parameters N and Q were defined as a function of μ and η . However, in order to examine the stabilizing influence of various factors separately as well as simultaneously, we redefine the Taylor number, the magnetic field parameter and the temperature gradient parameter as follows:

$$\begin{aligned}\bar{T}_a &= \frac{\Omega_1^2 R_2^4}{\nu^2} \\ \bar{Q} &= \frac{\mu_e H^2 R_2^2}{4\pi\rho\nu e} \\ \bar{N} &= \text{Pr} \propto (T_2 - T_1) \quad .\end{aligned}\tag{1}$$

Hence for a given fluid and fixed outer cylinder radius R_2 , the critical Taylor number $\bar{T}_C$, determine, independently of μ and η , the angular velocity of Ω_{1C} at which the flow becomes unstable. This simplifies the analysis of the effect of various gap sizes and velocities ratios in the stability of

the flow. We should mention here that this definition of the Taylor number was used before Sparrow et al (1964) and Astill and Chang (1976) in their studies about the effect of finite gap in the stability of isothermal flow.

In terms of the new quantities (1), the governing equations (91) - (93) (Section 3.7) take the following form,

$$[(DD^* - a^2)^2 + \bar{Q} (1 - \eta)^2 a^2] u = a^2 \bar{T}_a \cdot 4A_1 (1 - \eta)^4 [G(x)(DD^* - a^2) g + \frac{1}{4A_1 \ln \eta} \bar{N} \bar{\epsilon} (G(x))^2 \theta] \quad (2)$$

$$[(DD^* - a^2)^2 + \bar{Q} (1 - \eta)^2 a^2] g = u \quad (3)$$

$$(DD^* - a^2) \theta = \frac{1}{\bar{\epsilon}} u \quad (4)$$

The boundary conditions are:

a. Conducting Walls

$$u = Du = D^*g = (DD^* - a^2) g = \theta = 0 \quad \text{at } x = 0, 1 \quad (5)$$

b. Non-Conducting Walls

$$u = Du = g = (DD^* - a^2) g = \theta = 0 \quad \text{at } x = 0, 1 \quad (6)$$

where the variables have the same meaning as given in Chapter Three.

The equations (2) - (4) and the boundary conditions (5) or (6) define an eigenvalue problem of the form

$$s(\bar{T}_a, a, \mu, \eta, \bar{N}, \bar{Q}) = 0 \quad (7)$$

For fixed values of μ , η , $\bar{N}$ and $\bar{Q}$, we determined the minimum positive $\bar{T}_a$ over all real positive a for which there exists a non-trivial solution of the eigenvalue problem. The critical value of $\bar{T}_a$ determined the velocity Ω_1 at the onset of instability, and the corresponding critical value of a determined the spacing of the vortices in the z -direction.

4.3 Method of Solution

To solve the eigenvalue problem (7), we adopt the numerical procedure described by Harris and Reid for isothermal non-magnetic Couette flow. The procedure starts by rewriting the governing equations as a system of first order equations. For that, we put

$$\begin{aligned}
 A &= u, & B &= Du, \\
 C &= (DD^* - a^2) u, & E &= D^*(DD^* - a^2) u, \\
 F &= g, & G &= D^*g, \\
 H &= (DD^* - a^2) g, & I &= D^*(DD^* - a^2) g, \\
 J &= \theta, & K &= D\theta
 \end{aligned} \tag{8}$$

In view of (8), we may easily prove the following relations:

$$(i) \quad C = DB + mB - (m^2 + a^2) A$$

$$(ii) \quad E = DC + mC$$

$$(iii) \quad (DD^* - a^2)^2 u = DE - a^2 C \quad (9)$$

$$(iv) \quad G = DF + mF$$

$$(v) \quad H = DG - a^2 F$$

$$(vi) \quad I = DH + mH$$

$$(vii) \quad (DD^* - a^2)^2 g = DI - a^2 H$$

$$(ix) \quad (D^*D - a^2) \theta = DK + mK - a^2 J ,$$

$$\text{where} \quad m = \frac{1 - \eta}{\xi} . \quad (10)$$

Equations (2) - (4) can now be transformed into a system of first order equations as follows:

$$DA = B$$

$$DB = C - mB + (m^2 + a^2) A$$

$$DC = E - mC$$

$$\begin{aligned} DE = a^2 C - \bar{Q} (1 - \eta)^2 a^2 A + a^2 \bar{T}_a \cdot 4A_1 (1 - \eta)^4 [G(x) \cdot H \\ + \frac{1}{4A_1 \ln \eta} \bar{N} \xi (G(x))^2 J] \end{aligned} \quad (11)$$

$$DF = G - mF$$

$$DG = H + a^2 F$$

$$DH = I - mH$$

$$DI = a^2 H - \bar{Q} (1 - \eta)^2 a^2 F + A$$

$$DJ = K$$

$$DK = -mK + a^2 J + \frac{1}{\xi} A$$

The boundary conditions for the case of conducting walls (the corresponding case of non-conducting walls is referred to in brackets) require that

$$[A = B = G (F) = H = J] = 0 \quad \text{at } x = 0, 1 \quad (12)$$

We now define five linearly independent solutions A_i , F_i and J_i , ($i = 1, 2, 3, 4, 5$) of equations (2) - (4) by imposing the initial conditions

$$[A_i = B_i = G_i (F_i) = H_i = J_i] = 0 \quad (i = 1, 2, 3, 4, 5) \quad (13)$$

and

$$[C_i, E_i, F_i (G_i), H_i] = \begin{cases} (1, 0, 0, 0, 0) & i=1 \\ (0, 1, 0, 0, 0) & i=2 \\ (0, 0, 1, 0, 0) & i=3 \\ (0, 0, 0, 1, 0) & i=4 \\ (0, 0, 0, 0, 1) & i=5 \end{cases} \quad (14)$$

at $x = 0$.

A solution of equations (2) - (4), which automatically satisfies the boundary conditions at $x = 0$ can be now written in the form

$$\begin{aligned} u &= \sum_{i=1}^5 M_i A_i \\ g &= \sum_{i=1}^5 N_i F_i \\ \theta &= \sum_{i=1}^5 P_i J_i \end{aligned} \tag{15}$$

where M_i , N_i and P_i are constants. The boundary conditions at $x = 1$ require that

$$\begin{aligned} \sum_{i=1}^5 M_i A_i &= 0 \\ \sum_{i=1}^5 M_i B_i &= 0 \\ \sum_{i=1}^5 N_i G_i (F_i) &= 0 \\ \sum_{i=1}^5 N_i H_i &= 0 \\ \sum_{i=1}^5 P_i J_i &= 0 \end{aligned} \tag{16}$$

For non-trivial solution, it is necessary that the determinant of the above system vanishes, i.e.,

$$s(\bar{T}_a, a, \mu, n, \bar{N}, \bar{Q}) = \begin{vmatrix} A_i \\ B_i \\ G_i (F_i) \\ H_i \\ J_i \end{vmatrix} = 0, \quad \text{at } x = 1 \quad (17)$$

Equation (17) is the characteristic equation from which the critical Taylor number $\bar{T}_C$ and the corresponding wave number can be obtained for any assigned values of μ , n , $\bar{Q}$ and $\bar{N}$. The solution procedures followed to integrate the fifty equations defined by (11) and to evaluate the determinant (17) are same as described before in Chapter Two, Section 2.7.

4.4 Discussion of the Results

Since for a given fluid and fixed R_2 , the values of $\bar{T}_C$, $\bar{Q}$ and $\bar{N}$ determine respectively the critical angular velocity Ω_{1C} the strength of the magnetic field and the quantity of heating, we find that the stability effect of various values of $\bar{Q}$, $\bar{N}$, μ and n are best measured through a comparison with the stability condition of a Couette flow with $\bar{Q} = 0$ (no axial magnetic field is present), $\bar{N} = 0$ (isothermal flow), $\mu = 0$ (stationary outer cylinder) and $n \sim 1$ (narrow gap between the cylinders). For such flow, it has been reported previously and in Chapter Three, that the critical Taylor number $T_C = 3390$. (If we take $n = 0.95$, then the corresponding value of $\bar{T}_C$ equals 1.517×10^7 and we will denote this number by $\bar{\bar{T}}_C$). The calculated changes on $\bar{\bar{T}}_C$ due to the action of various factors are presented in Tables 1-17 and in Figs. 1-38 for the case of conducting walls and in Tables 18-34 and Figs. 39-76 for the case of non-conducting walls (in these Tables and Figures, we have dropped the bars from $\bar{T}_C$, $\bar{Q}$ and $\bar{N}$). From a general inspection of the results presented, we may state that

for various values of η , the type of reaction of $\bar{T}_C$ to the presence of heating and axial magnetic field is the same as described before in Chapter Three for the narrow gap case (i.e., $\eta \sim 1$). However, the quantitative influences of $\bar{Q}$, $\bar{N}$ and μ are strongly dependent on the value of η .

If we give our attention first to the case of conducting walls, then a close examination of the corresponding Tables and Figures reveals the following.

1. In general, a fixed negative temperature gradient between the cylinders (i.e. temperature of the outer cylinder is less than that of the inner one) has a greater effect on the stability of the flow than the equivalent positive temperature gradient. This difference between the effects of negative $\bar{N}$ and positive $\bar{N}$ is more pronounced in the presence of (i) a large gap between the cylinders (small η), (ii) co-rotating cylinders with high velocities ratio μ and (iii) strong magnetic field. However, for negative μ and large η , equal magnitudes of $+\bar{N}$ and $-\bar{N}$ destabilize and stabilize the flow respectively by nearly the same amount. These features are demonstrated in Figs. 1-30. These figures also indicate that the effect of heating on $\bar{T}_C$ is enhanced when it is accompanied by the above three factors, (i), (ii) and (iii). As an example in Fig.6 and Table 4, for $\eta = 0.85$, $\mu = 0.0$ and $\bar{Q} = 100$, the presence of a temperature gradient that corresponds to $\bar{N} = -0.1$ induces a variation in the $\bar{T}_C$ of $\bar{N} = 0$ by about 0.3% more than that induced by $\bar{N} = 0.1$ whereas, as shown in Table 14, for $\eta = 0.2$, $\mu = 0.01$ and $\bar{Q} = 1000$, the $\bar{T}_C$ for $\bar{N} = 0$ varies by about 190% more when $\bar{N} = -0.1$ than when $\bar{N} = 0.1$. The dependence of the stability contribution of heating on the gap size between the cylinders can be realized if we note, for example, that as $\bar{N}$ is raised from 0 to 0.1 (when $\mu = 0$ and $\bar{Q} = 500$) the $\bar{T}_C$ suffers a reduction by about 4% when $\eta = 0.85$ (Table 4 and Fig.6) and by more than 35% when $\eta = 0.2$ (Table 13 and

Fig.24). This effect of heating on $\bar{T}_C$ increases with co-rotating cylinders ($\mu > 0$) and decreases with counter-rotating cylinders ($\mu < 0$).

2. In the absence of heating, magnetic field and rotation of the outer cylinder (i.e., $\bar{N} = \bar{Q} = \mu = 0$), the critical angular velocity Ω_{1C} can be controlled by adjusting the ratio of the radii of the two cylinders. It is found that the most stable state of the flow is achieved by having η close to one (i.e., narrow gap between the cylinders). However, as η then slightly decreases, the flow suffers a great reduction in its stability condition and this is reflected by the changes in $\bar{T}_C$. The $\bar{T}_C$ loses about 20% of its original value as η drops by 0.1 from $\eta = 0.95$ to $\eta = 0.85$. This fast rate of destabilization slows down as η gets smaller and when η equals about 0.5 (i.e. $R_1 = \frac{1}{2} R_2$) the flow reaches its minimum stable state. Any further decrease in η will then bring stability, rather than instability in the flow. For $\eta < 0.5$, the rate of increase of $\bar{T}_C$ becomes higher as η becomes smaller. When η drops from 0.5 to 0.35, the $\bar{T}_C$ rises by about 27%, whereas it rises by about seven times when η drops from 0.2 to 0.1. In Fig.35, the stability curves for μ between $\eta = 0.35$ and $\eta = 0.75$ show that the minimum value of $\bar{T}_C$, for $\bar{Q} = \bar{N} = 0$, occurs around $\eta = 0.5$, not only for $\eta = 0$ but also for some other values of $\mu > 0$ and $\mu < 0$.

The presence of the magnetic field affects the stability action of η in two ways.

- (a) It increases the stability potential of the fluid for all values of η .
- (b) It shifts the minimum value of $\bar{T}_C$ towards a value of η larger than 0.5, this shifting being greater, the greater the strength of the magnetic field.

The phenomenon (a) can be readily seen from Figs.36-38. These Figures also indicate that the degree of stabilization of the fluid with $\bar{Q}$ is not the same for different values of η , but it increases with lower values of η .

Inspection of Fig.6 and Fig.19 shows that for $\mu = 0$ and $\bar{N} = 0$, raising $\bar{Q}$ from 0 to 500 stabilizes the flow by about 60% for $\eta = 0.85$ and by more than twenty times for $\eta = 0.35$. As regards the dependence of minimum $\bar{T}_C$ on $\bar{Q}$, Figs.35-38 reveal that this minimum, which occurs about $\eta = 0.5$ when $\bar{Q} = 0$, moves to a value of η between 0.65 and 0.75 when $\bar{Q} = 500$ and to η between 0.7 and 0.8 when $\bar{Q} = 1000$.

3. The variation of $\bar{T}_C$ with μ shown in Figs.31-34 clearly suggests that, as compared with the stationary outer cylinder, driving the two cylinders in the same direction always leads to a more stable flow. The stability curves are seen to rise monotonically with increasing values of positive μ . Hence, with large $\mu > 0$, greater angular velocity Ω_1 is needed to bring about instability in the flow. On the other hand, for counter-rotating cylinders, the stabilization of the flow with larger magnitudes of μ occurs, provided that $|\mu| > |\mu_{\min}|$, where $\mu_{\min}$ denotes the value of μ at which the flow is in its lowest stable state. This $\mu_{\min}$, as can be deduced from the tabular results and Figs.31-34, is a function of η and $\bar{Q}$. For large η (narrow gap size), $\mu_{\min}$ occurs at about $\mu = 0$. But as η gradually decreases $\mu_{\min}$ slowly moves towards the negative values of μ . The parameter $\bar{Q}$ has a similar effect to that of η on $\mu_{\min}$. The departure of $\mu_{\min}$ from $\mu = 0$ is accelerated in the presence of a stronger magnetic field. This effect can be seen if we note, as an example, from Tables 24 - 26, that for $\bar{N} = 0$ and $\eta = 0.75$, the $\mu_{\min}$ is predicted at about $\mu = -0.1$ when $\bar{Q} = 0$ and at about $\mu = -0.25$ when $\bar{Q} = 100$.

Some other interesting and inter-connected relations between the flow stability and μ can be outlined as follows.

1. For any fixed $\bar{Q}$ and $\bar{N}$, the rate of stabilization of the flow with larger magnitudes of positive μ is greater than that of negative μ . Consequently, rotating the two cylinders in the same direction with a given

ratio μ of the angular velocities always contributes to a more stable flow than rotating them with the same magnitude of μ in the opposite direction. As can be deduced from Tables 7, 8 and 9, with $Q = 0$, $\bar{N} = 0$ and $\eta = 0.75$, the $\bar{T}_C$ for $\mu = 0$ increases by about 33% when $\mu = 0.25$ and by less than 7% when $\mu = -0.25$.

2. As the negative value of μ is increased in magnitude, the tendencies of $\bar{Q}$ and $\bar{N}$ to bring about changes in the stability of the flow are reduced considerably. In Tables 5 and 6, for $\eta = 0.85$ and $\bar{N} = 0.1$, raising $\bar{Q}$ from 0 to 1000 causes $\bar{T}_C$ to increase by about 130% when $\mu = -0.1$ and by less than 80% when $\mu = -0.75$.

We now turn to the situation when the cylindrical walls are made of non-conducting materials. The variations of $\bar{T}_C$ with η , μ , $\bar{Q}$ and $\bar{N}$, shown in Tables 18 - 34 and Figs. 9 - 18, indicate that the reaction of the flow stability to the presence of various factors is of the same nature as that described before for the case of conducting walls. The type of stability contribution of radii ratio, velocities ratio, magnetic field and temperature gradient, mentioned above in (1), (2) and (3) for conducting walls is also predicted for non-conducting walls. However, for any fixed values of η , μ and $\bar{N}$, the degree of stabilization of flow due to the presence of magnetic field of a given strength is smaller when the cylinders are electrically non-conducting than when they are perfectly conducting. For example, for $\eta = 0.5$, $\mu = 0.0$ and $\bar{N} = 0.1$, the effect of applying an axial magnetic field that corresponds to $\bar{Q} = 1000$ is to increase the critical angular velocity Ω_1 by about twenty five times when the walls are conducting and by less than fourteen times when they are non-conducting.

Finally, we discuss the effects of different values of η , μ , $\bar{Q}$ and $\bar{N}$ on the critical wave number a_C . From Tables 1 - 17, we may state the

following related conclusions for the case of conducting walls.

1. For fixed μ , $\bar{Q}$, η , the a_c exhibits a gradual increase or decrease with larger magnitudes of negative or positive temperature gradients respectively. The rate of variation of a_c with $\bar{N}$ is greater in the presence of (i) large $\bar{Q}$, (ii) positive μ and (iii) small η . Also, with (i), (ii) and (iii) it becomes more evident that a given positive temperature gradient has a smaller effect on the value a_c of the isothermal flow than the equivalent negative temperature gradient. In Table 7, the a_c value for $\eta = 0.75$, $\mu = 0.0$, $\bar{Q} = 1000$ and $\bar{N} = 0.0$ increases by about 3% when $\bar{N} = 0.1$ and by about 3.8% when $\bar{N} = -0.1$. Whereas, it is evident from Table 14 that the a_c value for $\eta = 0.2$, $\bar{Q} = 1000$ and $\bar{N} = 0.0$ increases by about (a) 53% when $\mu = 0.0$ and $\bar{N} = 0.1$, (b) 99% when $\mu = 0.0$ and $\bar{N} = -0.1$, (c) 56% when $\mu = 0.01$ and $\bar{N} = 0.1$ and (d) 176% when $\mu = 0.01$ and $\bar{N} = -0.1$.
2. For fixed η , μ and $\bar{N}$, the a_c slowly increases with higher values of $\bar{Q}$ provided that η is large. But as η gets smaller, the a_c decreases with $\bar{Q}$ when (a) $\bar{N} \geq 0$, (b) $\mu \geq 0$ or slightly negative and (c) $\bar{Q} > \bar{\bar{Q}}$, where $\bar{\bar{Q}}$ depends on η . For $\eta \approx 0.75$, $\bar{\bar{Q}} \approx 700$. But as η decreases, $\bar{\bar{Q}}$ moves toward smaller values. For $\eta \approx 0.5$, $\bar{\bar{Q}} \approx 300$ and for $\eta \approx 0.35$, $\bar{\bar{Q}} \approx 100$.
3. For fixed η , $\bar{Q}$ and $\bar{N}$, the increasing values of positive μ cause the a_c to decrease when $\bar{N}$ is positive and to increase when $\bar{N}$ is negative, provided that $\bar{Q} > \bar{\bar{Q}}$. This $\bar{\bar{Q}}$ is a function of η . For large η ($\eta \sim 0.85$) $\bar{\bar{Q}}$ is around 1000. But as η becomes smaller, $\bar{\bar{Q}}$ also becomes smaller. For $\eta \approx 0.35$, $\bar{\bar{Q}} \approx 100$. On the other hand, with increasing magnitudes of negative μ , the a_c value always increases except when η is small and $\bar{Q}$ is large. In that case, the a_c value increases when $\bar{N} \geq 0$ and decreases when $\bar{N} < 0$. In Table 15, as μ goes from -0.01 to -0.1 , the a_c value for $\eta = 0.2$, $\bar{Q} = 100$ increases by about 120% at $\bar{N} = 0.1$ and decreases by 23% at $\bar{N} = -0.1$.

When the cylindrical walls are electrically non-conducting, the a_c

values exhibit small and smooth variation with different values of η , μ , $\bar{Q}$ and $\bar{N}$. We outline the variation of a_c with each parameter as follows.

1. For fixed η , μ and $\bar{Q}$ increasing the magnitudes of positive or negative $\bar{N}$ causes the a_c (in contrast to the case of conducting walls) to increase or decrease respectively. The only exception to this occurs at large negative μ . Here, the opposite phenomenon is predicted. For examples, when (i) $\eta = 0.85$ and $\mu = -0.75$ (Table 23), (ii) $\eta = 0.5$ and $\mu = -0.5$ (Table 28) and (iii) $\eta = 0.35$ and $\mu = -0.5$ (Table 30), the a_c value decreases or increases according as positive $\bar{N}$ or negative $\bar{N}$ increases in magnitude.
2. For fixed η , μ and $\bar{N}$, the a_c continuously decreases as the axial magnetic field gets stronger. The rate of this decrease is accelerated as η gets smaller.
3. For fixed η , $\bar{Q}$, and $\bar{N}$, driving the two cylinders with larger magnitudes of μ causes the a_c to increase when (a) $\mu < 0$ and (b) $\mu > 0$ provided that $\bar{Q} \geq 300$, $\bar{N} > 0$ and $\eta \leq 0.5$. The a_c decreases under all other situations.

4.5 Conclusions

If the contribution of each factor to the stability of the Couette flow is measured according to its effect on $\widetilde{T}_C$ (i.e. $\bar{T}_C$ for $\mu = 0$, $\bar{Q} = 0$, $\bar{N} = 0$ and $\eta \sim 1$), then we find the following statements to be true.

1. Applying a magnetic field in the axial direction always leads to a more stable flow. The greater the strength of the magnetic field, the greater is the angular velocity Ω_1 needed to bring about instability in the flow.
2. The presence of a positive temperature gradient between the cylinders (temperature of the outer cylinder greater than that of the inner one) destabilizes the flow, while the negative temperature gradient stabilizes it.
3. The stability of a flow is continuously enhanced when the two cylinders

are rotated in the same or the opposite direction with an increasing magnitude of μ . However, in the case of counter-rotating cylinders, the rate of the stabilization is slower and it starts after the flow attains its maximum stable state. This minimum point is a function of η and $\bar{Q}$. For large η ($\eta \sim 1$) it occurs at about $\mu = 0$. But as η becomes smaller and $\bar{Q}$ becomes larger, the minimum point moves gradually toward the negative values of μ .

4. In relation to the effect of various gap sizes between the cylinders on the critical Taylor number $\bar{T}_C$, we have established that the maximum and the minimum $\bar{T}_C$ are achieved when $\eta \sim 1$ (narrow gap size) and $\eta \sim \frac{1}{2}$ ($R_1 \sim \frac{1}{2} R_2$) respectively. The $\bar{T}_C$ decreases at decelerating rate from $\eta \sim 1$ to $\eta \sim \frac{1}{2}$. It then increases at an accelerating rate with smaller values of η .

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TABLE 1: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.95	0.0	0	0.1	3.128	1.475 (7)*	0.95	0.1	0	0.1	3.125	1.494 (7)
			0.05	3.128	1.495 (7)				0.05	3.124	1.519 (7)
			0.0	3.128	1.517 (7)				0.0	3.124	1.545 (7)
			-0.05	3.127	1.539 (7)				-0.05	3.124	1.572 (7)
			-0.1	3.127	1.561 (7)				-0.1	3.122	1.600 (7)
		100	0.1	3.135	1.494 (7)			100	0.1	3.132	1.513 (7)
			0.05	3.135	1.515 (7)				0.05	3.132	1.539 (7)
			0.0	3.135	1.537 (7)				0.0	3.132	1.566 (7)
			-0.05	3.135	1.560 (7)				-0.05	3.131	1.593 (7)
			-0.1	3.135	1.583 (7)				-0.1	3.131	1.622 (7)
	0.0	300	0.1	3.149	1.533 (7)		0.0	300	0.1	3.146	1.553 (7)
			0.05	3.149	1.555 (7)				0.05	3.146	1.580 (7)
			0.0	3.150	1.578 (7)				0.0	3.146	1.607 (7)
			-0.05	3.150	1.602 (7)				-0.05	3.146	1.636 (7)
			-0.1	3.150	1.626 (7)				-0.1	3.146	1.666 (7)
		500	0.1	3.163	1.572 (7)			500	0.1	3.158	1.592 (7)
			0.05	3.163	1.595 (7)				0.05	3.159	1.620 (7)
			0.0	3.163	1.619 (7)				0.0	3.159	1.649 (7)
			-0.05	3.164	1.644 (7)				-0.05	3.160	1.680 (7)
			-0.1	3.164	1.669 (7)				-0.1	3.161	1.711 (7)
	0.0	700	0.1	3.175	1.611 (7)		0.0	700	0.1	3.171	1.632 (7)
			0.05	3.176	1.635 (7)				0.05	3.171	1.661 (7)
			0.0	3.177	1.660 (7)				0.0	3.173	1.692 (7)
			-0.05	3.178	1.686 (7)				-0.05	3.174	1.725 (7)
			-0.1	3.178	1.713 (7)				-0.1	3.175	1.756 (7)
		1000	0.1	3.193	1.670 (7)			1000	0.1	3.188	1.692 (7)
			0.05	3.194	1.696 (7)				0.05	3.190	1.723 (7)
			0.0	3.195	1.723 (7)				0.0	3.191	1.755 (7)
			-0.05	3.197	1.750 (7)				-0.05	3.193	1.789 (7)
			-0.1	3.198	1.779 (7)				-0.1	3.195	1.824 (7)

* Numbers in the parenthesis denote powers of 10

TABLE 2: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.95	0.5	0	0.1	3.118	2.049 (7)*	0.95	-0.1	0	0.1	3.134	1.487 (7)
			0.05	3.117	2.136 (7)				0.05	3.133	1.505 (7)
			0.0	3.117	2.231 (7)				0.0	3.133	1.523 (7)
			-0.05	3.117	2.334 (7)				-0.05	3.133	1.542 (7)
			-0.1	3.117	2.447 (7)				-0.1	3.132	1.561 (7)
		100	0.1	3.125	2.075 (7)			100	0.1	3.141	1.506 (7)
			0.05	3.125	2.164 (7)				0.05	3.141	1.525 (7)
			0.0	3.125	2.261 (7)				0.0	3.141	1.543 (7)
			-0.05	3.126	2.367 (7)				-0.05	3.141	1.562 (7)
			-0.1	3.126	2.483 (7)				-0.1	3.140	1.582 (7)
		300	0.1	3.137	2.127 (7)			300	0.1	3.155	1.546 (7)
			0.05	3.138	2.220 (7)				0.05	3.155	1.565 (7)
			0.0	3.140	2.321 (7)				0.0	3.155	1.584 (7)
			-0.05	3.141	2.433 (7)				-0.05	3.155	1.604 (7)
			-0.1	3.142	2.555 (7)				-0.1	3.155	1.625 (7)
		500	0.1	3.149	2.179 (7)			500	0.1	3.169	1.585 (7)
			0.05	3.151	2.276 (7)				0.05	3.169	1.605 (7)
			0.0	3.153	2.382 (7)				0.0	3.169	1.625 (7)
			-0.05	3.155	2.499 (7)				-0.05	3.170	1.646 (7)
			-0.1	3.158	2.627 (7)				-0.1	3.170	1.668 (7)
		700	0.1	3.160	2.231 (7)			700	0.1	3.182	1.624 (7)
			0.05	3.163	2.332 (7)				0.05	3.182	1.645 (7)
			0.0	3.166	2.444 (7)				0.0	3.183	1.689 (7)
			-0.05	3.169	2.566 (7)				-0.05	3.183	1.689 (7)
			-0.1	3.173	2.700 (7)				-0.1	3.184	1.711 (7)
		1000	0.1	3.176	2.310 (7)			1000	0.1	3.200	1.684 (7)
			0.05	3.180	2.418 (7)				0.05	3.201	1.706 (7)
			0.0	3.184	2.536 (7)				0.0	3.202	1.729 (7)
			-0.05	3.189	2.667 (7)				-0.05	3.203	1.753 (7)
			-0.1	3.194	2.811 (7)				-0.1	3.203	1.777 (7)

* numbers in the parenthesis denote powers of 10

TABLE 3: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.95	-0.5	0	0.1	3.216	1.931 (7)*		0.95	-0.75	0	0.1	3.498	2.850 (7)
			0.05	3.216	1.946 (7)					0.05	3.501	2.874 (7)
			0.0	3.216	1.962 (7)					0.0	3.501	2.898 (7)
			-0.05	3.216	1.978 (7)					-0.05	3.504	2.923 (7)
			-0.1	3.216	1.995 (7)					-0.1	3.509	2.949 (7)
		100	0.1	3.225	1.954 (7)				100	0.1	3.507	2.879 (7)
			0.05	3.224	1.970 (7)					0.05	3.509	2.903 (7)
			0.0	3.224	1.987 (7)					0.0	3.512	2.928 (7)
			-0.05	3.224	2.003 (7)					-0.05	3.515	2.953 (7)
			-0.1	3.224	2.020 (7)					-0.1	3.517	2.979 (7)
		300	0.1	3.240	2.003 (7)				300	0.1	3.523	2.937 (7)
			0.05	3.240	2.019 (7)					0.05	3.526	2.962 (7)
			0.0	3.241	2.036 (7)					0.0	3.530	2.988 (7)
			-0.05	3.241	2.053 (7)					-0.05	3.533	3.014 (7)
			-0.1	3.241	2.071 (7)					-0.1	3.536	3.040 (7)
		500	0.1	3.256	2.051 (7)				500	0.1	3.539	2.995 (7)
			0.05	3.256	2.068 (7)					0.05	3.558	3.021 (7)
			0.0	3.256	2.086 (7)					0.0	3.562	3.047 (7)
			-0.05	3.257	2.104 (7)					-0.05	3.566	3.074 (7)
			-0.1	3.257	2.122 (7)					-0.1	3.569	3.101 (7)
		700	0.1	3.270	2.099 (7)				700	0.1	3.555	3.053 (7)
			0.05	3.271	2.117 (7)					0.05	3.558	3.080 (7)
			0.0	3.271	2.135 (7)					0.0	3.562	3.107 (7)
			-0.05	3.272	2.154 (7)					-0.05	3.566	3.135 (7)
			-0.1	3.272	2.173 (7)					-0.1	3.569	3.163 (7)
		1000	0.1	3.291	2.172 (7)				1000	0.1	3.577	3.140 (7)
			0.05	3.292	2.191 (7)					0.05	3.581	3.168 (7)
			0.0	3.292	2.221 (7)					0.0	3.585	3.196 (7)
			-0.05	3.293	2.230 (7)					-0.05	3.589	3.225 (7)
			-0.1	3.294	2.251 (7)					-0.1	3.593	3.255 (7)

* numbers in the parenthesis denote powers of 10

TABLE 4: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.85	0.0	0	0.1	3.132	7.007	(5)*	0.85	0.1	0	0.1	3.127	7.205 (5)
			0.05	3.131	7.111	(5)				0.05	3.126	7.337 (5)
			0.0	3.131	7.217	(5)				0.0	3.126	7.474 (5)
			-0.05	3.130	7.326	(5)				-0.05	3.126	7.616 (5)
			-0.1	3.129	7.439	(5)				-0.1	3.125	7.764 (5)
		100	0.1	3.190	7.841	(5)			100	0.1	3.185	8.060 (5)
			0.05	3.192	7.967	(5)				0.05	3.186	8.221 (5)
			0.0	3.193	8.097	(5)				0.0	3.188	8.389 (5)
			-0.05	3.194	8.232	(5)				-0.05	3.189	8.564 (5)
			-0.1	3.195	8.371	(5)				-0.1	3.191	8.746 (5)
		300	0.1	3.277	9.565	(5)			300	0.1	3.268	9.828 (5)
			0.05	3.280	9.744	(5)				0.05	3.273	1.006 (6)
			0.0	3.284	9.930	(5)				0.0	3.278	1.029 (6)
			-0.05	3.288	1.012	(6)				-0.05	3.283	1.054 (6)
			-0.1	3.292	1.032	(6)				-0.1	3.288	1.081 (6)
		500	0.1	3.334	1.137	(6)			500	0.1	3.324	1.168 (6)
			0.05	3.342	1.162	(6)				0.05	3.332	1.199 (6)
			0.0	3.348	1.187	(6)				0.0	3.341	1.231 (6)
			-0.05	3.356	1.213	(6)				-0.05	3.350	1.265 (6)
			-0.1	3.363	1.240	(6)				-0.1	3.360	1.301 (6)
		700	0.1	3.376	1.327	(6)			700	0.1	3.362	1.362 (6)
			0.05	3.385	1.358	(6)				0.05	3.374	1.402 (6)
			0.0	3.395	1.391	(6)				0.0	3.386	1.444 (6)
			-0.05	3.405	1.425	(6)				-0.05	3.400	1.489 (6)
			-0.1	3.417	1.461	(6)				-0.1	3.414	1.536 (6)
		1000	0.1	3.415	1.628	(6)			1000	0.1	3.397	1.670 (6)
			0.05	3.429	1.672	(6)				0.05	3.414	1.726 (6)
			0.0	3.444	1.719	(6)				0.0	3.433	1.786 (6)
			-0.05	3.460	1.768	(6)				-0.05	3.454	1.850 (6)
			-0.1	3.477	1.820	(6)				-0.1	3.476	1.919 (6)

* numbers in the parenthesis denote powers of 10

TABLE 5: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.85	0.5	0	0.1	3.119	1.278 (6)*		0.85	-0.1	0	0.1	3.138	7.029 (5)
			0.05	3.119	1.361 (6)					0.05	3.138	7.115 (5)
			0.0	3.119	1.457 (6)					0.0	3.138	7.202 (5)
			-0.05	3.119	1.567 (6)					-0.05	3.137	7.292 (5)
			-0.1	3.119	1.695 (6)					-0.1	3.137	7.384 (5)
		100	0.1	3.168	1.418 (6)				100	0.1	3.199	7.864 (5)
			0.05	3.173	1.519 (6)					0.05	3.200	7.969 (5)
			0.0	3.179	1.636 (6)					0.0	3.201	8.076 (5)
			-0.05	3.187	1.772 (6)					-0.05	3.201	8.186 (5)
			-0.1	3.195	1.933 (6)					-0.1	3.202	8.299 (5)
		300	0.1	3.233	1.705 (6)				300	0.1	3.287	9.592 (5)
			0.05	3.249	1.845 (6)					0.05	3.291	9.740 (5)
			0.0	3.268	2.010 (6)					0.0	3.293	9.892 (5)
			-0.05	3.290	2.206 (6)					-0.05	3.297	1.005 (6)
			-0.1	3.318	2.445 (6)					-0.1	3.300	1.021 (6)
		500	0.1	3.270	2.000 (6)				500	0.1	3.349	1.140 (6)
			0.05	3.297	2.184 (6)					0.05	3.354	1.160 (6)
			0.0	3.329	2.406 (6)					0.0	3.360	1.181 (6)
			-0.05	3.369	2.676 (6)					-0.05	3.365	1.202 (6)
			-0.1	3.418	3.013 (6)					-0.1	3.371	1.224 (6)
		700	0.1	3.289	2.303 (6)				700	0.1	3.392	1.330 (6)
			0.05	3.326	2.538 (6)					0.05	3.400	1.356 (6)
			0.0	3.372	2.825 (6)					0.0	3.408	1.383 (6)
			-0.05	3.430	3.183 (6)					-0.05	3.417	1.411 (6)
			-0.1	3.505	3.640 (6)					-0.1	3.425	1.440 (6)
		1000	0.1	3.295	2.773 (6)				1000	0.1	3.436	1.632 (6)
			0.05	3.348	3.096 (6)					0.05	3.448	1.668 (6)
			0.0	3.416	3.499 (6)					0.0	3.460	1.706 (6)
			-0.05	3.504	4.018 (6)					-0.05	3.473	1.746 (6)
			-0.1	3.623	4.703 (6)					-0.1	3.487	1.787 (6)

* numbers in the parenthesis denote powers of 10

TABLE 6: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.85	-0.5	0	0.1	3.275	9.877 (5)*		0.85	-0.75	0	0.1	3.819	1.633 (6)
			0.05	3.276	9.958 (5)					0.05	3.824	1.647 (6)
			0.0	3.276	1.004 (6)					0.0	3.829	1.661 (6)
			-0.05	3.276	1.013 (6)					-0.05	3.834	1.676 (6)
			-0.1	3.276	1.021 (6)					-0.1	3.840	1.690 (6)
		100	0.1	3.347	1.094 (6)				100	0.1	3.877	1.755 (6)
			0.05	3.348	1.104 (6)					0.05	3.883	1.771 (6)
			0.0	3.350	1.114 (6)					0.0	3.889	1.787 (6)
			-0.05	3.351	1.124 (6)					-0.05	3.895	1.803 (6)
			-0.1	3.352	1.134 (6)					-0.1	3.901	1.819 (6)
		300	0.1	3.458	1.311 (6)				300	0.1	3.971	2.002 (6)
			0.05	3.461	1.324 (6)					0.05	3.979	2.021 (6)
			0.0	3.464	1.337 (6)					0.0	3.986	2.041 (6)
			-0.05	3.467	1.351 (6)					-0.05	3.994	2.060 (6)
			-0.1	3.470	1.365 (6)					-0.1	4.002	2.080 (6)
		500	0.1	3.540	1.533 (6)				500	0.1	4.044	2.253 (6)
			0.05	3.546	1.550 (6)					0.05	4.054	2.276 (6)
			0.0	3.550	1.568 (6)					0.0	4.063	2.299 (6)
			-0.05	3.555	1.586 (6)					-0.05	4.072	2.323 (6)
			-0.1	3.560	1.604 (6)					-0.1	4.081	2.347 (6)
		700	0.1	3.606	1.762 (6)				700	0.1	4.104	2.510 (6)
			0.05	3.613	1.784 (6)					0.05	4.114	2.537 (6)
			0.0	3.619	1.806 (6)					0.0	4.126	2.564 (6)
			-0.05	3.626	1.828 (6)					-0.05	4.137	2.592 (6)
			-0.1	3.633	1.851 (6)					-0.1	4.148	2.620 (6)
		1000	0.1	3.682	2.119 (6)				1000	0.1	4.177	2.906 (6)
			0.05	3.691	2.148 (6)					0.05	4.190	2.939 (6)
			0.0	3.701	2.163 (6)					0.0	4.203	2.973 (6)
			-0.05	3.711	2.208 (6)					-0.05	4.217	3.007 (6)
			-0.1	3.722	2.240 (6)					-0.1	4.230	3.043 (6)

* numbers in the parenthesis denote powers of 10

TABLE 7: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.75	0.0	0	0.1	3.136	2.029	(5)*	0.75	0.1	0	0.1	3.131	2.139 (5)
			0.05	3.136	2.060	(5)				0.05	3.130	2.182 (5)
			0.0	3.136	2.093	(5)				0.0	3.130	2.228 (5)
			-0.05	3.136	2.126	(5)				-0.05	3.129	2.275 (5)
			-0.1	3.136	2.161	(5)				-0.1	3.129	2.325 (5)
		100	0.1	3.274	2.709	(5)			100	0.1	3.263	2.852 (5)
			0.05	3.277	2.762	(5)				0.05	3.268	2.925 (5)
			0.0	3.281	2.816	(5)				0.0	3.273	3.001 (5)
			-0.05	3.284	2.873	(5)				-0.05	3.278	3.082 (5)
			-0.1	3.288	2.932	(5)				-0.1	3.284	3.167 (5)
		300	0.1	3.402	4.210	(5)			300	0.1	3.381	4.422 (5)
			0.05	3.414	4.321	(5)				0.05	3.397	4.576 (5)
			0.0	3.427	4.438	(5)				0.0	3.414	4.740 (5)
			-0.05	3.440	4.562	(5)				-0.05	3.433	4.916 (5)
			-0.1	3.454	4.692	(5)				-0.1	3.453	5.105 (5)
		500	0.1	3.448	5.902	(5)			500	0.1	3.417	6.188 (5)
			0.05	3.470	6.097	(5)				0.05	3.445	6.457 (5)
			0.0	3.494	6.306	(5)				0.0	3.476	6.749 (5)
			-0.05	3.519	6.528	(5)				-0.05	3.510	7.067 (5)
			-0.1	3.546	6.766	(5)				-0.1	3.548	7.414 (5)
		700	0.1	3.456	7.775	(5)			700	0.1	3.413	8.137 (5)
			0.05	3.489	8.083	(5)				0.05	3.453	8.559 (5)
			0.0	3.525	8.416	(5)				0.0	3.501	9.025 (5)
			-0.05	3.563	8.774	(5)				-0.05	3.554	9.539 (5)
			-0.1	3.606	9.162	(5)				-0.1	3.613	1.011 (6)
		1000	0.1	3.428	1.089	(6)			1000	0.1	3.366	1.136 (6)
			0.05	3.478	1.143	(6)				0.05	3.429	1.210 (6)
			0.0	3.535	1.202	(6)				0.0	3.501	1.292 (6)
			-0.05	3.597	1.266	(6)				-0.05	3.586	1.385 (6)
			-0.1	3.668	1.337	(6)				-0.1	3.685	1.491 (6)

* numbers in the parenthesis denote powers of 10

TABLE 8: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.75	0.25	0	0.1	3.126	2.592 (5)*	0.75	-0.1	0	0.1	3.146	2.019 (5)
			0.05	3.125	2.680 (5)				0.05	3.146	2.044 (5)
			0.0	3.125	2.774 (5)				0.0	3.145	2.069 (5)
			-0.05	3.125	2.875 (5)				-0.05	3.144	2.095 (5)
			-0.1	3.124	2.984 (5)				-0.1	3.144	2.122 (5)
		100	0.1	3.250	3.438 (5)			100	0.1	3.288	2.695 (5)
			0.05	3.258	3.583 (5)				0.05	3.291	2.736 (5)
			0.0	3.266	3.741 (5)				0.0	3.294	2.779 (5)
			-0.05	3.276	3.913 (5)				-0.05	3.296	2.823 (5)
			-0.1	3.286	4.102 (5)				-0.1	3.299	2.868 (5)
	300	300	0.1	3.349	5.279 (5)		300	300	0.1	3.428	4.184 (5)
			0.05	3.375	5.581 (5)				0.05	3.437	4.272 (5)
			0.0	3.404	5.918 (5)				0.0	3.447	4.363 (5)
			-0.05	3.436	6.298 (5)				-0.05	3.457	4.458 (5)
			-0.1	3.474	6.727 (5)				-0.1	3.468	4.558 (5)
	500	500	0.1	3.365	7.320 (5)		500	500	0.1	3.486	5.861 (5)
			0.05	3.409	7.842 (5)				0.05	3.503	6.016 (5)
			0.0	3.461	8.440 (5)				0.0	3.552	6.177 (5)
			-0.05	3.522	9.130 (5)				-0.05	3.541	6.348 (5)
			-0.1	3.593	9.933 (5)				-0.1	3.562	6.527 (5)
	700	700	0.1	3.340	9.542 (5)		700	700	0.1	3.506	7.718 (5)
			0.05	3.404	1.035 (6)				0.05	3.533	7.961 (5)
			0.0	3.480	1.130 (6)				0.0	3.561	8.218 (5)
			-0.05	3.573	1.243 (6)				-0.05	3.592	8.491 (5)
			-0.1	3.687	1.377 (6)				-0.1	3.624	8.781 (5)
	1000	1000	0.1	3.263	1.316 (6)		1000	1000	0.1	3.499	1.081 (6)
			0.05	3.355	1.455 (6)				0.05	3.540	1.123 (6)
			0.0	3.472	1.622 (6)				0.0	3.586	1.168 (6)
			-0.05	3.622	1.828 (6)				-0.05	3.635	1.217 (6)
			-0.1	3.817	2.082 (6)				-0.1	3.689	1.269 (6)

* numbers in the parenthesis denote powers of 10

TABLE 9: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.75	-0.25	0	0.1	3.177	2.183	(5)*	0.75	-0.5	0	0.1	3.410	3.233 (5)
			0.05	3.176	2.204	(5)				0.05	3.411	3.262 (5)
			0.0	3.176	2.225	(5)				0.0	3.413	3.290 (5)
			-0.05	3.175	2.247	(5)				-0.05	3.414	3.319 (5)
			-0.1	3.175	2.269	(5)				-0.1	3.416	3.349 (5)
		100	0.1	3.330	2.900	(5)			100	0.1	3.593	4.120 (5)
			0.05	3.332	2.934	(5)				0.05	3.597	4.162 (5)
			0.0	3.334	2.969	(5)				0.0	3.601	4.204 (5)
			-0.05	3.336	3.005	(5)				-0.05	3.606	4.248 (5)
			-0.1	3.338	3.042	(5)				-0.1	3.610	4.292 (5)
		300	0.1	3.491	4.463	(5)			300	0.1	3.808	5.962 (5)
			0.05	3.499	4.534	(5)				0.05	3.817	6.038 (5)
			0.0	3.506	4.608	(5)				0.0	3.828	6.114 (5)
			-0.05	3.514	4.684	(5)				-0.05	3.838	6.193 (5)
			-0.1	3.522	4.762	(5)				-0.1	3.849	6.274 (5)
		500	0.1	3.573	6.205	(5)			500	0.1	3.940	7.932 (5)
			0.05	3.587	6.328	(5)				0.05	3.955	8.048 (5)
			0.0	3.602	6.456	(5)				0.0	3.971	8.168 (5)
			-0.05	3.617	6.589	(5)				-0.05	3.987	8.292 (5)
			-0.1	3.633	6.728	(5)				-0.1	4.003	8.418 (5)
		700	0.1	3.619	8.118	(5)			700	0.1	4.030	1.004 (6)
			0.05	3.641	8.309	(5)				0.05	4.052	1.020 (6)
			0.0	3.662	8.510	(5)				0.0	4.074	1.038 (6)
			-0.05	3.686	8.719	(5)				-0.05	4.096	1.055 (6)
			-0.1	3.711	8.938	(5)				-0.1	4.119	1.074 (6)
		1000	0.1	3.649	1.128	(6)			1000	0.1	4.126	1.344 (6)
			0.05	3.683	1.161	(6)				0.05	4.157	1.370 (6)
			0.0	3.720	1.195	(6)				0.0	4.190	1.398 (6)
			-0.05	3.758	1.232	(6)				-0.05	4.223	1.426 (6)
			-0.1	3.800	1.270	(6)				-0.1	4.257	1.455 (6)

* numbers in the parenthesis denote powers of 10

TABLE 10: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.5	0.0	0	0.1	3.164	7.168 (4)*		0.5	0.1	0	0.1	3.152	9.283 (4)
			0.05	3.163	7.301 (4)					0.05	3.151	9.441 (4)
			0.0	3.163	7.439 (4)					0.0	3.151	9.949 (4)
			-0.05	3.162	7.582 (4)					-0.05	3.150	1.013 (5)
			-0.1	3.161	7.731 (4)					-0.1	3.150	1.072 (5)
		100	0.1	3.463	1.740 (5)				100	0.1	3.397	2.208 (5)
			0.05	3.482	1.802 (5)					0.05	3.431	2.352 (5)
			0.0	3.502	1.867 (5)					0.0	3.469	2.515 (5)
			-0.05	3.524	1.938 (5)					-0.05	3.514	2.701 (5)
			-0.1	3.548	2.014 (5)					-0.1	3.566	2.915 (5)
		300	0.1	3.463	4.456 (5)				300	0.1	3.272	5.464 (5)
			0.05	3.535	4.748 (5)					0.05	3.382	6.116 (5)
			0.0	3.620	5.076 (5)					0.0	3.529	6.926 (5)
			-0.05	3.719	5.448 (5)					-0.05	3.726	7.945 (5)
			-0.1	3.834	5.869 (5)					-0.1	3.990	9.242 (5)
		500	0.1	3.304	7.813 (5)				500	0.1	3.016	9.269 (5)
			0.05	3.434	8.552 (5)					0.05	3.178	1.085 (6)
			0.0	3.605	9.423 (5)					0.0	3.441	1.298 (6)
			-0.05	3.833	1.046 (6)					-0.05	3.886	1.594 (6)
			-0.1	4.128	1.168 (6)					-0.1	4.586	1.996 (6)
		700	0.1	3.107	1.157 (6)				700	0.1	2.774	1.331 (6)
			0.05	3.284	1.299 (6)					0.05	2.951	1.619 (6)
			0.0	3.556	1.475 (6)					0.0	3.308	2.046 (6)
			-0.05	3.987	1.692 (6)					-0.05	4.161	2.693 (6)
			-0.1	4.605	1.956 (6)					-0.1	5.575	3.581 (6)
		1000	0.1	2.825	1.763 (6)				1000	0.1	2.492	1.950 (6)
			0.05	3.036	2.049 (6)					0.05	2.651	2.485 (6)
			0.0	3.473	2.431 (6)					0.0	3.091	3.393 (6)
			-0.05	5.002	3.212 (6)					-0.05	4.605	4.965 (6)
			-0.1	5.570	3.514 (6)					-0.1	6.205	1.536 (7)

* numbers in the parenthesis denote powers of 10

TABLE 11: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.5	-0.1	0	0.1	3.191	6.839 (4)*	0.5	-0.5	0	0.1	4.791	2.055 (5)
			0.05	3.191	6.922 (4)				0.05	4.796	2.072 (5)
			0.0	3.189	7.008 (4)				0.0	4.801	2.089 (5)
			-0.05	3.188	7.095 (4)				-0.05	4.806	2.106 (5)
			-0.1	3.188	7.185 (4)				-0.1	4.811	2.123 (5)
		100	0.1	3.546	1.650 (5)			100	0.1	5.053	3.189 (5)
			0.05	3.558	1.688 (5)				0.05	5.064	3.222 (5)
			0.0	3.571	1.728 (5)				0.0	5.077	3.256 (5)
			-0.05	3.585	1.770 (5)				-0.05	5.089	3.291 (5)
			-0.1	3.599	1.813 (5)				-0.1	5.101	3.326 (5)
		300	0.1	3.677	4.194 (5)			300	0.1	5.442	5.838 (5)
			0.05	3.731	4.731 (5)				0.05	5.468	5.921 (5)
			0.0	3.791	4.562 (5)				0.0	5.495	6.007 (5)
			-0.05	3.856	4.769 (5)				-0.05	5.522	6.094 (5)
			-0.1	3.929	4.994 (5)				-0.1	5.548	6.184 (5)
		500	0.1	3.648	7.360 (5)			500	0.1	5.718	8.906 (5)
			0.05	3.759	7.803 (5)				0.05	5.759	9.064 (5)
			0.0	3.891	8.294 (5)				0.0	5.804	9.227 (5)
			-0.05	4.044	8.839 (5)				-0.05	5.846	9.395 (5)
			-0.1	4.223	9.444 (5)				-0.1	5.889	9.567 (5)
		700	0.1	3.555	1.097 (6)			700	0.1	5.914	1.233 (6)
			0.05	3.732	1.182 (6)				0.05	5.975	1.259 (6)
			0.0	3.958	1.279 (6)				0.0	6.037	1.286 (6)
			-0.05	4.241	1.388 (6)				-0.05	6.100	1.314 (6)
			-0.1	4.577	1.511 (6)				-0.1	6.163	1.343 (6)
		1000	0.1	3.356	1.698 (6)			1000	0.1	6.112	1.806 (6)
			0.05	3.630	1.873 (6)				0.05	6.204	1.853 (6)
			0.0	4.034	2.077 (6)				0.0	6.299	1.901 (6)
			-0.05	4.576	2.313 (6)				-0.05	6.396	1.952 (6)
			-0.1	5.192	2.579 (6)				-0.1	6.497	2.005 (6)

* numbers in the parenthesis denote powers of 10

TABLE 12: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c	n	μ	Q	N	a_c	T_c
0.35	0.0	0	0.1	3.199	9.062 (4)*	0.35	0.05	0	0.1	3.186	1.230 (5)
			0.05	3.198	9.259 (4)				0.05	3.185	1.279 (5)
			0.0	3.198	9.464 (4)				0.0	3.184	1.331 (5)
			-0.05	3.197	9.679 (4)				-0.05	3.183	1.388 (5)
			-0.1	3.196	9.903 (4)				-0.1	3.183	1.450 (5)
		100	0.1	3.547	3.145 (5)			100	0.1	3.428	4.122 (5)
			0.05	3.586	3.311 (5)				0.05	3.493	4.511 (5)
			0.0	3.630	3.495 (5)				0.0	3.572	4.976 (5)
			-0.05	3.679	3.699 (5)				-0.05	3.668	5.537 (5)
			-0.1	3.734	3.926 (5)				-0.1	3.789	6.224 (5)
		300	0.1	3.472	9.364 (5)			300	0.1	3.100	1.157 (6)
			0.05	3.585	1.038 (6)				0.05	3.285	1.377 (6)
			0.0	3.799	1.161 (6)				0.0	3.603	1.687 (6)
			-0.05	4.090	1.311 (6)				-0.05	4.173	2.132 (6)
			-0.1	4.472	1.492 (6)				-0.1	5.049	2.761 (6)
		500	0.1	3.158	1.676 (6)			500	0.1	2.750	1.970 (6)
			0.05	3.425	1.943 (6)				0.05	2.964	2.501 (6)
			0.0	3.915	2.294 (6)				0.0	3.548	3.380 (6)
			-0.05	4.729	2.744 (6)				-0.05	5.247	4.799 (6)
			-0.1	5.699	3.295 (6)				-0.1	7.108	6.752 (6)
		700	0.1	2.895	2.461 (6)			700	0.1	2.496	2.781 (6)
			0.05	3.217	2.966 (6)				0.05	2.687	3.706 (6)
			0.0	4.052	3.683 (6)				0.0	3.549	5.486 (6)
			-0.05	5.462	4.616 (6)				-0.05	6.399	8.453 (6)
			-0.1	6.879	5.743 (6)				-0.1	8.865	1.246 (7)
		1000	0.1	2.587	3.661 (6)			1000	0.1	2.243	3.967 (7)
			0.05	2.909	4.624 (6)				0.05	2.386	5.557 (7)
			0.0	4.155	6.170 (6)				0.0	3.775	9.299 (7)
			-0.05	6.426	8.189 (6)				-0.05	7.840	1.568 (8)
			-0.1	8.454	1.060 (7)				-0.1	11.195	2.430 (8)

* numbers in the parenthesis denote powers of 10

TABLE 13: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.35	-0.05	0	0.1	3.225	8.138 (4)*		0.35	-0.5	0	0.1	6.596	4.160 (5)
			0.05	3.225	8.258 (4)					0.05	6.602	4.189 (5)
			0.0	3.224	8.381 (4)					0.0	6.607	4.218 (5)
			-0.05	3.223	8.507 (4)					-0.05	6.613	4.248 (5)
			-0.1	3.222	8.637 (4)					-0.1	6.619	4.278 (5)
		100	0.1	3.672	2.813 (5)				100	0.1	6.984	6.202 (5)
			0.05	3.699	2.913 (5)					0.05	6.998	6.256 (5)
			0.0	3.730	3.020 (5)					0.0	7.010	6.310 (5)
			-0.05	3.762	3.135 (5)					-0.05	7.023	6.366 (5)
			-0.1	3.797	3.258 (5)					-0.1	7.035	6.422 (5)
		300	0.1	3.783	8.397 (5)				300	0.1	7.542	1.086 (6)
			0.05	3.918	8.997 (5)					0.05	7.570	1.099 (6)
			0.0	4.080	9.674 (5)					0.0	7.595	1.111 (6)
			-0.05	4.273	1.044 (6)					-0.05	7.622	1.125 (6)
			-0.1	4.499	1.131 (6)					-0.1	7.648	1.138 (6)
		500	0.1	3.687	1.524 (6)				500	0.1	7.939	1.617 (6)
			0.05	3.966	1.682 (6)					0.05	7.980	1.640 (6)
			0.0	4.345	1.868 (6)					0.0	8.023	1.664 (6)
			-0.05	4.821	2.085 (6)					-0.05	8.064	1.688 (6)
			-0.1	5.365	2.335 (6)					-0.1	8.109	1.713 (6)
		700	0.1	3.498	2.280 (6)				700	0.1	8.238	2.205 (6)
			0.05	3.913	2.586 (6)					0.05	8.292	2.242 (6)
			0.0	4.549	2.957 (6)					0.0	8.356	2.281 (6)
			-0.05	5.351	3.397 (6)					-0.05	8.418	2.320 (6)
			-0.1	6.213	3.905 (6)					-0.1	8.482	2.360 (6)
		1000	0.1	3.179	3.483 (6)				1000	0.1	8.559	3.183 (6)
			0.05	3.721	4.100 (6)					0.05	8.647	3.249 (6)
			0.0	4.751	4.893 (6)					0.0	8.743	3.316 (6)
			-0.05	6.060	5.839 (6)					-0.05	8.838	3.386 (6)
			-0.1	7.387	6.928 (6)					-0.1	8.935	3.458 (6)

* numbers in the parenthesis denote powers of 10

TABLE 14: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.2	0.0	0	0.1	3.266	2.871	(5)*	0.2	0.01	0	0.1	3.254	3.441 (5)
			0.05	3.265	2.950	(5)				0.05	3.255	3.570 (5)
			0.0	3.263	3.035	(5)				0.0	3.253	3.709 (5)
			-0.05	3.262	3.123	(5)				-0.05	3.252	3.859 (5)
			-0.1	3.261	3.217	(5)				-0.1	3.251	4.021 (5)
		100	0.1	3.673	1.316	(6)			100	0.1	3.560	1.540 (6)
			0.05	3.747	1.425	(6)				0.05	3.653	1.712 (6)
			0.0	3.838	1.553	(6)				0.0	3.772	1.923 (6)
			-0.05	3.947	1.704	(6)				-0.05	3.925	2.185 (6)
			-0.1	4.077	1.881	(6)				-0.1	4.119	2.517 (6)
		300	0.1	3.595	4.036	(6)			300	0.1	3.288	4.548 (6)
			0.05	3.941	4.740	(6)				0.05	3.620	5.601 (6)
			0.0	4.514	5.682	(6)				0.0	4.326	7.178 (6)
			-0.05	5.330	6.938	(6)				-0.05	5.535	9.519 (6)
			-0.1	6.282	8.589	(6)				-0.1	6.915	1.288 (7)
		500	0.1	3.320	6.984	(6)			500	0.1	2.966	7.641 (6)
			0.05	3.871	8.711	(6)				0.05	3.393	1.010 (7)
			0.0	5.081	1.126	(7)				0.0	4.877	1.438 (7)
			-0.05	6.761	1.485	(7)				-0.05	7.296	2.116 (7)
			-0.1	8.517	1.974	(7)				-0.1	9.684	3.134 (7)
		700	0.1	3.048	9.925	(6)			700	0.1	2.725	1.063 (7)
			0.05	3.642	1.299	(7)				0.05	3.121	1.478 (7)
			0.0	5.451	1.797	(7)				0.0	5.236	2.306 (7)
			-0.05	8.037	2.522	(7)				-0.05	8.871	3.684 (7)
			-0.1	10.564	3.530	(7)				-0.1	12.168	5.809 (7)
		1000	0.1	2.745	1.422	(7)			1000	0.1	2.483	1.491 (7)
			0.05	3.261	1.962	(7)				0.05	2.784	2.182 (7)
			0.0	5.848	2.987	(7)				0.0	5.630	3.851 (7)
			-0.05	9.793	4.525	(7)				-0.05	11.028	6.784 (7)
			-0.1	11.626	6.704	(7)				-0.1	15.526	1.143 (8)

* numbers in the parenthesis denote powers of 10

TABLE 15: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.2	-0.01	0	0.1	3.279	2.553 (5)*		0.2	-0.1	0	0.1	4.237	3.353 (5)
			0.05	3.278	2.609 (5)					0.05	4.243	3.394 (5)
			0.0	3.277	2.668 (5)					0.0	4.250	3.436 (5)
			-0.05	3.276	2.729 (5)					-0.05	4.257	3.479 (5)
			-0.1	3.274	2.793 (5)					-0.1	4.263	3.523 (5)
		100	0.1	3.780	1.177 (6)				100	0.1	5.197	9.695 (5)
			0.05	3.844	1.255 (6)					0.05	5.228	9.907 (5)
			0.0	3.918	1.343 (6)					0.0	5.260	1.013 (6)
			-0.05	4.003	1.442 (6)					-0.05	5.292	1.036 (6)
			-0.1	4.100	1.556 (6)					-0.1	5.326	1.059 (6)
		300	0.1	3.908	3.668 (6)				300	0.1	6.077	2.544 (6)
			0.05	4.239	4.169 (6)					0.05	6.181	2.639 (6)
			0.0	4.701	4.796 (6)					0.0	6.292	2.741 (6)
			-0.05	5.294	5.580 (6)					-0.05	6.409	2.850 (6)
			-0.1	5.983	6.556 (6)					-0.1	6.532	2.966 (6)
		500	0.1	3.725	6.463 (6)				500	0.1	6.500	4.415 (6)
			0.05	4.313	7.720 (6)					0.05	6.712	4.640 (6)
			0.0	5.282	9.406 (6)					0.0	6.931	4.885 (6)
			-0.05	6.520	1.161 (7)					-0.05	7.168	5.152 (6)
			-0.1	7.860	1.445 (7)					-0.1	7.421	5.442 (6)
		700	0.1	3.447	9.334 (6)				700	0.1	6.718	6.510 (6)
			0.05	4.172	1.163 (7)					0.05	7.036	6.924 (6)
			0.0	5.664	1.493 (7)					0.0	7.386	7.381 (6)
			-0.05	7.603	1.937 (7)					-0.05	7.768	7.885 (6)
			-0.1	9.584	2.517 (7)					-0.1	8.182	8.440 (6)
		1000	0.1	3.083	1.361 (7)				1000	0.1	6.792	9.980 (6)
			0.05	3.819	1.788 (7)					0.05	7.299	1.079 (7)
			0.0	6.067	2.470 (7)					0.0	7.876	1.170 (7)
			-0.05	9.090	3.410 (7)					-0.05	8.512	1.273 (7)
			-0.1	11.966	4.654 (7)					-0.1	9.206	1.387 (7)

* numbers in the parenthesis denote powers of 10

TABLE 16: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.1	0.0	0	0.1	3.345	2.265 (6)*		0.1	0.001	0	0.1	3.340	2.431 (6)
			0.05	3.341	2.348 (6)					0.05	3.337	2.529 (6)
			0.0	3.340	2.437 (6)					0.0	3.335	2.636 (6)
			-0.05	3.338	2.532 (6)					-0.05	3.332	2.751 (6)
			-0.1	3.335	2.635 (6)					-0.1	3.332	2.875 (6)
		100	0.1	3.877	1.121 (7)				100	0.1	3.821	1.191 (7)
			0.05	3.991	1.257 (7)					0.05	3.940	1.351 (7)
			0.0	4.137	1.427 (7)					0.0	4.099	1.555 (7)
			-0.05	4.325	1.642 (7)					-0.05	4.312	1.819 (7)
			-0.1	4.565	1.917 (7)					-0.1	4.588	2.168 (7)
		300	0.1	4.170	3.179 (7)				300	0.1	4.017	3.331 (7)
			0.05	4.711	3.943 (7)					0.05	4.565	4.211 (7)
			0.0	5.613	5.094 (7)					0.0	5.559	5.596 (7)
			-0.05	6.889	6.874 (7)					-0.05	7.018	7.845 (7)
			-0.1	8.350	9.608 (7)					-0.1	8.643	1.144 (8)
		500	0.1	4.040	5.182 (7)				500	0.1	3.859	5.375 (7)
			0.05	4.858	6.865 (7)					0.05	4.647	7.281 (7)
			0.0	6.603	9.752 (7)					0.0	6.550	1.074 (8)
			-0.05	9.173	1.148 (8)					-0.05	9.463	1.717 (8)
			-0.1	11.763	2.323 (8)					-0.1	12.194	2.829 (8)
		700	0.1	3.820	7.082 (7)				700	0.1	3.656	7.293 (7)
			0.05	4.733	9.862 (7)					0.05	4.480	1.039 (8)
			0.0	7.329	1.519 (8)					0.0	7.276	1.676 (8)
			-0.05	11.281	2.534 (8)					-0.05	11.725	2.978 (8)
			-0.1	14.765	4.310 (8)					-0.1	15.131	5.294 (8)
		1000	0.1	3.537	9.756 (7)				1000	0.1	3.391	9.973 (7)
			0.05	4.383	1.435 (8)					0.05	4.120	1.499 (8)
			0.0	8.148	2.461 (8)					0.0	8.095	2.717 (8)
			-0.05	14.224	4.617 (8)					-0.05	14.874	5.498 (8)
			-0.1	18.443	8.454 (8)					-0.1	19.325	1.039 (9)

* numbers in the parenthesis denote powers of 10

TABLE 17: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c	n	μ	Q	N	a_c	T_c
0.1	-0.001	0	0.1	3.349	2.130 (6)*	0.1	-0.1	0	0.1	8.071	4.100 (6)
			0.05	3.347	2.201 (6)				0.05	8.080	4.140 (6)
			0.0	3.345	2.277 (6)				0.0	8.088	4.181 (6)
			-0.05	3.342	2.358 (6)				-0.05	8.096	4.223 (6)
			-0.1	3.340	2.444 (6)				-0.1	8.105	4.265 (6)
		100	0.1	3.932	1.062 (7)			100	0.1	8.605	6.657 (6)
			0.05	4.040	1.179 (7)				0.05	8.866	6.738 (6)
			0.0	4.175	1.324 (7)				0.0	8.887	6.825 (6)
			-0.05	4.346	1.502 (7)				-0.05	8.908	6.915 (6)
			-0.1	4.557	1.725 (7)				-0.1	8.929	7.007 (6)
		300	0.1	4.317	3.042 (7)			300	0.1	9.429	1.240 (7)
			0.05	4.843	3.711 (7)				0.05	9.994	1.259 (7)
			0.0	5.666	4.684 (7)				0.0	10.044	1.283 (7)
			-0.05	6.796	6.127 (7)				-0.05	10.095	1.307 (7)
			-0.1	8.106	8.270 (7)				-0.1	10.146	1.332 (7)
		500	0.1	4.221	5.002 (7)			500	0.1	10.646	1.874 (7)
			0.05	5.051	6.498 (7)				0.05	10.815	1.917 (7)
			0.0	6.656	8.942 (7)				0.0	10.901	1.962 (7)
			-0.05	8.945	1.300 (8)				-0.05	10.991	2.009 (7)
			-0.1	11.358	1.955 (8)				-0.1	11.083	2.058 (7)
		700	0.1	3.999	6.882 (7)			700	0.1	11.332	2.566 (7)
			0.05	4.971	9.385 (7)				0.05	11.458	2.637 (7)
			0.0	7.382	1.391 (8)				0.0	11.589	2.711 (7)
			-0.05	10.923	2.204 (8)				-0.05	11.725	2.789 (7)
			-0.1	14.290	3.587 (8)				-0.1	11.867	2.871 (7)
		1000	0.1	3.691	9.546 (7)			1000	0.1	12.013	3.694 (7)
			0.05	4.646	1.376 (8)				0.05	12.212	3.819 (7)
			0.0	8.201	2.250 (8)				0.0	12.422	3.951 (7)
			-0.05	13.685	3.970 (8)				-0.05	12.642	4.092 (7)
			-0.1	18.106	7.003 (8)				-0.1	12.873	4.241 (7)

* numbers in the parenthesis denote powers of 10

TABLE 18: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.95	0.0	0	0.1	3.128	1.475 (7)*		0.95	0.1	0	0.1	3.125	1.494 (7)
			0.05	3.128	1.495 (7)					0.05	3.124	1.519 (7)
			0.0	3.128	1.517 (7)					0.0	3.124	1.545 (7)
			-0.05	3.127	1.539 (7)					-0.05	3.124	1.572 (7)
			-0.1	3.127	1.561 (7)					-0.1	3.122	1.600 (7)
		100	0.1	3.125	1.488 (7)				100	0.1	3.121	1.507 (7)
			0.05	3.124	1.509 (7)					0.05	3.121	1.533 (7)
			0.0	3.124	1.531 (7)					0.0	3.121	1.559 (7)
			-0.05	3.124	1.553 (7)					-0.05	3.121	1.586 (7)
			-0.1	3.123	1.576 (7)					-0.1	3.120	1.615 (7)
		300	0.1	3.119	1.515 (7)				300	0.1	3.115	1.534 (7)
			0.05	3.119	1.536 (7)					0.05	3.115	1.560 (7)
			0.0	3.118	1.559 (7)					0.0	3.115	1.588 (7)
			-0.05	3.117	1.582 (7)					-0.05	3.114	1.616 (7)
			-0.1	3.117	1.605 (7)					-0.1	3.114	1.645 (7)
		500	0.1	3.113	1.541 (7)				500	0.1	3.109	1.561 (7)
			0.05	3.112	1.564 (7)					0.05	3.109	1.588 (7)
			0.0	3.112	1.587 (7)					0.0	3.108	1.616 (7)
			-0.05	3.112	1.611 (7)					-0.05	3.108	1.645 (7)
			-0.1	3.110	1.635 (7)					-0.1	3.108	1.676 (7)
		700	0.1	3.106	1.568 (7)				700	0.1	3.103	1.589 (7)
			0.05	3.105	1.592 (7)					0.05	3.102	1.617 (7)
			0.0	3.105	1.615 (7)					0.0	3.102	1.645 (7)
			-0.05	3.105	1.640 (7)					-0.05	3.101	1.675 (7)
			-0.1	3.105	1.665 (7)					-0.1	3.101	1.706 (7)
		1000	0.1	3.097	1.609 (7)				1000	0.1	3.092	1.630 (7)
			0.05	3.096	1.633 (7)					0.05	3.092	1.659 (7)
			0.0	3.096	1.658 (7)					0.0	3.091	1.689 (7)
			-0.05	3.096	1.684 (7)					-0.05	3.092	1.721 (7)
			-0.1	3.095	1.710 (7)					-0.1	3.091	1.753 (7)

* numbers in the parenthesis denote powers of 10

TABLE 19: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.95	0.5	0	0.1	3.118	2.049	(7)*	0.95	-0.1	0	0.1	3.134	1.487 (7)
			0.05	3.117	2.136	(7)				0.05	3.133	1.505 (7)
			0.0	3.117	2.231	(7)				0.0	3.133	1.523 (7)
			-0.05	3.117	2.334	(7)				-0.05	3.133	1.542 (7)
			-0.1	3.117	2.447	(7)				-0.1	3.132	1.561 (7)
		100	0.1	3.115	2.067	(7)			100	0.1	3.131	1.500 (7)
			0.05	3.114	2.155	(7)				0.05	3.130	1.518 (7)
			0.0	3.114	2.251	(7)				0.0	3.129	1.537 (7)
			-0.05	3.114	2.356	(7)				-0.05	3.129	1.556 (7)
			-0.1	3.114	2.472	(7)				-0.1	3.129	1.575 (7)
		300	0.1	3.109	2.103	(7)			300	0.1	3.125	1.527 (7)
			0.05	3.108	2.194	(7)				0.05	3.124	1.546 (7)
			0.0	3.108	2.293	(7)				0.0	3.123	1.565 (7)
			-0.05	3.108	2.401	(7)				-0.05	3.123	1.585 (7)
			-0.1	3.108	2.520	(7)				-0.1	3.123	1.605 (7)
		500	0.1	3.102	2.139	(7)			500	0.1	3.118	1.554 (7)
			0.05	3.102	2.232	(7)				0.05	3.118	1.574 (7)
			0.0	3.102	2.334	(7)				0.0	3.117	1.593 (7)
			-0.05	3.101	2.446	(7)				-0.05	3.117	1.613 (7)
			-0.1	3.101	2.569	(7)				-0.1	3.117	1.634 (7)
		700	0.1	3.097	2.175	(7)			700	0.1	3.113	1.582 (7)
			0.05	3.096	2.271	(7)				0.05	3.112	1.601 (7)
			0.0	3.095	2.376	(7)				0.0	3.111	1.622 (7)
			-0.05	3.095	2.492	(7)				-0.05	3.111	1.642 (7)
			-0.1	3.094	2.619	(7)				-0.1	3.111	1.664 (7)
		1000	0.1	3.086	2.230	(7)			1000	0.1	3.103	1.623 (7)
			0.05	3.085	2.330	(7)				0.05	3.103	1.643 (7)
			0.0	3.085	2.440	(7)				0.0	3.102	1.665 (7)
			-0.05	3.084	2.560	(7)				-0.05	3.101	1.686 (7)
			-0.1	3.084	2.694	(7)				-0.1	3.101	1.709 (7)

* numbers in the parenthesis denote powers of 10

TABLE 20: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.95	-0.5	0	0.1	3.216	1.931 (7)*		0.95	-0.75	0	0.1	3.498	2.805 (7)
			0.05	3.216	1.946 (7)					0.05	3.501	2.874 (7)
			0.0	3.216	1.962 (7)					0.0	3.501	2.898 (7)
			-0.05	3.216	1.978 (7)					-0.05	3.504	2.923 (7)
			-0.1	3.216	1.995 (7)					-0.1	3.509	2.949 (7)
		100	0.1	3.214	1.947 (7)				100	0.1	3.497	2.871 (7)
			0.05	3.214	1.963 (7)					0.05	3.500	2.895 (7)
			0.0	3.214	1.979 (7)					0.0	3.503	2.920 (7)
			-0.05	3.214	1.996 (7)					-0.05	3.506	2.945 (7)
			-0.1	3.214	2.012 (7)					-0.1	3.509	2.971 (7)
		300	0.1	3.210	1.981 (7)				300	0.1	3.497	2.914 (7)
			0.05	3.210	1.997 (7)					0.05	3.500	2.938 (7)
			0.0	3.210	2.014 (7)					0.0	3.503	2.964 (7)
			-0.05	3.210	2.031 (7)					-0.05	3.505	2.989 (7)
			-0.1	3.209	2.048 (7)					-0.1	3.509	3.016 (7)
		500	0.1	3.206	2.015 (7)				500	0.1	3.496	2.956 (7)
			0.05	3.205	2.031 (7)					0.05	3.499	2.982 (7)
			0.0	3.205	2.048 (7)					0.0	3.502	3.007 (7)
			-0.05	3.205	2.066 (7)					-0.05	3.505	3.034 (7)
			-0.1	3.205	2.083 (7)					-0.1	3.509	3.060 (7)
		700	0.1	3.201	2.049 (7)				700	0.1	3.495	2.999 (7)
			0.05	3.201	2.066 (7)					0.05	3.498	3.025 (7)
			0.0	3.200	2.083 (7)					0.0	3.502	3.051 (7)
			-0.05	3.200	2.101 (7)					-0.05	3.505	3.078 (7)
			-0.1	3.200	2.119 (7)					-0.1	3.508	3.105 (7)
		1000	0.1	3.194	2.100 (7)				1000	0.1	3.494	3.063 (7)
			0.05	3.194	2.118 (7)					0.05	3.497	3.090 (7)
			0.0	3.195	2.136 (7)					0.0	3.501	3.117 (7)
			-0.05	3.194	2.155 (7)					-0.05	3.504	3.145 (7)
			-0.1	3.194	2.174 (7)					-0.1	3.507	3.173 (7)

* numbers in the parenthesis denote powers of 10

TABLE 21: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- NON-CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c		η	μ	Q	N	a_c	T_c
0.85	0.0	0	0.1	3.132	7.007 (5)*		0.85	0.1	0	0.1	3.127	7.205 (5)
			0.05	3.131	7.111 (5)					0.05	3.126	7.337 (5)
			0.0	3.131	7.217 (5)					0.0	3.126	7.474 (5)
			-0.05	3.130	7.326 (5)					-0.05	3.126	2.616 (5)
			-0.1	3.129	7.439 (5)					-0.1	3.125	7.764 (5)
		100	0.1	3.103	7.581 (5)				100	0.1	3.098	7.793 (5)
			0.05	3.103	7.699 (5)					0.05	3.098	7.944 (5)
			0.0	3.103	7.821 (5)					0.0	3.097	8.101 (5)
			-0.05	3.102	7.946 (5)					-0.05	3.097	8.264 (5)
			-0.1	3.101	8.076 (5)					-0.1	3.097	8.434 (5)
		300	0.1	3.045	8.776 (5)				300	0.1	3.038	9.018 (5)
			0.05	3.044	8.927 (5)					0.05	3.038	9.211 (5)
			0.0	3.043	9.083 (5)					0.0	3.037	9.412 (5)
			-0.05	3.043	9.245 (5)					-0.05	3.037	9.622 (5)
			-0.1	3.042	9.412 (5)					-0.1	3.036	9.842 (5)
		500	0.1	2.983	1.003 (6)				500	0.1	2.976	1.030 (6)
			0.05	2.982	1.022 (6)					0.05	2.975	1.054 (6)
			0.0	2.981	1.042 (6)					0.0	2.974	1.080 (6)
			-0.05	2.980	1.062 (6)					-0.05	2.972	1.106 (6)
			-0.1	2.978	1.083 (6)					-0.1	2.971	1.134 (6)
		700	0.1	2.919	1.134 (6)				700	0.1	2.911	1.164 (6)
			0.05	2.918	1.157 (6)					0.05	2.909	1.194 (6)
			0.0	2.916	1.181 (6)					0.0	2.907	1.225 (6)
			-0.05	2.914	1.207 (6)					-0.05	2.905	1.258 (6)
			-0.1	2.911	1.233 (6)					-0.1	2.902	1.292 (6)
		1000	0.1	2.820	1.340 (6)				1000	0.1	2.812	1.375 (6)
			0.05	2.817	1.371 (6)					0.05	2.807	1.414 (6)
			0.0	2.813	1.403 (6)					0.0	2.803	1.455 (6)
			-0.05	2.809	1.436 (6)					-0.05	2.798	1.499 (6)
			-0.1	2.805	1.471 (6)					-0.1	2.793	1.545 (6)

* numbers in the parenthesis denote powers of 10

TABLE 22: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- NON-CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.85	0.5	0	0.1	3.119	1.278 (6)*	0.8	-0.1	0	0.1	3.138	7.029 (5)
			0.05	3.119	1.361 (6)				0.05	3.138	7.115 (5)
			0.0	3.119	1.457 (6)				0.0	3.138	7.202 (5)
			-0.05	3.119	1.567 (6)				-0.05	3.137	7.292 (5)
			-0.1	3.119	1.695 (6)				-0.1	3.137	7.384 (5)
		100	0.1	3.090	1.376 (6)			100	0.1	3.111	7.604 (5)
			0.05	3.090	1.471 (6)				0.05	3.111	7.702 (5)
			0.0	3.090	1.580 (6)				0.0	3.110	7.802 (5)
			-0.05	3.089	1.706 (6)				-0.05	3.110	7.905 (5)
			-0.1	3.089	1.855 (6)				-0.1	3.109	8.010 (5)
		300	0.1	3.029	1.577 (6)			300	0.1	3.054	8.802 (5)
			0.05	3.029	1.697 (6)				0.05	3.054	8.927 (5)
			0.0	3.028	1.836 (6)				0.0	3.053	9.056 (5)
			-0.05	3.027	2.000 (6)				-0.05	3.052	9.188 (5)
			-0.1	3.025	2.197 (6)				-0.1	3.052	9.324 (5)
		500	0.1	2.968	1.786 (6)			500	0.1	2.994	1.006 (6)
			0.05	2.965	1.934 (6)				0.05	2.993	1.022 (6)
			0.0	2.962	2.108 (6)				0.0	2.992	1.038 (6)
			-0.05	2.959	2.316 (6)				-0.05	2.991	1.054 (6)
			-0.1	2.954	2.570 (6)				-0.1	2.990	1.072 (6)
		700	0.1	2.905	2.001 (6)			700	0.1	2.932	1.137 (6)
			0.05	2.900	2.179 (6)				0.05	2.930	1.157 (6)
			0.0	2.894	2.393 (6)				0.0	2.929	1.177 (6)
			-0.05	2.887	2.652 (6)				-0.05	2.927	1.197 (6)
			-0.1	2.876	2.974 (6)				-0.1	2.925	1.218 (6)
		1000	0.1	2.810	2.334 (6)			1000	0.1	2.836	1.344 (6)
			0.05	2.800	2.564 (6)				0.05	2.832	1.370 (6)
			0.0	2.787	2.844 (6)				0.0	2.829	1.396 (6)
			-0.05	2.769	3.193 (6)				-0.05	2.826	1.423 (6)
			-0.1	2.745	3.639 (6)				-0.1	2.822	1.451 (6)

* numbers in the parenthesis denote powers of 10

TABLE 23: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c	n	μ	Q	N	a_c	T_c
0.85	-0.5	0	0.1	3.275	9.877 (5)*	0.85	-0.75	0	0.1	3.819	1.633 (6)
			0.05	3.276	9.958 (5)				0.05	3.824	1.647 (6)
			0.0	3.276	1.004 (6)				0.0	3.829	1.661 (6)
			-0.05	3.276	1.013 (6)				-0.05	3.834	1.676 (6)
			-0.1	3.276	1.021 (6)				-0.1	3.840	1.690 (6)
	100		0.1	3.261	1.063 (6)		100		0.1	3.818	1.725 (6)
			0.05	3.261	1.072 (6)				0.05	3.824	1.740 (6)
			0.0	3.261	1.082 (6)				0.0	3.829	1.756 (6)
			-0.05	3.262	1.091 (6)				-0.05	3.835	1.771 (6)
			-0.1	3.262	1.110 (6)				-0.1	3.841	1.787 (6)
	300		0.1	3.229	1.218 (6)		300		0.1	3.813	1.912 (6)
			0.05	3.230	1.230 (6)				0.05	3.820	1.930 (6)
			0.0	3.231	1.241 (6)				0.0	3.826	1.948 (6)
			-0.05	3.231	1.253 (6)				-0.05	3.832	1.966 (6)
			-0.1	3.232	1.266 (6)				-0.1	3.839	1.985 (6)
	500		0.1	3.196	1.379 (6)		500		0.1	3.804	2.103 (6)
			0.05	3.197	1.393 (6)				0.05	3.810	2.124 (6)
			0.0	3.198	1.408 (6)				0.0	3.818	2.144 (6)
			-0.05	3.199	1.423 (6)				-0.05	3.825	2.165 (6)
			-0.1	3.199	1.438 (6)				-0.1	3.832	2.187 (6)
	700		0.1	3.161	1.546 (6)		700		0.1	3.790	2.298 (6)
			0.05	3.162	1.563 (6)				0.05	3.798	2.321 (6)
			0.0	3.163	1.580 (6)				0.0	3.806	2.345 (6)
			-0.05	3.164	1.598 (6)				-0.05	3.814	2.369 (6)
			-0.1	3.165	1.616 (6)				-0.1	3.822	2.394 (6)
	1000		0.1	3.105	1.805 (6)		1000		0.1	3.765	2.597 (6)
			0.05	3.106	1.827 (6)				0.05	3.773	2.625 (6)
			0.0	3.108	1.850 (6)				0.0	3.783	2.654 (6)
			-0.05	3.109	1.873 (6)				-0.05	3.792	2.683 (6)
			-0.1	3.110	1.896 (6)				-0.1	3.801	2.712 (6)

* numbers in the parenthesis denote powers of 10

TABLE 24: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c	n	μ	Q	N	a_c	T_c
0.75	0.0	0	0.1	3.136	2.029 (5)*	0.75	0.1	0	0.1	3.131	2.139 (5)
			0.05	3.136	2.060 (5)				0.05	3.130	2.182 (5)
			0.0	3.135	2.093 (5)				0.0	3.130	2.228 (5)
			-0.05	3.135	2.126 (5)				-0.05	3.129	2.275 (5)
			-0.1	3.135	2.161 (5)				-0.1	3.129	2.325 (5)
		100	0.1	3.057	2.499 (5)			100	0.1	3.050	2.632 (5)
			0.05	3.056	2.544 (5)				0.05	3.049	2.694 (5)
			0.0	3.056	2.590 (5)				0.0	3.048	2.759 (5)
			-0.05	3.055	2.638 (5)				-0.05	3.047	2.827 (5)
			-0.1	3.054	2.688 (5)				-0.1	3.046	2.899 (5)
		300	0.1	2.883	3.535 (5)			300	0.1	2.873	3.716 (5)
			0.05	2.880	3.615 (5)				0.05	2.870	3.827 (5)
			0.0	2.877	3.698 (5)				0.0	2.866	3.944 (5)
			-0.05	2.874	3.786 (5)				-0.05	2.862	4.068 (5)
			-0.1	2.871	3.878 (5)				-0.1	2.856	4.201 (5)
		500	0.1	2.698	4.677 (5)			500	0.1	2.685	4.910 (5)
			0.05	2.691	4.804 (5)				0.05	2.677	5.084 (5)
			0.0	2.683	4.938 (5)				0.0	2.668	5.271 (5)
			-0.05	2.675	5.079 (5)				-0.05	2.657	5.472 (5)
			-0.1	2.666	5.228 (5)				-0.1	2.645	5.688 (5)
		700	0.1	2.512	5.903 (5)			700	0.1	2.501	6.186 (5)
			0.05	2.498	6.087 (5)				0.05	2.483	6.438 (5)
			0.0	2.483	6.282 (5)				0.0	2.464	6.711 (5)
			-0.05	2.467	6.490 (5)				-0.05	2.442	7.007 (5)
			-0.1	2.449	6.712 (5)				-0.1	2.417	7.330 (5)
		1000	0.1	2.251	7.852 (5)			1000	0.1	2.244	8.209 (5)
			0.05	2.225	8.137 (5)				0.05	2.211	8.598 (5)
			0.0	2.196	8.442 (5)				0.0	2.174	9.024 (5)
			-0.05	2.164	8.769 (5)				-0.05	2.132	9.490 (5)
			-0.1	2.128	9.119 (5)				-0.1	2.083	1.000 (6)

* numbers in the parenthesis denote powers of 10

TABLE 25: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.75	0.25	0	0.1	3.126	2.592	(5)*	0.75	-0.1	0	0.1	3.146	2.019 (5)
			0.05	3.125	2.680	(5)				0.05	3.146	2.044 (5)
			0.0	3.125	2.774	(5)				0.0	3.145	2.069 (5)
			-0.05	3.125	2.875	(5)				-0.05	3.144	2.095 (5)
			-0.1	3.124	2.984	(5)				-0.1	3.144	2.122 (5)
		100	0.1	3.044	3.178	(5)			100	0.1	3.070	2.486 (5)
			0.05	3.043	3.303	(5)				0.05	3.069	2.522 (5)
			0.0	3.042	3.437	(5)				0.0	3.068	2.558 (5)
			-0.05	3.041	3.583	(5)				-0.05	3.067	2.595 (5)
			-0.1	3.040	3.742	(5)				-0.1	3.066	2.634 (5)
	300	300	0.1	2.865	4.459	(5)		300	300	0.1	2.901	3.517 (5)
			0.05	2.861	4.677	(5)				0.05	2.899	3.580 (5)
			0.0	2.856	4.919	(5)				0.0	2.897	3.645 (5)
			-0.05	2.850	5.186	(5)				-0.05	2.894	3.713 (5)
			-0.1	2.844	5.484	(5)				-0.1	2.891	3.783 (5)
	500	500	0.1	2.682	5.835	(5)		500	500	0.1	2.721	4.654 (5)
			0.05	2.669	6.195	(5)				0.05	2.716	4.754 (5)
			0.0	2.655	6.579	(5)				0.0	2.709	4.859 (5)
			-0.05	2.637	7.013	(5)				-0.05	2.703	4.967 (5)
			-0.1	2.615	7.507	(5)				-0.1	2.696	5.081 (5)
	700	700	0.1	2.504	7.332	(5)		700	700	0.1	2.540	5.876 (5)
			0.05	2.479	7.822	(5)				0.05	2.528	6.021 (5)
			0.0	2.448	8.381	(5)				0.0	2.516	6.174 (5)
			-0.05	2.410	9.024	(5)				-0.05	2.503	6.334 (5)
			-0.1	2.363	9.768	(5)				-0.1	2.489	6.503 (5)
	1000	1000	0.1	2.261	9.657	(5)		1000	1000	0.1	2.280	7.823 (5)
			0.05	2.214	1.041	(6)				0.05	2.258	8.049 (5)
			0.0	2.155	1.128	(6)				0.0	2.235	8.287 (5)
			-0.05	2.082	1.229	(6)				-0.05	2.209	8.539 (5)
			-0.1	1.994	1.347	(6)				-0.1	2.181	8.806 (5)

* numbers in the parenthesis denote powers of 10

TABLE 26: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.75	-0.25	0	0.1	3.177	2.183	(5)*	0.75	-0.5	0	0.1	3.410	3.233 (5)
			0.05	3.176	2.204	(5)				0.05	3.411	3.263 (5)
			0.0	3.176	2.225	(5)				0.0	3.413	3.290 (5)
			-0.05	3.175	2.247	(5)				-0.05	3.414	3.319 (5)
			-0.1	3.175	2.269	(5)				-0.1	3.416	3.349 (5)
	100		0.1	3.110	2.682	(5)			100	0.1	3.390	3.876 (5)
			0.05	3.109	2.711	(5)				0.05	3.392	3.914 (5)
			0.0	3.108	2.741	(5)				0.0	3.395	3.952 (5)
			-0.05	3.107	2.772	(5)				-0.05	3.397	3.991 (5)
			-0.1	3.107	2.803	(5)				-0.1	3.400	4.031 (5)
	300		0.1	2.962	3.776	(5)			300	0.1	3.336	5.239 (5)
			0.05	2.959	3.828	(5)				0.05	3.340	5.300 (5)
			0.0	2.957	3.881	(5)				0.0	3.344	5.362 (5)
			-0.05	2.955	3.936	(5)				-0.05	3.348	5.425 (5)
			-0.1	2.953	3.993	(5)				-0.1	3.352	5.490 (5)
	500		0.1	2.800	4.979	(5)			500	0.1	3.267	6.691 (5)
			0.05	2.796	5.061	(5)				0.05	3.272	6.780 (5)
			0.0	2.792	5.146	(5)				0.0	3.277	6.871 (5)
			-0.05	2.787	5.233	(5)				-0.05	3.282	6.965 (5)
			-0.1	2.783	5.324	(5)				-0.1	3.288	7.061 (5)
	700		0.1	2.635	6.270	(5)			700	0.1	3.185	8.219 (5)
			0.05	2.628	6.389	(5)				0.05	3.191	8.342 (5)
			0.0	2.620	6.513	(5)				0.0	3.197	8.468 (5)
			-0.05	2.610	6.641	(5)				-0.05	3.204	8.599 (5)
			-0.1	2.601	6.774	(5)				-0.1	3.211	8.733 (5)
	1000		0.1	2.393	8.329	(5)			1000	0.1	3.047	1.063 (6)
			0.05	2.376	8.515	(5)				0.05	3.053	1.082 (6)
			0.0	2.359	8.708	(5)				0.0	3.058	1.101 (6)
			-0.05	2.340	8.910	(5)				-0.05	3.065	1.120 (6)
			-0.1	2.320	9.121	(5)				-0.1	3.072	1.141 (6)

* numbers in the parenthesis denote powers of 10

TABLE 27: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_C	T_C		n	μ	Q	N	a_C	T_C
0.5	0.0	0	0.1	3.164	7.168	(4)*	0.5	0.1	0	0.1	3.152	9.283 (4)
			0.05	3.163	7.301	(4)				0.05	3.151	9.441 (4)
			0.0	3.163	7.439	(4)				0.0	3.151	9.949 (4)
			-0.05	3.162	7.582	(4)				-0.05	3.150	1.013 (5)
			-0.1	3.161	7.731	(4)				-0.1	3.150	1.072 (5)
		100	0.1	2.828	1.419	(5)			100	0.1	2.807	1.814 (5)
			0.05	2.824	1.461	(5)				0.05	2.798	1.912 (5)
			0.0	2.817	1.505	(5)				0.0	2.789	2.022 (5)
			-0.05	2.811	1.552	(5)				-0.05	2.777	2.145 (5)
			-0.1	2.804	1.602	(5)				-0.1	2.764	2.284 (5)
		300	0.1	2.161	3.136	(5)			300	0.1	2.169	3.920 (5)
			0.05	2.121	3.280	(5)				0.05	2.106	4.250 (5)
			0.0	2.079	3.437	(5)				0.0	2.030	4.635 (5)
			-0.05	2.025	3.608	(5)				-0.05	1.937	5.086 (5)
			-0.1	1.970	3.793	(5)				-0.1	1.826	5.615 (5)
		500	0.1	1.714	5.011	(5)			500	0.1	1.761	6.182 (5)
			0.05	1.656	5.284	(5)				0.05	1.659	6.803 (5)
			0.0	1.591	5.580	(5)				0.0	1.548	7.526 (5)
			-0.05	1.524	5.899	(5)				-0.05	1.432	8.363 (5)
			-0.1	1.459	6.243	(5)				-0.1	1.321	9.327 (5)
		700	0.1	1.441	6.930	(5)			700	0.1	1.506	8.487 (5)
			0.05	1.379	7.336	(5)				0.05	1.392	9.413 (5)
			0.0	1.308	7.772	(5)				0.0	1.277	1.048 (6)
			-0.05	1.251	8.240	(5)				-0.05	1.173	1.170 (6)
			-0.1	1.192	8.740	(5)				-0.1	1.079	1.309 (6)
		1000	0.1	1.201	9.835	(5)			1000	0.1	1.267	1.197 (6)
			0.05	1.127	1.044	(6)				0.05	1.149	1.336 (6)
			0.0	1.068	1.109	(6)				0.0	1.046	1.494 (6)
			-0.05	1.014	1.177	(6)				-0.05	0.955	1.673 (6)
			-0.1	0.965	1.250	(6)				-0.1	0.877	1.875 (6)

* numbers in the parenthesis denote powers of 10

TABLE 28: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- NON-CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.5	-0.1	0	0.1	3.191	6.839 (4)*	0.5	-0.5	0	0.1	4.791	2.055 (5)
			0.05	3.191	6.922 (4)				0.05	4.796	2.072 (5)
			0.0	3.189	7.008 (4)				0.0	4.801	2.089 (5)
			-0.05	3.188	7.095 (4)				-0.05	4.806	2.106 (5)
			-0.1	3.188	7.185 (4)				-0.1	4.811	2.123 (5)
		100	0.1	2.886	1.350 (5)			100	0.1	4.665	2.897 (5)
			0.05	2.883	1.376 (5)				0.05	4.673	2.926 (5)
			0.0	2.879	1.403 (5)				0.0	4.681	2.955 (5)
			-0.05	2.874	1.431 (5)				-0.05	4.691	2.985 (5)
			-0.1	2.870	1.461 (5)				-0.1	4.699	3.015 (5)
	0	300	0.1	2.248	2.985 (5)			300	0.1	4.413	4.813 (5)
			0.05	2.220	3.076 (5)				0.05	4.429	4.876 (5)
			0.0	2.187	3.173 (5)				0.0	4.444	4.941 (5)
			-0.05	2.153	3.275 (5)				-0.05	4.459	5.008 (5)
			-0.1	2.118	3.383 (5)				-0.1	4.474	5.075 (5)
		500	0.1	1.781	4.787 (5)			500	0.1	4.152	6.958 (5)
			0.05	1.733	4.962 (5)				0.05	4.170	7.070 (5)
			0.0	1.682	5.147 (5)				0.0	4.189	7.186 (5)
			-0.05	1.630	5.342 (5)				-0.05	4.207	7.304 (5)
			-0.1	1.577	5.546 (5)				-0.1	4.225	7.426 (5)
	0	700	0.1	1.489	6.638 (5)			700	0.1	3.880	9.266 (5)
			0.05	1.436	6.900 (5)				0.05	3.898	9.441 (5)
			0.0	1.382	7.175 (5)				0.0	3.916	9.622 (5)
			-0.05	1.332	7.462 (5)				-0.05	3.933	9.809 (5)
			-0.1	1.285	7.761 (5)				-0.1	3.950	1.000 (6)
		1000	0.1	1.222	9.444 (5)			1000	0.1	3.481	1.294 (6)
			0.05	1.172	9.835 (5)				0.05	3.490	1.323 (6)
			0.0	1.125	1.024 (6)				0.0	3.497	1.353 (6)
			-0.05	1.081	1.067 (6)				-0.05	3.503	1.385 (6)
			-0.1	1.040	1.111 (6)				-0.1	3.508	1.418 (6)

* numbers in the parenthesis denote powers of 10

TABLE 29: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.35	0.0	0	0.1	3.199	9.062	(4)*	0.35	0.05	0	0.1	3.186	1.230 (5)
			0.05	3.198	9.259	(4)				0.05	3.185	1.279 (5)
			0.0	3.198	9.464	(4)				0.0	3.184	1.331 (5)
			-0.05	3.197	9.679	(4)				-0.05	3.183	1.388 (5)
			-0.1	3.197	9.903	(4)				-0.1	3.183	1.450 (5)
		100	0.1	2.638	2.417	(5)			100	0.1	2.618	3.206 (5)
			0.05	2.619	2.518	(5)				0.05	2.591	3.445 (5)
			0.0	2.602	2.627	(5)				0.0	2.558	3.721 (5)
			-0.05	2.578	2.745	(5)				-0.05	2.518	4.043 (5)
			-0.1	2.552	2.874	(5)				-0.1	2.468	4.424 (5)
		300	0.1	1.790	6.079	(5)			300	0.1	1.835	7.831 (5)
			0.05	1.711	6.167	(5)				0.05	1.721	8.723 (5)
			0.0	1.632	6.892	(5)				0.0	1.590	9.786 (5)
			-0.05	1.558	7.358	(5)				-0.05	1.454	1.164 (6)
			-0.1	1.471	7.866	(5)				-0.1	1.328	1.252 (6)
		500	0.1	1.385	9.921	(5)			500	0.1	1.468	1.262 (6)
			0.05	1.299	1.063	(6)				0.05	1.325	1.426 (6)
			0.0	1.221	1.139	(6)				0.0	1.191	1.617 (6)
			-0.05	1.156	1.222	(6)				-0.05	1.075	1.838 (6)
			-0.1	1.091	1.311	(6)				-0.1	0.977	2.094 (6)
		700	0.1	1.164	1.380	(6)			700	0.1	1.256	1.743 (6)
			0.05	1.083	1.482	(6)				0.05	1.110	1.983 (6)
			0.0	1.010	1.593	(6)				0.0	0.989	2.260 (6)
			-0.05	0.949	1.711	(6)				-0.05	0.888	2.576 (6)
			-0.1	0.894	1.838	(6)				-0.1	0.809	2.939 (6)
		1000	0.1	0.968	1.963	(6)			1000	0.1	1.064	2.467 (6)
			0.05	0.894	2.114	(6)				0.05	0.923	2.822 (6)
			0.0	0.832	2.275	(6)				0.0	0.815	3.226 (6)
			-0.05	0.781	2.446	(6)				-0.05	0.730	3.684 (6)
			-0.1	0.735	2.630	(6)				-0.1	0.663	4.207 (6)

* numbers in the parenthesis denote powers of 10

TABLE 30: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- NON-CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.35	-0.05	0	0.1	3.225	8.138 (4)*	0.35	-0.5	0	0.1	6.596	4.160 (5)
			0.05	3.225	8.258 (4)				0.05	6.602	4.189 (5)
			0.0	3.224	8.381 (4)				0.0	6.607	4.218 (5)
			-0.05	3.223	8.507 (4)				-0.05	6.613	4.248 (5)
			-0.1	3.222	8.637 (4)				-0.1	6.619	4.278 (5)
	100		0.1	2.706	2.168 (5)			100	0.1	6.468	5.666 (5)
			0.05	2.696	2.229 (5)				0.05	6.477	5.713 (5)
			0.0	2.682	2.294 (5)				0.0	6.487	5.760 (5)
			-0.05	2.668	2.362 (5)				-0.05	6.497	5.809 (5)
			-0.1	2.652	2.434 (5)				-0.1	6.502	5.858 (5)
	300		0.1	1.856	5.489 (5)			300	0.1	6.228	9.047 (5)
			0.05	1.795	5.986 (5)				0.05	6.243	9.143 (5)
			0.0	1.731	5.986 (5)				0.0	6.259	9.242 (5)
			-0.05	1.667	6.259 (5)				-0.05	6.274	9.342 (5)
			-0.1	1.604	6.549 (5)				-0.1	6.290	9.444 (5)
	500		0.1	1.416	8.998 (5)			500	0.1	5.967	1.280 (6)
			0.05	1.349	9.440 (5)				0.05	5.985	1.296 (6)
			0.0	1.284	9.905 (5)				0.0	6.003	1.313 (6)
			-0.05	1.233	1.039 (6)				-0.05	6.021	1.330 (6)
			-0.1	1.179	1.091 (6)				-0.1	6.038	1.348 (6)
	700		0.1	1.183	1.254 (6)			700	0.1	5.684	1.682 (6)
			0.05	1.127	1.319 (6)				0.05	5.701	1.707 (6)
			0.0	1.061	1.386 (6)				0.0	5.717	1.733 (6)
			-0.05	1.010	1.456 (6)				-0.05	5.734	1.759 (6)
			-0.1	0.964	1.529 (6)				-0.1	5.750	1.786 (6)
	1000		0.1	0.977	1.788 (6)			1000	0.1	5.246	2.323 (6)
			0.05	0.920	1.882 (6)				0.05	5.253	2.364 (6)
			0.0	0.870	1.980 (6)				0.0	5.260	2.407 (6)
			-0.05	0.827	2.082 (6)				-0.05	5.265	2.451 (6)
			-0.1	0.790	2.188 (6)				-0.1	5.270	2.497 (6)

* numbers in the parenthesis denote powers of 10

TABLE 31: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF η , μ , Q AND N
- NON-CONDUCTING WALLS -

η	μ	Q	N	a_c	T_c	η	μ	Q	N	a_c	T_c
0.2	0.0	0	0.1	3.266	2.871 (5)*	0.2	0.01	0	0.1	3.254	3.441 (5)
			0.05	3.265	2.950 (5)				0.05	3.255	3.570 (5)
			0.0	3.263	3.035 (5)				0.0	3.253	3.709 (5)
			-0.05	3.262	3.123 (5)				-0.05	3.252	3.859 (5)
			-0.1	3.261	3.217 (5)				-0.1	3.251	4.021 (5)
		100	0.1	2.497	9.694 (5)			100	0.1	2.480	1.144 (6)
			0.05	2.456	1.029 (6)				0.05	2.432	1.238 (6)
			0.0	2.402	1.096 (6)				0.0	2.368	1.348 (6)
			-0.05	2.342	1.171 (6)				-0.05	2.283	1.477 (6)
			-0.1	2.279	1.256 (6)				-0.1	2.185	1.630 (6)
		300	0.1	1.647	2.551 (6)			300	0.1	1.679	2.962 (6)
			0.05	1.515	2.783 (6)				0.05	1.515	3.323 (6)
			0.0	1.396	3.041 (6)				0.0	1.365	3.744 (6)
			-0.05	1.289	3.326 (6)				-0.05	1.236	4.228 (6)
			-0.1	1.198	3.641 (6)				-0.1	1.123	4.784 (6)
		500	0.1	1.294	4.168 (6)			500	0.1	1.353	4.807 (6)
			0.05	1.159	4.586 (6)				0.05	1.170	5.462 (6)
			0.0	1.050	5.041 (6)				0.0	1.028	6.204 (6)
			-0.05	0.964	5.536 (6)				-0.05	0.924	7.043 (6)
			-0.1	0.892	6.077 (6)				-0.1	0.841	7.993 (6)
		700	0.1	1.105	5.791 (6)			700	0.1	1.181	6.653 (6)
			0.05	0.971	6.396 (6)				0.05	0.984	7.609 (6)
			0.0	0.876	7.048 (6)				0.0	0.859	8.673 (6)
			-0.05	0.802	7.752 (6)				-0.05	0.767	9.863 (6)
			-0.1	0.741	8.517 (6)				-0.1	0.698	1.121 (7)
		1000	0.1	0.932	8.226 (6)			1000	0.1	1.010	9.418 (6)
			0.05	0.807	9.116 (6)				0.05	0.820	1.083 (7)
			0.0	0.722	1.006 (7)				0.0	0.712	1.238 (7)
			-0.05	0.662	1.108 (7)				-0.05	0.635	1.410 (7)
			-0.1	0.614	1.218 (7)				-0.1	0.578	1.603 (7)

* numbers in the parenthesis denote powers of 10

TABLE 32: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c	n	μ	Q	N	a_c	T_c
0.2	-0.01	0	0.1	3.279	2.553 (5)*	0.2	-0.1	0	0.1	4.237	3.353 (5)
			0.05	3.278	2.609 (5)				0.05	4.243	3.394 (5)
			0.0	3.277	2.668 (5)				0.0	4.250	3.436 (5)
			-0.05	3.276	2.729 (5)				-0.05	4.257	3.479 (5)
			-0.1	3.274	2.793 (5)				-0.1	4.263	3.523 (5)
		100	0.1	2.533	8.661 (5)			100	0.1	4.108	7.884 (5)
			0.05	2.495	9.084 (5)				0.05	4.121	8.036 (5)
			0.0	2.458	9.549 (5)				0.0	4.133	8.194 (5)
			-0.05	2.407	1.006 (6)				-0.05	4.146	8.358 (5)
			-0.1	2.359	1.062 (6)				-0.1	4.160	8.528 (5)
		300	0.1	1.654	2.298 (6)			300	0.1	3.541	1.830 (6)
			0.05	1.541	2.465 (6)				0.05	3.542	1.888 (6)
			0.0	1.437	2.646 (6)				0.0	3.542	1.949 (6)
			-0.05	1.342	2.843 (6)				-0.05	3.541	2.014 (6)
			-0.1	1.258	3.055 (6)				-0.1	3.538	2.083 (6)
		500	0.1	1.291	3.768 (6)			500	0.1	2.994	2.953 (6)
			0.05	1.173	4.069 (6)				0.05	2.955	3.072 (6)
			0.0	1.078	4.389 (6)				0.0	2.907	3.201 (6)
			-0.05	0.999	4.731 (6)				-0.05	2.849	3.340 (6)
			-0.1	0.938	5.096 (6)				-0.1	2.777	3.491 (6)
		700	0.1	1.094	5.244 (6)			700	0.1	2.585	4.104 (6)
			0.05	0.981	5.680 (6)				0.05	2.506	4.295 (6)
			0.0	0.895	6.139 (6)				0.0	2.413	4.502 (6)
			-0.05	0.830	6.625 (6)				-0.05	2.304	4.726 (6)
			-0.1	0.780	7.142 (6)				-0.1	2.183	4.968 (6)
		1000	0.1	0.916	7.461 (6)			1000	0.1	2.177	5.847 (6)
			0.05	0.809	8.099 (6)				0.05	2.061	6.153 (6)
			0.0	0.739	8.765 (6)				0.0	1.932	6.485 (6)
			-0.05	0.683	9.467 (6)				-0.05	1.801	6.840 (6)
			-0.1	0.641	1.021 (7)				-0.1	1.676	7.217 (6)

* numbers in the parenthesis denote powers of 10

TABLE 33: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

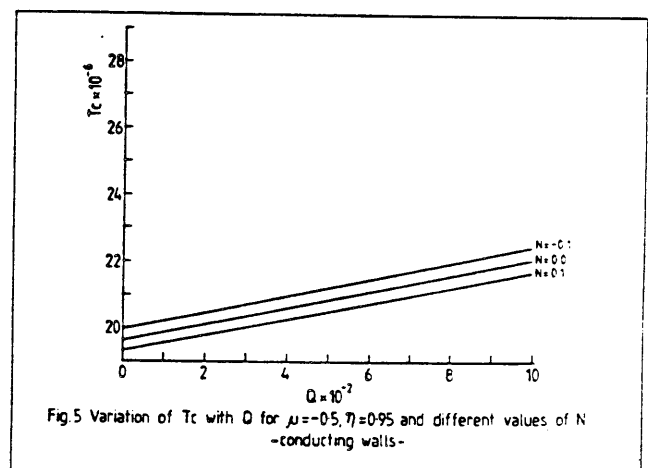
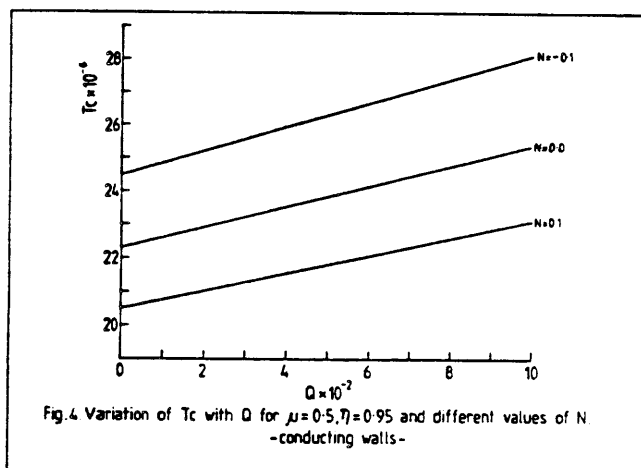
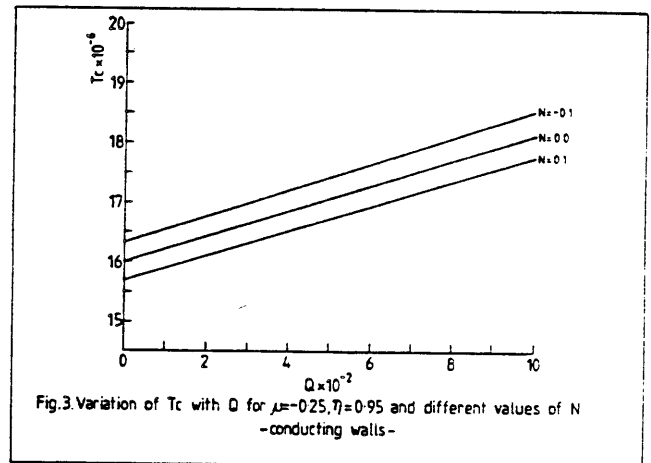
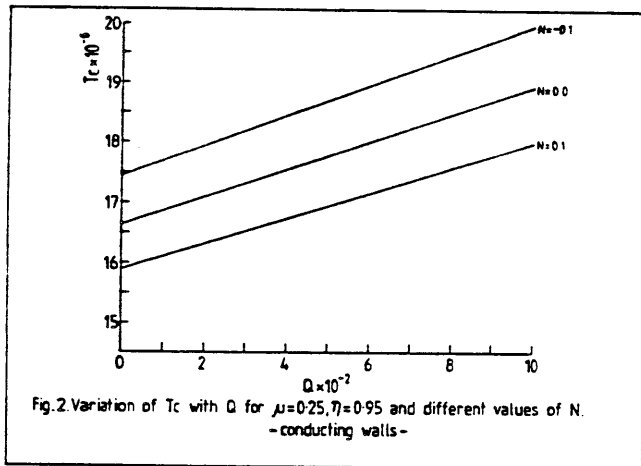
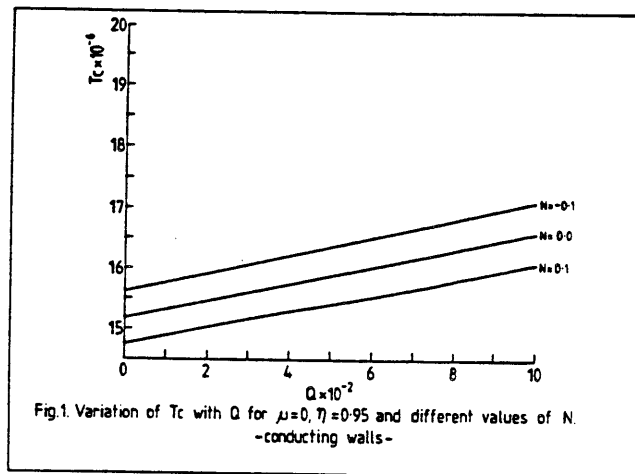
n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.1	0.0	0	0.1	3.345	2.265	(6)*	0.1	0.001	0	0.1	3.340	2.431 (6)
			0.05	3.341	2.348	(6)				0.05	3.337	2.529 (6)
			0.0	3.340	2.437	(6)				0.0	3.335	2.636 (6)
			-0.05	3.338	2.532	(6)				-0.05	3.332	2.751 (6)
			-0.1	3.335	2.635	(6)				-0.1	3.332	2.875 (6)
		100	0.1	2.534	8.268	(6)			100	0.1	2.529	8.817 (6)
			0.05	2.462	8.984	(6)				0.05	2.449	9.659 (6)
			0.0	2.368	9.819	(6)				0.0	2.340	1.066 (7)
			-0.05	2.258	1.080	(7)				-0.05	2.225	1.185 (7)
			-0.1	2.130	1.195	(7)				-0.1	2.085	1.327 (7)
		300	0.1	1.876	2.134	(7)			300	0.1	1.890	2.260 (7)
			0.05	1.583	2.419	(7)				0.05	1.592	2.593 (7)
			0.0	1.368	2.744	(7)				0.0	1.355	2.979 (7)
			-0.05	1.228	3.110	(7)				-0.05	1.185	3.421 (7)
			-0.1	1.088	3.520	(7)				-0.1	1.072	3.925 (7)
		500	0.1	1.741	3.425	(7)			500	0.1	1.768	3.612 (7)
			0.05	1.262	3.967	(7)				0.05	1.267	4.247 (7)
			0.0	1.032	4.544	(7)				0.0	1.021	4.933 (7)
			-0.05	0.906	5.175	(7)				-0.05	0.888	5.694 (7)
			-0.1	0.818	5.876	(7)				-0.1	0.797	6.553 (7)
		700	0.1	1.795	4.678	(7)			700	0.1	1.832	4.916 (7)
			0.05	1.066	5.518	(7)				0.05	1.082	5.903 (7)
			0.0	0.861	6.348	(7)				0.0	0.853	6.892 (7)
			-0.05	0.756	7.243	(7)				-0.05	0.743	7.972 (7)
			-0.1	0.683	8.236	(7)				-0.1	0.603	9.184 (7)
		1000	0.1	1.817	6.487	(7)			1000	0.1	1.854	6.792 (7)
			0.05	0.901	7.847	(7)				0.05	0.913	8.389 (7)
			0.0	0.714	9.057	(7)				0.0	0.706	9.832 (7)
			-0.05	0.626	1.035	(8)				-0.05	0.614	1.139 (8)
			-0.1	0.564	1.177	(8)				-0.1	0.552	1.313 (8)

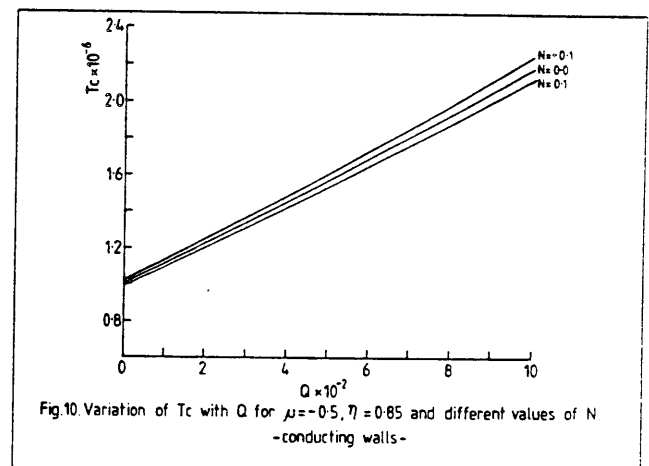
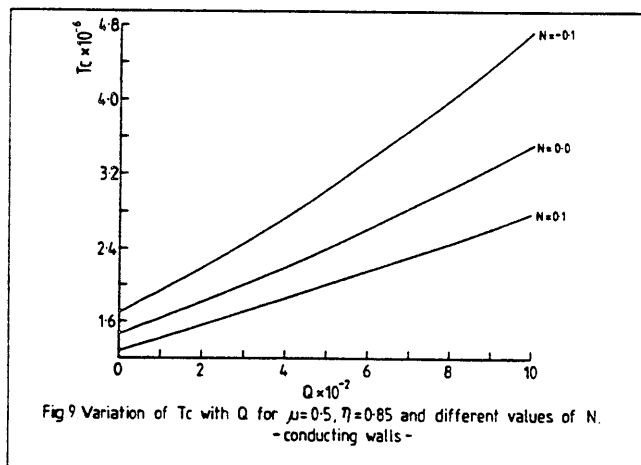
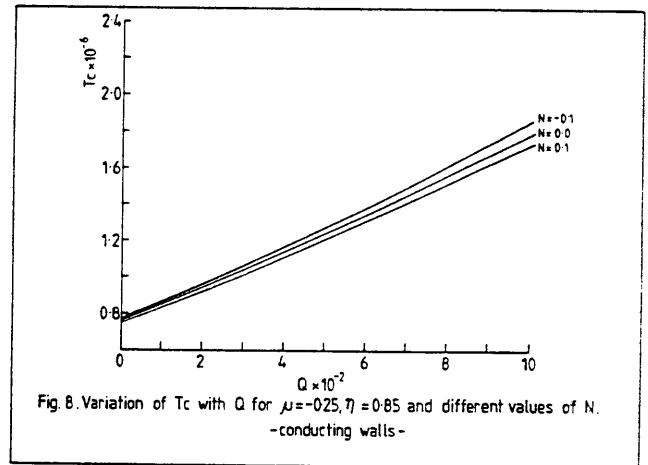
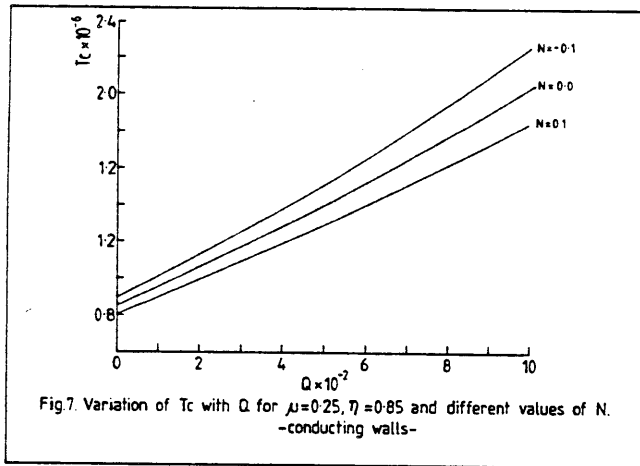
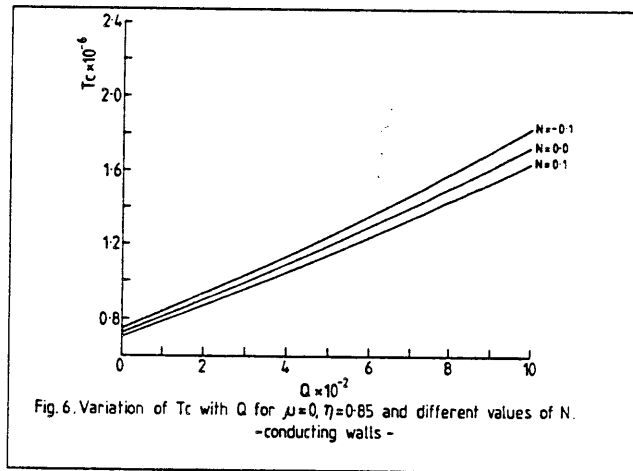
* numbers in the parenthesis denote powers of 10

TABLE 34: CRITICAL VALUES OF TAYLOR NUMBERS AND WAVE NUMBERS
FOR VARIOUS VALUES OF n , μ , Q AND N
- NON-CONDUCTING WALLS -

n	μ	Q	N	a_c	T_c		n	μ	Q	N	a_c	T_c
0.1	-0.001	0	0.1	3.349	2.130	(6)*	0.1	-0.1	0	0.1	8.071	4.100 (6)
			0.05	3.347	2.201	(6)				0.05	8.080	4.140 (6)
			0.0	3.345	2.277	(6)				0.0	8.088	4.181 (6)
			-0.05	3.342	2.358	(6)				-0.05	8.096	4.223 (6)
			-0.1	3.340	2.444	(6)				-0.1	8.105	4.265 (6)
	100		0.1	2.550	7.809	(6)			100	0.1	7.949	5.932 (6)
			0.05	2.478	8.428	(6)				0.05	7.963	6.003 (6)
			0.0	2.391	9.142	(6)				0.0	7.976	6.076 (6)
			-0.5	2.289	9.966	(6)				-0.05	7.990	6.150 (6)
			-0.1	2.164	1.092	(7)				-0.1	8.004	6.226 (6)
	300		0.1	1.871	2.027	(7)			300	0.1	7.722	9.963 (6)
			0.05	1.600	2.275	(7)				0.05	7.742	1.012 (7)
			0.0	1.388	2.553	(7)				0.0	7.761	1.029 (7)
			-0.05	1.232	2.863	(7)				-0.05	7.781	1.046 (7)
			-0.1	1.116	3.207	(7)				-0.1	7.800	1.063 (7)
	500		0.1	1.718	3.262	(7)			500	0.1	7.436	1.431 (7)
			0.05	1.063	3.734	(7)				0.05	7.455	1.459 (7)
			0.0	0.939	4.229	(7)				0.0	7.474	1.489 (7)
			-0.05	0.808	4.764	(7)				-0.05	7.493	1.519 (7)
			-0.1	0.733	5.352	(7)				-0.1	7.513	1.551 (7)
	700		0.1	1.790	4.467	(7)			700	0.1	7.102	1.431 (7)
			0.05	0.953	5.196	(7)				0.05	7.115	1.459 (7)
			0.0	0.762	5.912	(7)				0.0	7.127	1.489 (7)
			-0.05	0.675	6.670	(7)				-0.05	7.138	1.519 (7)
			-0.1	0.613	7.501	(7)				-0.1	7.150	1.551 (7)
	1000		0.1	1.789	6.214	(7)			1000	0.1	6.570	2.597 (7)
			0.05	0.890	7.392	(7)				0.05	6.562	2.669 (7)
			0.0	0.756	8.441	(7)				0.0	6.551	2.746 (7)
			-0.05	0.634	9.527	(7)				-0.05	6.537	2.826 (7)
			-0.1	0.579	1.072	(8)				-0.1	6.517	2.912 (7)

* numbers in the parenthesis denote powers of 10





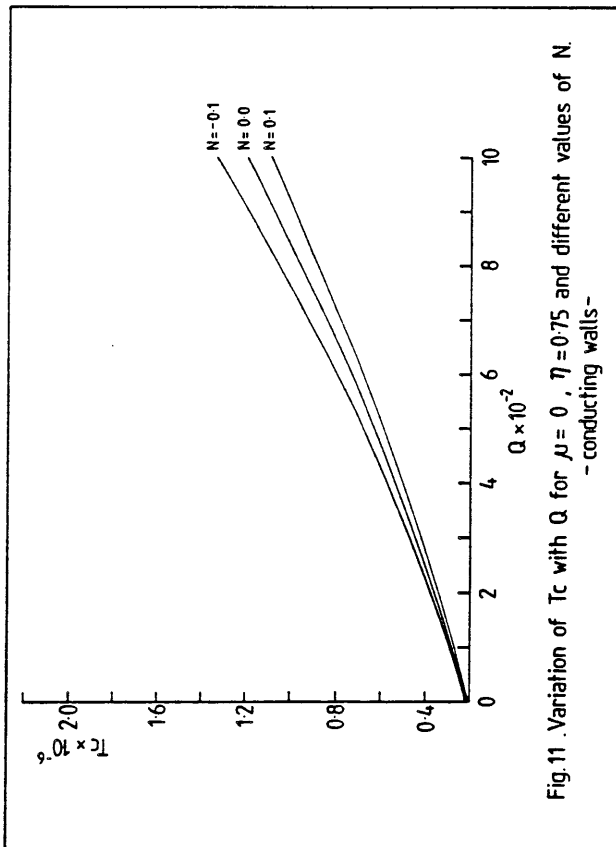


Fig. 11. Variation of T_c with Q for $\mu = 0$, $\eta = 0.75$ and different values of N . -conducting walls-

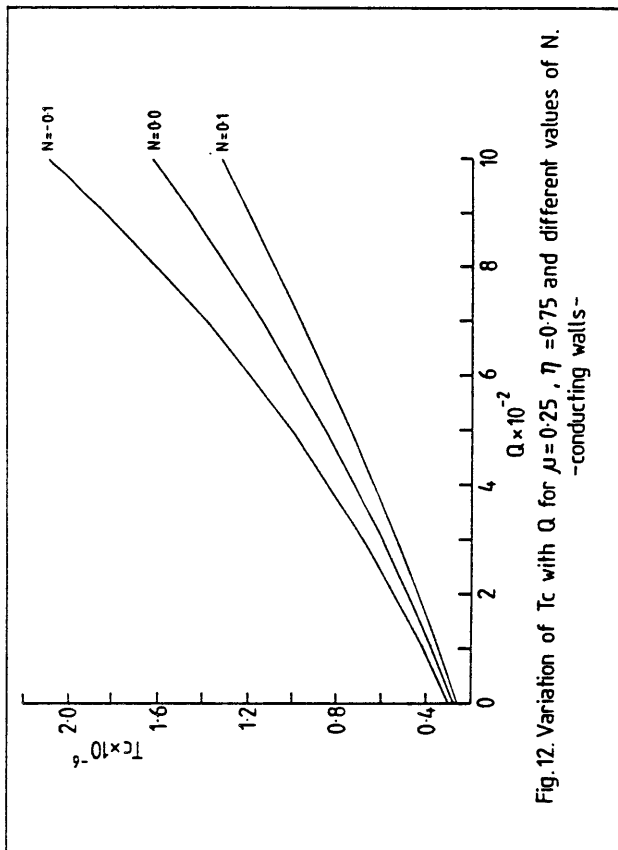


Fig. 12. Variation of T_c with Q for $\mu = 0.25$, $\eta = 0.75$ and different values of N . -conducting walls-

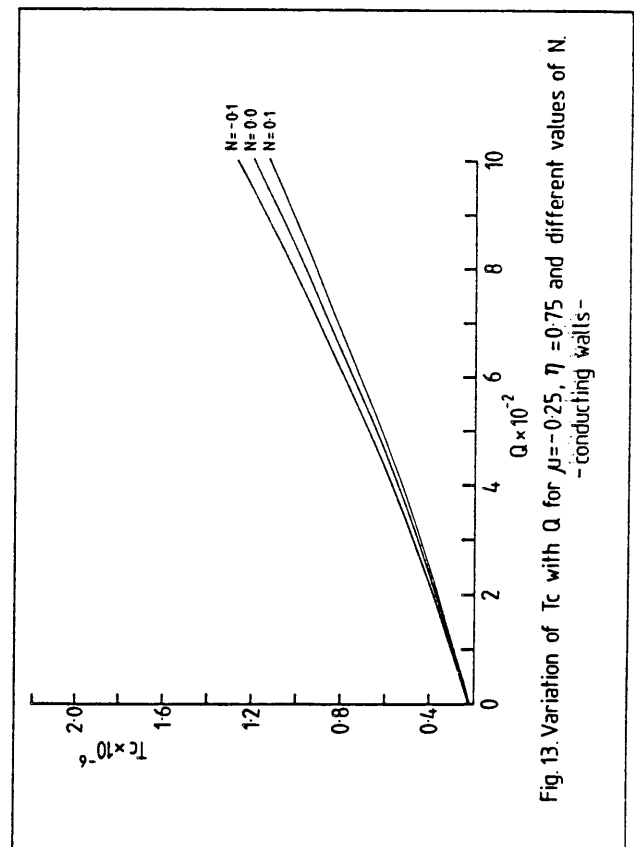


Fig. 13. Variation of T_c with Q for $\mu = 0.25$, $\eta = 0.75$ and different values of N . -conducting walls-

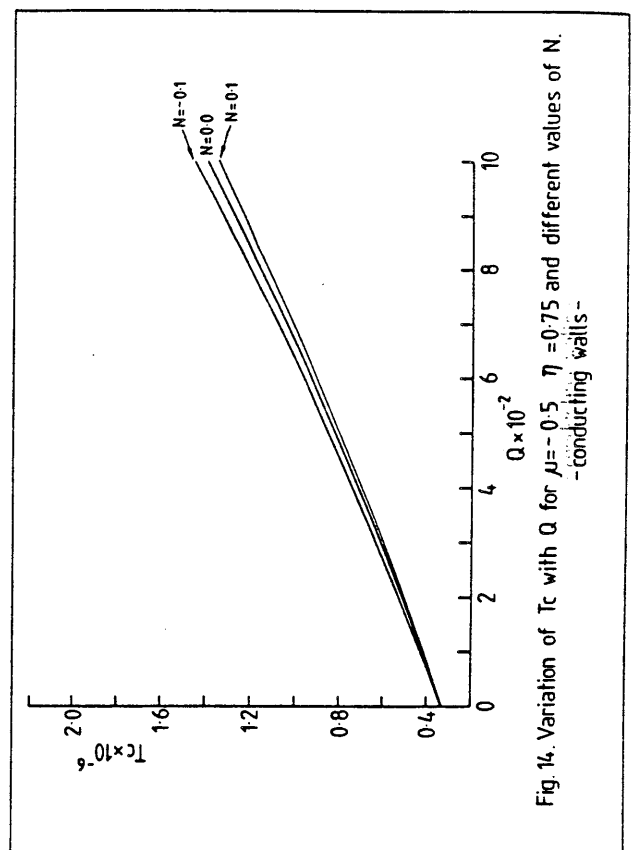
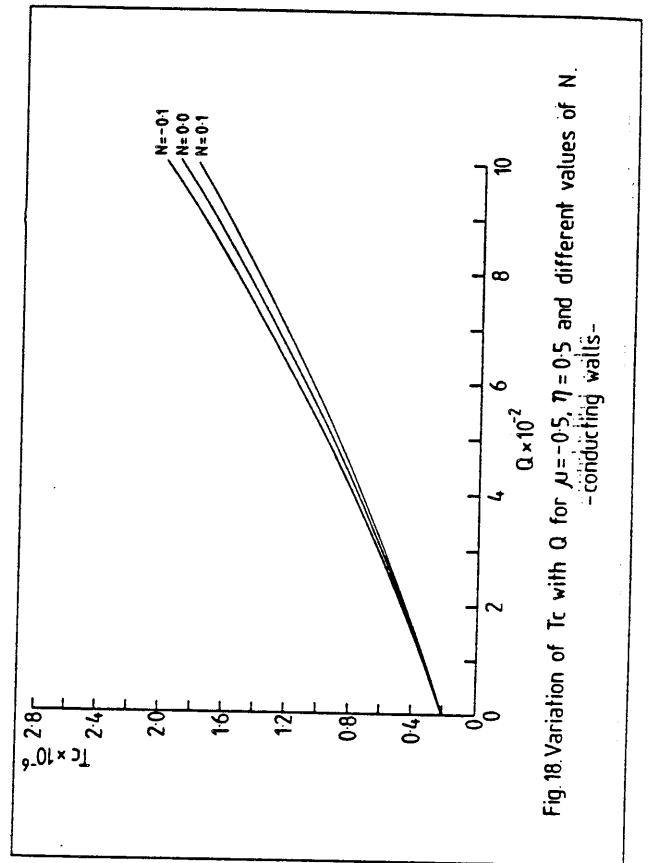
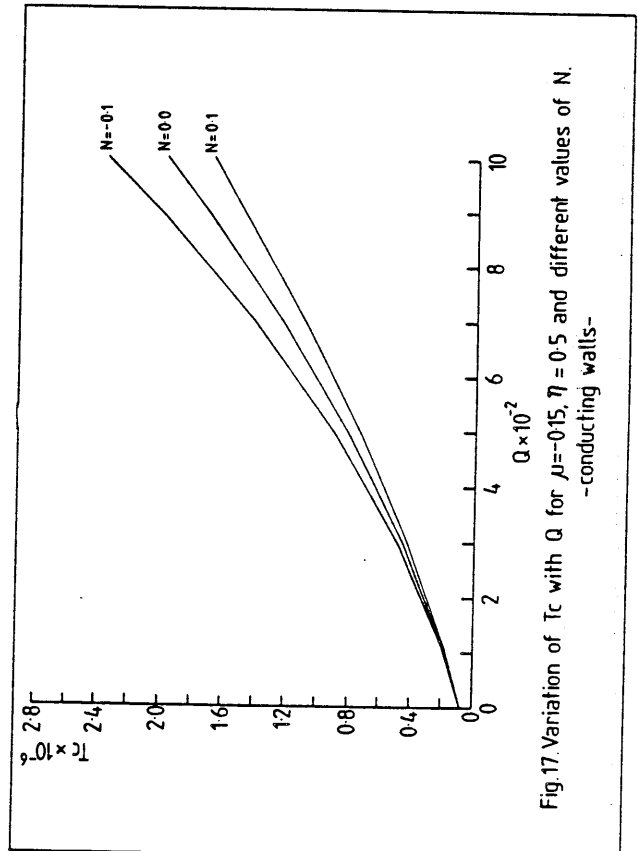
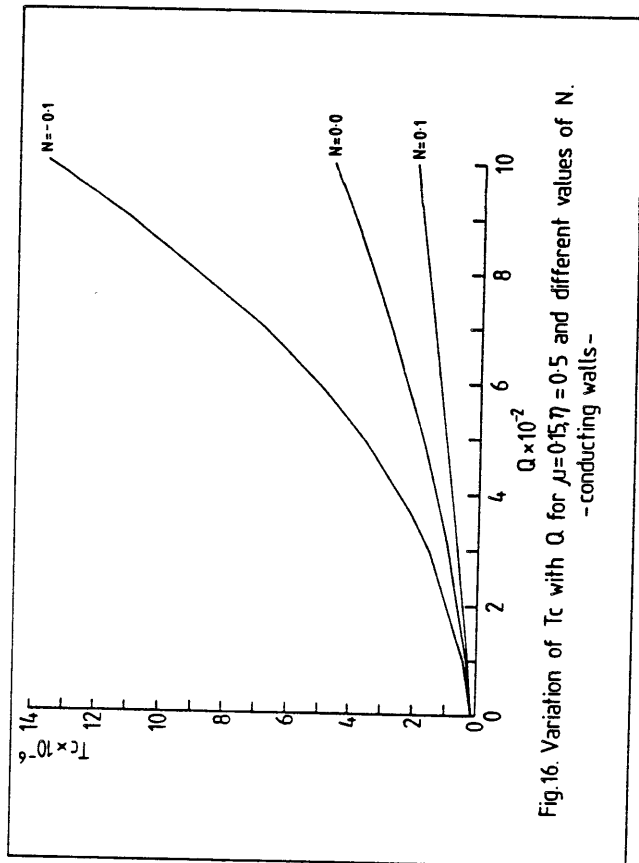
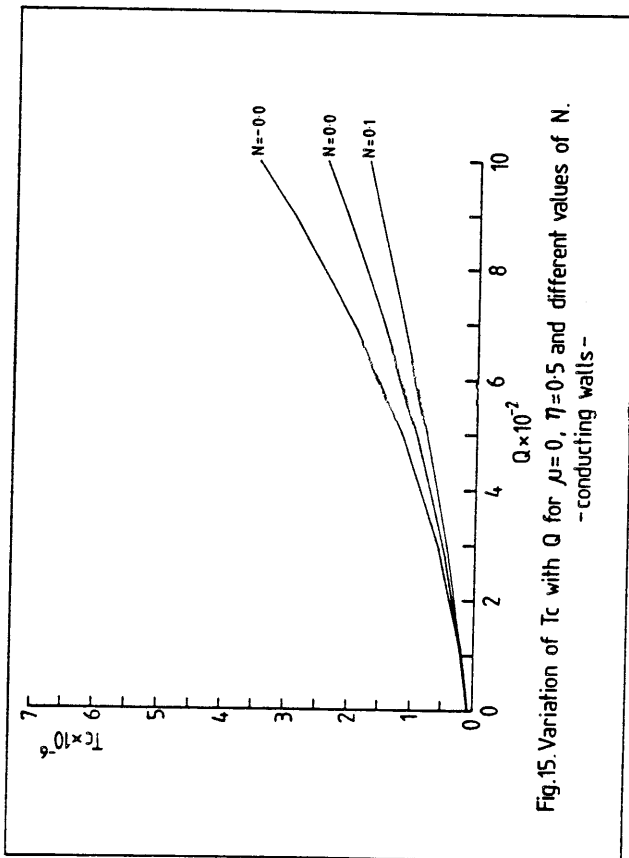
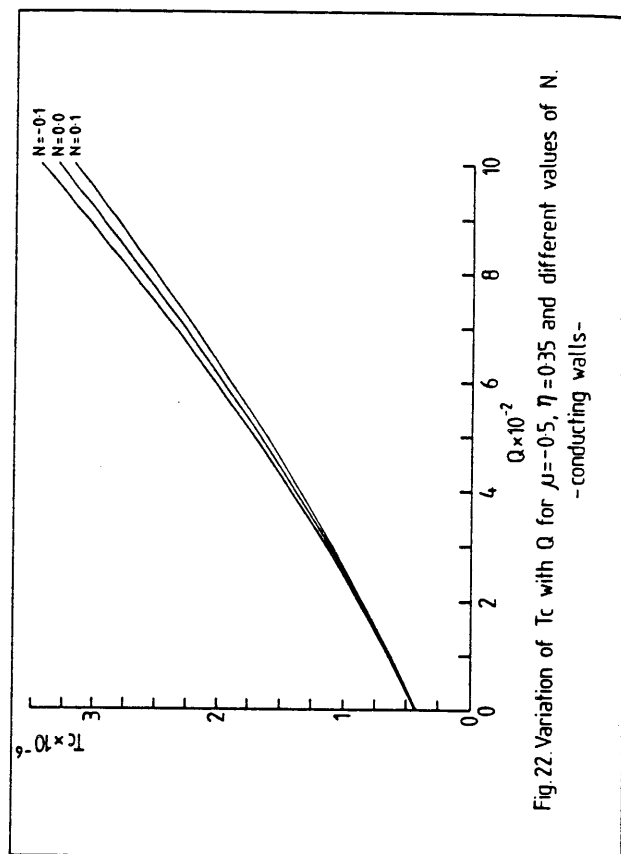
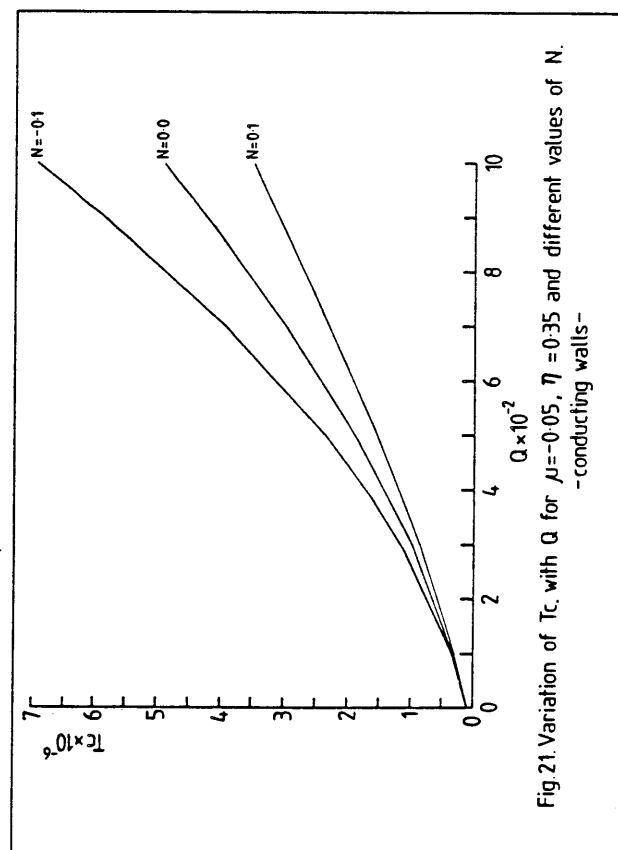
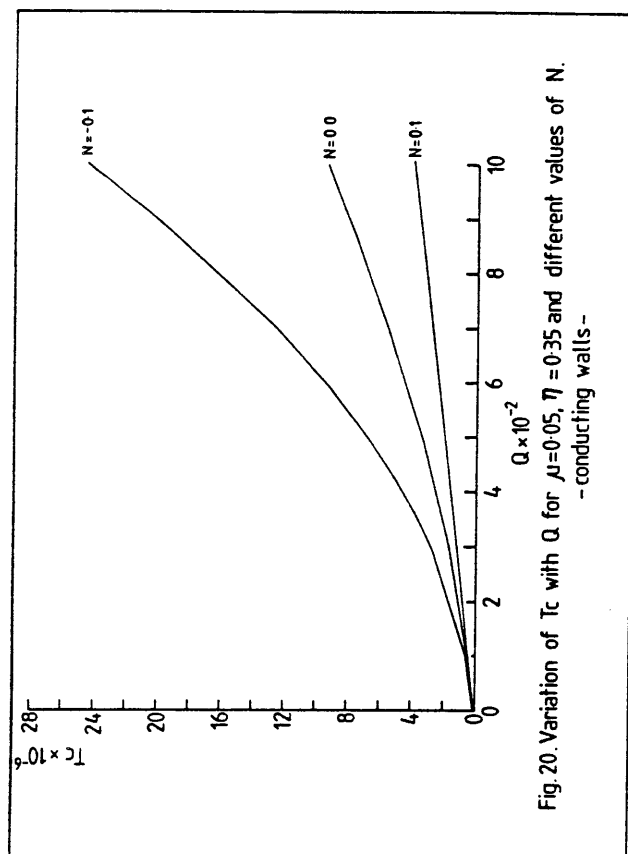
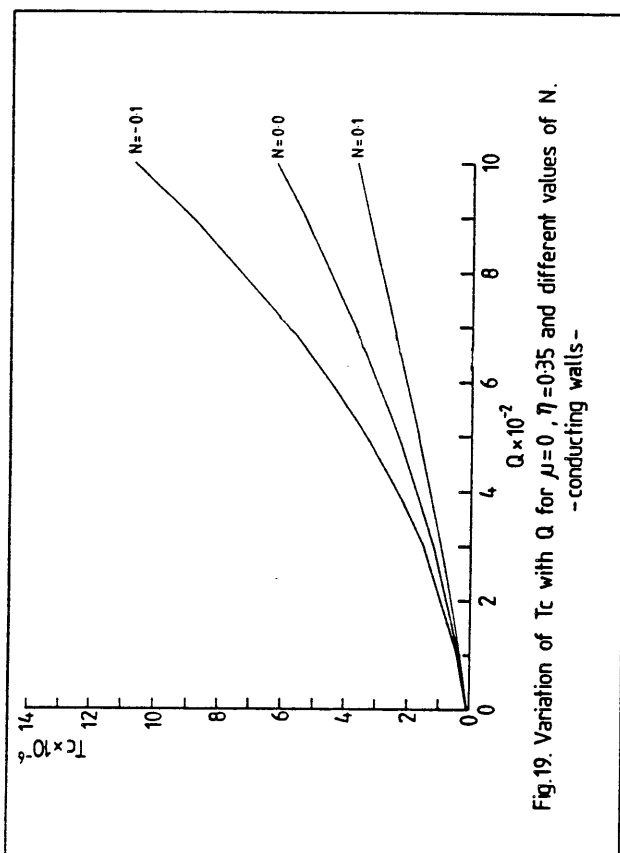
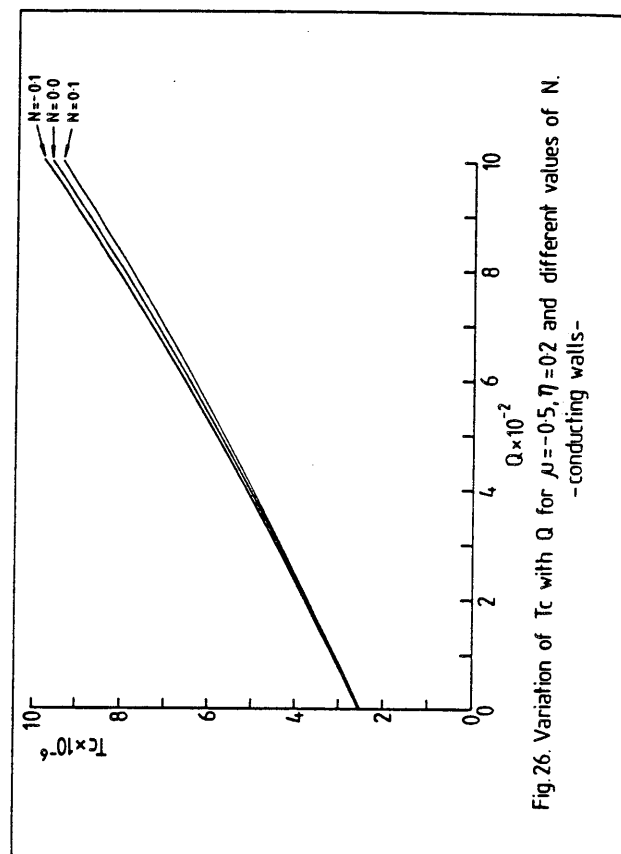
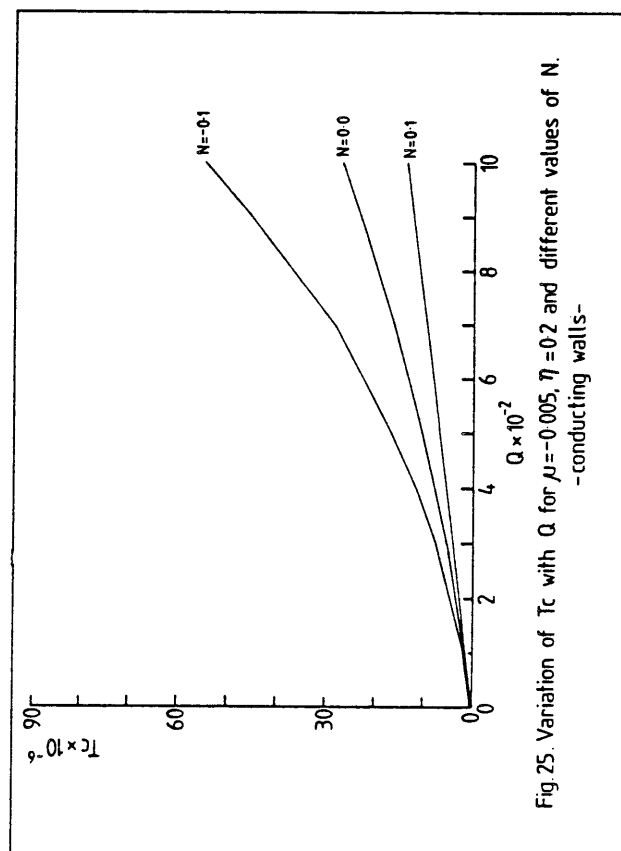
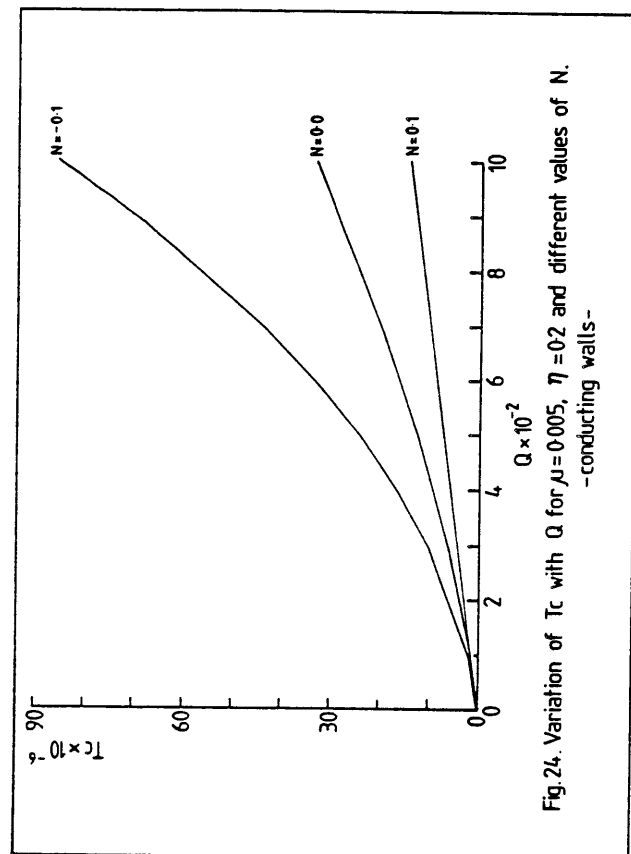
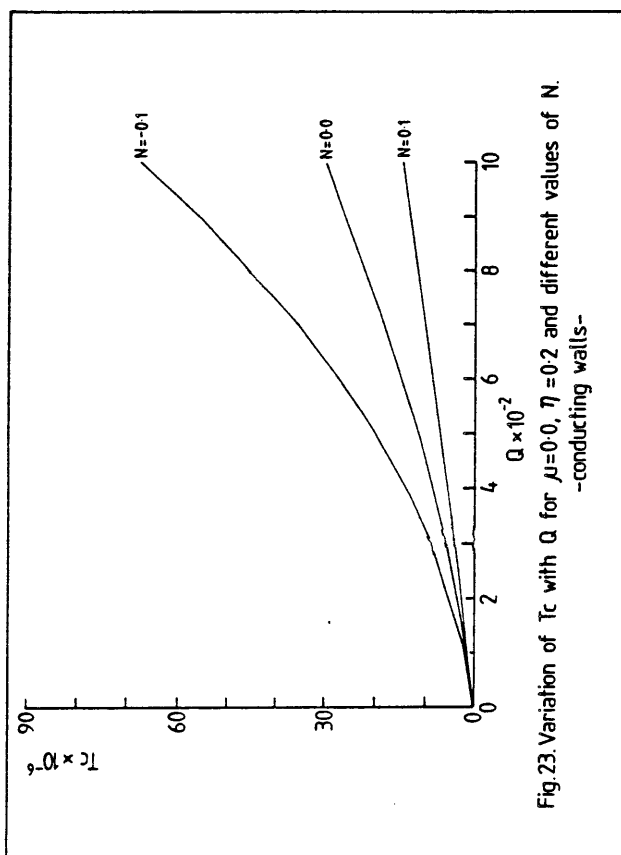


Fig. 14. Variation of T_c with Q for $\mu = -0.5$, $\eta = 0.75$ and different values of N . -conducting walls-







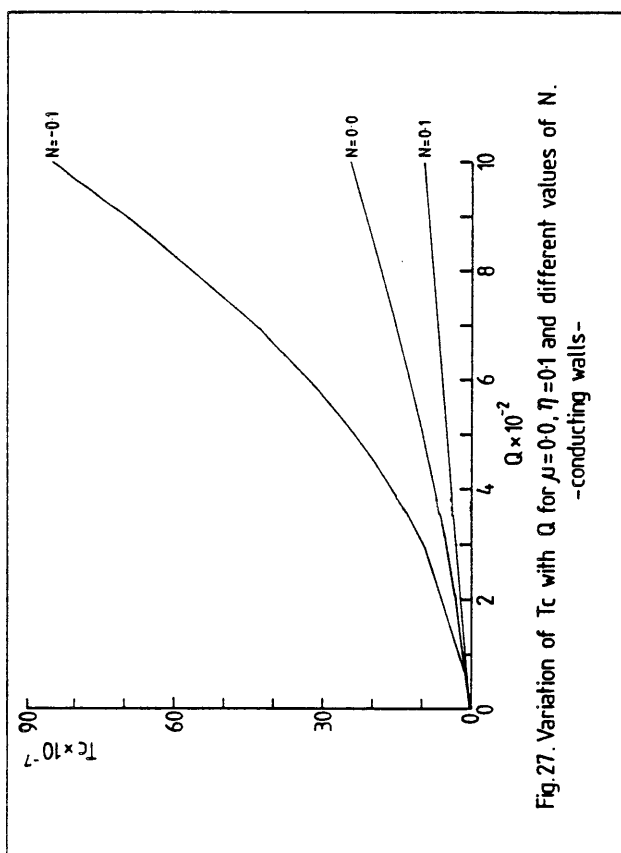


Fig. 27. Variation of T_c with Q for $\mu=0.0$, $\eta=0.1$ and different values of N . -conducting walls-

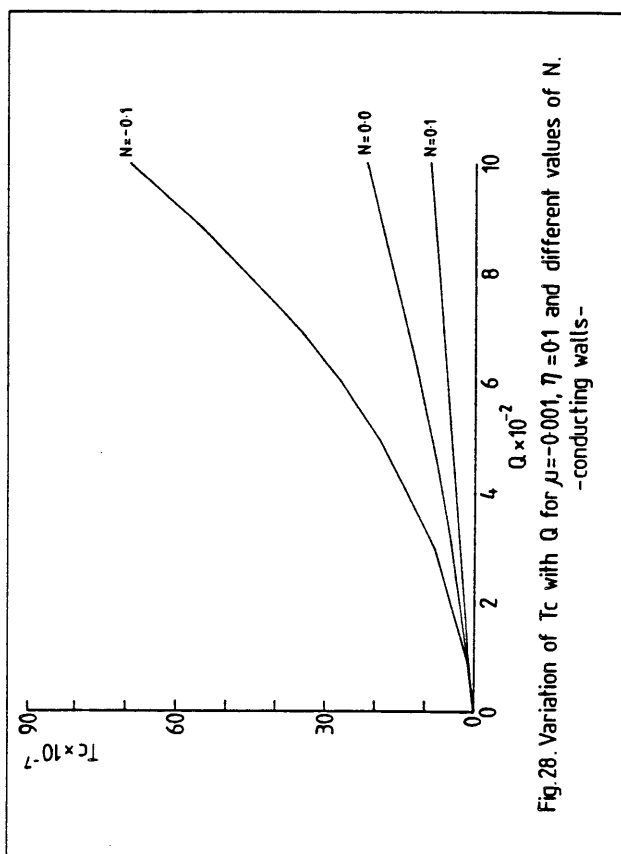


Fig. 28. Variation of T_c with Q for $\mu=-0.001$, $\eta=0.1$ and different values of N . -conducting walls-

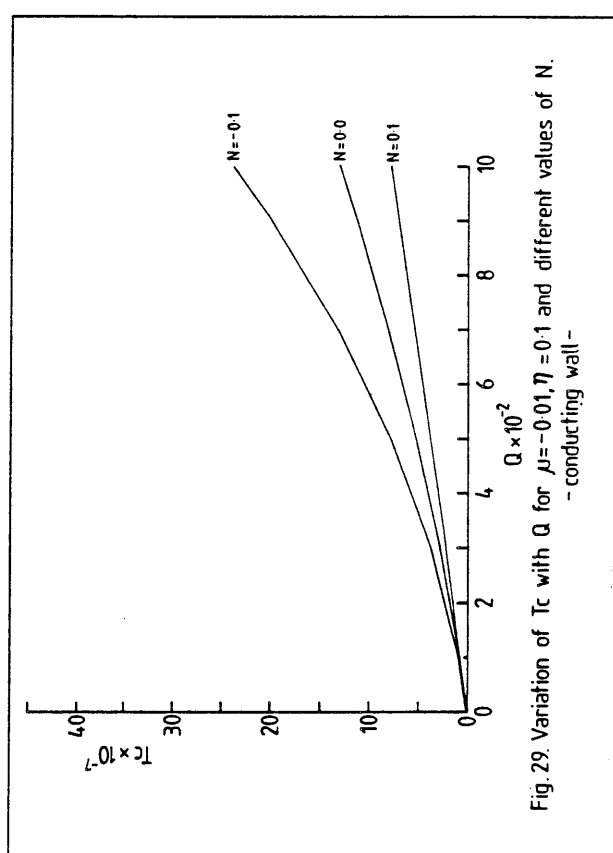


Fig. 29. Variation of T_c with Q for $\mu=-0.01$, $\eta=0.1$ and different values of N . -conducting wall-

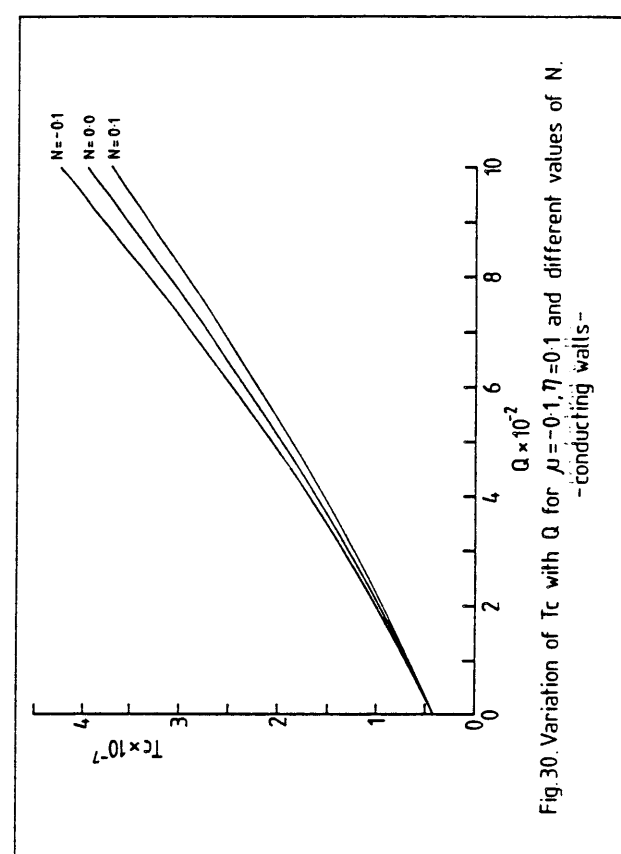
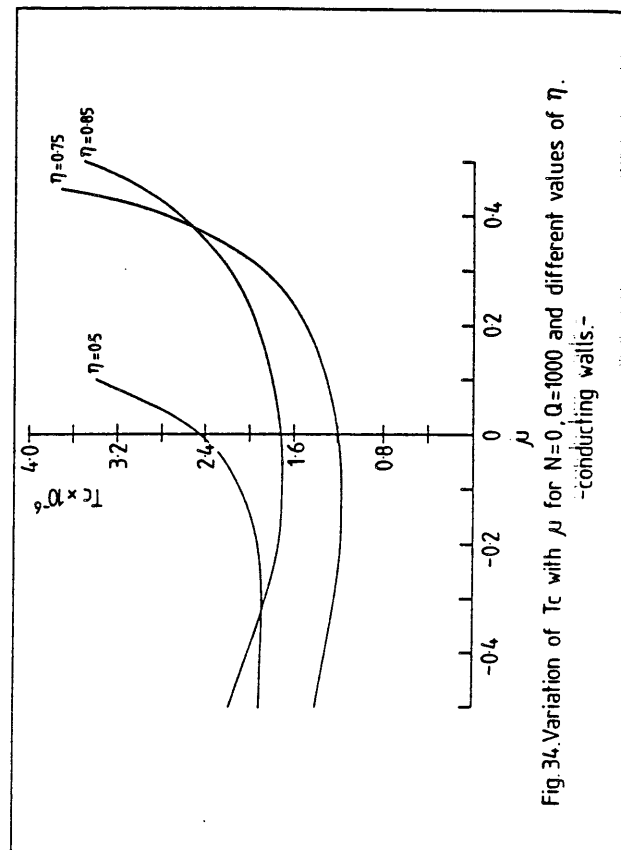
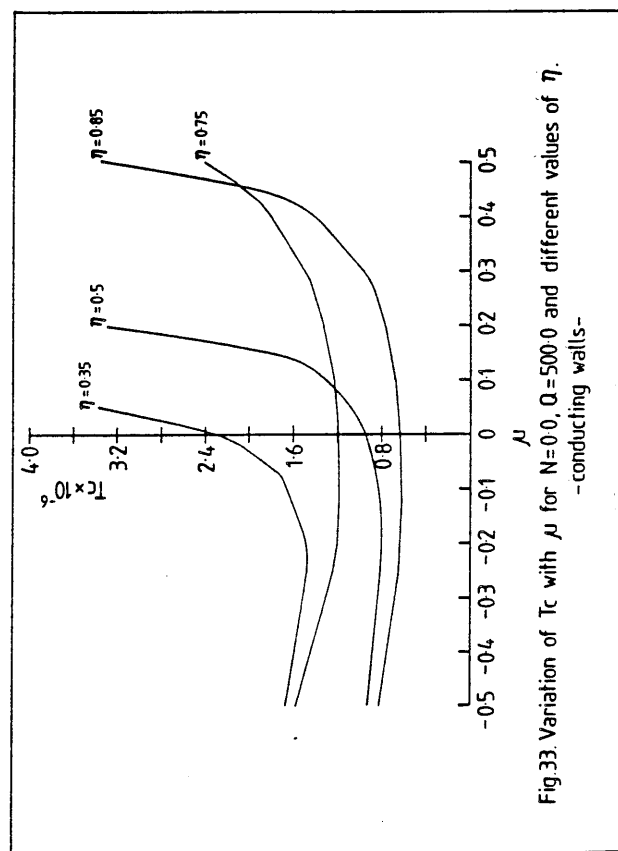
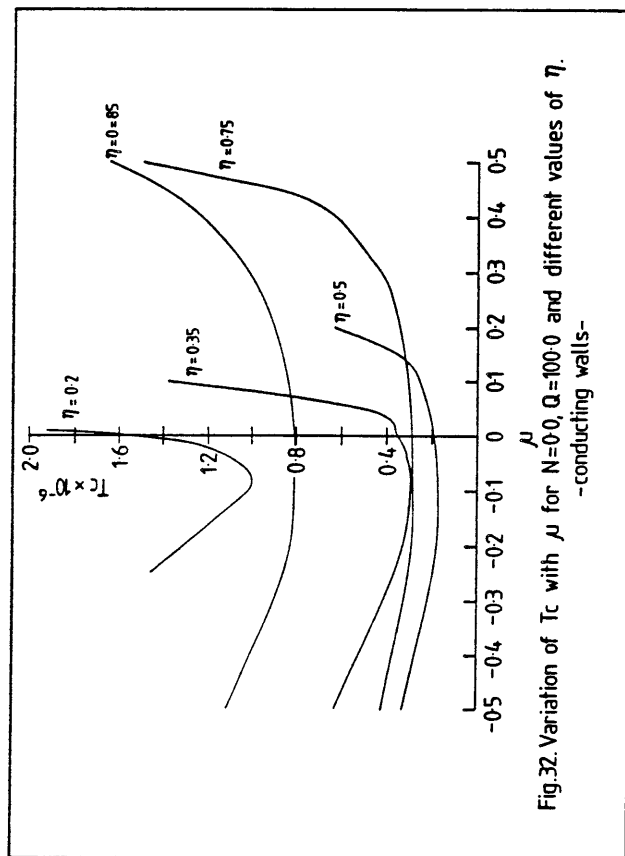
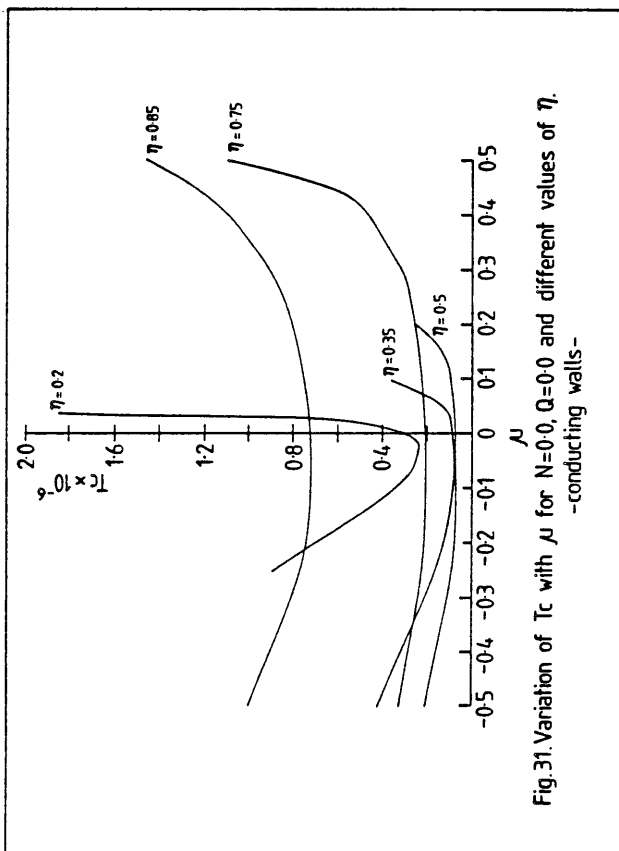
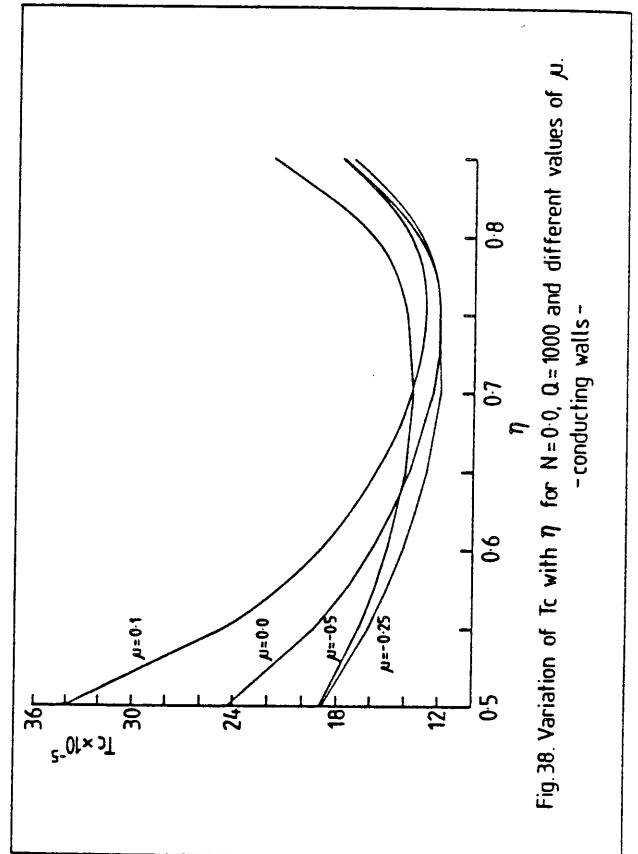
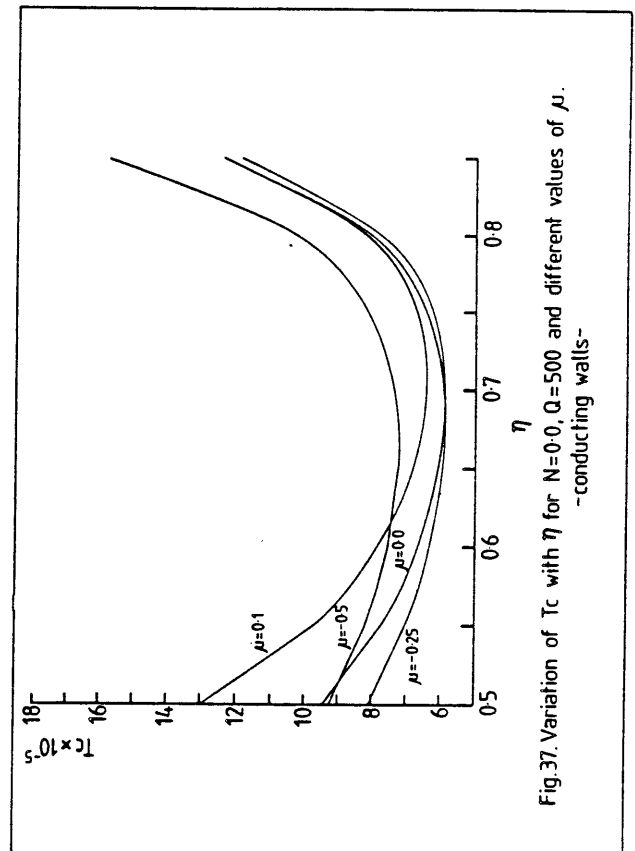
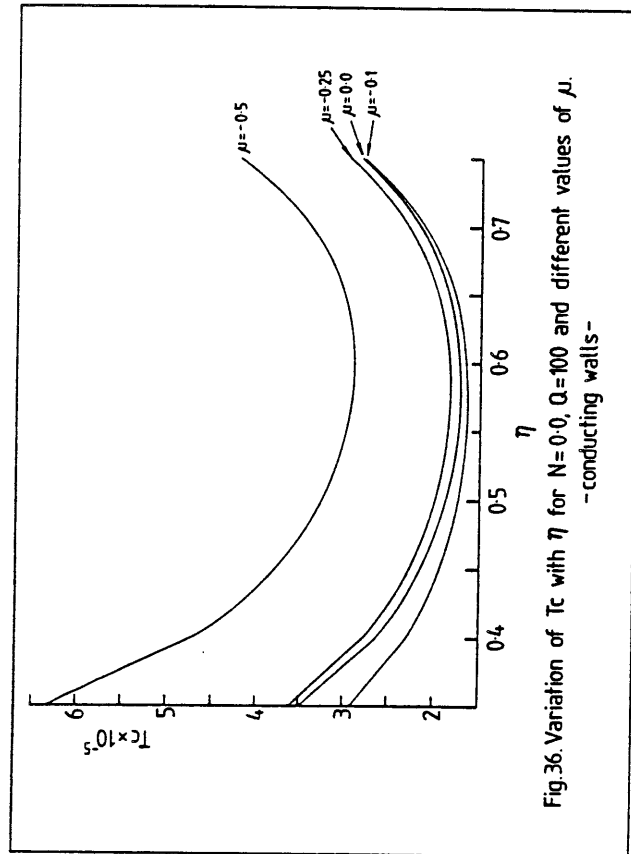
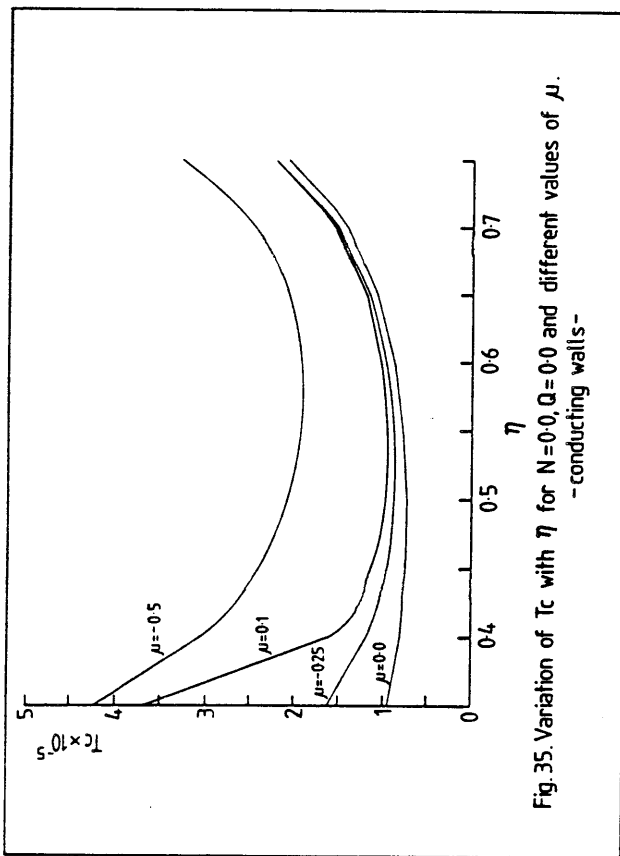
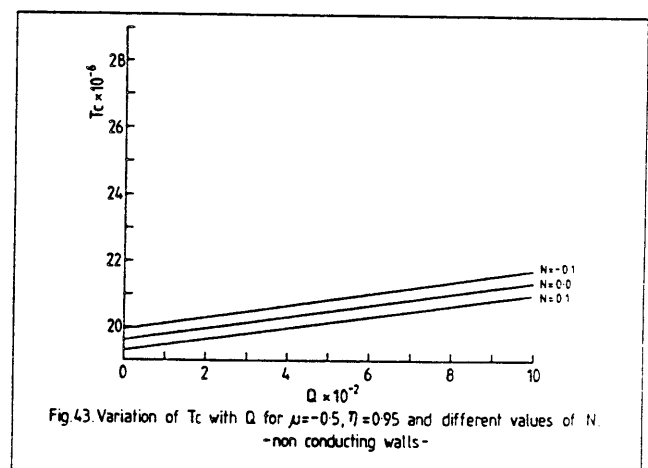
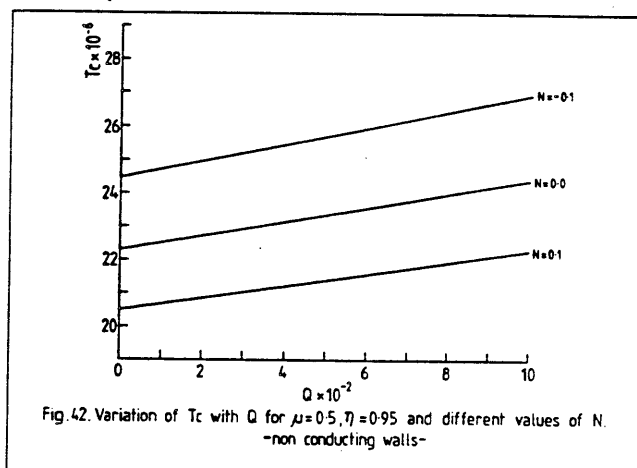
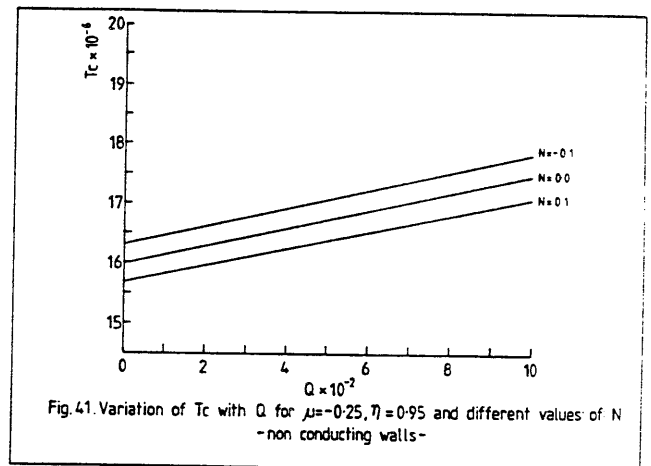
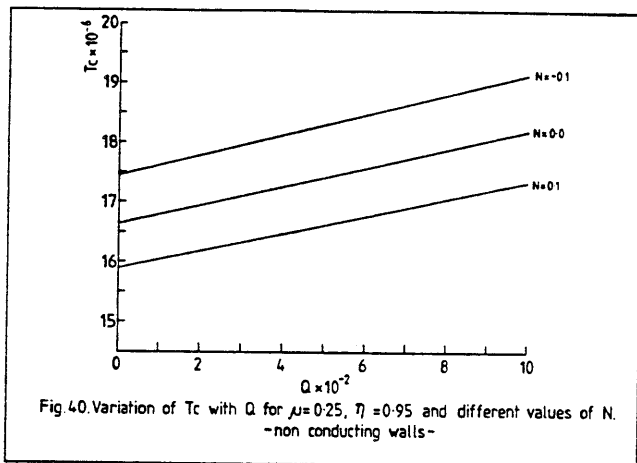
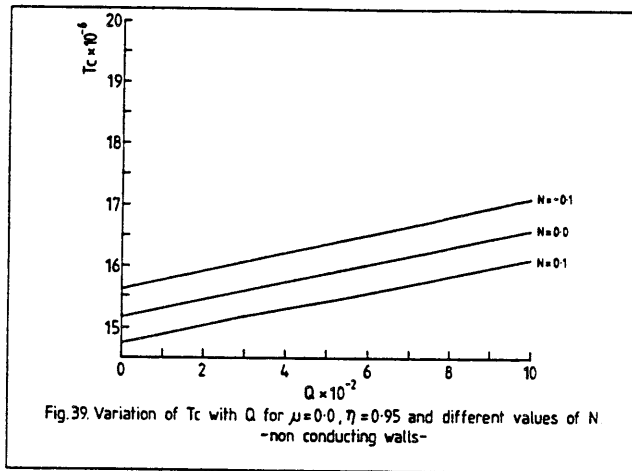
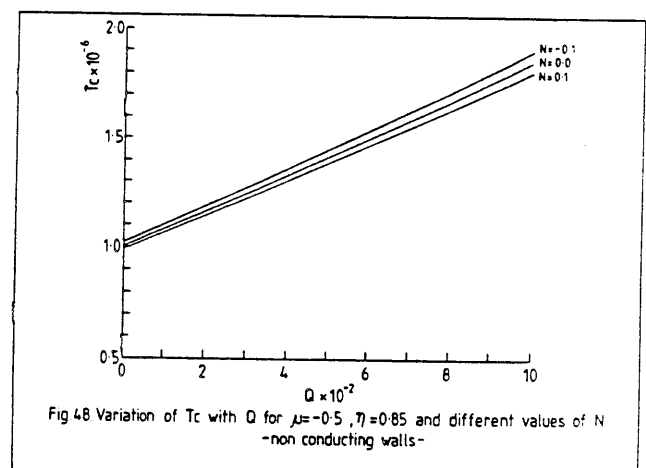
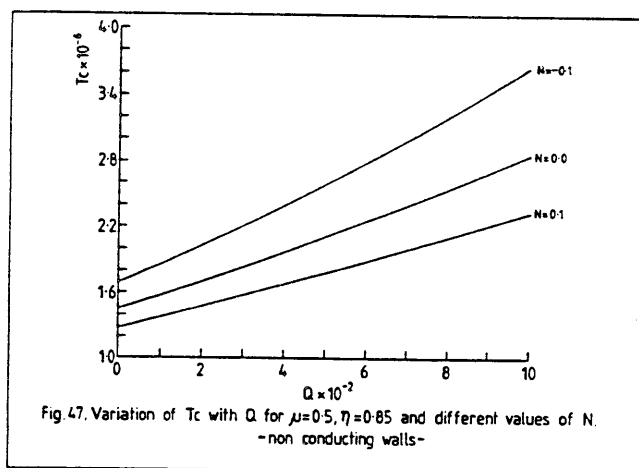
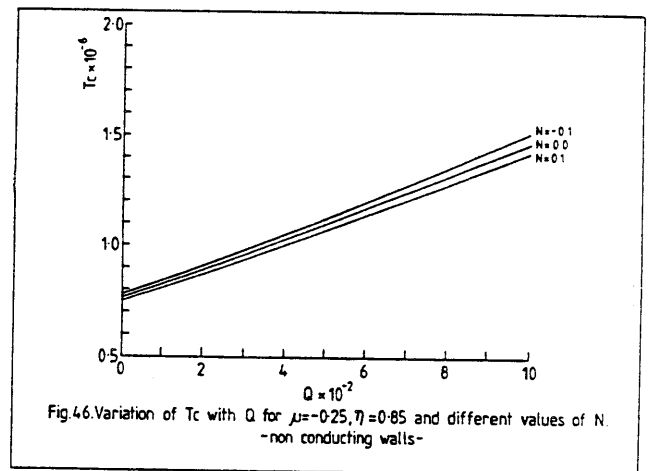
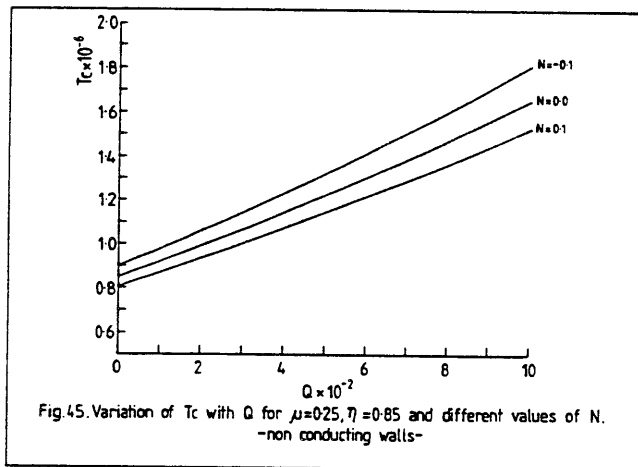
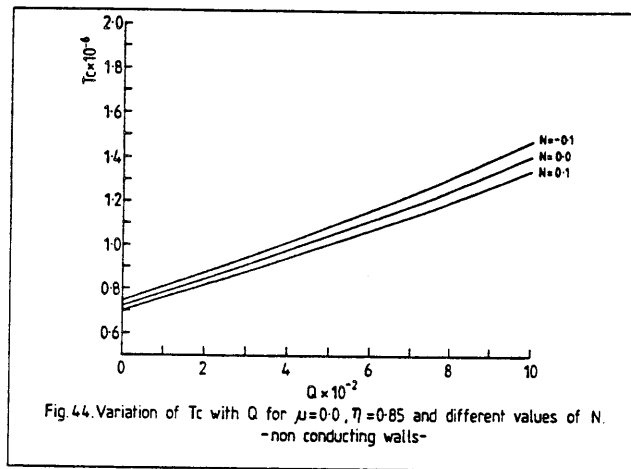


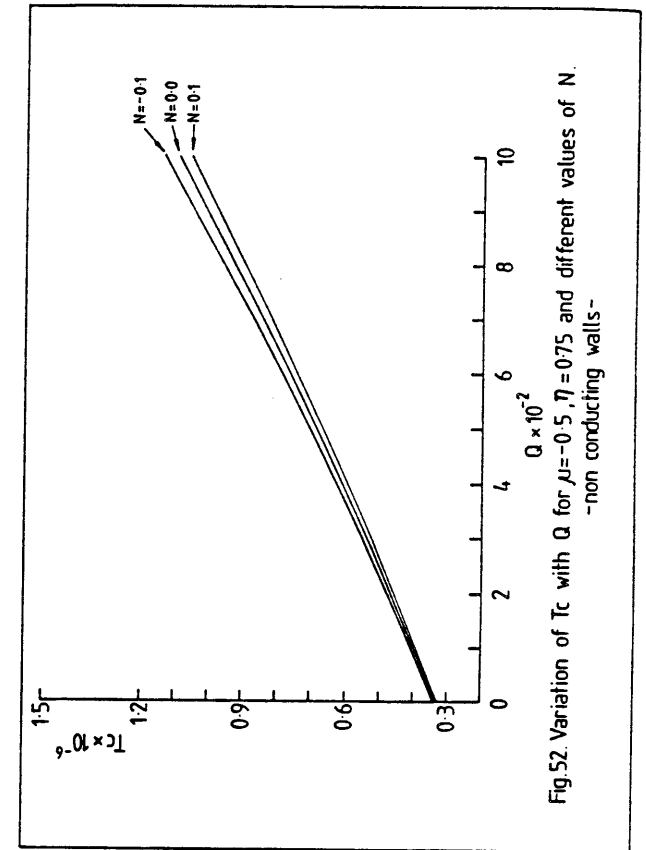
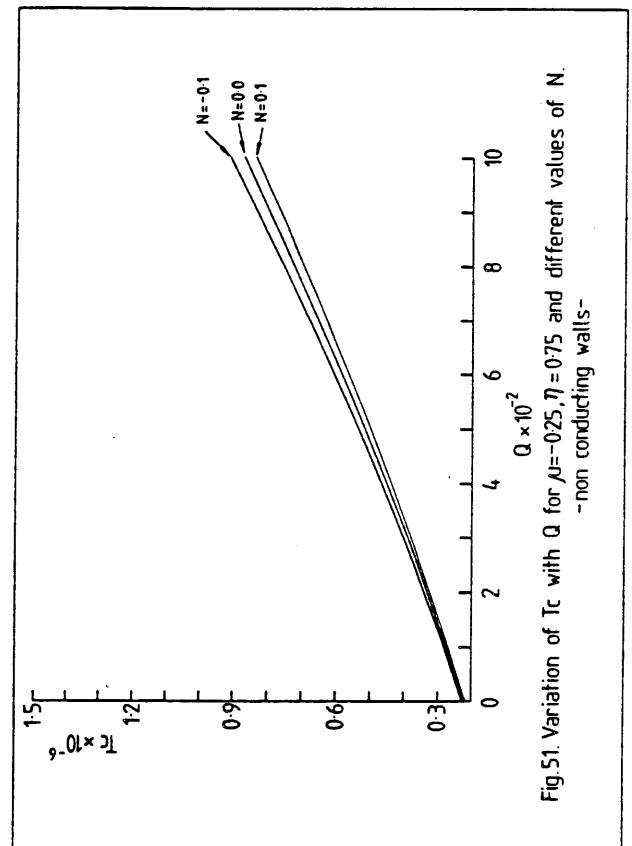
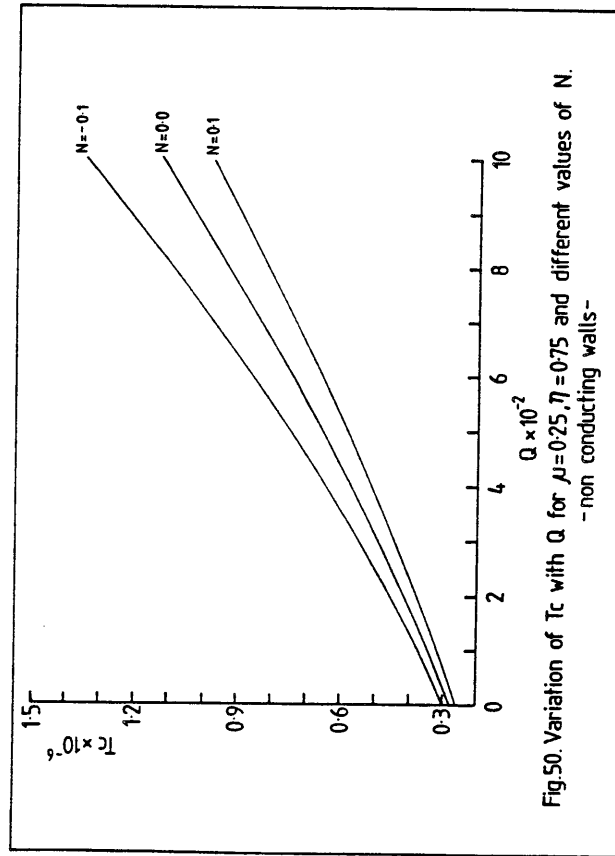
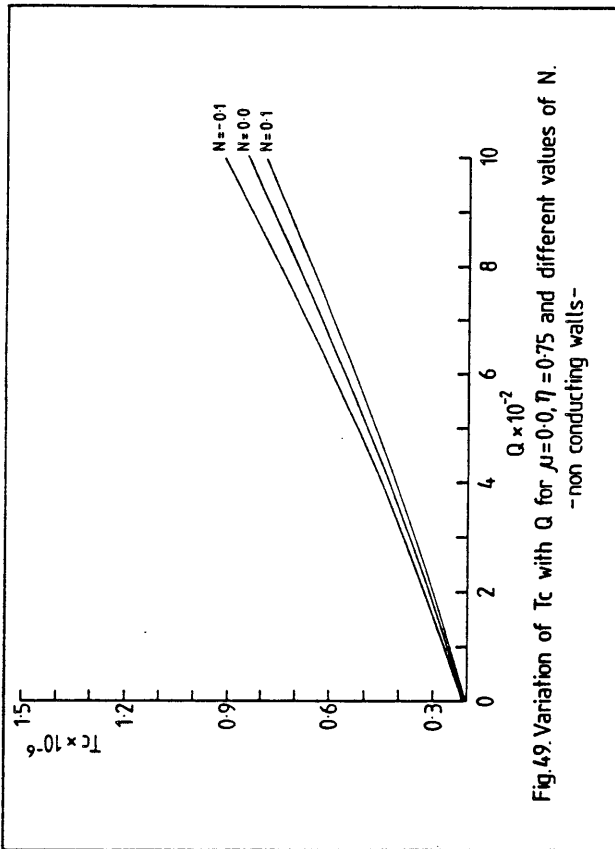
Fig. 30. Variation of T_c with Q for $\mu=-0.1$, $\eta=0.1$ and different values of N . -conducting walls-

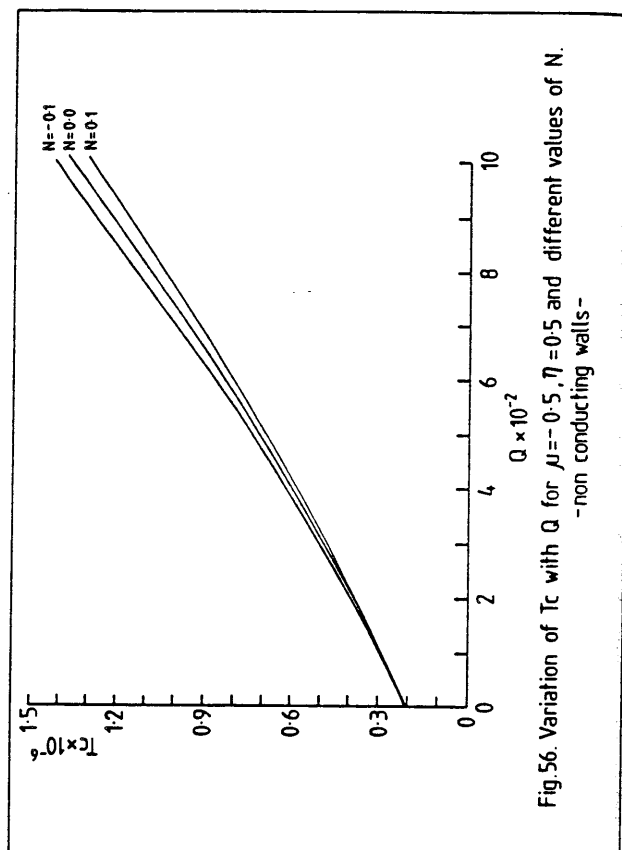
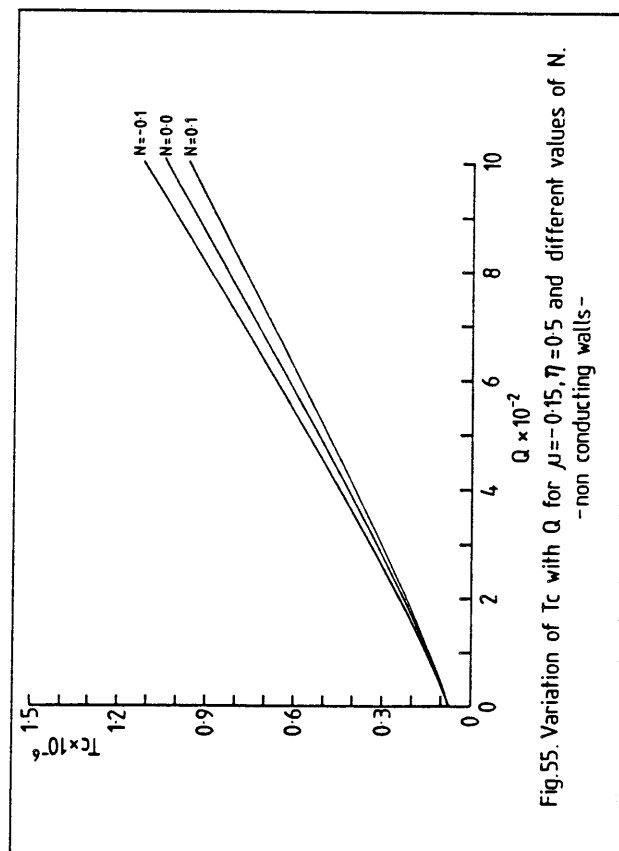
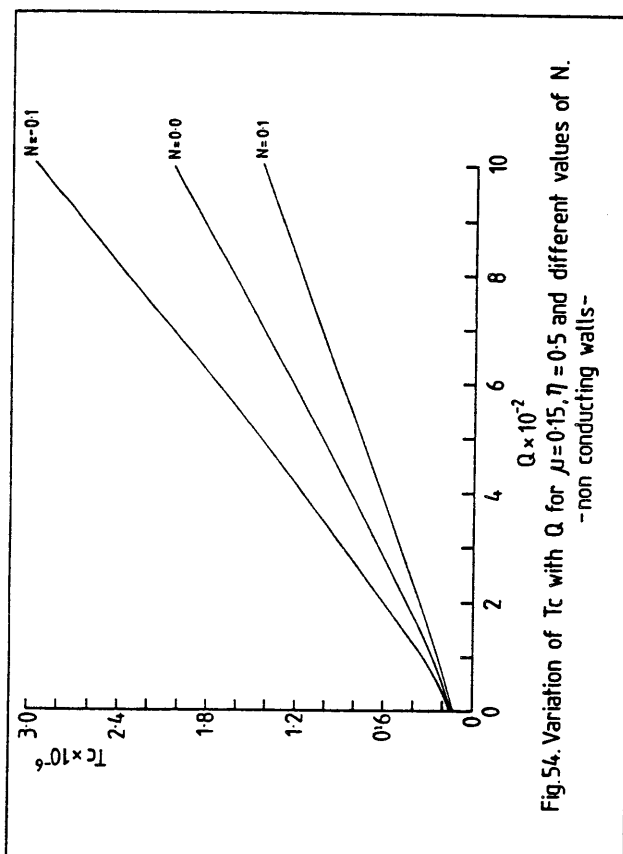
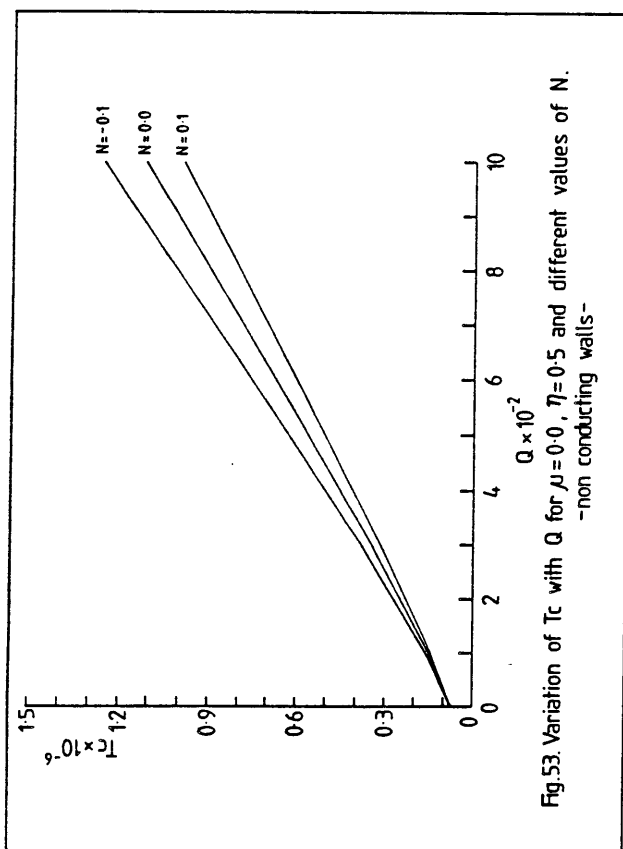


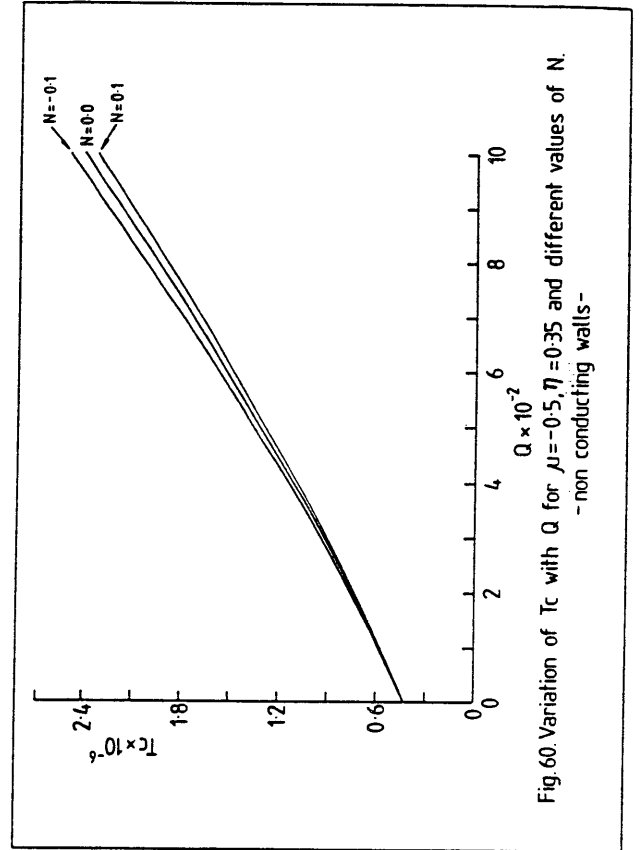
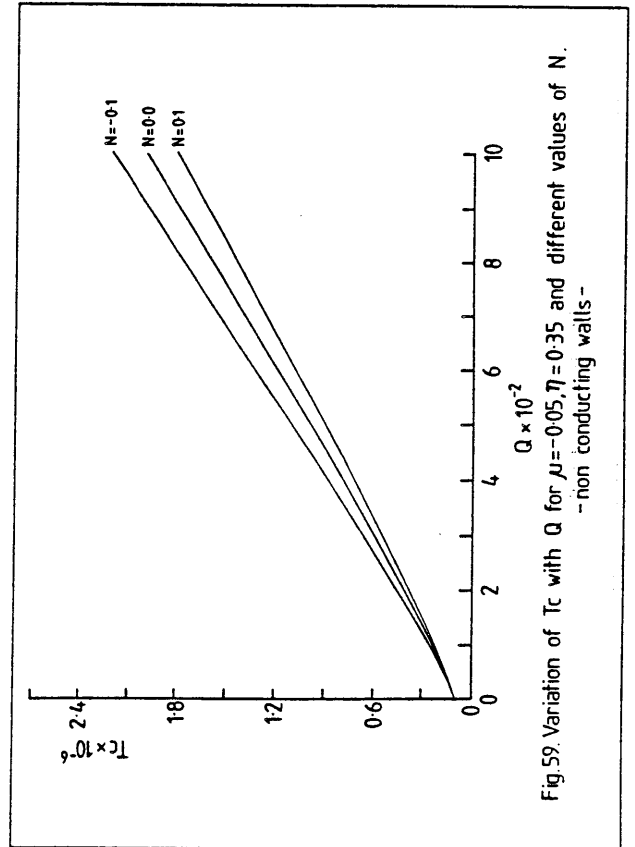
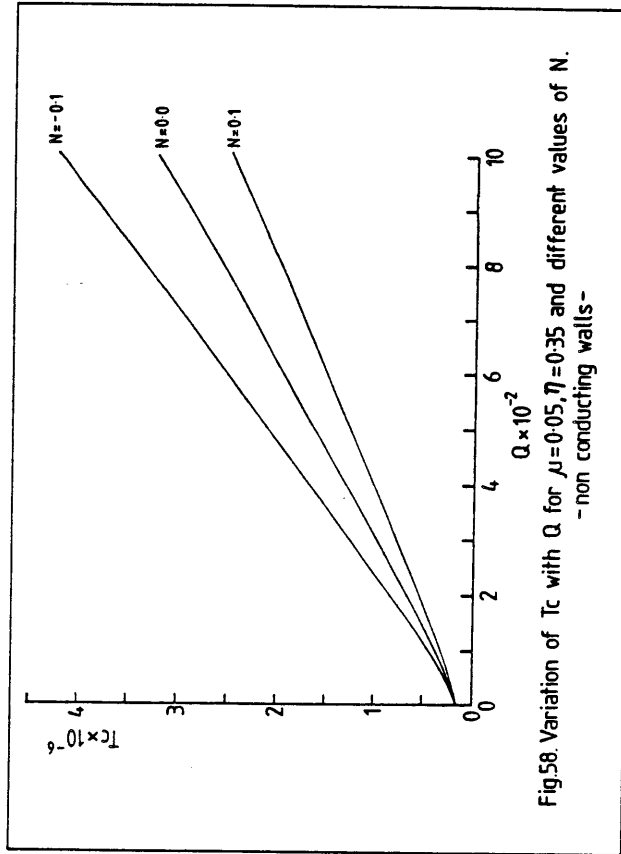
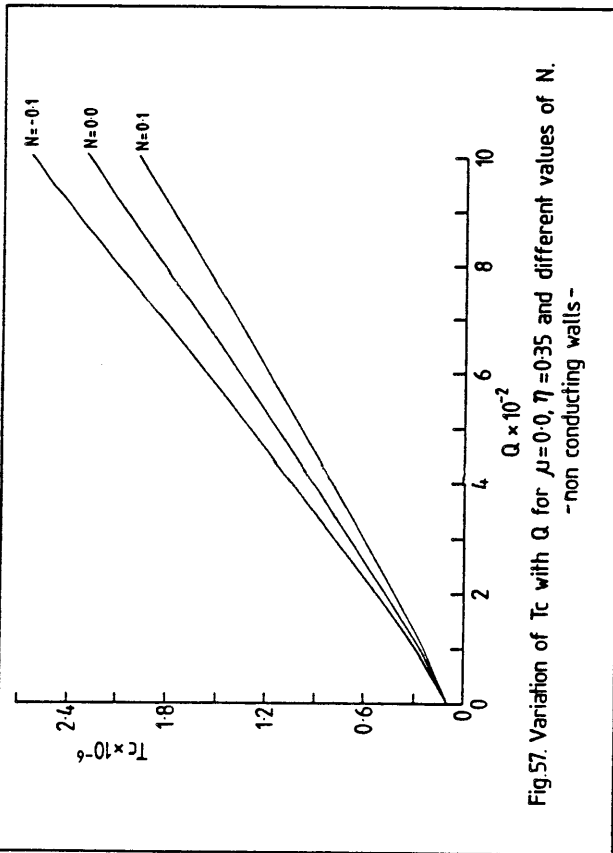


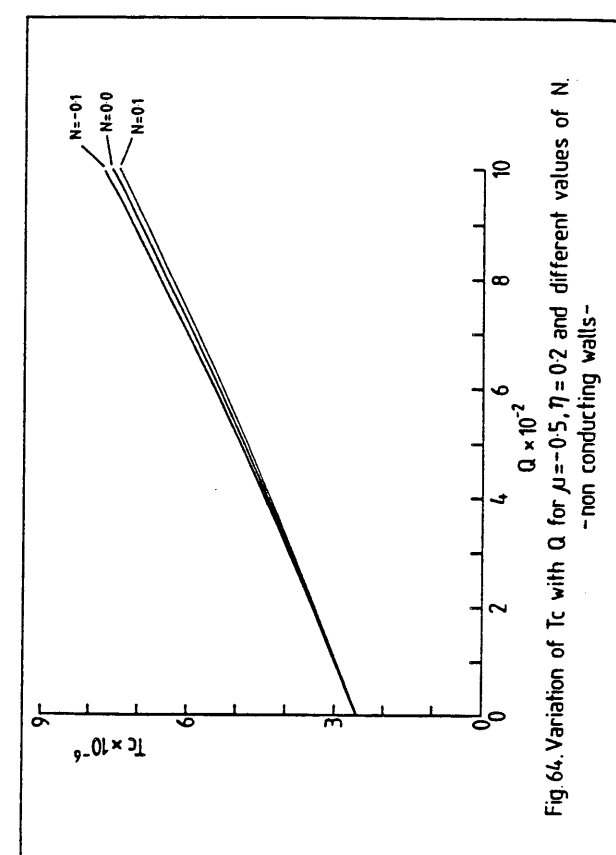
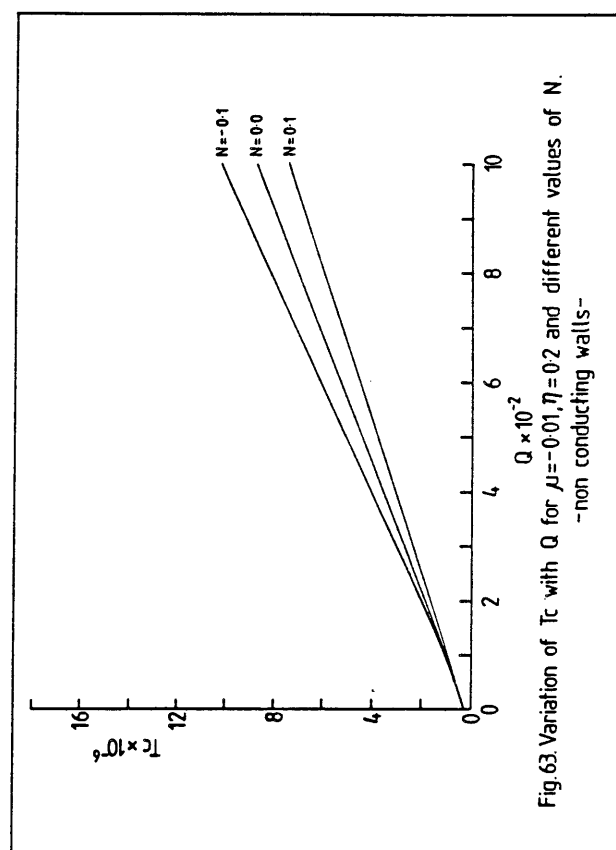
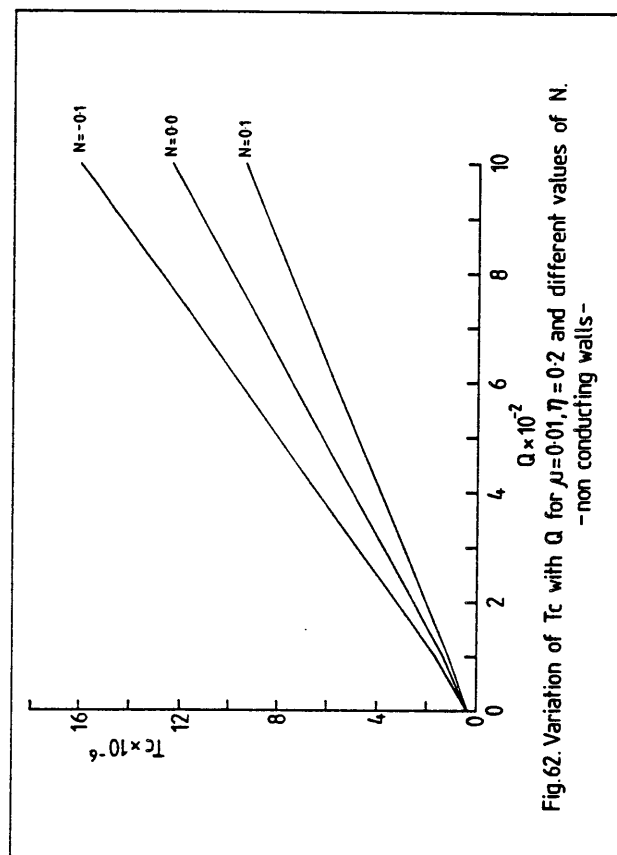
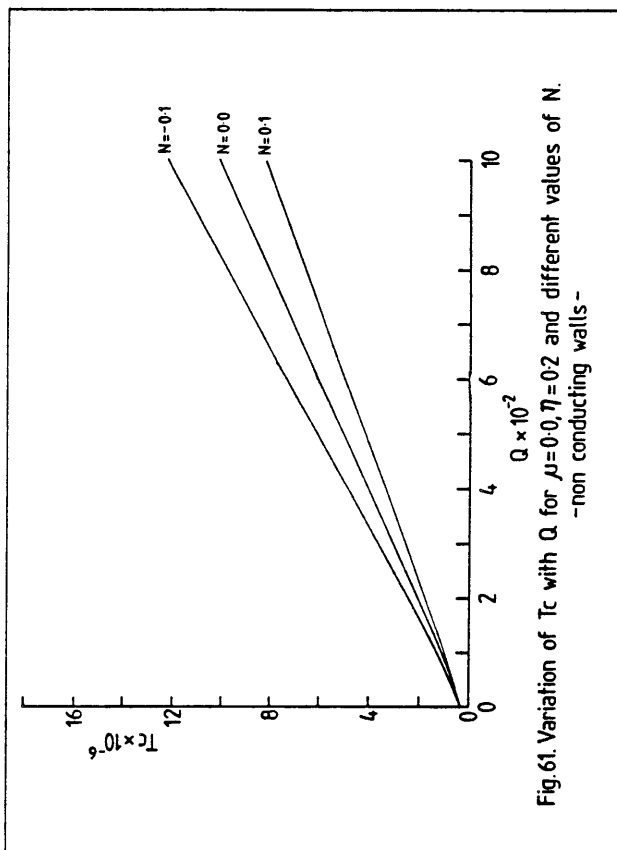


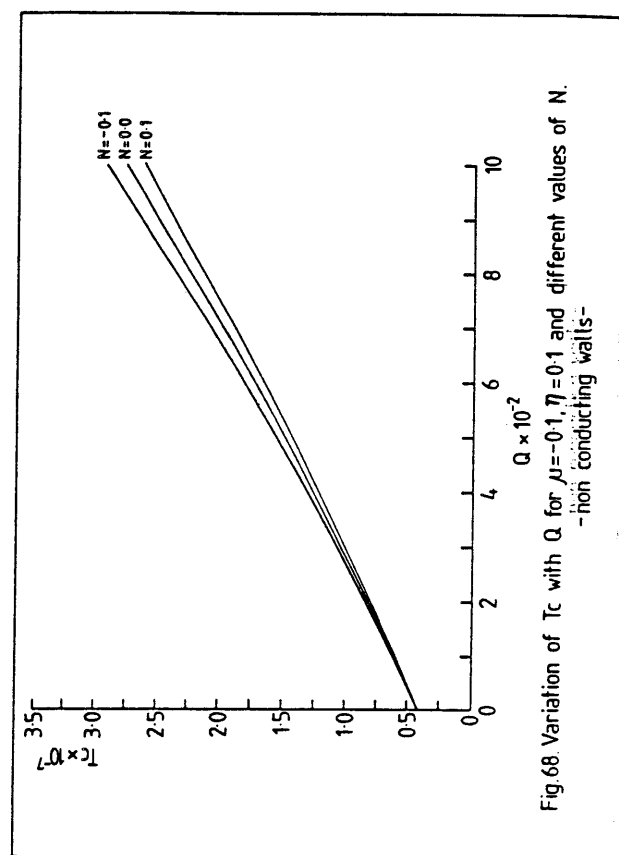
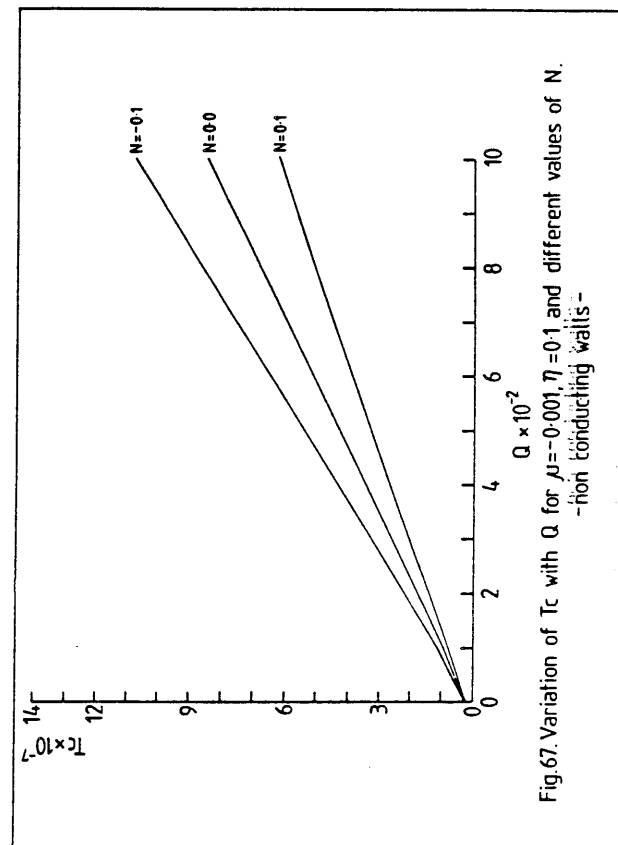
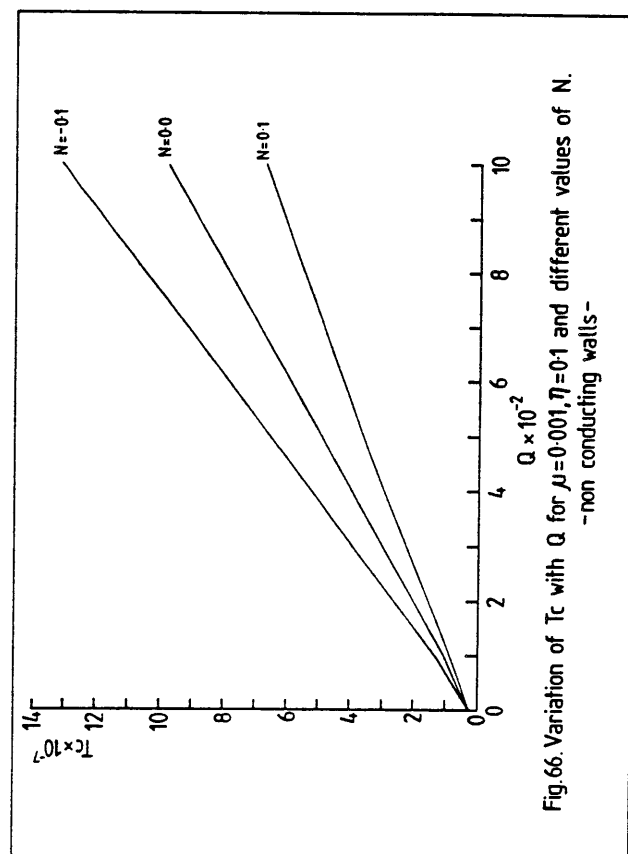
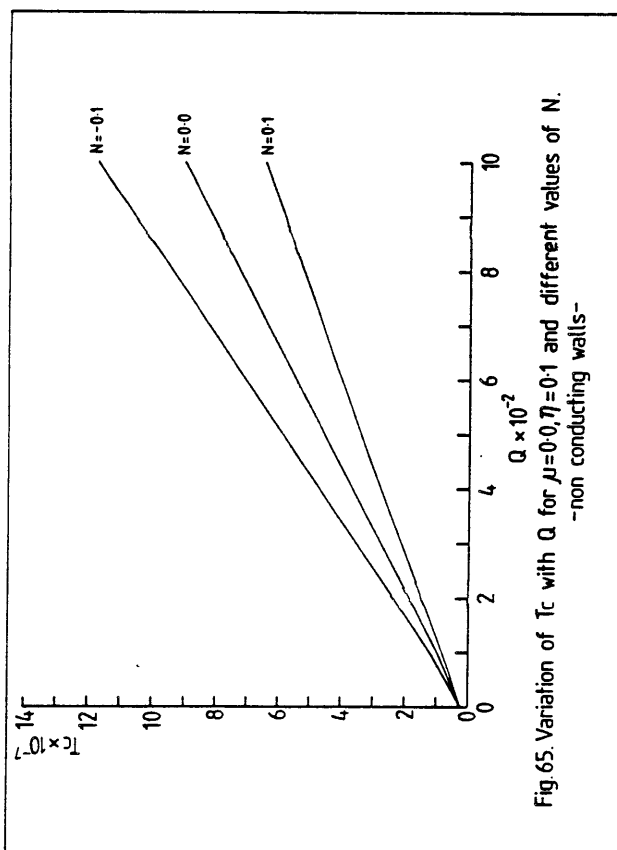


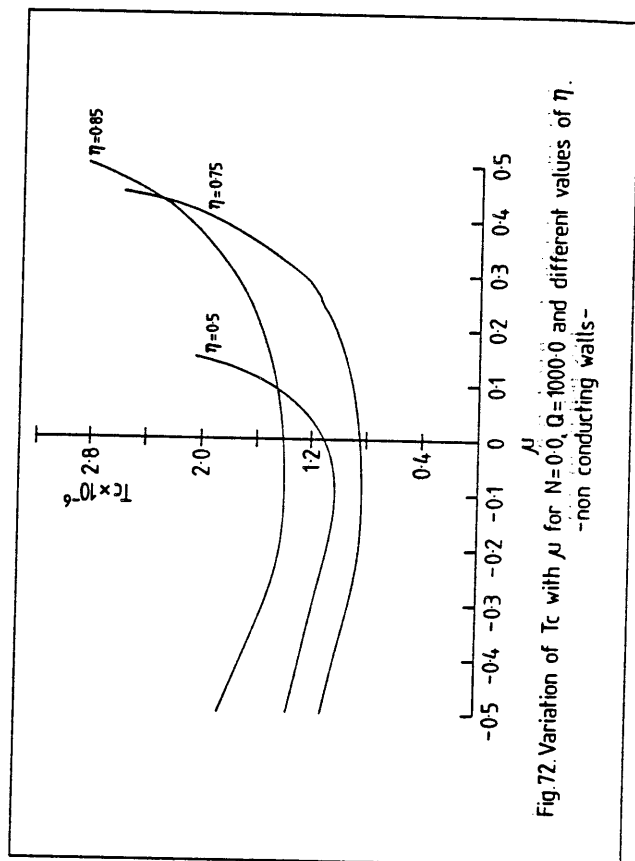
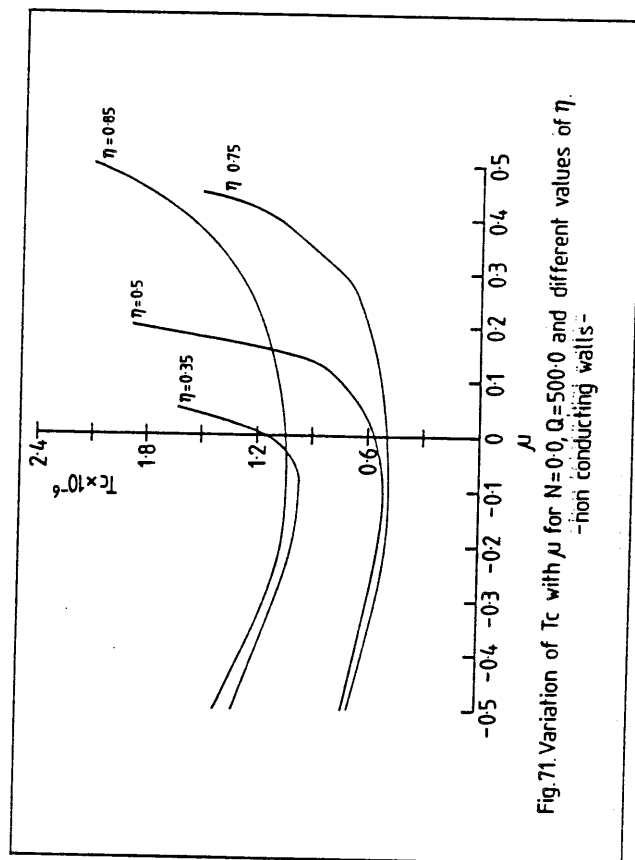
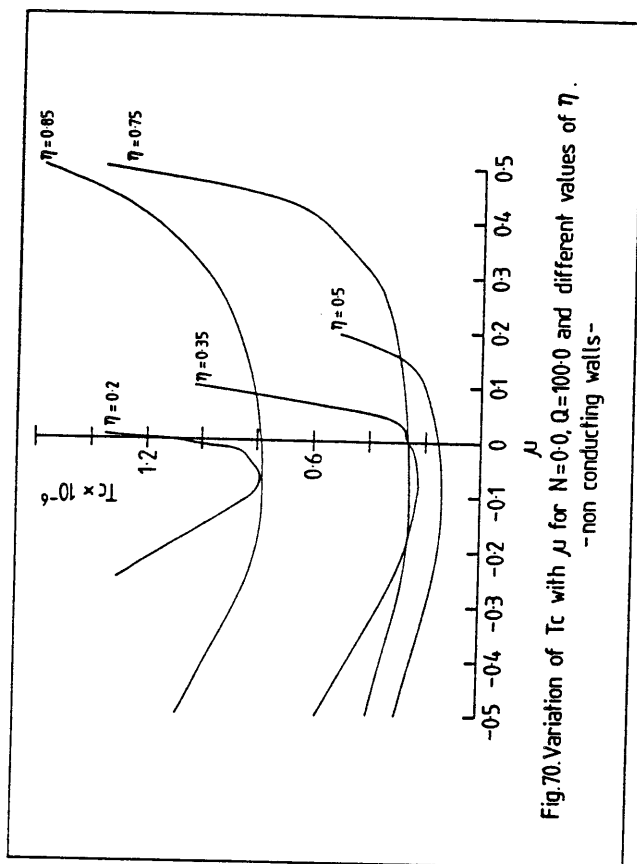
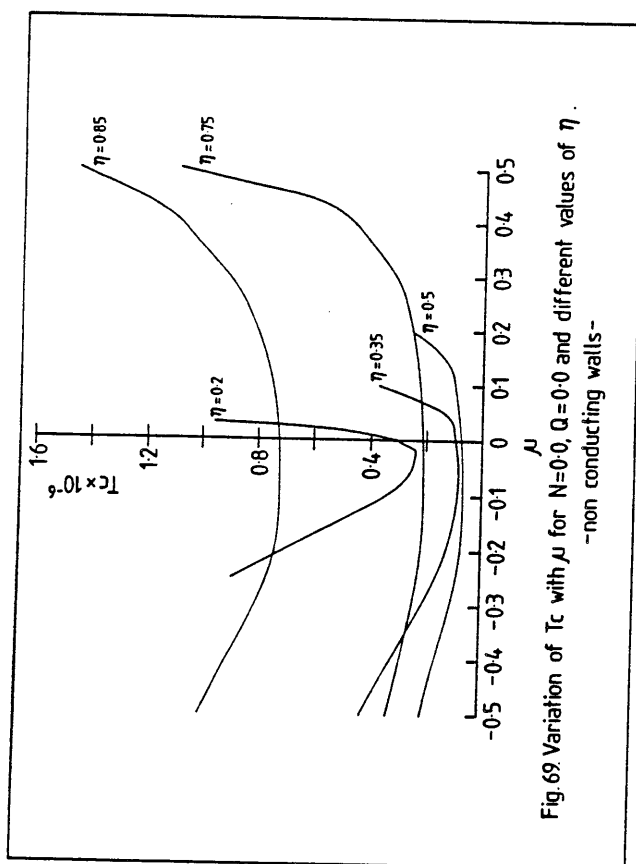


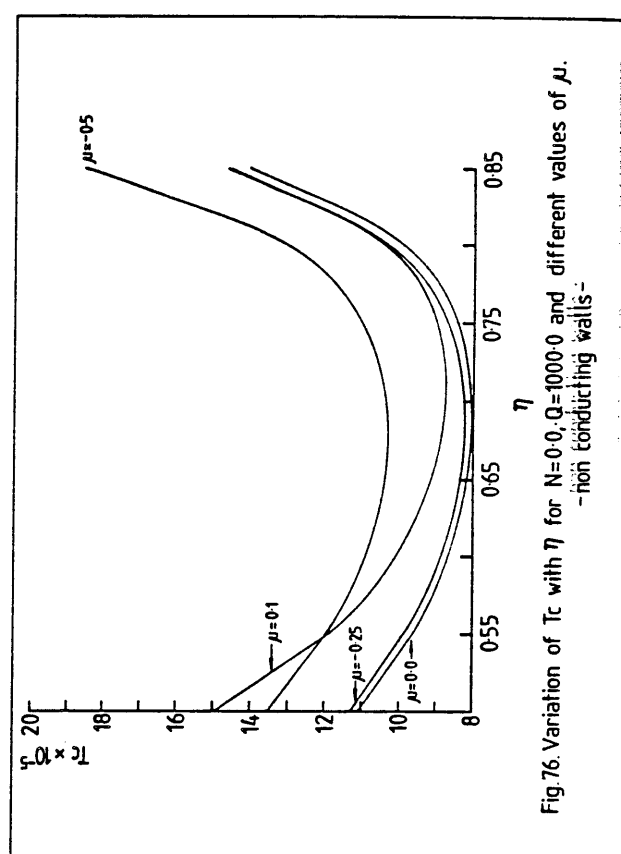
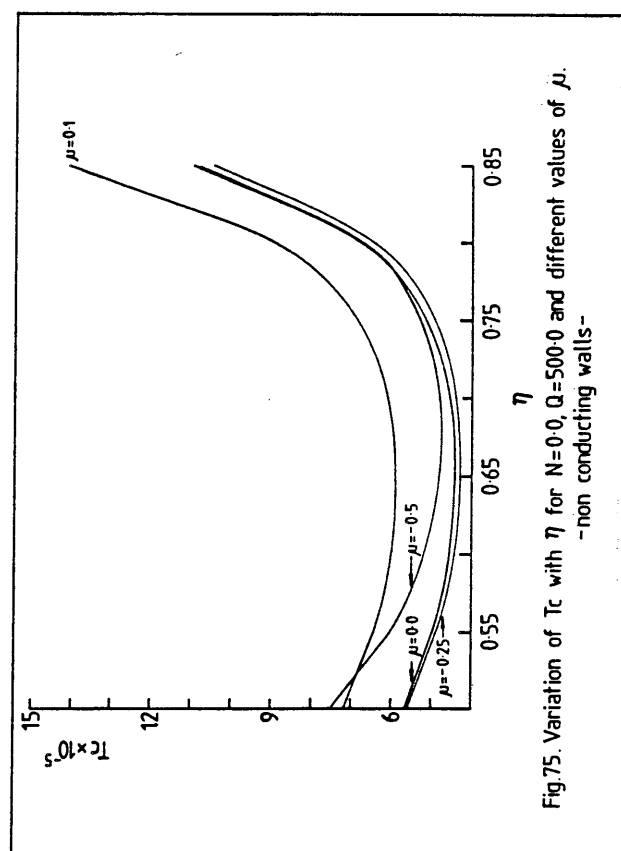
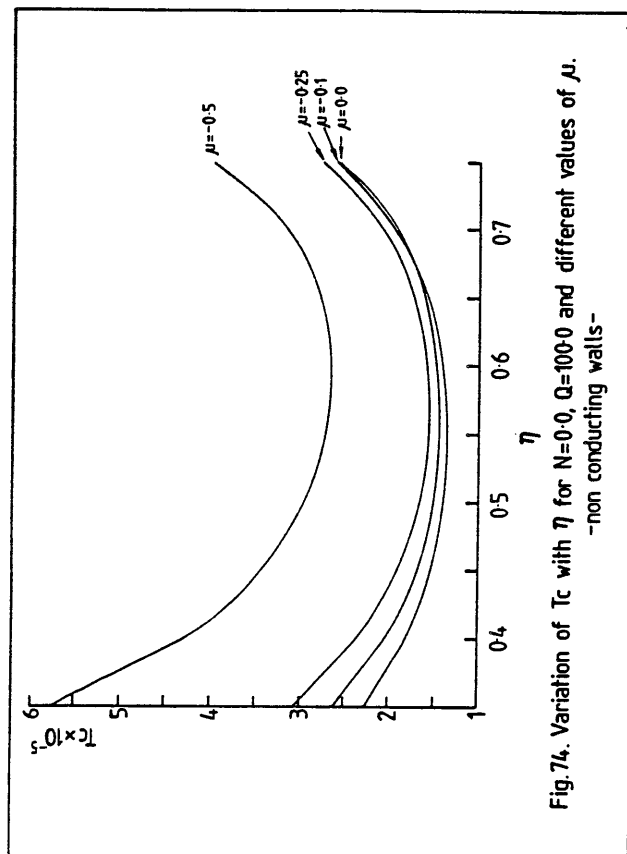
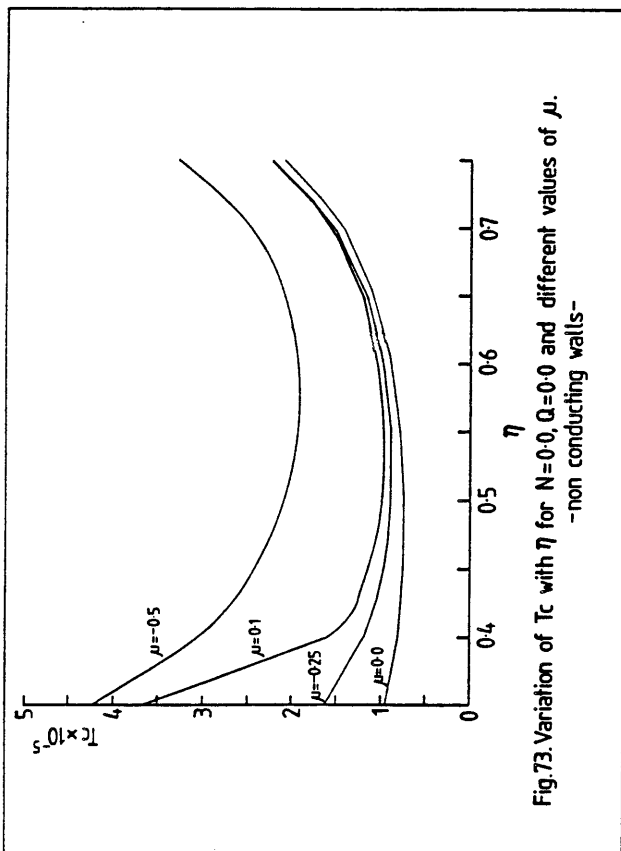












APPENDIX A

A.1 A PROGRAM USED TO STUDY THE EFFECT OF CONSTANT HEAT FLUX AT THE INNER CYLINDER ON THE STABILITY OF COUETTE FLOW

A.1.1 -NARROW GAP PROBLEM-

A PROGRAM TO CALCULATE THE CRITICAL TAYLOR NUMBER(TC) AND THE CORRESPONDING WAVE NUMBER(AC) FOR ASSIGNED VALUES OF EMU(M), AN(N)

PROGRAM CHFIMC(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

COMMON /AR7/ EMU,AN

EXTERNAL FUCH

THE ASSIGNED VALUES OF EMU,AN

EMU=0.5

AN=0.5

SET UP THE PARAMETERS TO FIND THE MINIMUM TC OVER ALL

REAL POSITIVE AC

AD=3.0

BD=5.0

E1=0.00001

E2=0.00001

MAX=50

IFAIL=1

CALLING A SUBROUTINE WHICH FIND THE MINIMUM TC

CALL DO4ABF(FUCH,E1,E2,AD,BD,MAX,AC,TC,IFAIL)

THE VALUES OF TC AND AC FOR THE ASSIGNED VALUES OF

EMU,AN

WRITE(6,11)EMU,AN,AC,TC

FORMAT(' EMU=',F7.4,2X,' AN=',F4.2,2X,' AC=',

+F5.3,2X,' TC=',F3.1)

END

SUBROUTINE FUCH(XC,FC)

REAL XC,FC

COMMON /AR3/ EX,N,T(3)

REAL Y(32),W(32,32),WKSPAC(80),D(5,5),DT(3)

COMMON /AR1/ XEND,R,ICN

COMMON /AR7/ EMU,AN

EXTERNAL FCN,OUT

INITIALIZATION OF THE PARAMETERS FOR BISECTION METHOD

D=1

DLI=800.0

T(1)=3000.0

ICI=1

T(2)=T(1)+DLI

ICI=ICT+1

SET UP THE PARAMETERS FOR INTEGRATING THE SYSTEM OF FIRST

ORDER EQUATIONS

X=0.0

XEND=1.0

R=(XEND-X)/20.0

ICN=19

TOL=1.0E-5

IRE=0

NEQ=32

THE INITIAL CONDITIONS

DO 2 I=1,NEQ

Y(I)=0.0

Y(9)=1.0

Y(14)=1.0

Y(23)=1.0

Y(28)=1.0

IFAIL=0

EX=XC

CALLING A SUBROUTINE WHICH INTEGRATE THE SYSTEM OF

FIRST ORDER EQUATIONS

CALL DO2DBF(X,XEND,NEQ,Y,TOL,IRE,FCN,OUT,W,IFAIL)

SET UP ARRAY FOR EVALUATION OF DETERMINANT

```
D(1,1)=Y(1)
D(1,2)=Y(2)
D(1,3)=Y(3)
D(1,4)=Y(4)
D(2,1)=Y(5)
D(2,2)=Y(6)
D(2,3)=Y(7)
D(2,4)=Y(8)
D(3,1)=Y(17)
D(3,2)=Y(18)
D(3,3)=Y(19)
D(3,4)=Y(20)
D(4,1)=Y(25)
D(4,2)=Y(26)
D(4,3)=Y(27)
D(4,4)=Y(28)
```

IB=5

IN=4

IFAIL=1

CALLING A SUBROUTINE WHICH EVALUATE THE DETERMINANT
CALL F03AAF(D,IB,IN,DET,WKSPAC,IFAIL)

IF (IFAIL .EQ. 1)DET=0.0

SET UP THE BISECTION METHOD TO DETERMINE THE TC FOR
A GIVEN WAVE NUMBER EX

DT(N)=DET

N=N+1

IF(N.EQ.2)GO TO 20

IF(ABS(DET) .LT. 0.000001)GO TO 30

IF(ICT .GT.120)GO TO 40

IF(DT(2) .GT. 0.0 .AND. DT(1) .LT. 0.0)GO TO 50

IF(DT(1) .GT. 0.0 .AND. DT(2) .LT. 0.0)GO TO 50

T(1)=T(2)

DT(1)=DT(2)

N=2

GO TO 20

DLT=0.5*DLT

IF(ABS(DET) .LT. 0.000001)GO TO 30

IF(DLT .LT. 0.01)GO TO 30

N=2

ICT=1

GO TO 20

WRITE THE VALUES OF THE PARAMETERS IF CONVERGENCE
OF THE DETERMINANT IS NOT OBTAINED

0 WRITE(6,222)ICT,I(N),DLT,DET,EX

2 FORMAT(' ICT=',I4,2X,' TA=',F14.4,2X,' DLT=',F18.5,
+2X,' DET=',F18.5,2X,' EX=',F6.3)

THE VALUE OF THE TAYLOR NUMBER FOR THE WAVE NUMBER XC

0 FC=T(2)

RETURN

END

SUBROUTINE FCN(S,Y,F)

REAL S

REAL F(32),Y(32)

COMMON /AR3/ EX,N,T(3)

COMMON /AR7/ EMU,AM

EX2=EX*EX

EX4=EX2*EX2

GX=1.0-(1.0-EMU)*S

PT=-EX2*T(N)*GX

RT=-PT*AM*GX

EVALUATING THE DERIVATIVES

F(1)=Y(5)

F(2)=Y(6)

```

F(3)=Y(7)
F(4)=Y(8)
F(5)=Y(9)
F(6)=Y(10)
F(7)=Y(11)
F(8)=Y(12)
F(9)=Y(13)
F(10)=Y(14)
F(11)=Y(15)
F(12)=Y(16)
F(13)=2.0*EX2*Y(9)-EX4*Y(1)+PT*Y(17)+RT*Y(25)
F(14)=2.0*EX2*Y(10)-EX4*Y(2)+PT*Y(18)+RT*Y(26)
F(15)=2.0*EX2*Y(11)-EX4*Y(3)+PT*Y(19)+RT*Y(27)
F(16)=2.0*EX2*Y(12)-EX4*Y(4)+PT*Y(20)+RT*Y(28)
F(17)=Y(21)
F(18)=Y(22)
F(19)=Y(23)
F(20)=Y(24)
F(21)=EX2*Y(17)+Y(1)
F(22)=EX2*Y(18)+Y(2)
F(23)=EX2*Y(19)+Y(3)
F(24)=EX2*Y(20)+Y(4)
F(25)=Y(29)
F(26)=Y(30)
F(27)=Y(31)
F(28)=Y(32)
F(29)=EX2*Y(25)+Y(1)
F(30)=EX2*Y(26)+Y(2)
F(31)=EX2*Y(27)+Y(3)
F(32)=EX2*Y(28)+Y(4)
RETURN
END
SUBROUTINE OUT(X,Y)
REAL Y(32)
COMMON /AR2/ XEND,R,ICN
X=XEND+FLOAT(ICN)*R
ICN=ICN-1
RETURN
END

```

A.1 A PROGRAM USED TO STUDY THE EFFECT OF CONSTANT HEAT FLUX AT THE INNER CYLINDER ON THE STABILITY OF COUETTE FLOW

A.1.2 -WIDE GAP PROBLEM-

A PROGRAM TO CALCULATE THE CRITICAL TAYLOR NUMBER(TC) AND THE CORRESPONDING WAVE NUMBER(AC) FOR ASSIGNED VALUES OF ETA(η), EMU(μ), AN(N)

PROGRAM CHFNCW(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

COMMON /AR7/ ETA,EMU,AN

EXTERNAL FUCN

THE ASSIGNED VALUES OF ETA,EMU,AN

ETA=0.35

EMU=0.0

AN=-0.5

SET UP THE PARAMETERS TO FIND THE MINIMUM TC OVER ALL REAL POSITIVE AC

AD=3.0

BD=4.0

E1=0.00001

E2=0.00001

MAX=50

IFAIL=1

CALLING A SUBROUTINE WHICH FIND THE MINIMUM TC

CALL EO4ASF(FUCN,E1,E2,AD,BD,MAX,AC,TC,IFAIL)

```

THE VALUES OF TC AND AC FOR THE ASSIGNED VALUES OF
ETA,EMU,AN
WRITE(6,111)ETA,EMU,AN,AC,TC
1 FORMAT(' ETA=',F4.2,2X,' EMU=',F7.4,2X,' AN=',
+F6.3,2X,' AC=',F6.3,2X,' TC=',E14.4)
END
SUBROUTINE FUCH(XC,FC)
REAL XC,FC
COMMON /AR3/ EX,M,I(3)
REAL Y(32),W(80,80),WKSPAC(80),D(5,5),DT(3)
COMMON /AR2/ XEND,R,ICM
COMMON /AR7/ ETA,EMU,AN
EXTERNAL FCN,OUT
INITIALIZATION OF THE PARAMETERS FOR BISECTION METHOD
M=1
DLT=20000.0
T(1)=80000.0
ICT=1
0 ICT(2)=T(1)+DLT
ICT=ICT+1
SET UP THE PARAMETERS FOR INTEGRATING THE SYSTEM OF FIRST
ORDER EQUATIONS
X=0.0
XEND=1.0
R=(XEND-X)/20.0
TCM=19
TOL=1.0E-5
IRE=0
NEQ=32
THE INITIAL CONDITIONS
DO 2 I=1,N+Q
2 Y(I)=0.0
Y(9)=1.0
Y(14)=1.0
Y(23)=1.0
Y(28)=1.0
IFAIL=0
EX=XC
CALLING A SUBROUTINE WHICH INTEGRATE THE SYSTEM OF
FIRST ORDER EQUATIONS
CALL DOZBBF(X,XEND,N+Q,Y,TOL,IRE,FCN,OUT,W,IFAIL)
SET UP ARRAY FOR EVALUATION OF DETERMINENT
D(1,1)=Y(1)
D(1,2)=Y(2)
D(1,3)=Y(3)
D(1,4)=Y(4)
D(2,1)=Y(5)
D(2,2)=Y(6)
D(2,3)=Y(7)
D(2,4)=Y(8)
D(3,1)=Y(17)
D(3,2)=Y(16)
D(3,3)=Y(19)
D(3,4)=Y(26)
D(4,1)=Y(25)
D(4,2)=Y(26)
D(4,3)=Y(27)
D(4,4)=Y(28)
ID=5
IN=4
IFAIL=1
CALLING A SUBROUTINE WHICH EVALUATE THE DETERMINENT
CALL FOJAAF(D,ID,IN,DET,WKSPAC,IFAIL)
IF (IFAIL .EQ. 1)DET=0.0
SET UP THE BISECTION METHOD TO DETERMINE THE TC FOR
A GIVEN WAVE NUMBER EX
DT(N)=DET

```

```

N=N+1
IF(N.EQ.2)GO TO 20
IF(ABS(DET) .LT. 0.000001)GO TO 30
IF(ICT .GT.120)GO TO 40
IF(DI(2) .GT. 0.0 .AND. DI(1) .LT. 0.0)GO TO 50
IF(DI(1) .GT. 0.0 .AND. DI(2) .LT. 0.0)GO TO 50
T(1)=T(2)
DT(1)=DT(2)
N=2
GO TO 20
DLT=0.5*DLT
IF(ABS(DET) .LT. 0.000001)GO TO 30
IF(DLT .LT. 0.0001)GO TO 30
N=2
ICT=1
GO TO 20
WRITE THE VALUES OF THE PARAMETERS IF CONVERGENCE
OF THE DETERMINANT IS NOT OBTAINED
0 WRITE(6,222)ICT,T(1),DLT,DET,EX
2 FORMAT(' ICT=' ,I4,2X,' TA=' ,F14.4,2X,' DLT=' ,F18.5,
+2X,' DET=' ,F18.5,2X,' EX=' ,F6.3)
THE VALUE OF THE TAYLOR NUMBER FOR THE WAVE NUMBER XC
0 FC=T(2)
RETURN
END
SUBROUTINE FCM(S,Y,F)
REAL S
REAL F(32),Y(32)
COMMON /AR5/ EX,M,T(2)
COMMON /AR7/ ETA,EMU,AM
EX2=EX*EX
EX4=EX2*EX2
AX=(1.0-ETA)*S+ETA
AY=1.0/AX
AY2=AY*AY
AM=(1.0-ETA)/AX
ET2=ETA*ETA
A1=(EMU-ET2)/(1.0-ET2)
B1=ET2*(1.0-EMU)/(1.0-ET2)
B2=B1*AY2
GX=A1+B2
GX2=GX*GX
PT=EX2*4*A1*((1.0-ETA)**4)*T(N)*GX
RT=-EX2*((1.0-ETA)**4)*T(N)*ETA*AM*AX*GX2
EVALUATING THE DERIVATIVES
F(1)=Y(5)-AM*Y(1)
F(2)=Y(6)-AM*Y(2)
F(3)=Y(7)-AM*Y(3)
F(4)=Y(8)-AM*Y(4)
F(5)=Y(9)+EX2*Y(1)
F(6)=Y(10)+EX2*Y(2)
F(7)=Y(11)+EX2*Y(3)
F(8)=Y(12)+EX2*Y(4)
F(9)=Y(12)-AM*Y(9)
F(10)=Y(14)-AM*Y(10)
F(11)=Y(15)-AM*Y(11)
F(12)=Y(16)-AM*Y(12)
F(13)=EX2*Y(9)+PT*Y(17)+RT*Y(25)
F(14)=EX2*Y(10)+PT*Y(18)+RT*Y(26)
F(15)=EX2*Y(11)+PT*Y(19)+RT*Y(27)
F(16)=EX2*Y(12)+PT*Y(20)+RT*Y(28)
F(17)=Y(21)-AM*Y(17)
F(18)=Y(22)-AM*Y(18)
F(19)=Y(23)-AM*Y(19)
F(20)=Y(24)-AM*Y(20)
F(21)=EX2*Y(17)+Y(1)

```

```

F(22)=_X2*Y(18)+Y(2)
F(23)=EX2*Y(19)+Y(3)
F(24)=_X2*Y(20)+Y(4)
F(25)=Y(29)
F(26)=Y(30)
F(27)=Y(31)
F(28)=Y(32)
F(29)=AM*Y(29)+EX2*Y(25)+AY*Y(1)
F(30)=AM*Y(30)+EX2*Y(26)+AY*Y(2)
F(31)=AM*Y(31)+EX2*Y(27)+AY*Y(3)
F(32)=AM*Y(32)+EX2*Y(28)+AY*Y(4)
RETURN
END
SUBROUTINE OUT(X,Y)
REAL Y(32)
COMMON /AR2/ XEND,R,ICM
X=XEND+FLOAT(ICM)*R
ICM=ICM-1
RETURN
END

```

A.2 A PROGRAM USED TO STUDY THE EFFECT OF RADIAL TEMPERATURE GRADIENT AND AXIAL MAGNETIC FIELD ON THE STABILITY OF COUETTE FLOW -CONDUCTING WALLS-

A.2.1 -NARROW GAP PROBLEM-

A PROGRAM TO CALCULATE THE CRITICAL TAYLOR NUMBER(IC) AND THE CORRESPONDING WAVE NUMBER(AC) FOR ASSIGNED VALUES OF EMU(μ), AN(N), Q "

PROGRAM MHSTGCM(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

COMMON /AR7/ EMU,AN,Q

EXTERNAL FUCH

THE ASSIGNED VALUES OF EMU,AN,Q

EMU=0.5

AN=0.1

Q=50.0

SET UP THE PARAMETERS TO FIND THE MINIMUM IC OVER ALL REAL POSITIVE AC

AD=3.0

BD=4.0

E1=0.00001

E2=0.00001

MAX=50

IFAIL=1

CALLING A SUBROUTINE WHICH FIND THE MINIMUM IC

CALL E04ABF(FUCH,E1,E2,AD,BD,MAX,AC,IC,IFAIL)

THE VALUES OF IC AND AC FOR THE ASSIGNED VALUES OF EMU,AN,Q

WRITE(6,111)EMU,AN,Q,AC,IC

1 FORMAT(' EMU=',F4.2,2X, ' AN=',F5.3,2X, ' Q=',F5.1,2X, ' AC=',F6.3,2X, ' IC=',F9.1)

END

SUBROUTINE FUCH(XC,FC)

REAL XC,FC

COMMON /AR3/ EX,N,T(3)

REAL Y(50),W(50,50),EXSPAC(80),D(6,6),DT(3)

COMMON /AR2/ XEND,R,ICM

COMMON /AR7/ EMU,AN,Q

EXTERNAL FCN,OLT

INITIALIZATION OF THE PARAMETERS FOR BISECTION METHOD

N=1

DET=0.0

DLT=2000.0

T(1)=4000.0

```

0 I(2)=Y(1)+DLI
  ICT=ICT+1
  SET UP THE PARAMETERS FOR INTEGRATING THE SYSTEM OF FIRST
  ORDER EQUATIONS
  X=0.0
  XEND=1.0
  R=(X-NC-X)/20.0
  ICM=19
  TOL=1.0E-5
  IRE=0
  NEQ=50
  THE INITIAL CONDITIONS
  DO 2 I=1,NEQ
2 Y(I)=0.0
  Y(11)=1.0
  Y(17)=1.0
  Y(23)=1.0
  Y(39)=1.0
  Y(50)=1.0
  IFAIL=0
  EX=XC
  CALLING A SUBROUTINE WHICH INTEGRATE THE SYSTEM OF
  FIRST ORDER EQUATIONS
  CALL D02BBF(X,XEND,NEQ,Y,TOL,IRE,FCN,OUT,W,IFAIL)
  SET UP ARRAY FOR EVALUATION OF DETERMINANT
  D(1,1)=Y(1)
  D(1,2)=Y(2)
  D(1,3)=Y(3)
  D(1,4)=Y(4)
  D(1,5)=Y(5)
  D(2,1)=Y(6)
  D(2,2)=Y(7)
  D(2,3)=Y(8)
  D(2,4)=Y(9)
  D(2,5)=Y(10)
  D(3,1)=Y(26)
  D(3,2)=Y(27)
  D(3,3)=Y(28)
  D(3,4)=Y(29)
  D(3,5)=Y(30)
  D(4,1)=Y(31)
  D(4,2)=Y(32)
  D(4,3)=Y(33)
  D(4,4)=Y(34)
  D(4,5)=Y(35)
  D(5,1)=Y(41)
  D(5,2)=Y(42)
  D(5,3)=Y(43)
  D(5,4)=Y(44)
  D(5,5)=Y(45)
  IB=6
  IN=5
  IFAIL=1
  CALLING A SUBROUTINE WHICH EVALUATE THE DETERMINANT
  CALL F03AAF(D,IB,IN,DET,WKSPAC,IFAIL)
  IF (IFAIL .EQ. 1) DET=0.0
  SET UP THE DISSECTION METHOD TO DETERMINE THE IC FOR
  A GIVEN WAVE NUMBER EX
  DT(N)=DET
  N=N+1
  IF(N.EQ.2) GO TO 20
  IF(ABS(DET) .LT. 0.000001) GO TO 30
  IF(ICT .GT. 120) GO TO 40
  IF(DT(2) .GT. 0.0 .AND. DT(1) .LT. 0.0) GO TO 50
  IF(DT(1) .GT. 0.0 .AND. DT(2) .LT. 0.0) GO TO 50

```

```

F(37)=EX2*Y(32)-Q*EX2*Y(22)+Y(2)
F(38)=EX2*Y(33)-Q*EX2*Y(23)+Y(3)
F(39)=EX2*Y(34)-Q*EX2*Y(24)+Y(4)
F(40)=EX2*Y(35)-Q*EX2*Y(25)+Y(5)
F(41)=Y(46)
F(42)=Y(47)
F(43)=Y(48)
F(44)=Y(49)
F(45)=Y(50)
F(46)=EX2*Y(41)+Y(1)
F(47)=EX2*Y(42)+Y(2)
F(48)=EX2*Y(43)+Y(3)
F(49)=EX2*Y(44)+Y(4)
F(50)=EX2*Y(45)+Y(5)
RETURN
END
SUBROUTINE OUT(X,Y)
REAL Y(50)
COMMON /AR2/ XEND,Q,ICM
X=XEND+FLOAT(ICM)*R
ICM=ICM-1
RETURN
END

```

A.2 A PROGRAM USED TO STUDY THE EFFECT OF RADIAL TEMPERATURE GRADIENT AND AXIAL MAGNETIC FIELD ON THE STABILITY OF COUETTE FLOW -CONDUCTING WALLS-

A.2.2 -WIDE GAP PROBLEM-

A PROGRAM TO CALCULATE THE CRITICAL TAYLOR NUMBER(TC) AND THE CORRESPONDING WAVE NUMBER(AC) FOR ASSIGNED VALUES OF ETA(η), EMU(μ), AN(N), Q

PROGRAM MHOTGCM(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

COMMON /AR7/ ETA,EMU,AN,Q

EXTERNAL FUCN

THE ASSIGNED VALUES OF ETA,EMU,AN,Q

ETA=0.5

EMU=0.0

AN=0.1

Q=100.0

SET UP THE PARAMETERS TO FIND THE MINIMUM TC OVER ALL REAL POSITIVE AC

AD=3.0

BD=5.0

E1=0.00001

E2=0.00001

MAX=50

IFAIL=1

CALLING A SUBROUTINE WHICH FIND THE MINIMUM TC

CALL E04ABF(FUCN,E1,E2,AD,BD,MAX,AC,TC,IFAIL)

THE VALUES OF TC AND AC FOR THE ASSIGNED VALUES OF

ETA,EMU,AN,Q

WRITE(6,111)ETA,EMU,AN,Q,AC,TC

1 FORMAT(' ETA=',F4.2,2X,' EMU=',F7.4,2X,' AN=',

+F6.3,2X,' Q=',F6.1,2X,' AC=',F6.3,2X,' TC=',E14.4)

END

SUBROUTINE FUCN(XC,FC)

REAL XC,FC

COMMON /AR3/ EX,M,I(3)

REAL Y(50),W(50,50),WK,PAC(80),D(6,6),DT(3)

COMMON /AR2/ XEND,R,ICM

COMMON /AR7/ ETA,EMU,AN,Q

EXTERNAL FCN,OUT

```

W=1
DLT=30000.0
T(1)=120000.0
ICT=1
0 T(2)=T(1)+DLT
ICT=ICT+1
SET UP THE PARAMETERS FOR INTEGRATING THE SYSTEM OF FIRST
ORDER EQUATIONS
X=0.0
XEND=1.0
R=(XEND-X)/20.0
ICB=19
TOL=1.0E-5
IRE=0
NEQ=50
THE INITIAL CONDITIONS
00 2 I=1,NLQ
2 Y(I)=0.0
Y(11)=1.0
Y(17)=1.0
Y(23)=1.0
Y(39)=1.0
Y(50)=1.0
IFAIL=0
EX=XC
CALLING A SUBROUTINE WHICH INTEGRATE THE SYSTEM OF
FIRST ORDER EQUATIONS
CALL D02BBF(X,XEND,NEQ,Y,TOL,IRE,FCN,OUT,W,IFAIL)
SET UP ARRAY FOR EVALUATION OF DETERMINANT
D(1,1)=Y(1)
D(1,2)=Y(2)
D(1,3)=Y(3)
D(1,4)=Y(4)
D(1,5)=Y(5)
D(2,1)=Y(6)
D(2,2)=Y(7)
D(2,3)=Y(8)
D(2,4)=Y(9)
D(2,5)=Y(10)
D(3,1)=Y(26)
D(3,2)=Y(27)
D(3,3)=Y(28)
D(3,4)=Y(29)
D(3,5)=Y(30)
D(4,1)=Y(31)
D(4,2)=Y(32)
D(4,3)=Y(33)
D(4,4)=Y(34)
D(4,5)=Y(35)
D(5,1)=Y(41)
D(5,2)=Y(42)
D(5,3)=Y(43)
D(5,4)=Y(44)
D(5,5)=Y(45)
IB=6
IH=5
IFAIL=1
CALLING A SUBROUTINE WHICH EVALUATE THE DETERMINANT
CALL F03AAF(D,IB,IH,DET,WKSPAC,IFAIL)
IF (IFAIL .EQ. 1) DLT=0.0
SET UP THE BISECTION METHOD TO DETERMINE THE IC FOR
A GIVEN WAVE NUMBER EX
DT(N)=DET
N=N+1
IF(N.EQ.2) GO TO 20
IF(ABS(DET) .LT. 0.000001) GO TO 30

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IF( ICT .GT. 120 ) GO TO 40
IF( DT(2) .GT. 0.0 .AND. DT(1) .LT. 0.0 ) GO TO 50
IF( DT(1) .GT. 0.0 .AND. DT(2) .LT. 0.0 ) GO TO 50
T(1)=T(2)
DT(1)=DT(2)
N=2
GO TO 20
DLT=0.5*DLT
IF( ABS( DET ) .LT. 0.000001 ) GO TO 30
IF( DLT .LT. 0.01 ) GO TO 30
N=2
ICT=1
GO TO 20
WRITE THE VALUES OF THE PARAMETERS IF CONVERGENCE
OF THE DETERMINANT IS NOT OBTAINED
0 WRITE(6,222) ICT, T(N), DLT, DET, EX
2 FORMAT( ' ICT= ', I4, 2X, ' TA= ', F14.4, 2X, ' DLT= ', F18.5,
+ 2X, ' DET= ', F18.5, 2X, ' EX= ', F6.3 )
THE VALUE OF THE TAYLOR NUMBER FOR THE WAVE NUMBER XC
0 FC=T(2)
RETURN
END
SUBROUTINE FCN(S,Y,F)
REAL S
REAL F(50),Y(50)
COMMON /AR5/ EX,N,T(3)
COMMON /AR7/ ETA,EMU,AM,Q
EX2=EX*EX
EX4=EX2*EX2
AX=(1.0-LTA)*S+ETA
AY=1.0/AX
AY2=AY*AY
AM=(1.0-ETA)/AX
AM2=AM*AM
ETL=ETA*ETA
A1=(EMU-ETL)/(1.0-ET2)
B1=ET2*(1.0-EMU)/(1.0-ET2)
B2=B1*AY
GX=A1+B2
GX2=GX*GX
PT=EX2*4*A1*((1.0-ETA)**4)*T(N)*GX
RT=EX2*((1.0-LTA)**4)/ALOG(ETA))*T(N)*AM*AX*GX2
EVALUATING THE DERIVATIVES
F(1)=Y(6)
F(2)=Y(7)
F(3)=Y(8)
F(4)=Y(9)
F(5)=Y(10)
F(6)=Y(11)-AM*Y(6)+(AM2+EX2)*Y(1)
F(7)=Y(12)-AM*Y(7)+(AM2+EX2)*Y(2)
F(8)=Y(13)-AM*Y(8)+(AM2+EX2)*Y(3)
F(9)=Y(14)-AM*Y(9)+(AM2+EX2)*Y(4)
F(10)=Y(15)-AM*Y(10)+(AM2+EX2)*Y(5)
F(11)=Y(16)-AM*Y(11)
F(12)=Y(17)-AM*Y(12)
F(13)=Y(18)-AM*Y(13)
F(14)=Y(19)-AM*Y(14)
F(15)=Y(20)-AM*Y(15)
F(16)=EX2*Y(11)-((1-ETA)**2)*Q*EX2*Y(1)+PT*Y(31)+RT*Y(41)
F(17)=EX2*Y(12)-((1-ETA)**2)*Q*EX2*Y(2)+PT*Y(32)+RT*Y(42)
F(18)=EX2*Y(13)-((1-ETA)**2)*Q*EX2*Y(3)+PT*Y(33)+RT*Y(43)
F(19)=EX2*Y(14)-((1-ETA)**2)*Q*EX2*Y(4)+PT*Y(34)+RT*Y(44)
F(20)=EX2*Y(15)-((1-ETA)**2)*Q*EX2*Y(5)+PT*Y(35)+RT*Y(45)
F(21)=Y(26)-AM*Y(21)
F(22)=Y(27)-AM*Y(22)
F(23)=Y(28)-AM*Y(23)

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F(24)=Y(29)-AM*Y(24)
F(25)=Y(30)-AM*Y(25)
F(26)=Y(31)+EX2*Y(21)
F(27)=Y(32)+EX2*Y(22)
F(28)=Y(33)+EX2*Y(23)
F(29)=Y(34)+EX2*Y(24)
F(30)=Y(35)+EX2*Y(25)
F(31)=Y(36)-AM*Y(31)
F(32)=Y(37)-AM*Y(32)
F(33)=Y(38)-AM*Y(33)
F(34)=Y(39)-AM*Y(34)
F(35)=Y(40)-AM*Y(35)
F(36)=EX2*Y(31)-((1-ETA)**2)*Q*EX2*Y(21)+Y(1)
F(37)=EX2*Y(32)-((1-ETA)**2)*Q*EX2*Y(22)+Y(2)
F(38)=EX2*Y(33)-((1-ETA)**2)*Q*EX2*Y(23)+Y(3)
F(39)=EX2*Y(34)-((1-ETA)**2)*Q*EX2*Y(24)+Y(4)
F(40)=EX2*Y(35)-((1-ETA)**2)*Q*EX2*Y(25)+Y(5)
F(41)=Y(46)
F(42)=Y(47)
F(43)=Y(48)
F(44)=Y(49)
F(45)=Y(50)
F(46)=-AM*Y(46)+EX2*Y(41)+AY*Y(1)
F(47)=-AM*Y(47)+EX2*Y(42)+AY*Y(2)
F(48)=-AM*Y(48)+EX2*Y(43)+AY*Y(3)
F(49)=-AM*Y(49)+EX2*Y(44)+AY*Y(4)
F(50)=-AM*Y(50)+EX2*Y(45)+AY*Y(5)
RETURN
END
SUBROUTINE OUT(X,Y)
REAL Y(50)
COMMON /AR2/ XEND,R,ICM
X=XEND+FLOAT(ICM)*R
ICM=ICM-1
RETURN
END

```