

UNIVERSITY OF BAHRAIN
COLLEGE OF SCIENCE
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**RESONANCE PROPERTIES
OF THE BUBBLES**

BY

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To
My Family
And
My Friends

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ABSTRACT

The resonance properties of the bubbles are considered in this thesis.

Firstly, the oscillation of the gas bubble is studied, and the resonance frequency of that bubble is investigated by linearization the radius of the bubble. It is assumed that the pressure is uniform inside the bubble, but that will not be the case for the temperature, which maintain us to investigate an analytical solutions for the amplitudes of pressure and temperature by the mean of perturbation method.

The calculation of the temperature profile inside oscillating gas bubble is studied and it is investigated to be non-monotonic. Also velocity profile of a gas bubble is determined to be almost linear.

The dynamics of soluble gas bubble in liquid, which controlled by diffusion is considered later in this thesis, by using the basic equations that describe the behavior of a soluble gas in liquid, and it is shown that this problem can reduced to a problem of dynamics of vapor bubbles with phase transitions, also formula is obtained for the second resonance of soluble gas bubble.

The capillary effects and diffusion taken together, are shown to produce a new resonant frequency of a small soluble gas bubbles which differs from that described by Minnaert.

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INTRODUCTION

The problem of dynamics and heat transfer of gas bubbles, and dynamics of vapor and soluble gas bubbles in liquid is considered. An analytical method is developed to calculate both the amplitudes and phase of an oscillating bubble in liquid subjected to an acoustic field. Thermal conduction, viscosity, and surface tension are all taken into account. Analytical solutions, which are valid for small – amplitude oscillations, are also obtained for the radius and temperature.

In chapter I : we derive Rayleigh – Plesset equation by using Euler's equation and by the mean of linearization the Rayleigh – Plesset equation we can obtain the amplitude of radius of the bubble. The amplitudes of pressure and temperature are obtained by linearization equation of uniform pressure and heat equation respectively, and we will study the resonance phenomenon of a gas bubble.

And we will also obtain the formula of the velocity profile of the gas inside the bubble, which describes the velocity distribution inside the bubble.

In chapter II : we will study the dynamics of soluble gas bubble and we will show that the problem can be reduced to a problem of the dynamics of a vapor bubble with phase transitions.

We shall also discuss the existence of a new resonance frequency for the vapor and soluble gas bubbles which differs from Minnaert frequency.

CHAPTER I

DYNAMICS AND HEAT TRANSFER OF GAS BUBBLES IN LIQUID

1.1 INTRODUCTION

The behavior of a single bubble in an infinite domain of liquid at rest far from bubble has been investigated in many papers (see for example Brennen 1995, Chapman & Plesset 1971, and Nigmatulin & Khabeev 1975). And for experimental bubble dynamics (Lauterborn 1982).

In this chapter we will consider a gas bubble and the thermodynamic behavior of the gas inside the bubble.

As a typical nonlinear system, it may pulsate with a combination of many different frequencies including the fundamental, higher harmonic, and subharmonic frequencies when forced by an external force of oscillatory nature.

In this chapter, a theory is developed which describes the characteristic of an oscillating gas bubble in liquid subjected to an acoustic field. Surface tension and viscosity are taken into account in the analysis. Analytical solutions are obtained by the means of the perturbation method. These solutions include the amplitude and phase of the bubble oscillation and the temperature and velocity distributions inside the gas bubbles.

1.2 RAYLEIGH-PLESSET EQUATION

Consider a spherical bubble of radius R in an infinite domain of a liquid whose pressure far from the bubble is $p(\infty)$ and the liquid density is ρ .

Rayleigh (1917) analyzed the problem of cavitation and dynamics of a bubble. He derived the equation (Brennen 1995 & Leighton 1993) :

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P(R) - P(\infty)}{\rho} \quad (*)$$

The formulation (*) was derived in the absence of the surface tension, and the viscosity. It was developed by Plesset (1949) when he involved the surface tension σ and the viscosity μ , then the equation became:

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P(R) - P(\infty)}{\rho} - \frac{2\sigma}{\rho R} - \frac{4\mu \dot{R}}{\rho R} \quad (**)$$

To derive equation (*) we will start with the continuity equation:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \bar{u}) = 0 \quad (1.2.1)$$

Since we are dealing with an incompressible fluid, then the density ρ of the liquid is constant, and hence $\text{div}(\bar{u}) = 0$

Since the bubble is spherical, we will transform $\text{div}(\bar{u}) = 0$ to spherical coordinates and it will take the form:

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) = 0$$

Thus: $\frac{\partial}{\partial r} (r^2 u) = 0$

Hence: $r^2 u(r,t) = f(t)$ (1.2.2)

Where $f(t)$ is related to $R(t)$ by a kinematic boundary condition at the bubble surface and it will be in the form: $f(t) = R^2 \dot{R}$

Since the liquid is incompressible, which means that the velocity at a distance r from the bubble center is given by:

$$u(r,t) = \frac{R^2 \dot{R}}{r^2}$$
 (1.2.3)

The total derivative $\frac{d u}{d t}$ is the rate of change of velocity of a given fluid particle as it moves in a space, and by using chain rule which take the form:

$$\frac{d u}{d t} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r}$$
 (1.2.4)

By substituting (1.2.3) into (1.2.4) after differentiating and simple algebra we can receive:

$$\frac{d u}{d t} = \frac{\ddot{R} R^2}{r^2} + \frac{2 R \dot{R}^2}{r^2} - \frac{2 R^4 \dot{R}^2}{r^5} \quad (1.2.5)$$

We shall substitute (1.2.5) into Euler's equation in one-dimensional motion:

$$\rho \frac{d \bar{u}}{d t} = - \frac{\partial P}{\partial r} \quad (1.2.6)$$

We will get:

$$\rho \left(\frac{\ddot{R} R^2}{r^2} + \frac{2 R \dot{R}^2}{r^2} - \frac{2 R^4 \dot{R}^2}{r^5} \right) = - \frac{\partial P}{\partial r} \quad (1.2.7)$$

Now we will integrate both sides of (1.2.7) with respect to r from R to ∞ as follow:

$$\rho \int_R^{\infty} \left(\frac{\ddot{R} R^2}{r^2} + \frac{2 R \dot{R}^2}{r^2} - \frac{2 R^4 \dot{R}^2}{r^5} \right) dr = - \int_R^{\infty} \frac{\partial P}{\partial r} dr$$

$$\rho \left[\frac{\ddot{R} R^2}{r^2} + \frac{2 R \dot{R}^2}{r^2} - \frac{2 R^4 \dot{R}^2}{r^5} \right]_R^{\infty} = - [P(\infty) - P(R)]$$

$$\rho \left[R \ddot{R} + 2 \dot{R}^2 - \frac{\dot{R}^2}{2} \right] = P(R) - P(\infty)$$

$$\text{Thus: } R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P_I(R) - P(\infty)}{\rho} \quad (1.2.8)$$

Which is equation (*).

To complete this part of analysis, a dynamic boundary condition on the bubble surface must be constructed. For this purpose consider a control volume consisting of a small infinitely thin lamina containing a segment of interface.

The net force per unit area balance at the bubble surface yields (Brennen 1995) :

$$P_I(R) = P(R) - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R} \quad (1.2.9)$$

Now substitute (1.2.9) into (1.2.8) we get:

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P(R) - P(\infty)}{\rho} - \frac{2\sigma}{\rho R} - \frac{4\mu\dot{R}}{\rho R} \quad (1.2.10)$$

1.3 LINEARIZATION OF THE RADIUS OF THE BUBBLE :

All parameters are changing around the equilibrium position. Hence for a very small parameter amplitudes, the response is linear.

It is important to note that these situations follows with changing amplitudes, and to note that other factors are important in determining the kind of response.

We will consider forced oscillation when the pressure far from the bubble is represented by the real part of the expression (Wang 1974 and Khabeev 1987)

$$P(\infty) = p_{\infty} + \mathbf{P}_A e^{i\omega t} \quad , \quad \left| \mathbf{P}_A \right| \ll p_{\infty} \quad (1.3.1)$$

Since the gas bubble execute small forced oscillations in the liquid, then the instantaneous bubble radius R can be described by the real part of the expression:

$$R = R_0 [1 + \alpha e^{i\omega t}] \quad (1.3.2)$$

Where α is a complex number and $\left| \alpha \right| \ll 1$

Now we will try to find an analytical solution for the amplitude of radius of a gas bubble α .

To investigate when it will be maximum and hence we have resonance.

The Rayleigh – Plesset equation (1.2.10) is :

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P(R) - P(\infty)}{\rho} - \frac{2\sigma}{\rho R} - \frac{4\mu \dot{R}}{\rho R}$$

We will use the fact that $|\alpha| \ll 1$ in (1.3.2) $R = R_0 [1 + \alpha e^{i\omega t}]$, then we will differentiate it with respect to t , we get :

$$\dot{R} = R_0 \alpha i \omega e^{i\omega t} \quad (1.3.3)$$

$$\ddot{R} = -R_0 \alpha \omega^2 e^{i\omega t}$$

Since $\alpha^2 \rightarrow 0$ (second order) $\Rightarrow \dot{R}^2 \rightarrow 0 \Rightarrow$ we can ignore $\dot{R}^2$, as all second order terms.

$$\text{Hence : } R \ddot{R} = R_0 (1 + \alpha e^{i\omega t}) (-R_0 \alpha \omega^2 e^{i\omega t})$$

$$= -R_0 (R_0 \alpha \omega^2 e^{i\omega t} + R_0 \alpha^2 \omega^2 e^{2i\omega t})$$

$$= -R_0^2 \alpha \omega^2 e^{i\omega t} \quad (1.3.4)$$

Now consider a small oscillation, the familiar relation for an adiabatic behavior of the gas bubble, we have (Leighton 1993) :

$$P R^{3\gamma} = \text{constant} \quad (\text{where } \gamma \text{ is the specific heat ratio})$$

$$\Rightarrow p_0 R_0^{3\gamma} = p_g R^{3\gamma}$$

$$\Rightarrow \frac{p_g}{p_0} = \left(\frac{R_0}{R} \right)^{3\gamma} \quad (1.3.5)$$

Substitute (1.3.2) into (1.3.5) we get :

$$\frac{p_g}{p_0} = \left(\frac{R_0}{R_0 (1 + \alpha e^{i\omega t})} \right)^{3\gamma}$$

$$\approx 1 - 3\gamma\alpha e^{i\omega t}$$

$$\Rightarrow p_g = P(R) \approx p_0 (1 - 3\gamma\alpha e^{i\omega t}) \quad (1.3.6)$$

Now substitute (1.3.1), (1.3.2), (1.3.3), (1.3.4) and (1.3.6) into (1.2.10) we get :

$$\begin{aligned} -R_0^2 \alpha \omega^2 e^{i\omega t} = \frac{1}{\rho} [p_0 (1 - 3\gamma\alpha e^{i\omega t}) - p_\infty - P_A e^{i\omega t} \\ - \frac{2\sigma}{R_0 (1 + \alpha e^{i\omega t})} - \frac{4\mu R_0 i \omega \alpha e^{i\omega t}}{R_0 (1 + \alpha e^{i\omega t})}] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\rho} \left[-3\gamma p_0 \alpha e^{i\omega t} - \mathbf{P}_A e^{i\omega t} - \frac{2\sigma}{R_0(1+\alpha e^{i\omega t})} \times \frac{1-\alpha e^{i\omega t}}{1-\alpha e^{i\omega t}} \right. \\
&\quad \left. - \frac{4\mu i \omega \alpha e^{i\omega t}}{1+\alpha e^{i\omega t}} \times \frac{1-\alpha e^{i\omega t}}{1-\alpha e^{i\omega t}} \right] \\
&= \frac{1}{\rho} \left[-3\gamma p_0 \alpha e^{i\omega t} - \mathbf{P}_A e^{i\omega t} - \frac{2\sigma}{R_0} + \frac{2\sigma}{R_0} \alpha e^{i\omega t} - 4\mu i \omega \alpha e^{i\omega t} \right]
\end{aligned}$$

Hence : $\rho R_0^2 \alpha \omega^2 = 3\gamma p_0 \alpha + \mathbf{P}_A - \frac{2\sigma}{R_0} \alpha + 4\mu i \omega \alpha$

$$\alpha \left(\rho R_0^2 \omega^2 - 3\gamma p_0 + \frac{2\sigma}{R_0} - 4\mu i \omega \right) = \mathbf{P}_A$$

$$\Rightarrow \alpha = \frac{\mathbf{P}_A}{\rho R_0^2 \omega^2 + \frac{2\sigma}{R_0} - 3\gamma p_0 - 4\mu i \omega} \quad (1.3.7)$$

Let us define S to be the denominator of α , as follow :

$$S = \rho R_0^2 \omega^2 + \frac{2\sigma}{R_0} - 3\gamma p_0 - 4\mu i \omega \quad (1.3.8)$$

The natural frequency of an adiabatic gas bubble in the absence of viscosity and capillary effects is determined by solving the equation:

$$\rho R_0^2 \omega^2 - 3\gamma p_0 = 0$$

$$\Rightarrow \omega = \frac{1}{R_0} \sqrt{\frac{3\gamma p_0}{\rho}} \quad (1.3.9)$$

and it is got by (Minnaert , 1933)

The resonant frequency is determined by solving the following equation :

$$\frac{\partial |S|}{\partial \omega} = 0 \quad (1.3.10)$$

And checking the result by the condition $\frac{\partial^2 |S|}{\partial \omega^2} > 0$

To investigate the resonant frequency of the gas bubble we will take the modulus of (1.3.8) to get :

$$|S| = \sqrt{(\rho R_0^2 \omega^2 + \frac{2\sigma}{R_0} - 3\gamma p_0)^2 + (4\mu\omega)^2} \quad (1.3.11)$$

Now differentiate (1.3.11) with respect to ω we get:

$$\frac{\partial |S|}{\partial \omega} = \frac{4\rho R_0^2 \omega \left(\rho R_0^2 \omega^2 + \frac{2\sigma}{R_0} - 3\gamma p_0 \right) + 32\mu^2 \omega}{2 \cdot \sqrt{\left(\rho R_0^2 \omega^2 + \frac{2\sigma}{R_0} - 3\gamma p_0 \right)^2 + 16\mu^2 \omega^2}} \quad (1.3.12)$$

If we apply (1.3.10) to (1.3.12) we can obtain the analytic formula for the resonant frequency of the gas bubble and it will be in the form :

$$\omega = \frac{1}{R_0} \sqrt{\frac{3\gamma p_0}{\rho} - \frac{2\sigma}{\rho R_0} - \frac{8\mu^2}{\rho^2 R_0^2}} \quad (1.3.13)$$

And now we will represent the graph of the amplitude and phase for the oscillation of a gas bubbles in water as a function of bubble radius and frequency.

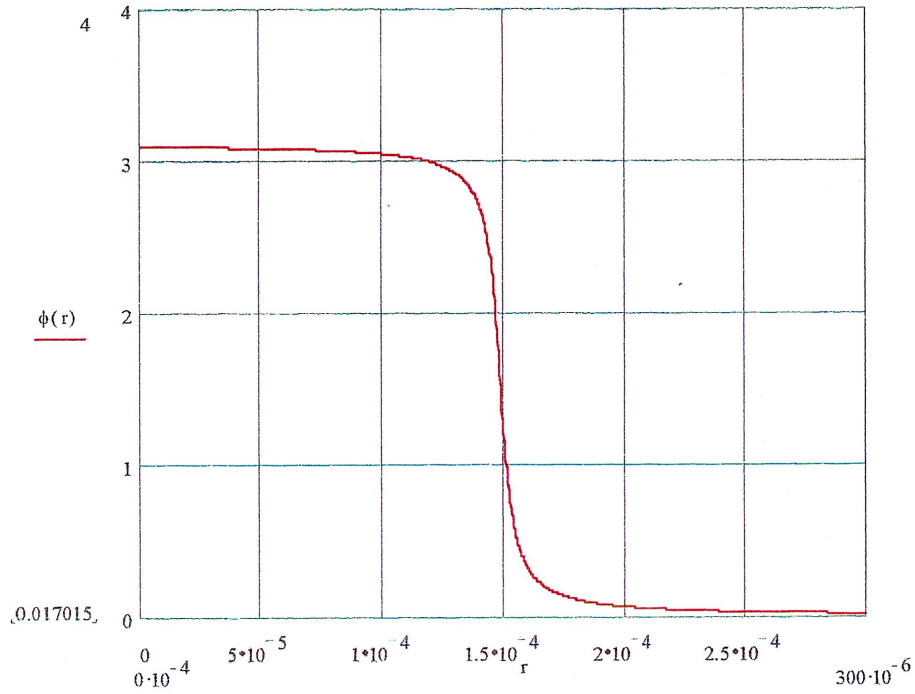
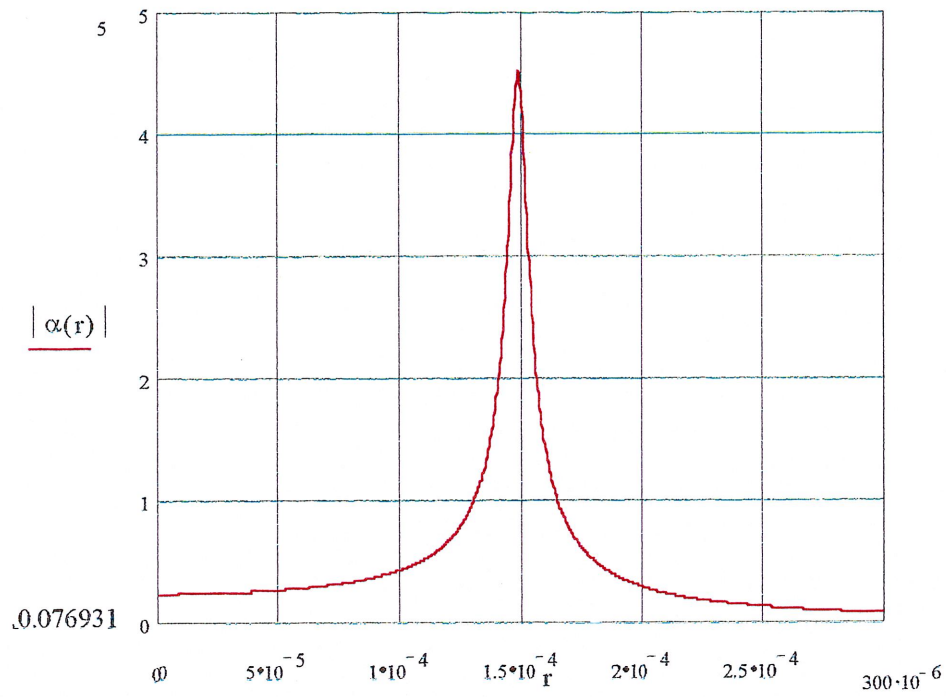


Fig 1 : Amplitude and phase for the oscillation of gas bubbles in water at 22 kHz as a function of bubble radius.

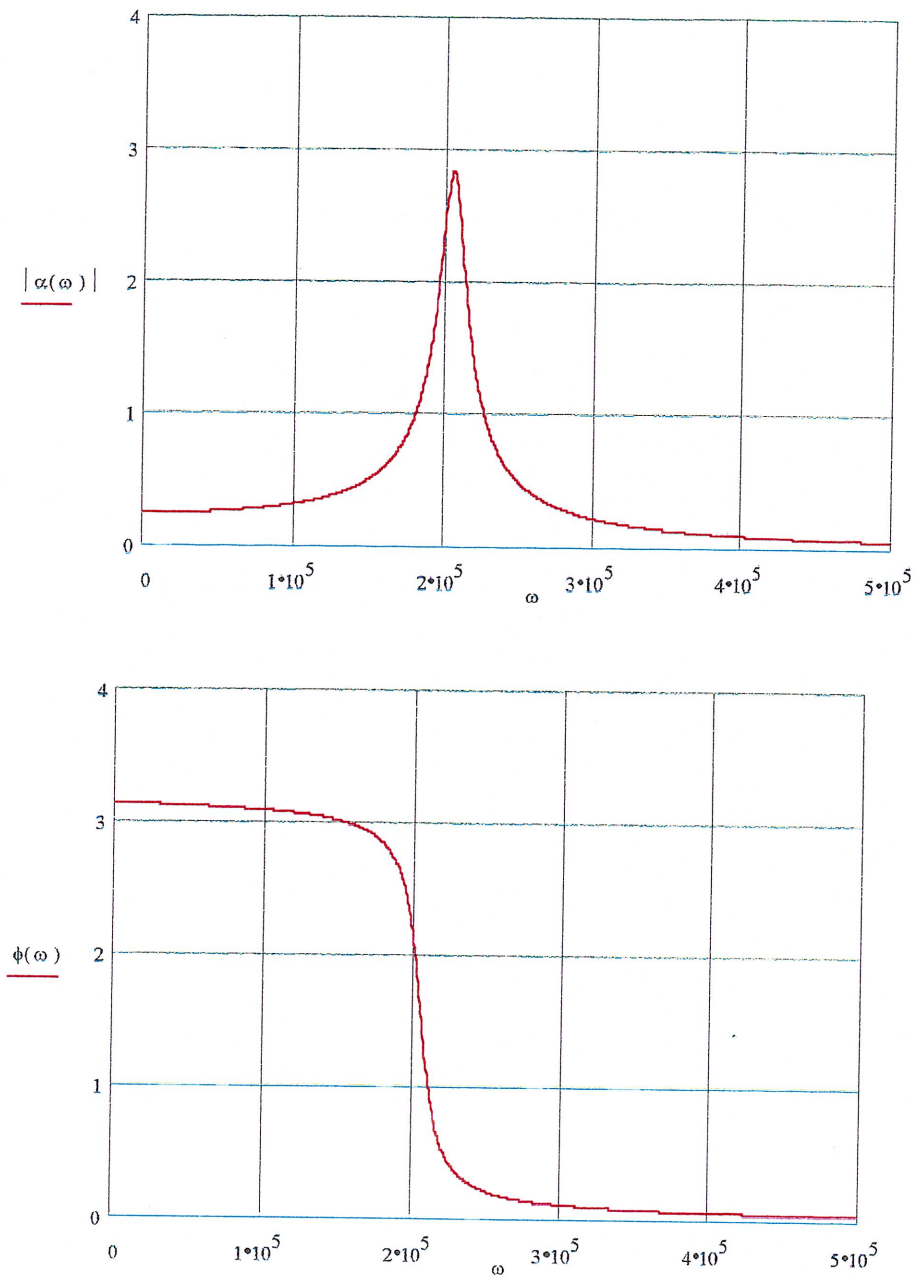


Fig 2 : Amplitude and phase for the oscillation of gas bubble of $R_0 = 10^{-4}$ m in water as a function of frequency.

In fig 1, the dimensionless amplitude $\frac{p_{\infty} |\alpha|}{P_A}$ and phase ϕ for the oscillation of gas bubble in water at 300 °K are calculated as a function of the bubble radius using equation (1.3.7). It is seen in the figures that for certain bubble radii, the pulsation amplitudes are grossly amplified, which is called resonance phenomenon. The resonant amplitude occurs at a bubble radius of size approximately at 150 microns.

A special characteristic of the phase curve around the resonant size is a sharp decrease from 180° to the minimum value of nearly 0°.

In fig 2, the amplitude and the phase distribution for oscillation of a gas bubble of radius $R_0 = 10^{-4}$ m in water as a function of the frequency.

The resonant phenomena in amplitude is seen to occur at the frequency of about $\frac{\omega}{2\pi} \approx 33$ kHz, at which the phase difference between the bubble pulsation and the exciting pressure is approach 90°. The phase undergoes a sharp decrease from 180° to 0° at the resonant frequency.

1.4 THE AMPLITUDE OF TEMPERATURE AND PRESSURE:

Let us consider the thermal behavior of the gas inside the bubble is given by :

$$\rho_g c_p \frac{dT}{dt} = \frac{\lambda}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial T}{\partial r}) + \frac{dp_g}{dt} \quad (1.4.1)$$

Where c_p is the specific heat at constant pressure, and λ is the thermal conductivity of the liquid (Haberman 1987).

We suppose that the pressure is uniform but the temperature is not, this assumption is adopted in the Rayleigh formulation for the dynamics of a single bubble, viz a spherical symmetry of the process and pressure uniformity $p_v(t)$ (homobaricity) within the bubble. In the course of bubble oscillations, the homobaricity prevails when the size of the bubble is much less than the length of a sound wave in the gas, $\omega R \ll C$ (where C is the speed of sound). For the gas bubble without phase transitions it is valid to make the simplifying assumption $T_R = T_0$ (Nigmatulin & Khabeev 1974), which make it possible to get by solving only the interior thermal problem in the gas. This is connected with the fact that the thermal conductivity of the liquid is much greater than that of the gas, while the thermal diffusivity of the liquid is much less than of the gas.

The need to consider the exterior thermal problem for a gas bubble and to take into account the fact that $T_R \neq T_0$ can only arise in the pressure of intense overcompression of the gas in the bubble.

The boundary conditions for the heat supply equation (1.4.1) is of the form:

$$T(R) = T_0 = \text{const} \quad , \quad T(0) < \infty$$

Suppose that the pressure inside the bubble and temperature are described by the real parts of the expressions (Chapman – Plesset 1971) :

$$p = p_0 (1 + \mathbf{P} e^{i\omega t}) \quad (1.4.2)$$

$$T = T_0 (1 + \theta(r) e^{i\omega t}) \quad (1.4.3)$$

Where $\mathbf{P}$ and $\theta(r)$ are a complex number which represent the amplitude of pressure and temperature respectively such that : $|\mathbf{P}| \ll 1$ and $|\theta(r)| \ll 1$.

Now we will find the analytic solution for the amplitude of temperature and the amplitude of pressure.

We will start by differentiating (1.4.2) with respect to t , and differentiating (1.4.3) with respect to r and t to get :

$$\frac{dp}{dt} = p_0 \mathbf{P} i\omega e^{i\omega t} \quad (1.4.4)$$

$$\frac{\partial T}{\partial r} = T_0 \frac{d\theta}{dr} e^{i\omega t} \quad (1.4.5)$$

$$\frac{\partial T}{\partial t} = T_0 \theta(r) i\omega e^{i\omega t} \quad (1.4.6)$$

Substitute (1.4.4) , (1.4.5) , and (1.4.6) into (1.4.1) to get :

$$\rho_g c_p (T_0 \theta(r) i\omega e^{i\omega t}) = \frac{T_0 \lambda}{r^2} \frac{\partial}{\partial r} (r^2 \frac{d\theta}{dr}) e^{i\omega t} + p_0 \mathbf{P} i\omega e^{i\omega t}$$

$$\Rightarrow \rho_g c_p T_0 \theta(r) i \omega = \frac{T_0 \lambda}{r^2} \frac{d}{dr} \left(r^2 \frac{d\theta}{dr} \right) + p_0 \mathbf{P} i \omega$$

$$\Rightarrow \rho_g c_p T_0 \theta(r) i \omega = \frac{T_0 \lambda}{r} \frac{d^2}{dr^2} (r \theta(r)) + p_0 \mathbf{P} i \omega \quad (1.4.7)$$

Since from equation of state we have :

$$p_0 = \rho_0 B T_0 = \frac{\gamma - 1}{\gamma} \rho_0 c_p T_0$$

Hence (1.4.7) becomes :

$$\rho_0 c_p T_0 i \omega \theta(r) = \frac{T_0 \lambda}{r} \frac{d^2}{dr^2} (r \theta(r)) + \frac{\gamma - 1}{\gamma} \rho_0 c_p T_0 i \omega \mathbf{P}$$

$$\Rightarrow i \omega \theta(r) = \frac{\lambda}{\rho_0 c_p} \frac{1}{r} \frac{d^2}{dr^2} (r \theta(r)) + \frac{\gamma - 1}{\gamma} \mathbf{P} i \omega$$

$$\text{Put : } a = \frac{\lambda}{\rho_0 c_p} \Rightarrow i \omega \theta(r) = \frac{a}{r} \frac{d^2}{dr^2} (r \theta(r)) + \frac{\gamma - 1}{\gamma} \mathbf{P} i \omega$$

$$\Rightarrow \frac{d^2}{dr^2} (r \theta(r)) - \frac{i \omega}{a} r \theta(r) = -\frac{i \omega}{a} \frac{\gamma - 1}{\gamma} \mathbf{P} i \omega \quad (1.4.8)$$

$$\text{Let } r \theta(r) = y(r) \quad , \quad \frac{i \omega}{a} = k$$

$$\Rightarrow y''(r) - k y(r) = -k \frac{\gamma - 1}{\gamma} \mathbf{P} r \quad (1.4.9)$$

$$\Rightarrow y_h(r) = C \cosh \sqrt{k} r + D \sinh \sqrt{k} r$$

$$\text{And } \left. \begin{aligned} y_p(r) &= A r + B \\ y_p'(r) &= A \\ y_p''(r) &= 0 \end{aligned} \right\} \quad (1.4.10)$$

Substitute (1.4.10) into (1.4.9) we get :

$$-k (A r + B) = -k \frac{\gamma - 1}{\gamma} \mathbf{P} r$$

$$\Rightarrow A = \frac{\gamma - 1}{\gamma} \mathbf{P} \quad , \quad B = 0$$

$$\Rightarrow y_c(r) = C \cosh \sqrt{k} r + D \sinh \sqrt{k} r + \frac{\gamma - 1}{\gamma} \mathbf{P} r$$

$$\Rightarrow r \theta(r) = C \cosh \sqrt{k} r + D \sinh \sqrt{k} r + \frac{\gamma - 1}{\gamma} \mathbf{P} r$$

$$\Rightarrow \theta(r) = \frac{C}{r} \cosh \sqrt{k} r + \frac{D}{r} \sinh \sqrt{k} r + \frac{\gamma - 1}{\gamma} \mathbf{P}$$

$$\text{Since : } \lim_{r \rightarrow 0^+} \frac{\cosh \sqrt{k} r}{r} = \infty \quad \text{which gives that } C = 0$$

$$\Rightarrow \theta(r) = \frac{D}{r} \sinh \sqrt{k} r + \frac{\gamma - 1}{\gamma} \mathbf{P} \quad (1.4.11)$$

Now let us use this substitution $\eta(r) = \frac{r}{R}$ corresponding to the boundary

condition : $\eta(0) = 0$, $\eta(R) = 1$

$$\Rightarrow \theta(\eta) = \frac{D}{R \eta} \sinh \sqrt{k} R \eta + \frac{\gamma-1}{\gamma} \mathbf{P}$$

$$\text{Since : } \theta(1) = 0 \Rightarrow 0 = \frac{D}{R} \sinh \sqrt{k} R + \frac{\gamma-1}{\gamma} \mathbf{P}$$

$$\Rightarrow D = -\frac{\gamma-1}{\gamma} \mathbf{P} \frac{R}{\sinh \sqrt{\frac{i \omega R^2}{a}}}$$

$$\text{Hence : } \theta(\eta) = -\frac{\gamma-1}{\gamma} \frac{\mathbf{P}}{\sinh \sqrt{\frac{i \omega R^2}{a}}} \frac{\sinh \sqrt{\frac{i \omega R^2}{a}} \eta}{\eta} + \frac{\gamma-1}{\gamma} \mathbf{P}$$

$$\text{Let } \frac{i \omega R^2}{a} = G$$

$$\therefore \theta(\eta) = -\frac{\gamma-1}{\gamma} \cdot \frac{\mathbf{P}}{\sinh \sqrt{G}} \cdot \frac{\sinh \sqrt{G} \eta}{\eta} + \frac{\gamma-1}{\gamma} \mathbf{P} \quad (1.4.12)$$

$$\Rightarrow \theta(r) = -\frac{(\gamma-1) \mathbf{P} R}{\gamma \sinh \sqrt{G}} \cdot \frac{\sinh \sqrt{k} r}{r} + \frac{\gamma-1}{\gamma} \mathbf{P} \quad (1.4.13)$$

To investigate the amplitude of the pressure P , we shall start by deriving the equation of the uniform pressure, and to derive this equation we will use the equations of heat influx, continuity equation, and equation of state which takes the form (Nigmatulin & Khabeev 1974) :

$$P_g(t) = \rho_g(r, t) B T(r, t) \quad \text{which can be written in the form :}$$

$$T = \frac{P_g}{\rho_g B} \quad (1.4.14)$$

Take the total derivative for T with respect to t , we have :

$$\frac{dT}{dt} = \frac{1}{\rho_g B} \frac{dP_g}{dt} - \frac{P_g}{B \rho_g^2} \frac{d\rho_g}{dt} \quad (1.4.15)$$

Substitute (1.4.15) into (1.4.1) we get :

$$\rho_g c_p \left(\frac{1}{\rho_g B} \frac{dP_g}{dt} - \frac{P_g}{B \rho_g^2} \frac{d\rho_g}{dt} \right) = \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{dP_g}{dt} \quad (1.4.16)$$

Multiplying both sides of (1.4.16) by r and integrating with respect to r from 0 to R , we get :

$$\begin{aligned} \int_0^R r^2 \rho_g c_p \left(\frac{1}{\rho_g B} \frac{dP_g}{dt} - \frac{P_g}{B \rho_g^2} \frac{d\rho_g}{dt} \right) dr &= \int_0^R \lambda \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) dr + \int_0^R r^2 \frac{dP_g}{dt} dr \\ \Rightarrow \frac{c_p R^3}{3B} \frac{dP_g}{dt} - \frac{c_p P_g}{B} \int_0^R \frac{r^2}{\rho_g} \left(\frac{d\rho_g}{dt} \right) dr &= R^2 \left(\lambda \frac{\partial T}{\partial r} \right)_R + \frac{R^3}{3} \frac{dP_g}{dt} \quad (1.4.17) \end{aligned}$$

The boundary condition for the equation of gas heat influx (1.4.1) may be represented in the form:

$$r=0 : \frac{\partial T}{\partial r} = 0, \quad u=0$$

$$r=R : T = T_o = \text{constant}, \quad u = \dot{R}$$

Multiplying both sides of the gas continuity equation,

$$\frac{d\rho_g}{dt} + \frac{\rho_g}{r^2} \frac{\partial}{\partial r}(r^2 u) = 0 \quad (1.4.18)$$

by $\frac{r^2}{\rho}$ and integrating with respect to r from 0 to R and using the boundary

condition, we get:

$$\int_0^R \frac{r^2}{\rho_g} \left(\frac{d\rho_g}{dt} \right) dr = -R^2 u(R) = -R^2 \dot{R} \quad (1.4.19)$$

Substitute (1.4.19) into (1.4.17) gives:

$$\begin{aligned} \frac{c_p R^3}{3B} \frac{dP_g}{dt} + \frac{c_p P_g}{B} R^2 \dot{R} &= R^2 \left(\lambda \frac{\partial T}{\partial r} \right)_R + \frac{R^3}{3} \frac{dP_g}{dt} \\ \Rightarrow \frac{R^3}{3} \left(1 - \frac{c_p}{B} \right) \frac{dP_g}{dt} &= \frac{c_p}{B} P_g R^2 \dot{R} - R^2 \left(\lambda \frac{\partial T}{\partial r} \right)_R \end{aligned} \quad (1.4.20)$$

$$\text{Since : } \frac{c_p}{B} = \frac{\gamma}{\gamma-1} \quad \text{hence : } 1 - \frac{c_p}{B} = 1 - \left(\frac{\gamma}{\gamma-1} \right) = \frac{\gamma-1-\gamma}{\gamma-1} = \frac{-1}{\gamma-1}$$

Therefore : (1.4.20) becomes :

$$\begin{aligned}
 -\frac{R^3}{3(\gamma-1)} \frac{d P_g}{d t} &= \frac{c_p}{B} P_g R^2 \dot{R} - R^2 \left(\lambda \frac{\partial T}{\partial r} \right)_R \\
 \Rightarrow \frac{d P_g}{d t} &= \frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3 \gamma P_g \dot{R}}{R}
 \end{aligned} \tag{1.4.21}$$

That is the equation for uniform pressure, which is used to derive the amplitude of the pressure inside the bubbles.

Now we are seeking for the amplitude of pressure **P** .

We will start by the substitution : $\eta = \frac{r}{R}$

By chain rule , we have :

$$\frac{\partial}{\partial r} = \frac{1}{R} \frac{\partial}{\partial \eta} \quad \Rightarrow \quad \frac{\partial T}{\partial r} = \frac{1}{R} \frac{\partial T}{\partial \eta}$$

But from (1.4.3) we have :

$$T = T_0 (1 + \theta(r) e^{i\omega t}) \quad \Rightarrow \quad \frac{\partial T}{\partial \eta} = T_0 \frac{d\theta}{d\eta} e^{i\omega t} \tag{1.4.22}$$

From (1.4.12) we have :

$$\theta(\eta) = -\frac{\gamma-1}{\gamma} \cdot \frac{\mathbf{P}}{\sinh \sqrt{G}} \cdot \frac{\sinh \sqrt{G} \eta}{\eta} + \frac{\gamma-1}{\gamma} \mathbf{P}$$

Differentiate (1.4.12) with respect to η , we get :

$$\frac{d\theta}{d\eta} = -\frac{\gamma-1}{\gamma} \frac{\mathbf{P}}{\sinh \sqrt{G}} \cdot \frac{\sqrt{G} \eta \cosh \sqrt{G} \eta - \sinh \sqrt{G} \eta}{\eta^2} \quad (1.4.23)$$

Substitute (1.4.23) into (1.4.22) we have :

$$\frac{\partial T}{\partial \eta} = -\frac{(\gamma-1)T_0 \mathbf{P} e^{i\omega t}}{\gamma \sinh \sqrt{G}} \left(\frac{\sqrt{G} \cosh \sqrt{G} \eta}{\eta} - \frac{\sinh \sqrt{G} \eta}{\eta^2} \right) \quad (1.4.24)$$

Consider (1.4.2) $p = p_0 (1 + \mathbf{P} e^{i\omega t})$ differentiate it with respect to t we get

$$\frac{dp}{dt} = p_0 \mathbf{P} i \omega e^{i\omega t} \quad (1.4.25)$$

Consider the (1.4.21) in the surface :

$$\frac{dp}{dt} = \frac{3(\gamma-1)}{R_0} \left(\lambda \frac{1}{R_0} \frac{\partial T}{\partial \eta} \right)_{\eta=1} - \frac{3\gamma p_0 \dot{R}}{R_0} \quad (1.4.26)$$

From (1.4.25) and (1.4.21) we have :

$$p_0 \mathbf{P} i \omega e^{i\omega t} = \frac{3(\gamma-1)}{R_0} \left(\lambda \frac{1}{R_0} \frac{\partial T}{\partial \eta} \right)_{\eta=1} - \frac{3\gamma p_0 \dot{R}}{R_0} \quad (1.4.27)$$

Substitute (1.5.24) into (1.5.27) we have :

$$p_0 \mathbf{P} i \omega e^{i\omega t} = -\frac{3(\gamma-1)}{R_0} \left[\frac{\lambda}{R_0} \frac{(\gamma-1)T_0 \mathbf{P} e^{i\omega t}}{\gamma \sinh \sqrt{G}} (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G}) \right] \\ - \frac{3\gamma p_0 R_0 \alpha i \omega e^{i\omega t}}{R_0}$$

$$\Rightarrow p_0 \mathbf{P} i \omega = -\frac{3(\gamma-1)^2 \lambda T_0 \mathbf{P}}{R_0^2 \gamma \sinh \sqrt{G}} (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})$$

$$-3 \gamma p_0 \alpha i \omega$$

But we know that :

$$\lambda T_0 = a c_p \rho_0 T_0 = a \left(\frac{\gamma}{\gamma-1} \right) B \rho_0 T_0 = a \left(\frac{\gamma}{\gamma-1} \right) p_0$$

$$\Rightarrow p_0 \mathbf{P} i \omega = -\frac{3(\gamma-1)^2 a \gamma p_0 \mathbf{P}}{R_0^2 \gamma (\gamma-1) \sinh \sqrt{G}} (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})$$

$$-3 \gamma p_0 \alpha i \omega \quad (1.4.28)$$

Substitute (1.3.7) into (1.4.28) to get :

$$\mathbf{P} i \omega = -\frac{3(\gamma-1) a \mathbf{P}}{R_0^2 \sinh \sqrt{G}} (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})$$

$$- \frac{3 \gamma \mathbf{P}_A i \omega}{\rho \omega^2 R_0^2 - 3 \gamma p_0 + \frac{2 \sigma}{R_0} - 4 \mu i \omega}$$

$$\Rightarrow \mathbf{P} (i \omega + \frac{3(\gamma-1) a (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})}{R_0^2 \sinh \sqrt{G}})$$

$$= - \frac{3 \gamma \mathbf{P}_A i \omega}{\rho \omega^2 R_0^2 - 3 \gamma p_0 + \frac{2 \sigma}{R_0} - 4 \mu i \omega}$$

$$\Rightarrow \mathbf{P} \left(\frac{i \omega R_0^2 \sinh \sqrt{G} + 3(\gamma-1) a (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})}{R_0^2 \sinh \sqrt{G}} \right)$$

$$= - \frac{3 \gamma \mathbf{P}_A i \omega}{\rho \omega^2 R_0^2 - 3 \gamma p_0 + \frac{2 \sigma}{R_0} - 4 \mu i \omega}$$

$$\mathbf{P} = \frac{-3 \gamma R_0^2 i \omega \sinh \sqrt{G} \mathbf{P}_A}{(i \omega R_0^2 \sinh \sqrt{G} + 3(\gamma-1) a (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})) (\rho \omega^2 R_0^2 - 3 \gamma p_0 + \frac{2 \sigma}{R_0} - 4 \mu i \omega)} \quad (1.4.29)$$

Which is the amplitude of the pressure inside the bubble at equilibrium radius R_0

Substitute (1.4.29) into (1.4.12) we get :

$$\Theta(\eta) = \frac{3(\gamma-1) R_0^2 i \omega \mathbf{P}_A}{(i \omega R_0^2 \sinh \sqrt{G} + 3(\gamma-1) a (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})) (\rho \omega^2 R_0^2 - 3 \gamma p_0 + \frac{2 \sigma}{R_0} - 4 \mu i \omega)} \frac{\sinh \sqrt{G} \eta}{\eta} \\ - \frac{3(\gamma-1) R_0^2 i \omega \sinh \sqrt{G} \mathbf{P}_A}{(i \omega R_0^2 \sinh \sqrt{G} + 3(\gamma-1) a (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G})) (\rho \omega^2 R_0^2 - 3 \gamma p_0 + \frac{2 \sigma}{R_0} - 4 \mu i \omega)} \quad (1.4.30)$$

1.5 THE RELATION BETWEEN THE AMPLITUDE OF THE TEMPERATURE AND THE AMPLITUDE OF RADIUS :

The amplitude of the temperature in the gas bubble will now be illustrated graphically for the oscillation of three different size of air bubbles in water under the acoustic frequency of 20 kHz. The mean radii of the bubbles are taken to be 500 micron, 100 micron, and 50 micron and it will be matched with the amplitude of radius.

Since $\theta(\eta)$ is a complex valued function :

$$\Rightarrow \theta(\eta) = \theta_1(\eta) + i \theta_2(\eta) \quad (1.5.1)$$

Where $\theta_1(\eta)$ and $\theta_2(\eta)$ are real valued function , that represent the real part , and imaginary part of $\theta(\eta)$ respectively.

Substitute (1.5.1) into (1.4.3) we get :

$$\begin{aligned} T &= T_0 (1 + (\theta_1(\eta) + i \theta_2(\eta)) e^{i\omega t}) \\ &= T_0 (1 + [\theta_1(\eta) + i \theta_2(\eta)] [\cos \omega t + i \sin \omega t]) \\ &= T_0 (1 + (\theta_1(\eta) \cos \omega t - \theta_2(\eta) \sin \omega t) \\ &\quad + i (\theta_1(\eta) \sin \omega t + \theta_2(\eta) \cos \omega t)) \end{aligned}$$

Since the temperature is taken to be the real part of T , which means that the behavior of the temperature during the time t can be represented by :

$$\Theta(\eta) = \theta_1(\eta) \cos \omega t - \theta_2(\eta) \sin \omega t \quad (1.5.2)$$

And we will graph that function by taking the ratio $\frac{p_{\infty}\Theta(\eta)}{P_A}$ for the times :

$$\omega t = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \text{ and } \frac{8\pi}{5}$$

And for same manners , we will represent the radius α by the following equation

$$\alpha = \alpha_1 \cos \omega t - \alpha_2 \sin \omega t \quad (1.5.3)$$

For familiar values of the parameters in $T = 300K$, the curves are illustrated as follows in the figures :

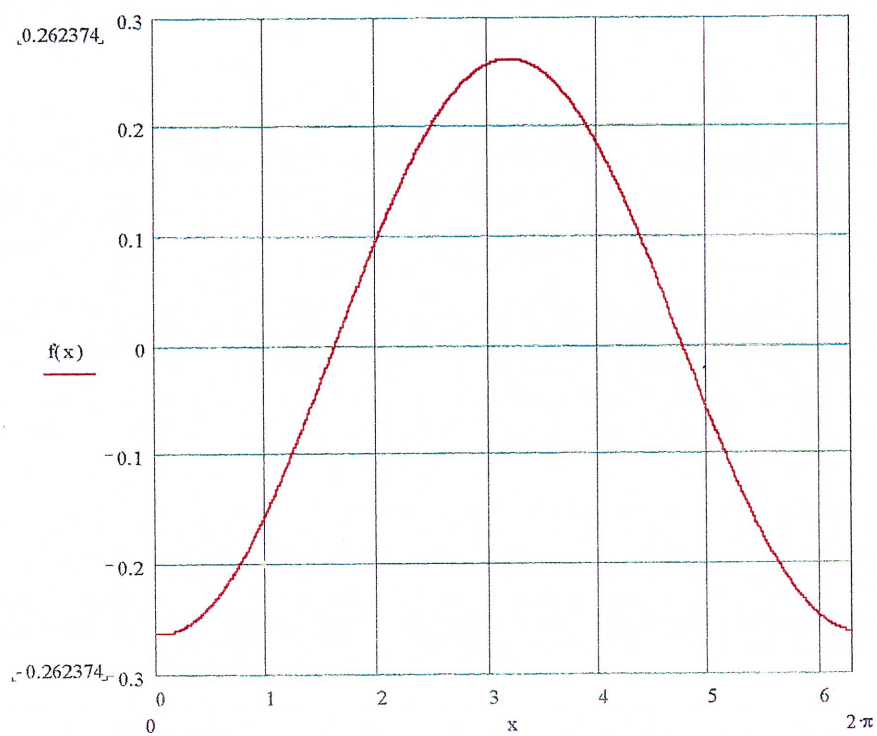
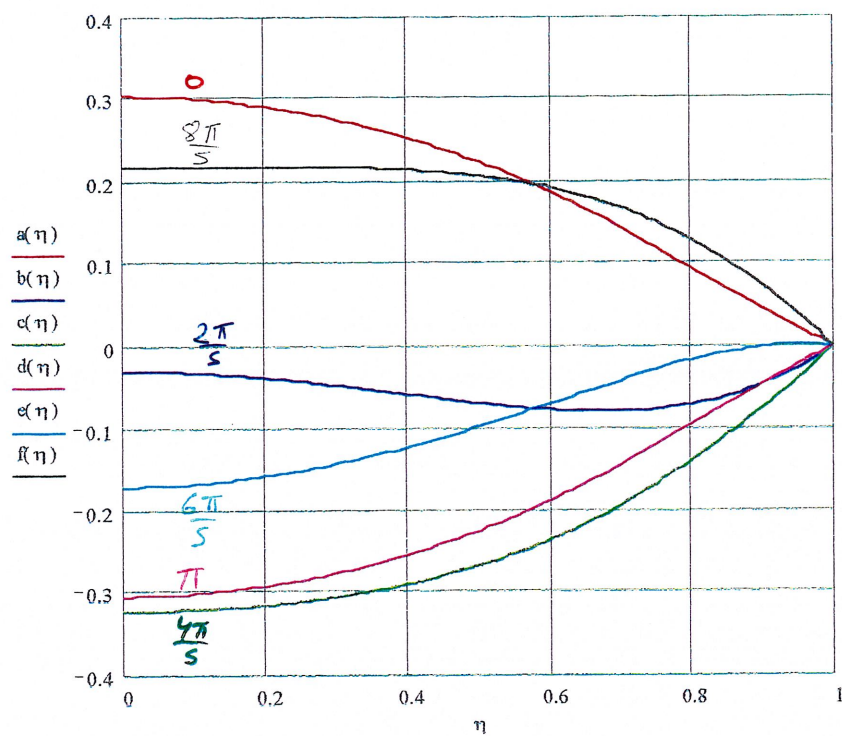


Fig 3. Comparison between the temperature distribution and the amplitude of the radius the oscillation of a gas bubble in water at the radius $R_0 = 50$ micron.

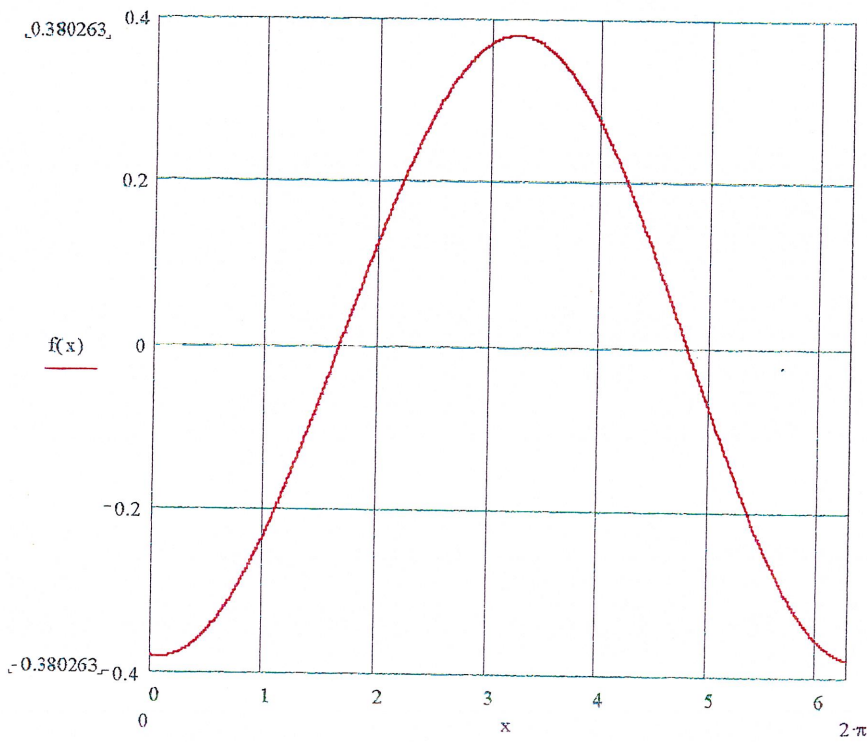
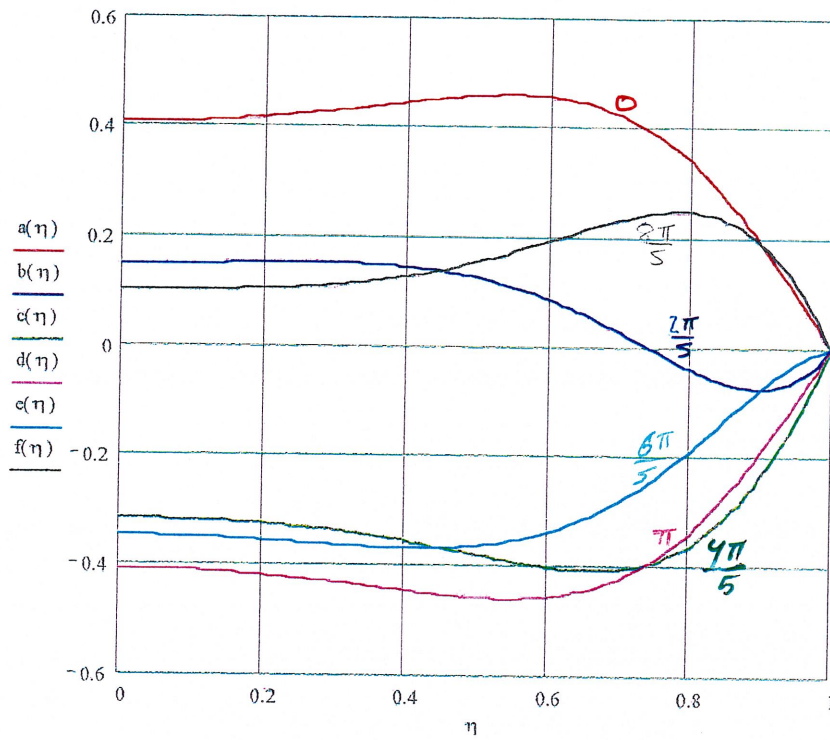


Fig 4. Comparison between the temperature distribution and the amplitude of the radius the oscillation of a gas bubble in water at the radius $R_0 = 100$ micron.

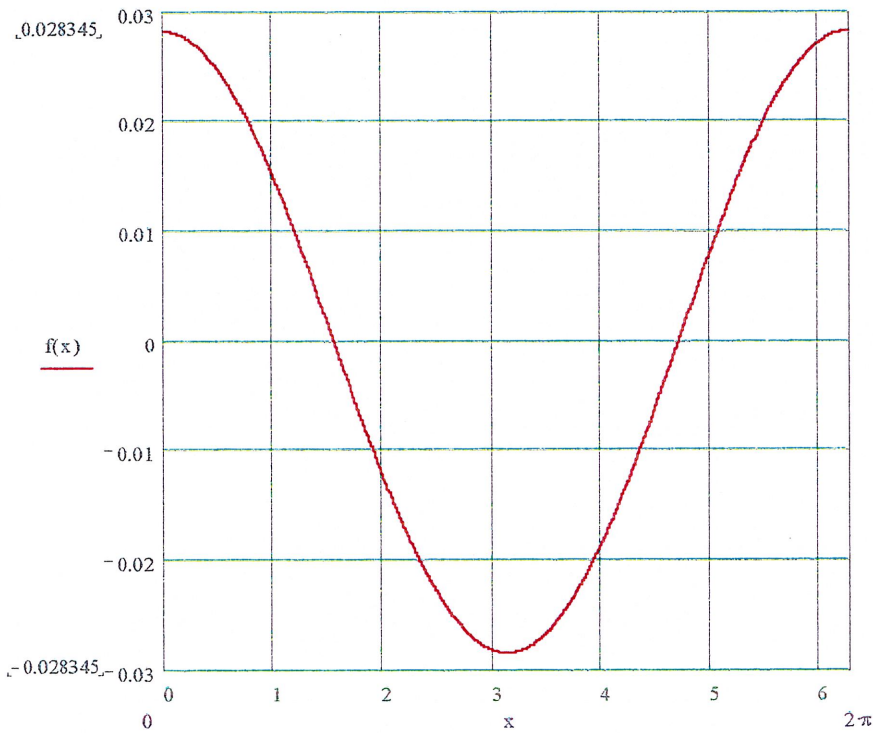
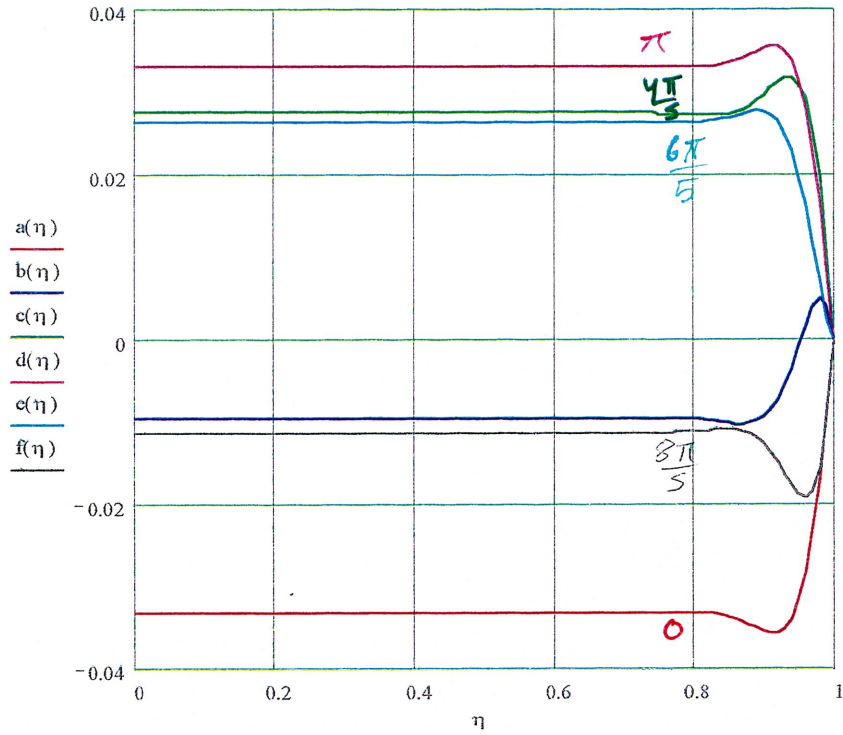


Fig 5. Comparison between the temperature distribution and the amplitude of the radius the oscillation of a gas bubble in water at the radius $R_0 = 500$ micron.

The illustrated figures show the temperature distribution $\theta^* = \frac{\text{Re}\{\theta(\eta)\} p_\infty}{P_A}$ during one pulsation period of time for a gas bubble pulsations in water for thermal diffusivity $a = 2 \times 10^{-5} \text{ m}^2/\text{s}$ under the acoustic frequency of 20 kHz.

Mainly, the temperature profile inside the bubble is non-monotonic, an interesting phenomenon is that the temperature varies in the region near the bubble surface, that is due to the gas temperature gradients at the bubble surface vary with time periodically from positive value to a negative one for the small bubble and vice versa for the large bubble.

And if we compare θ^* with the amplitude of radius $\alpha^* = \frac{p_\infty \text{Re}\{\alpha\}}{P_A}$ we observe that as we increase the radius of the bubble, the temperature will decrease.

1.6 THE VELOCITY DISTRIBUTION INSIDE THE BUBBLE:

In this section we will discuss the velocity distribution inside the bubble , and we will begin by deriving the velocity profile of the gas bubble (Nigmatulin & Khabeev 1974).

From (1.4.15) we get:

$$\frac{dT}{dt} = \frac{1}{\rho_g} \left(\frac{1}{B} \frac{dP_g}{dt} - \frac{P_g}{B \rho_g} \frac{d\rho_g}{dt} \right) \quad (1.6.1)$$

$$\frac{dT}{dt} = \frac{1}{\rho_g} \left(\frac{1}{B} \frac{dP_g}{dt} - T \frac{d\rho_g}{dt} \right) \quad (1.6.2)$$

Substitute (1.6.2) into (1.4.1) we get:

$$\begin{aligned} \rho_g c_p \frac{1}{\rho_g} \left(\frac{1}{B} \frac{dP_g}{dt} - T \frac{d\rho_g}{dt} \right) &= \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{dP_g}{dt} \\ \frac{c_p}{B} \frac{dP_g}{dt} - c_p T \frac{d\rho_g}{dt} &= \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{dP_g}{dt} \\ \therefore \frac{d\rho_g}{dt} &= \frac{1}{c_p T} \left[\left(\frac{c_p}{B} - 1 \right) \frac{dP_g}{dt} - \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right] \end{aligned} \quad (1.6.3)$$

Substitute (1.4.21) into (1.6.3), we get:

$$\frac{d\rho_g}{dt} = \frac{1}{c_p T} \left[\left(\frac{c_p}{B} - 1 \right) \left[\frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma P_g \dot{R}}{R} \right] - \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right]$$

$$= \frac{1}{c_p T} \left(\frac{c_p}{B} - 1 \right) \left[\frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma P_g \dot{R}}{R} \right] - \frac{\lambda}{r^2 c_p T} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right)$$

$$\text{But : } \frac{1}{c_p T} \left(\frac{c_p}{B} - 1 \right) = \frac{1}{c_p T} \left(\frac{c_p - B}{B} \right) = \frac{1}{BT} \left(1 - \frac{\gamma-1}{\gamma} \right) = \frac{1}{BT\gamma}$$

Therefore we get that :

$$\frac{d \rho_g}{dt} = \frac{1}{BT\gamma} \left[\frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma P_g \dot{R}}{R} \right] - \frac{1}{r^2 c_p T} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} \right) \quad (1.6.4)$$

Substitute (1.6.4) into (1.4.18), we get:

$$\begin{aligned} \frac{r^2}{\rho_g} \left[\frac{1}{BT\gamma} \left[\frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma P_g \dot{R}}{R} \right] - \frac{1}{r^2 c_p T} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} \right) \right] + \frac{\partial}{\partial r} (r^2 u) &= 0 \\ \frac{r^2}{\rho_g BT\gamma} \frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{r^2}{\rho_g BT\gamma} \frac{3\gamma P_g \dot{R}}{R} - \frac{r^2}{\rho_g} \frac{1}{r^2 c_p T} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial r} (r^2 u) &= 0 \\ \Rightarrow \frac{3r^2(\gamma-1)}{P_g \gamma R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - 3r^2 \frac{\dot{R}}{R} - \frac{\gamma-1}{\gamma P_g} \frac{\partial}{\partial r} \left(r^2 \lambda \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial r} (r^2 u) &= 0 \end{aligned} \quad (1.6.5)$$

Integrating both sides of (1.6.5) with respect to r from 0 to r, we get:

$$\frac{r^3(\gamma-1)}{P_g \gamma R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - r^3 \frac{\dot{R}}{R} - \frac{\gamma-1}{\gamma P_g} r^2 \left(\lambda \frac{\partial T}{\partial r} \right)_r + r^2 u(r, t) = 0 \quad (1.6.6)$$

Dividing both sides of (1.6.6) by r^2 , we get :

$$u(r, t) = r \frac{\dot{R}}{R} + \frac{\gamma - 1}{\gamma} \left[\left(\lambda \frac{\partial T}{\partial r} \right)_r - \frac{r}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R \right] \quad (1.6.7)$$

Now since : $\eta = \frac{r}{R} \Rightarrow \frac{\partial}{\partial r} = \frac{1}{R} \frac{\partial}{\partial \eta}$

Apply this rule to the derivative of (1.4.3) with respect to r we get :

$$\frac{\partial T}{\partial r} = \frac{1}{R} \frac{\partial T}{\partial \eta} = \frac{1}{R} T_0 \frac{d\theta}{d\eta} e^{i\omega t} \quad (1.6.8)$$

Substitute (1.4.23) into (1.6.8) we get :

$$\begin{aligned} \frac{\partial T}{\partial r} &= \frac{T_0}{R} e^{i\omega t} \left[-\frac{(\gamma - 1) \mathbf{P}}{\gamma \sinh \sqrt{G}} \left(\frac{\sqrt{G} \cosh \sqrt{G} \eta}{\eta} - \frac{\sinh \sqrt{G} \eta}{\eta^2} \right) \right] \\ &= -\frac{(\gamma - 1) T_0 \mathbf{P} e^{i\omega t}}{\gamma R \sinh \sqrt{G}} \left[\frac{\sqrt{G} \cosh \sqrt{G} \eta}{\eta} - \frac{\sinh \sqrt{G} \eta}{\eta^2} \right] \end{aligned}$$

Back to (1.6.7) in equilibrium state :

$$\begin{aligned} u(R_0 \eta) &= \eta \dot{R} + \frac{\gamma - 1}{\gamma p_0} \left[-\frac{\lambda(\gamma - 1) T_0 \mathbf{P} e^{i\omega t}}{\gamma R_0 \sinh \sqrt{G}} \left(\frac{\sqrt{G} \cosh \sqrt{G} \eta}{\eta} - \frac{\sinh \sqrt{G} \eta}{\eta^2} \right) \right. \\ &\quad \left. + \eta \frac{\lambda(\gamma - 1) T_0 \mathbf{P} e^{i\omega t}}{\gamma R_0 \sinh \sqrt{G}} \left(\frac{\sqrt{G} \cosh \sqrt{G} \eta}{\eta} - \frac{\sinh \sqrt{G} \eta}{\eta^2} \right)_{\eta=1} \right] \end{aligned}$$

Since : $p_0 = \rho_0 B T_0$

$$\begin{aligned}
\therefore u(R_0\eta) &= \eta \dot{R} + \frac{\lambda(\gamma-1)^2 T_0 \mathbf{P} e^{i\omega t}}{\rho_0 B T_0 \gamma^2 \sinh \sqrt{G}} \left[\frac{\sqrt{G} \cosh \sqrt{G} \eta}{\eta} - \frac{\sinh \sqrt{G} \eta}{\eta^2} \right. \\
&\quad \left. + \eta (\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G}) \right] \\
&= \eta \dot{R} + \frac{a(\gamma-1) T_0 \mathbf{P} e^{i\omega t}}{\gamma \sinh \sqrt{G}} \left[-\frac{\sqrt{G} \cosh \sqrt{G} \eta}{R_0 \eta} + \frac{\sinh \sqrt{G} \eta}{R_0 \eta^2} \right. \\
&\quad \left. + \frac{\eta}{R_0} \sqrt{G} \cosh \sqrt{G} - \frac{\eta}{R_0} \sinh \sqrt{G} \right] \\
&= \eta \dot{R} + \frac{3a(\gamma-1) \gamma \alpha i \omega \sinh \sqrt{G} e^{i\omega t}}{\gamma \sinh \sqrt{G} A} \left[-\frac{\sqrt{G} \cosh \sqrt{G} \eta}{R_0 \eta} \right. \\
&\quad \left. + \frac{\sinh \sqrt{G} \eta}{R_0 \eta^2} + \frac{\eta}{R_0} \sqrt{G} \cosh \sqrt{G} - \frac{\eta}{R_0} \sinh \sqrt{G} \right]
\end{aligned}$$

Where : $A = 3a(\gamma-1) \left[\frac{\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G}}{R_0^2} \right] + i \omega \sinh \sqrt{G}$

$$\begin{aligned}
\therefore u(R_0\eta) &= \eta R_0 i \omega \alpha e^{i\omega t} - \frac{3a(\gamma-1) i \omega e^{i\omega t}}{A} \left[-\sqrt{G} \frac{\cosh \sqrt{G} \eta}{R_0 \eta} \right. \\
&\quad \left. + \frac{\sinh \sqrt{G} \eta}{R_0 \eta^2} + \frac{\eta}{R_0} \sqrt{G} \cosh \sqrt{G} - \frac{\eta}{R_0} \sinh \sqrt{G} \right]
\end{aligned}$$

$$\begin{aligned}
&= \eta R_0 i \omega \alpha e^{i \omega t} - \frac{3a(\gamma-1)R_0^2 \alpha i \omega e^{i \omega t}}{3a(\gamma-1)(\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G}) + i \omega R_0^2 \sinh \sqrt{G}} \\
&\quad \cdot \left[-\sqrt{G} \frac{\cosh \sqrt{G} \eta}{R_0 \eta^2} + \frac{\sinh \sqrt{G} \eta}{R_0 \eta^2} + \frac{\eta}{R_0} \sqrt{G} \cosh \sqrt{G} - \frac{\eta}{R_0} \sinh \sqrt{G} \right] \\
&= R_0 i \omega \alpha \left[\eta - \frac{3a(\gamma-1)}{3a(\gamma-1)(\sqrt{G} \cosh \sqrt{G} - \sinh \sqrt{G}) + i \omega R_0^2 \sinh \sqrt{G}} \right. \\
&\quad \left. \cdot \left(-\sqrt{G} \frac{\cosh \sqrt{G} \eta}{\eta} + \frac{\sinh \sqrt{G} \eta}{\eta^2} + \eta \sqrt{G} \cosh \sqrt{G} - \eta \sinh \sqrt{G} \right) \right] e^{i \omega t}
\end{aligned}
\tag{1.6.9}$$

Like what we did for the amplitude of radius and temperature , we will illustrate the characteristic velocity distribution within an oscillating gas bubble at different of time , by using the function :

$$U(\eta) = u_1(\eta) \cos \omega t - u_2(\eta) \sin \omega t$$

Fig 6. presents the characteristic velocity distribution within an oscillation gas bubble at different time at the radius at the radius of $R_0 = 100$ micron.

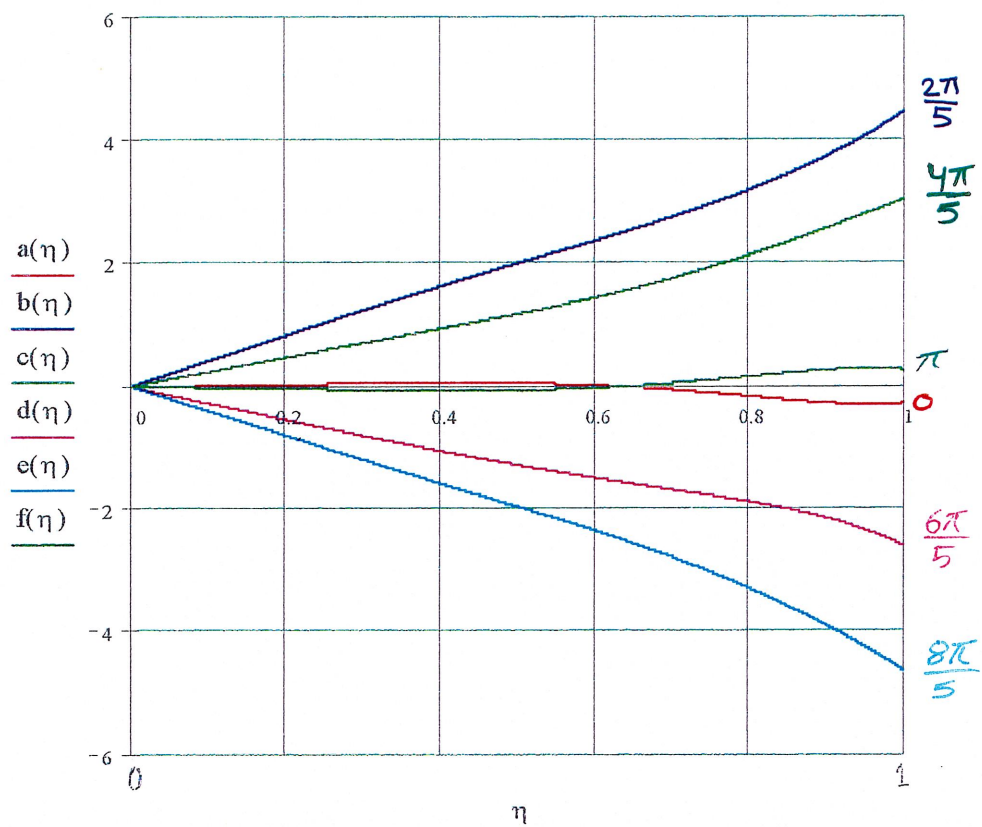


Fig 6. The characteristic velocity distribution within an oscillation gas bubble at different time at the radius of $R_0 = 100$ micron

Fig 6. shows the plot of the characteristic velocity distribution of oscillating gas bubble at different time corresponding to the sequence of angles

$\omega t = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \text{ and } \frac{8\pi}{5}$ that illustrate the time for thermal diffusivity $\alpha = 2 \times 10^{-5} \text{ m}^2/\text{s}$ under the acoustic frequency of 20 kHz with specific heat ratio $\gamma = 1.4$ and radius $R_0 = 100$ microns.

The velocity profile is almost linear, with maximum deviation of the velocity from the basic linear dependence is about 10 % and this deviation is due to the temperature gradient.

CHAPTER II

DYNAMICS OF VAPOR AND SOLUBLE GAS BUBBLES

2.1 INTRODUCTION :

The behavior of oscillations of vapor and soluble gas bubbles in liquid has been examined in a number of studies (Nigmatulin & Khabeev 1975 and Nagiev & Khabeev 1985).

It was shown by (Khabeev 1981) that the resonance frequency of large vapor bubbles practically coincides with the natural frequency of adiabatic gas bubbles which we described in the last chapter, while in the case of small bubble, the presence of capillary effects and phase transitions leads to a new resonance frequency that differs from the Minnaert frequency.

By using the analogy of similarity between vapor and soluble gas bubbles a simple analytic formula is obtained that relates the resonance frequency of the soluble gas bubble and its radius.

It is shown in this chapter that the problem of the mass, thermal and dynamic interaction between a bubble containing a soluble gas and liquid can be reduced to a problem of the behavior of a vapor bubble with phase transitions.

2.2 RESONANCE OF VAPOR BUBBLES :

The formulation of the oscillation of a gas bubbles in viscous fluid has been previously presented in sec 1.3 within the framework of a linear representation, analytic solution has been obtained for the oscillations of bubbles.

In this section, we will use this solution to study the resonance properties of a vapor bubbles.

In the case of vapor bubbles in an incompressible fluid, the amplitude of radius of bubbles is given by the formula (Khabeev 1981) :

$$\alpha = \frac{\mathbf{P_A}}{S} \quad \text{where} \quad S = \rho_l \omega^2 R_0^2 + \frac{2\sigma}{R_0} - 4i\omega\mu_l - \frac{3}{Q} \quad (2.2.1)$$

$$\text{Here} \quad Q = \frac{1 - ia_v[BH(1-b) + k(1+E\sqrt{G})]/(\omega R_0^2)}{\gamma p_0 + i\omega a_v \rho_{v_0}[BHb - b(1+E\sqrt{G})]/3} \quad (2.2.2)$$

And we will define the parameters as follow :

$$b = \frac{c_p T_0}{\ell}, \quad G = \frac{i \omega R_0^2}{a_v}, \quad E = \sqrt{\frac{a_v}{a_l}}, \quad B = \sqrt{G} \coth \sqrt{G} - 1$$

$$a_v = \frac{\lambda_v}{\rho_{v_0} c_p}, \quad a_l = \frac{\lambda_l}{\rho_l c_l}, \quad p_0 = p_\infty + \frac{2\sigma}{R_0}, \quad T_0 = T_s(p_0)$$

$$H = 3(\gamma - 1)(1 - b) \quad , \quad k = 3(\gamma - 1) b^2 \frac{\lambda_l}{\lambda_v}$$

Here Q is the compressibility of a bubble, $T_s(p)$ is the saturation temperature, and the subscripts “ l ” and “ v ” refer to the liquid and vapor parameters respectively, while the subscript “ 0 ” refers to parameters in the equilibrium state.

Let us estimate the size of a bubble dimensions and frequencies of the acoustic field, such that the resonance frequencies and dimensions of a large bubble are determined by considering the conditions :

$$4\omega\mu_l \ll 3\gamma p_\infty \quad , \quad \frac{2\sigma}{R_0} \ll 3\gamma p_\infty \quad (2.2.3)$$

In the other limiting case of sufficiently small bubbles, in which we have the conditions satisfied:

$$|G| \ll k|1 + E\sqrt{G}| \quad , \quad E|\sqrt{G}| \gg 1$$

$$\rho_l \omega^2 R_0^2 \ll \frac{2\sigma}{R_0} \quad , \quad 4\omega\mu_l \ll A R_0 \omega^{\frac{1}{2}} \quad (2.3.4)$$

Then, the expression for the resonance function is of the form :

$$S = \frac{2\sigma}{R_0} - A R_0 \omega^{\frac{1}{2}} (1 + i) \quad , \quad A = \frac{\ell^2 \rho_v^2}{\lambda_l T_0} \sqrt{\frac{a_l}{2}} \quad (2.3.5)$$

To obtain the resonant frequency of the vapor bubble we begin with taking the modulus of S .

$$\begin{aligned}
|S| &= \sqrt{\left(\frac{2\sigma}{R_0} - A R_0 \omega^{\frac{1}{2}}\right)^2 + \left(A R_0 \omega^{\frac{1}{2}}\right)^2} \\
&= \sqrt{\frac{4\sigma^2}{R_0^2} - 4\sigma \omega^{\frac{1}{2}} A + 2A^2 R_0^2 \omega} \quad (2.2.6)
\end{aligned}$$

Now differentiate w . r . t ω we get :

$$\frac{\partial |S|}{\partial \omega} = \frac{-\frac{2\sigma A}{\omega^{1/2}} + 2A^2 R_0^2}{2 \cdot \sqrt{\frac{4\sigma^2}{R_0^2} - 4\sigma \omega^{1/2} A + 2A^2 R_0^2 \omega}}$$

Put $\frac{\partial |S|}{\partial \omega} = 0$, we get that the resonance frequency of vapor bubble is :

$$\omega = \left(\frac{\sigma}{A}\right)^2 R_0^{-4} \quad (2.2.7)$$

Now we will illustrate the amplitude $\frac{p_\infty}{P_A} \propto |$ to determine resonance values of the oscillation.

R_0 , micron	$\frac{ \alpha p_\infty}{P_A}$	R_0 , micron	$\frac{ \alpha p_\infty}{P_A}$
0.000	0.000	6.000	1.696
0.025	0.025	8.000	1.300
0.050	0.195	10.000	1.073
0.075	0.293	30.000	0.524
0.100	0.391	40.000	0.474
0.250	0.978	45.000	0.462
0.500	1.993	48.000	0.458
0.750	3.143	49.000	0.457
1.000	4.497	50.000	0.456
1.200	5.605	51.000	0.456
1.400	6.391	55.000	0.455
1.500	6.557	60.000	0.459
1.540	6.577	100.000	0.682
1.550	6.579	120.000	1.164
1.560	6.578	130.000	1.650
1.570	6.577	132.000	1.729
1.580	6.574	133.000	1.757
1.600	6.563	134.000	1.775
1.620	6.548	135.000	1.781
1.700	6.440	136.000	1.775
1.800	6.229	138.000	1.726
2.000	5.691	140.000	1.636
3.000	3.598	150.000	1.066
4.000	2.590	200.000	0.251

Table 1 : The amplitude of radius of a vapor bubble versus the bubble radius

ω , Hz	$\frac{ \alpha p_{\infty}}{P_A}$	ω , Hz	$\frac{ \alpha p_{\infty}}{P_A}$
0.010	116.824	1.00×10^5	0.453
0.020	116.855	1.10×10^5	0.488
0.050	116.775	1.20×10^5	0.537
0.100	116.167	1.30×10^5	0.603
0.200	113.191	1.40×10^5	0.695
0.500	95.993	1.50×10^5	0.830
1.000	68.284	1.60×10^5	1.031
2.000	41.053	1.70×10^5	1.324
3.000	29.544	1.75×10^5	1.487
4.000	23.377	1.80×10^5	1.613
5.000	19.540	1.81×10^5	1.628
8.500	13.574	1.82×10^5	1.639
10.000	11.514	1.83×10^5	1.646
100.00	3.245	1.84×10^5	1.643
1000.0	1.163	1.85×10^5	1.638
5000.0	0.630	1.90×10^5	1.537
10000	0.509	2.00×10^5	1.168
20000	0.429	2.50×10^5	0.352
50000	0.382	3.00×10^5	0.182
80000	0.406	4.00×10^5	0.081

Table 2 : The amplitude of radius of a vapor bubble versus the bubble frequency

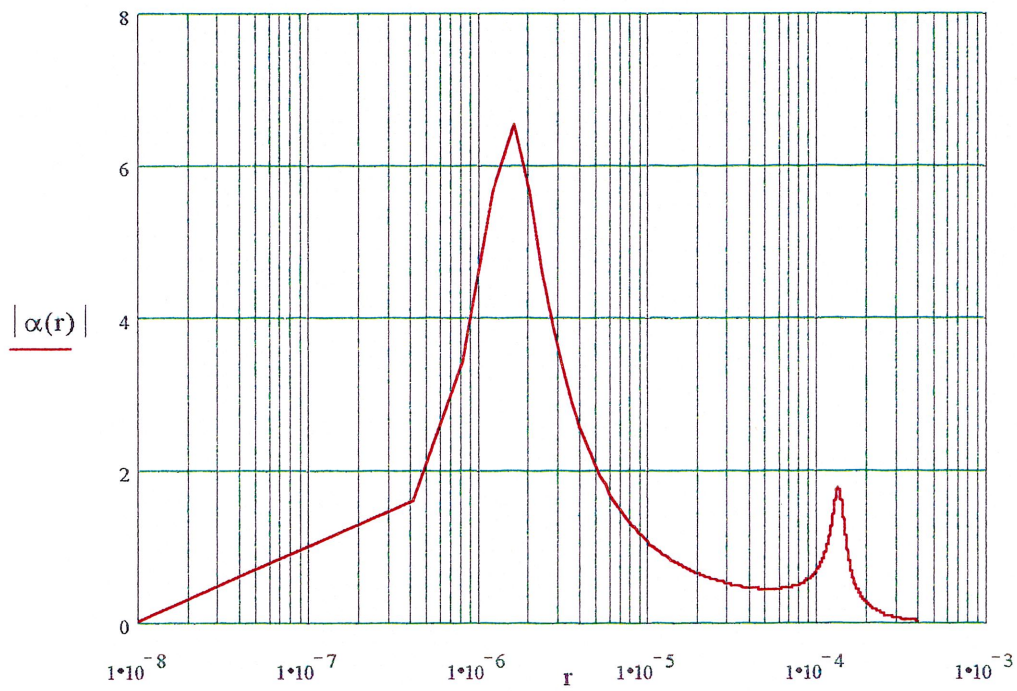
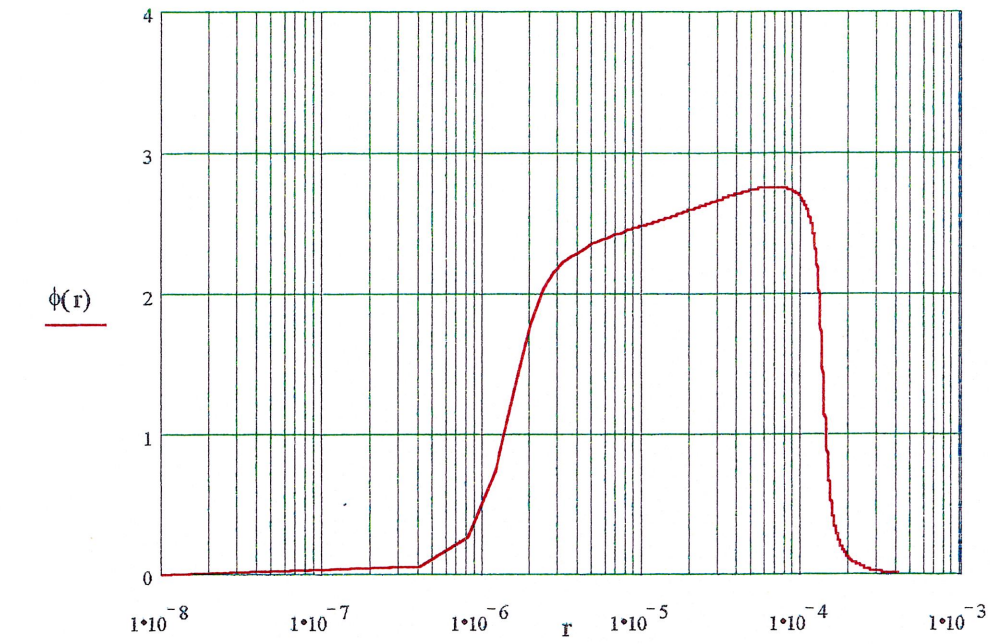


Fig 7. The phase and amplitude of a vapor bubble pulsations in water at oscillation frequency $\frac{\omega}{2\pi} = 22$ kHz versus the bubble radius.

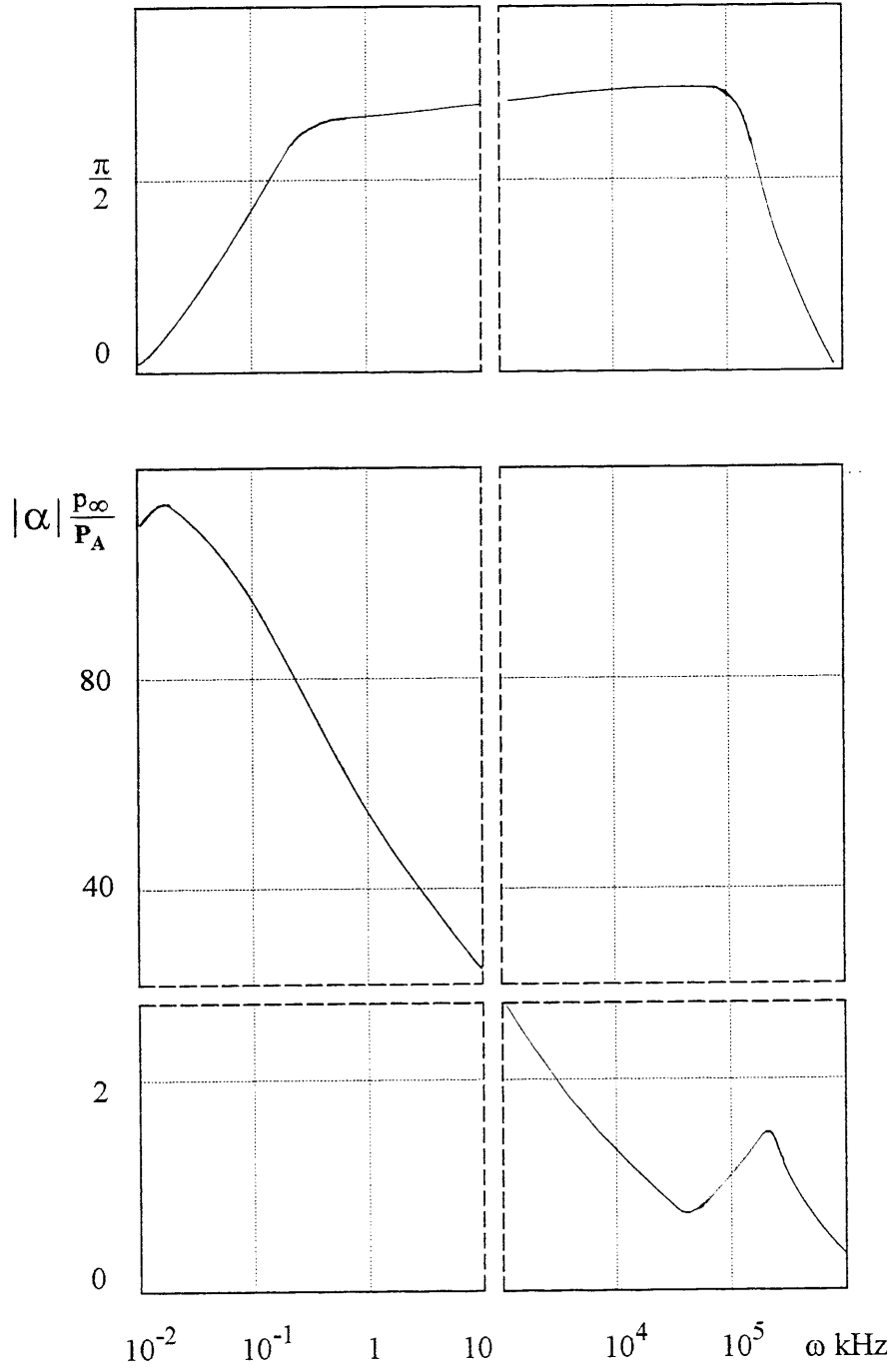


Fig. 8 : The phase and relative amplitude of a vapor bubble pulsations in water at radius $R_0 = 10^{-4}$ m versus frequency ω .

2.3 BASIC EQUATIONS FOR SOLUBLE GAS BUBBLE :

In this section we will state the main equations which describe the dynamics of the soluble gas bubbles in liquid , and the first equation is the diffusion equation for the gas in a liquid in the spherical Euler coordinates (r , t) which take the form :

$$\frac{\partial c}{\partial t} + \dot{R} \frac{R^2}{r^2} \frac{\partial c}{\partial r} = \frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right), \quad r > R \quad (2.3.1)$$

Where c is the concentration of the gas in the liquid , and D is the diffusion coefficient (Plesset & Prosperetti 1977).

The boundary condition at the surface and remote from the bubble are :

$$r = R(t) : c = c_s = \Gamma p, \quad r = \infty : c = c_0 = \Gamma p_0 \quad (2.3.2)$$

Where c_s is the saturation concentration , and Γ is the Henry's coefficient.

The velocity of the bubble surface $\dot{R}$ and the mass velocity of the gas at that surface w are related by the relation :

$$\dot{R} = w + \frac{j}{\rho_R}, \quad j = D \left(\frac{\partial c}{\partial r} \right)_R \quad (2.3.3)$$

Where j is the rate of mass transfer per unit interphase surface.

The heat supply equation and the equation of state of the gas take the form :

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(\lambda r^2 \frac{\partial T}{\partial r} \right) + \frac{dp}{dt}, \quad 0 < r < R$$

$$p(t) = \rho(r, t) B T(r, t) \quad (2.3.4)$$

The boundary conditions for the heat supply equation (2.3.4) is of the form :

$$R = R(t) : T = T_0, \quad r = 0 : \frac{\partial T}{\partial r} = 0 \quad (2.3.5)$$

The equation for the uniform pressure (1.4.21) and the velocity profile of gas (1.6.7) which we mentioned before are :

$$\frac{dp}{dt} = \frac{3(\gamma-1)}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R - \frac{3\gamma p}{R} \dot{R},$$

$$u(r, t) = \frac{r}{R} \dot{R} + \frac{\gamma-1}{\gamma p} \left[\lambda \frac{\partial T}{\partial r} - \frac{r}{R} \left(\lambda \frac{\partial T}{\partial r} \right)_R \right]$$

The Rayleigh – Plesset equation (1.2.10) and the equation of change of mass of the bubble are of the form :

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{p - p_\infty}{\rho_l} - \frac{2\sigma}{\rho_l R} - \frac{4\mu \dot{R}}{\rho_l R}$$

$$\dot{m} = 4\pi R^2 j \quad (2.3.6)$$

The behavior of a soluble gas bubble in a liquid for a given law of action $p_{\infty}(t)$ is determined by the following physical characteristics (Nagiev & Khabeev 1985) :

$$R_0 , p_0 , \rho_g , \rho_l , \gamma , \lambda_g , D , c_p , \sigma , \mu , \text{ and } \Gamma$$

Which form the following seven independent non dimensional parameters :

$$\gamma , A = \frac{\rho_g}{\rho_l} , L = \frac{D \rho_g c_p}{\lambda_g} , P_e = \frac{R_0}{D} \sqrt{\frac{p_0}{\rho_l}} , S = \frac{\sigma}{R_0 p_0}$$

$$M = \frac{\mu}{\rho_l D} , \text{ and } K = \frac{\Gamma p_0}{\rho_l}$$

Thus , the entire range of solutions of the problem of the behavior of a soluble gas bubble is determined by this set of the seven independent dimensionless parameters, which verify that there are several possible conditions of solution of the problem.

2.4 SMALL OSCILLATIONS, RESONANCES :

Let us consider the damping of free oscillation of a bubble which caused by the solubility of the gas in the liquid. In this case the radius of the bubble is given by the real part of the expression (Khabeev 1981) :

$$R = R_0 [1 + \delta e^{ht}] \quad , \quad | \delta | \ll 1$$

The frequency of the oscillation is :

$$f = \frac{\omega}{2\pi} = \frac{\text{Im}(h)}{2\pi}$$

System of equations (2.3.1) – (2.3.6) is linearized. The condition of existence of a nontrivial solution of the system of a linear equations leads to a characteristic equation in h .

And the system of equation (2.3.1) – (2.3.5) describing the behavior of a soluble gas bubble in a liquid is analogous to the system of equations describing the behavior of an equilibrium vapor bubble with phase transitions (Nigmatulin & Khabeev 1975) :

$$\begin{aligned} \frac{\partial T_l}{\partial t} + \dot{R} \frac{R^2}{r^2} \frac{\partial T_l}{\partial r} &= \frac{a_\ell}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T_l}{\partial r} \right) \\ \frac{dT_s}{dp} &= \frac{T_s}{\ell p_g} \quad , \quad j = \frac{\lambda_l}{\ell} \left(\frac{\partial T_l}{\partial r} \right)_R \end{aligned} \quad (2.4.1)$$

Where a_ℓ is the thermal diffusivity , T_s is the vapor saturation temperature, for which Clapeyron – Clausius relation hold.

In the case of small oscillations, after linearization (2.3.1) – (2.3.3) equation (2.4.1) can be written in the dimensionless form :

$$\frac{\partial C}{\partial \tau} = \frac{1}{\eta^2} \frac{\partial}{\partial \eta} \left(\eta^2 \frac{\partial C}{\partial \eta} \right), \quad \frac{\partial C_s}{\partial P} = 1, \quad J = \left(\frac{\partial C}{\partial \eta} \right)_1 \quad (2.4.2)$$

$$\frac{\partial \theta}{\partial \tau} = \frac{a_l}{D \eta^2} \frac{\partial}{\partial \eta} \left(\eta^2 \frac{\partial \theta}{\partial \eta} \right), \quad \frac{\partial \theta_s}{\partial P} = \frac{p_0}{\ell \rho_{g0}}, \quad J = \frac{\rho_l c_l T_0 a_l}{\Gamma p_0 \ell D} \left(\frac{\partial \theta}{\partial \eta} \right)_1 \quad (2.4.3)$$

Where : $C = \frac{c}{\Gamma p_0}$, $\tau = \frac{t D}{R_0^2}$, $\theta = \frac{T_l}{T_0}$, $\eta = \frac{r}{R}$

$$J = \frac{j R_0}{\Gamma p_0 D} , \quad P = \frac{p}{p_0}$$

If we make the identification $C = \theta$ and require that the dimensionless combinations in these equations be the same , then systems (2.4.2) and (2.4.3) will completely coincide.

Hence it is necessary to satisfy the conditions :

$$\frac{a_l}{D} = 1 , \quad \frac{p_0}{\ell \rho_{g0}} = 1 , \quad \frac{\rho_l c_l T_0}{\Gamma p_0 \ell} = 1 \quad (2.4.4)$$

Thus , all the analytical results for linear problems obtained for equilibrium vapor bubble with phase transitions can be taken to the case of soluble bubbles.

For this it is necessary to make substitutions in the corresponding equations from (2.4.4). Following the above procedure and suitable adjusting for the expression for natural frequency of vapor bubble (1.3.8), the natural frequency of a soluble bubble can take the form :

$$\omega = \frac{1}{R_0} \sqrt{\frac{3\gamma p_0}{\rho_l}} \left(1 - 3\gamma \frac{\Gamma p_0}{2\rho_{g_0} \sqrt{P_{eD}}} \right),$$

$$P_{eD} = \frac{2R_0}{D} \sqrt{\frac{3\gamma p_0}{\rho_l}} \quad (2.4.5)$$

Where P_{eD} is the diffusion Péclet number.

The steady – state oscillation of vapor bubbles in an acoustic field is associated with the interesting phenomenon of second resonance. The second resonance frequency of a vapor bubble is caused by frequency dispersion, since, as distinct from the case of gas bubbles, the compressibility of vapor bubbles depends to a considerable extent on the frequency of the oscillations and small frequencies increases rapidly as the frequency decreases.

At low frequencies, when phase transitions (*dissolving of the gas in the liquid*) can take place, the change in the volume of the bubble is accompanied by a change in the pressure in it due only to capillary effects.

The expression for the second resonance frequency of vapor bubble (2.2.7) can take the form :

$$\omega = \frac{2F^2}{a_l R_0^4} \quad , \quad F = \frac{\sigma \lambda_l T_0}{\ell^2 \rho_v^2} \quad (2.4.6)$$

Using substitution (2.3.4) , (2.4.6) can be written for the case of a soluble gas bubble in the form :

$$\omega_R = \frac{2D}{R_0^4} \left(\frac{\sigma \Gamma}{\rho_{g0}} \right)^2 \quad (2.4.7)$$

R_0 , micron	$\frac{ \alpha p_\infty}{P_A}$	R_0 , micron	$\frac{ \alpha p_\infty}{P_A}$
0.000	0.000	4.000	1.368
0.025	0.098	5.000	1.141
0.050	0.195	7.500	0.824
0.075	0.293	10.000	0.666
0.100	0.391	15.000	0.517
0.250	0.978	20.000	0.448
0.500	1.980	30.000	0.387
0.750	3.045	50.000	0.363
1.000	4.015	75.000	0.408
1.100	4.256	100.00	0.561
1.150	4.326	120.00	0.953
1.175	4.347	130.00	1.551
1.200	4.359	135.00	2.175
1.210	4.361	138.00	2.656
1.220	4.362	139.00	2.793
1.230	4.362	140.00	2.892
1.240	4.360	141.00	2.935
1.250	4.357	142.00	2.913
1.300	4.322	143.00	2.829
1.400	4.175	145.00	2.537
1.500	3.962	150.00	1.724
1.750	3.358	160.00	0.914
2.000	2.893	180.00	0.430
3.000	1.779	200.00	0.267

Table 3 : The amplitude of radius of a soluble bubble versus the bubble radius

ω , Hz	$\frac{ \alpha p_{\infty}}{P_A}$	ω , Hz	$\frac{ \alpha p_{\infty}}{P_A}$
0.010	115.842	1000	1.476
0.020	115.863	2500	1.000
0.030	115.877	5000	0.765
0.035	115.883	7500	0.662
0.040	115.882	10^4	0.602
0.050	115.878	2.5×10^4	0.469
0.100	115.671	4×10^4	0.433
0.200	114.289	5×10^4	0.426
0.250	113.138	6×10^4	0.428
0.500	104.067	8×10^4	0.450
0.750	92.640	1.0×10^5	0.503
1.000	81.712	1.20×10^5	0.600
2.000	52.696	1.25×10^5	0.635
2.500	44.533	1.5×10^5	0.932
5.000	25.798	1.6×10^5	1.130
7.500	18.862	1.7×10^5	1.331
10.00	15.246	1.75×10^5	1.382
20.00	9.553	1.8×10^5	1.365
50.00	5.795	2.0×10^5	1.160
100.0	4.256	3.0×10^5	0.902

Table 4 : The amplitude of radius of soluble bubble versus the bubble frequency

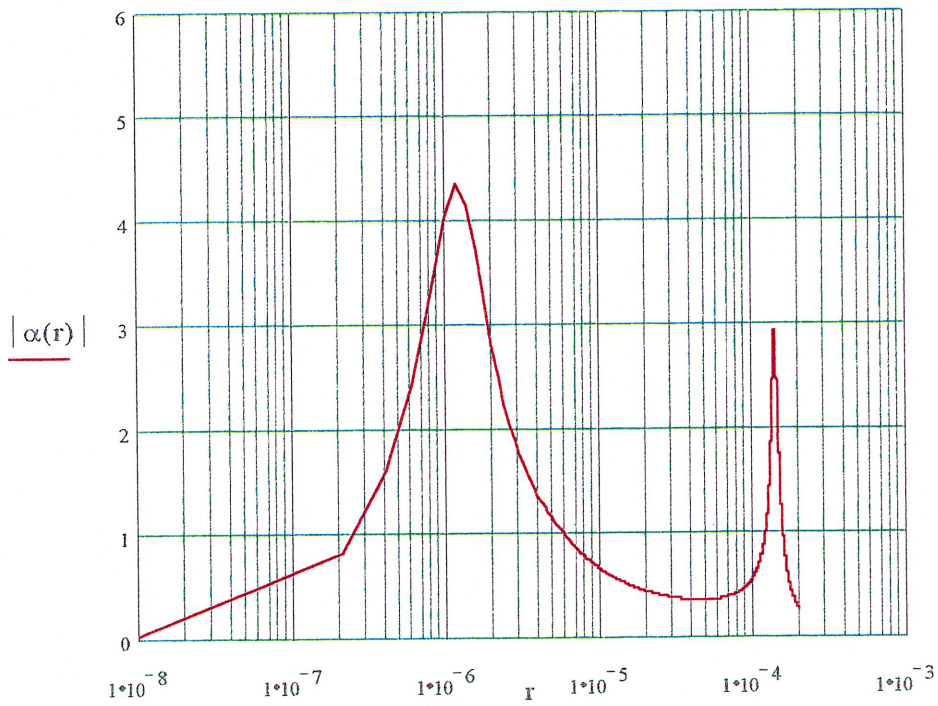
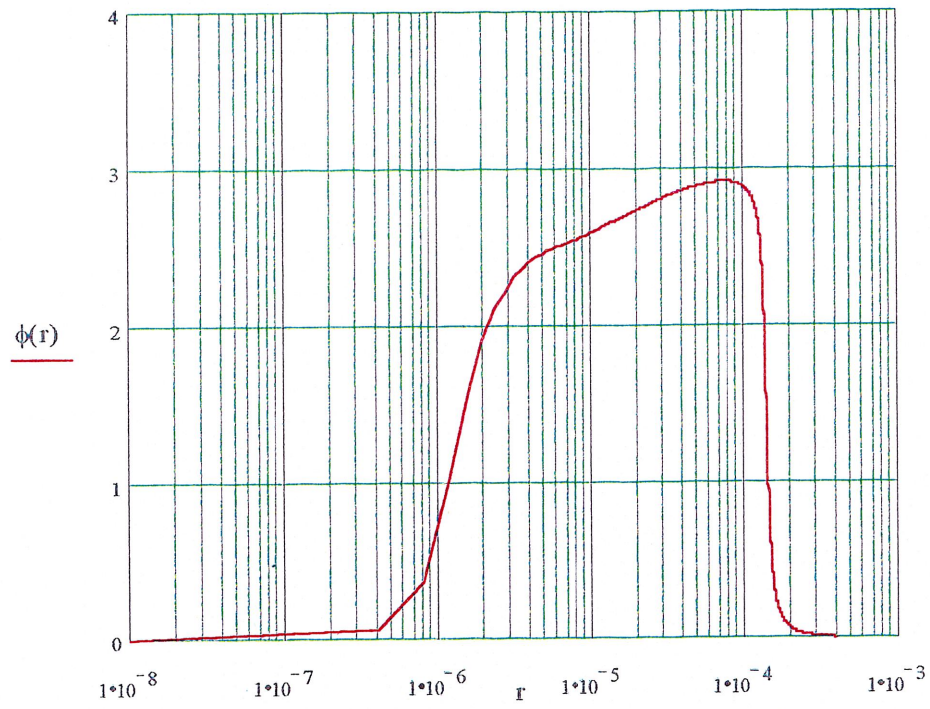


Fig 9. The phase and amplitude of a soluble gas bubble pulsations in water at oscillation frequency $\frac{\omega}{2\pi} = 22$ kHz versus the bubble radius.

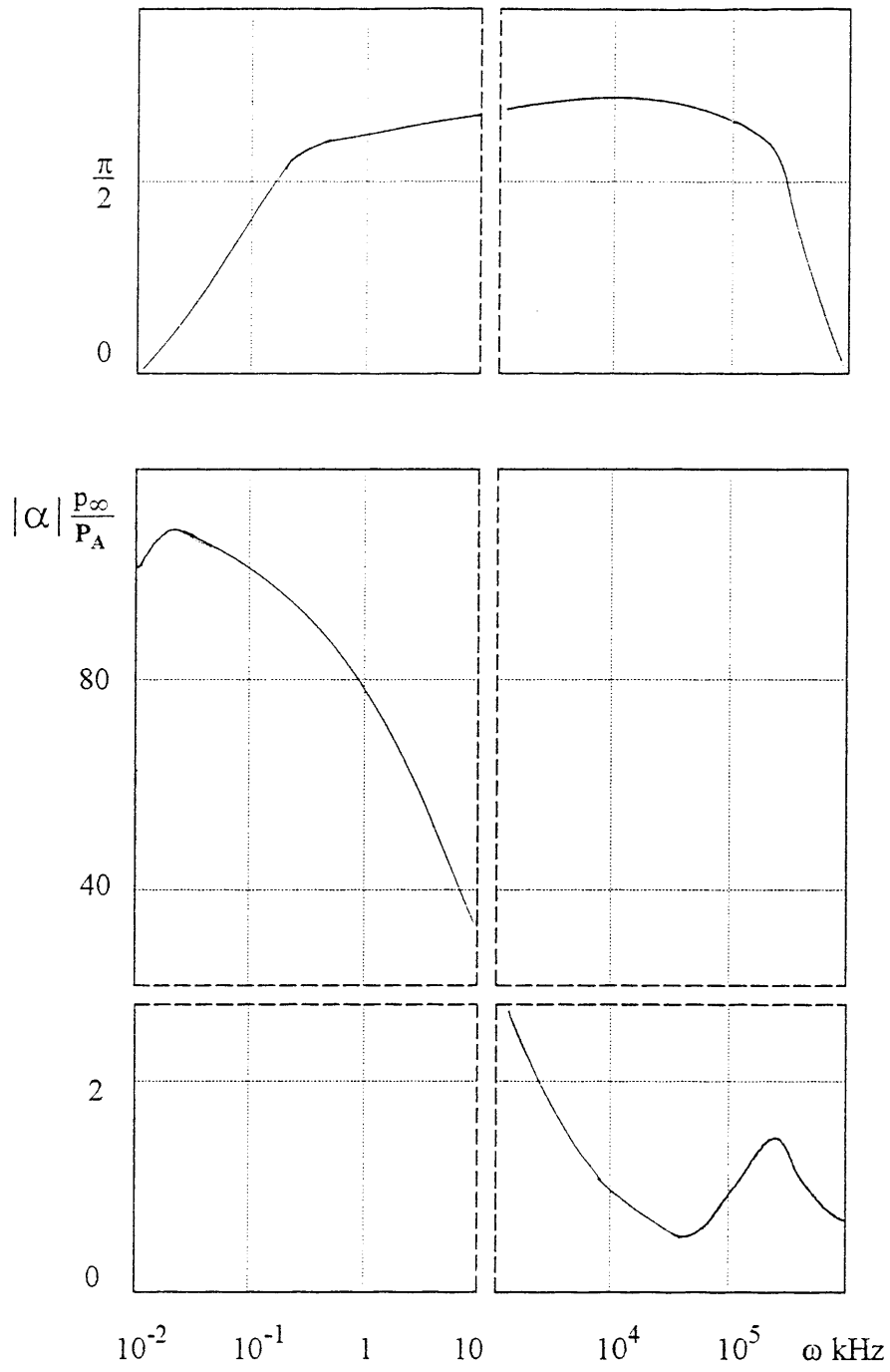


Fig. 10 : The phase and relative amplitude of a soluble gas bubble pulsations in water at radius $R_0 = 10^{-4}$ m versus frequency ω .

2.5 DISCUSSION OF THE RESULTS:

As seen in the figures 7, 8, 9, and 10, unlike the case of gas bubbles, there exist two resonance frequencies for each vapor and soluble gas bubbles.

Fig 7 and fig 9 present the dependence of the oscillation amplitude $\left| \alpha \right| \frac{p_{\infty}}{p_A}$ and phase on bubble radius for the vapor and soluble gas bubbles respectively.

The bubble radius oscillating in water at atmospheric pressure at a frequency of 22 kHz.

Computations demonstrated that there exists only the Minnaert resonance for large vapor and soluble gas bubbles ($R_0 > 10^{-3}$ m) in water, while for bubbles with $R_0 < 10^{-4}$ m there are two resonance frequencies and the second resonance frequency of the vapor bubbles induced by capillary effect and phase transition, while the second resonance frequency of the soluble gas bubbles is due to capillary effect and diffusion.

Fig 8 and fig 10 present the dependence of the bubble oscillation amplitude and phase in water for the vapor and soluble gas bubbles respectively at the atmospheric pressure $p_{\infty} = 0.1$ Mpa as a function of the acoustic field frequency with $R_0 = 10^{-4}$ m.

There exist two weak resonances, the resonance due to the capillary effect and phase transitions occurs at very low acoustic field frequencies.

NOMENCLATURE

R - bubble radius;
 $\dot{R}$ - time derivative of the radius;
 $\ddot{R}$ - acceleration of the radius;
 r - radial coordinate;
 t - time;
 p - pressure;
 T - temperature;
 ρ - density;
 u - velocity;
 w - radial velocity of the gas ;
 λ - thermal conductivity;
 a - thermal diffusivity;
 D - interdiffusion coefficient;
 B - gas constant;
 j - rate of phase transition
 μ - viscosity;
 - specific heat of vaporization;
 γ - specific heat ratio;
 c_p - heat capacity of a gas (vapor) at constant pressure;
 f - frequency of oscillation;
 $\omega = 2 \pi f$ external frequency;
 m - mass of a bubble;
 S - resonance function;
 η - non – dimensional coordinate, $\eta = \frac{r}{R}$

P_A - acoustic pressure amplitude;
 α - amplitude of the radius of the bubble;
 P - amplitude of the pressure of the bubble;
 $\theta(r)$ - amplitude of the temperature of the bubble;
 Q - bubble compressibility;
 σ - surface tension coefficient;
 Γ - Henry's coefficient;
 c - concentration of the gas in the liquid;
 C - non - dimensional concentration, $C = \frac{c}{\Gamma p_0}$;
 Pe - Péclet number;
 Pe_D - diffusion Péclet number;
 δ - amplitude of free oscillation;
 h - exponential index.

Subscripts

l - liquid
 v - vapor
 g - gas
 s - at saturation
 0 - at equilibrium
 ∞ - conditions far away from the bubble.

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الخلاصة

تهتم هذه الأطروحة بدراسة ظاهرة الرنين التي تحدث للفقاعات بأنواعها الثلاثة :
الفقاعة الغازية ، الفقاعة البخارية و الفقاعة المذابة في السائل.
فبينما يوجد تردد رنيني واحد للفقاعة الغازية ، فإنه يوجد ترددين رنينيين لكل من
الفقاعة البخارية والفقاعة المذابة في السائل.

فقد تم التوصل إلى التردد الرنيني للفقاعة الغازية عن طريق تحويل معادلة
(Rayleigh) إلى معادلة خطية باستخدام طريقة الاضطراب.

وعند الأخذ بعين الاعتبار كل من التوتر السطحي و انتقال الطور بالنسبة للفقاعة
البخارية ، والتوتر السطحي مع الانتشار بالنسبة للفقاعة المذابة فإنه يكون هنالك
ترددان رنينيين يختلفان عن التردد الرنيني للفقاعة الغازية الذي تم التوصل إليه عن
طريق العالم (Minnaert).

هذا وقد تم تناول موضوع الانتقال الحراري داخل الفقاعة الغازية باستخدام
المعامل الابعدي $\eta = \frac{r}{R}$ ، وتم التوصل إلى أنه كلما زاد نصف قطر الفقاعة فإنه
يقل مقدار الانتقال الحراري فيها والعكس ، كما تمت دراسة توزيع السرعات داخل
الفقاعة باستخدام نفس المعامل η ، وتم التوصل إلى أن نسبة التشتت في السرعات
لا تتجاوز ١٠ %.