



Influence of Magnetic Saturation Modelling on the Stability of Synchronous Machine

By

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DEDICATION



Dedicated
to
my parents, brothers
and sisters

DECLARATION

No part of the work included in this dissertation has been submitted in support of an application for another degree or qualification of this or any other institute of learning.

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ABSTRACT

This dissertation presents a small disturbance stability analysis of unsaturated and saturated models of a salient pole synchronous machine. A procedure for linearisation of saturated models is proposed. The linearised unsaturated and saturated models are used to present a single-machine-infinite bus system. The turbine governor and excitation voltage controller are not modelled so they can be treated as independent inputs. Simulations of non-linear and linearised unsaturated and saturated models are performed using MATLAB. Results of the simulation obtained from the saturated models are compared with each other and with the unsaturated models. In addition, Monte Carlo simulation is performed using MATLAB, in which the real power, saturation factor and saliency factor are randomly chosen and uniformly distributed between certain limits.

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NOMENCLATURE

$\psi_a \ \psi_b \ \psi_c$	-Instantaneous stator flux linkages.
ψ_{qs}	-Instantaneous stator q-axis flux linkage.
ψ_{ds}	-Instantaneous stator d-axis flux linkage.
ψ_f	-Instantaneous field winding flux linkage.
ψ_{kq1}	-Instantaneous q-axis primary damper winding flux linkage.
ψ_{kq2}	-Instantaneous q-axis secondary damper winding flux linkage.
ψ_{kd1}	-Instantaneous d-axis damper winding flux linkage.
$\psi_m = \psi_u$	-Magnetising flux.
ψ_{md}	-d axis magnetising flux component.
ψ_{mq}	-q axis magnetising flux component.
$i_m = i_u$	-Magnetising current.
i_{md}	-d axis magnetising current component.
i_{mq}	-q axis magnetising current component.
L_{ls}	-Stator leakage inductance.
L_{lf}	-Field leakage inductance.
L_{lkq1}	-q-axis primary damper winding leakage inductance.
L_{lkq2}	-q-axis secondary damper winding leakage inductance.
L_{lkd1}	-d-axis damper winding leakage inductance.
L_{md}	-d-axis magnetising inductance.
L_{mq}	-q-axis magnetising inductance.
f_e	-System frequency in Hertz.
$\omega_b = 2\pi f_e$	-Base synchronous speed in rad/sec.
ω_r	-Instantaneous machine rotor angular speed.
ω	-Reference frame speed.
δ	-Machine rotor angle.
H	-Machine inertia.
p	-Time derivative.

CHAPTER 1

INTRODUCTION

Power system networks have been developed from the original generating station concept to a modern highly interconnected system with improved technologies influencing each part of the network to satisfy the increasing demand. This growth increases the complexity of the network and the need to maintain and secure continuous and stable supply of power. The reliability of the power supply depends on the characteristics of its various components (synchronous generators, transmission and distribution lines, transformers, reactive power compensators, protection devices...etc) during normal operating condition and during transient disturbances. Therefore, examination of these components and their effects on the overall system performance has to be done to be able to predict the performance of such complex system during normal and abnormal conditions. The most important requirement of reliable service is to keep synchronous generators, which are sources of power in the network, running in synchronism. This is because the generators are subjected to small and large disturbances due to continuous load changes and different types of faults (short and open circuits). As the demand of power varies with the condition throughout the system, the amount of power drawn off from the generator fluctuates, and so does the magnetic drag at the generators. As a result, the characteristics of those synchronous generators obviously play an important part in power stability problem. It is necessary to understand the features of the machines and develop an accurate mathematical model of their dynamic performance.

1.1 Literature Review

The operation of synchronous machines involves electrical and the electromechanical processes that play a significant role in the estimation and valuation of protection standards and reliable operation of power systems. In addition an analysis of these processes yields important information for machine characteristics and performance. Consequently the simulation of the synchronous machine considers wide range of application from power system analysis to electric drives systems. In recent years, improvement in modelling synchronous machine has been an important issue to which much effort has been allocated. A great deal of this effort has been pertained to proper accounting and elucidating iron magnetic saturation effects. Many mathematical models

of saturation in synchronous machine have been introduced over the years. Most of these are presented or addressed as mentioned in references. Although these models may vary in level of complexity and accuracy, most of them are based on the well known two axis theory, (the direct and quadrature axis theory), and simulated using different programming languages and packages.

Harley *et al.*(1980) suggested a model for the saturated synchronous machine using d and q saturation factors, which are functions of the flux in their respective axes. With this model they showed that for a micro alternator, saturation in the q axis is important in the determination of the initial value of the load angle after short circuit faults.

In 1983, Vas *et al.* derived a new set of generalised equations describing the transient and steady state performance of an ac machine with currents deliberately chosen as the state variables, which take account of saturation in main flux path. They utilised these equations to calculate the starting performance of the induction motor and compared with those obtained by two sets of calculation from the conventional equations in which the level of magnetisation has been assumed constant and other in which it has been represented by continuous magnetisation curve. Furthermore, they adopted the synchronous rotating reference frame for stability analysis.

Ramshaw and Xie (1984) presented models for round rotor and salient pole synchronous generators using concepts of “static” and “dynamic” permeances to represent the saturation of the air gap flux. Considering saturation effects on the d-axis magnetising path, they showed that for a line to ground fault on a double circuit transmission line connected to synchronous generator, the calculated power angle swing was reduced with saturated model compared to the constant parameter model. Also they pointed out greater critical clearing fault time for the saturated model.

In 1986, DeMello and Hannett took account of the saturation due to the field of the synchronous generator using an approach that adjusted the main fluxes in the d and q axes for both steady state and transient operation. They obtained a good correlation between experimental and simulated results.

El-Serafi *et al.*(1988) presented a model for a small salient pole synchronous machine in the d and q axes frame, which includes both the saturated d and q axis synchronous reactances and the cross-magnetising effect. They verified experimentally the effect of the cross-magnetising phenomenon on power/load angle relationship and introduced this phenomenon in the phasor diagram of the synchronous machine.

In 1989, Lipo and Ojo suggested an improved and more accurate model, which accounts for the saturation in the stator core, rotor core as well as stator teeth and rotor teeth and rotor pole face using separate saturation factors, for transient analysis of salient pole synchronous motor. They calculated the saturation factor using finite element method that provided accurate results. With this model, they calculated the starting performance of synchronous motor for a direct on line start as well as transient performance. They showed that neglecting stator core saturation during load change would provide good results only during low voltage starting.

In more recent papers, El-Serafi and Abdulla (1992) proposed a model for the d and q axis synchronous reactances of a salient pole machine in which effects of both the d and q axis saturation factors and the variation of the d and q axis mutual reactances versus ampere-turns are also shown. They showed that the accuracy of the machine reactances, which include the impact of the cross-magnetising phenomenon in addition to the d and q axis saturation factors, is good. Moreover, they found the results of the machine active and reactive outputs power which are calculated using these reactances to be in a good agreement with the measured results.

In 1993, El Serafi and Wu defined analytically the parameters, which represent the cross-magnetising effect in round rotor and salient pole synchronous machines and proposed techniques for determining these parameters. They verified the analytical determination of the parameters representing the cross-magnetising effect in the saturated synchronous machine by experimental investigation.

Takeda and Tamura (1995) developed a new method to express the main flux saturation in synchronous machine in which they expressed the saturation by auxiliary currents and unsaturated magnetising inductances instead of the saturated inductances.

They presented the results of simulation of the developed model for transient stability analysis.

In 1997 and 1998, based on “*generalised flux linkage*” and “*generalised inductance*” concepts, Levi introduced a procedure of main flux saturation modelling to the modelling of saturated salient pole and round rotor synchronous machine. The procedure enabled derivation of saturated synchronous machine models for different selections of set of state-space variables (winding current or fluxes or both) and thus a number of novel model can be derived. He discussed the suitability of models for different application of synchronous machine. Also he verified that the accuracy of saturation representation in all the novel models is the same.

Finally in 1998, Corzine *et al.* proposed an improved technique for incorporating saturation into d-q axis model of synchronous machine. They presented the saturation function by an arctangent function with four parameters. They verified the model for steady state and transient conditions using a laboratory synchronous machine-rectifier system.

From the brief review of the previous research, which has been done during last 20 years, two important points are extracted. First, any practical method of saturation representation has to be established based on pragmatic assumption and empirical approximations. The important factor in developing a successful model is that the model structure has to be simple, based on the machine characteristic data such as open circuit characteristics and reasonable approximations. Second, the importance of including magnetic saturation in synchronous machine modelling comes from the influence of this phenomenon in a number of transient and steady state operating regimes of synchronous machine. Some of these are presented in the references and involve synchronous generator swing curves during temporary three-phase short circuit, determination the behaviour of the generator voltage during load rejection, transient behaviour of a self commutated synchronous motor drive, phasor diagram of the synchronous machine and power/load angle relationship.

1.2 Objectives

This project aims to:

1. Study some methods of incorporating magnetic saturation in synchronous machine modelling.
2. Develop linearised models of saturated synchronous machine. Perform the small disturbance analysis for a single-machine infinite bus system using MATLAB as simulation tool.
3. Discuss the effects of saturation on the dynamic behaviour of the synchronous machine.

1.3 Content

In chapter 2 of this dissertation, the basic operation principles of synchronous machine are explained. The mathematical model of salient pole synchronous machine is developed starting with basic machine equations using abc phase notation, then progresses to the rotor reference frame (Park's equations) assuming a linear magnetic system; saturation is not considered. Finally, the saturated machine models are discussed with derivation of some models for illustration. Chapter 3 introduces some of basic concepts of small disturbance stability analysis. Small disturbance unsaturated models of the synchronous machine are presented. Then a method of linearisation is proposed to linearise the saturated models of the machine. The method is used to develop different small disturbance saturated models of the machine. Results of simulation and discussion are given in chapter 4. In chapter 5, conclusions and recommendations are presented.

CHAPTER 2

SYNCHRONOUS MACHINE MODELLING

Synchronous machine, which is driven either by hydro or steam or combustion engines, is the source of electric power in any power system network. It is principal means of converting energy from mechanical to electrical. Synchronous machine is categorised into two principal types the round rotor machine and salient pole machine. The former (turbo-generator), which is driven by steam or gas turbine, has round rotor (rotating member) with slots in which distributed field winding is placed and has a number poles of two, four or six. However the later (hydro-generator), which is driven by hydraulic-turbines, has laminated salient pole rotor with concentrated field winding and a large number of poles. Some synchronous machines are equipped with one or two damper winding to damp out the oscillation that would occur during abnormal conditions. The stator winding of both types, which carry the current to the supplied load, is embedded around the inside circumference of the stationary part.

In general, a synchronous machine is connected to the transmission network through a step up transformer. In the case of a small unit, the generator and the transformer are connected by cables, while a large generator may be connected to its transformer by a single-phase, screened busbars.

2.1 Basic Operating Principals

The operation of synchronous machine is based on Faraday's Law of electromagnetic induction, which states that the e.m.f induced in a circuit is proportional to the rate of change of flux linkages. Assuming lossless circuit, e.m.f is given as

$$e.m.f = \frac{d\psi}{dt} \quad (2.1)$$

where,

$$\psi = N\phi \quad (2.2)$$

ψ is the flux linkage.

N is the number of turns.

ϕ is the instantaneous value of a time varying flux.

It has be noted that a traditional negative sign in equation (2.1) is used in deference to Lenz's law, which states that any current produced by the e.m.f tends to oppose the flux change.

If the field winding, which is mounted on the rotor, is exited with constant current and the rotor rotates at synchronous speed, e.m.f is induced in the stator winding which generates current supplied to the loads. This current develops a magnetic flux in the air gap, which rotates at synchronous speed. The synchronous speed is given as

$$n_s = \frac{120f}{P} \quad (2.3)$$

where, f_e is the system frequency and P is the number of poles.

This flux interact with the flux created by the field current, and electromagnetic torque produced from the desire of these two magnetic fields to align. This torque resists rotation, which is sustained because of the applied mechanical torque.

2.2 Basic Machine Model

The following basic equation and notation for synchronous machine can be used to model the behaviour of the machine (Kundur, 1992; Vas, 1992; Krause *et al.*, 1995). The mathematical model is developed based on the following assumptions (Kundur, 1994).

1. The stator windings are sinusoidally distributed along the air gap as far as the mutual effects with rotor are concerned.
2. The stator slots cause no appreciable variation of the rotor inductance with rotor position.
3. Magnetic hysteresis is negligible.
4. Magnetic saturation effects are negligible.

The phase stator and rotor voltages of a salient pole synchronous machine with one damper winding in the same axis as the field winding and two damper windings, which are displaced 90 degree ahead of the magnetic axis of the field winding, are defined as

$$\begin{aligned}v_a &= \frac{d\psi_a}{dt} - r_s i_a \\v_b &= \frac{d\psi_b}{dt} - r_s i_b \\v_c &= \frac{d\psi_c}{dt} + r_s i_c\end{aligned}\tag{2.4}$$

$$\begin{aligned}v_f &= \frac{d\psi_f}{dt} + i_f r_f \\v_{kd1} &= \frac{d\psi_{kd1}}{dt} + r_{kd1} i_{kd1} \\v_{kq1} &= \frac{d\psi_{kq1}}{dt} + r_{kq1} i_{kq1} \\v_{kq2} &= \frac{d\psi_{kq2}}{dt} + r_{kq2} i_{kq2}\end{aligned}\tag{2.5}$$

where ψ is the flux linking the winding indicated by the subscript, r_s is the stator winding resistance, r_f is the field winding resistance, r_{kd1} is the d- axis damper winding resistance, r_{kq1} is the q- axis first damper winding resistance and r_{kq2} is the q- axis second damper winding resistance. Using the standard abc phase reference the flux linkage is a function of the currents as follows:

$$\begin{aligned}\psi_a &= L_{aa} i_a + L_{ab} (i_b + i_c) + L_{ad} (i_f + i_{kd}) + L_{aq} (i_{kq1} + i_{kq2}) \\ \psi_{Ad} &= L_{AA} i_A + L_{Ad} (i_a + i_b + i_c) + L_{mdr} i_{kd1} \\ \psi_{Aq} &= L_{AA} i_A + L_{Aq} (i_a + i_b + i_c) + L_{mqr} i_{kq1}\end{aligned}\tag{2.6}$$

where, ψ_i is the flux linking winding i and L_{ij} is the inductance winding i and j .

Under balanced conditions, if no neutral winding exists, the vector sum of the stator currents must be zero:

$$i_a + i_b + i_c = 0\tag{2.7}$$

2.3 Reference Frame Theory

The voltage equations that describe the performance of the synchronous machine are established in the previous section. It is found that some of the machine inductances are functions of the rotor position, consequently the coefficients of the differential equations, which describe the machine performance are time varying. A change of variables is used to reduce the complexity of these equations, which was done by Park in the late 1920s. He refereed the variables of the stator winding of the synchronous machine to a frame of reference fixed in the rotor. Then several changes of variables, i.e. reference of frames, have been introduced to transfer the variables of electric machines and power system components to a reference frame that would simplify their analysis and simulation.

The transformation of the three phase variables of stationary circuit element, which can be a stator of a synchronous machine, to the arbitrary reference frame may be expressed (Krause *et al.*, 1995)

$$f_{qdos} = K_s f_{abcs} \quad (2.8)$$

where $f_{qdos}^T = (f_{qs} \ f_{ds} \ f_{os0})$, $f_{abcs}^T = (f_{as} \ f_{bs} \ f_{cs})$, the subscript s is used to represent the parameters in the stator, and

$$K_s = \frac{2}{3} \begin{pmatrix} \cos \theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin \theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (2.9)$$

where the angular displacement θ is defined as

$$\theta = \int_0^t \omega(\xi) d\xi + \theta(0) \quad (2.10)$$

where ξ is a dummy variable.

The above transformation can be applied to the stationary circuits parameters (machines, transmission lines, loads impedance) or to the rotor circuits parameters (synchronous generators, induction motors).

The important advantage of reference frame transformation is the ability to eliminate time varying components. For example, consider a balanced three phase voltage source,

$$v_a = V_m \cos(\theta_s) \quad (2.11)$$

$$v_b = V_m \cos\left(\theta_s - \frac{2\pi}{3}\right) \quad (2.12)$$

$$v_c = V_m \cos\left(\theta_s + \frac{2\pi}{3}\right) \quad (2.13)$$

where V_m is the peak voltage of the source, θ_s is the angular position of the voltage defined using equation (2.14) and ω_s is the frequency of the source.

$$\theta_s = \int_0^t \omega_s(\xi) d\xi + \theta(0) \quad (2.14)$$

Applying transformation given in equation (2.8):

$$\begin{aligned} \begin{pmatrix} v_q \\ v_d \\ v_o \end{pmatrix} &= \begin{pmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} V_m \cos(\theta_s) \\ V_m \cos\left(\theta_s - \frac{2\pi}{3}\right) \\ V_m \cos\left(\theta_s + \frac{2\pi}{3}\right) \end{pmatrix} \\ &= V_m \begin{pmatrix} \cos(\theta_s - \theta) \\ -\sin(\theta_s - \theta) \\ 0 \end{pmatrix} \end{aligned} \quad (2.15)$$

When the angular velocity ω of the transformation is set equal to the frequency of the source ω_s , then the transformed voltages have constant values. The same analysis can be applied to any balanced three phase electrical variable (current, voltage, flux linkage).

The angular displacement θ is the displacement associated with the qdo transformation's reference frame. Therefore, the angular velocity ω associated to the angular displacement is unspecified. The choice of ω determines the reference frame to which the parameters are transferred. The following table presents the possible reference frames (Krause *et al.*, 1995).

Table2.1.Speed of reference frames.

Reference Frame Speed	Interpretation
ω	Stator and rotor variables referred to an arbitrary reference of frame.
0	Stator and rotor variables referred to a stationary reference of frame.
ω_r	Stator and rotor variables referred to a reference of frame fixed in the rotor.
ω_e	Stator and rotor variables referred to Synchronously rotating reference of frame.

Normally, in studying the transient and dynamic stability analysis of power system, synchronously rotating reference frame is adopted (Vas *et al.*, 1983; Anderson and Fouad, 1994; Krause *et al.*, 1995)

2.3.1 qdo Transformation on Resistive Elements

Considering a three-phase circuit, the voltage across the resistors is

$$v_{abc} = Ri_{abc} \quad (2.16)$$

$$v_{qdo} = K_s R K_s^{-1} i_{qdo} \quad (2.17)$$

where,

$$v_{qdo}^T = (v_q \quad v_d \quad v_o) \quad (2.18)$$

$$v_{abc}^T = (v_a \quad v_b \quad v_c) \quad (2.19)$$

$$i_{qdo}^T = (i_{qs} \quad i_{ds} \quad i_{os}) \quad (2.20)$$

$$i_{abc}^T = (i_a \quad i_b \quad i_c) \quad (2.21)$$

and R is a diagonal matrix with the resistance of each phase in the diagonal entries, and off-diagonal entries are zeros. If the resistance of each phase is equal, then the nonzero elements of the matrix R are equal. Therefore, the resistive matrix is the same for both the actual circuit and the arbitrary reference frame circuit (Krause *et al.*, 1995).

$$K_s R K_s^{-1} = R \quad (2.22)$$

2.3.2 qdo Transformation on Inductive Elements

The application of qdo transformation to inductive elements is performed by considering the voltage across the inductors (Krause *et al.*, 1995):

$$v_{abc} = \frac{d\psi_{abc}}{dt} \quad (2.23)$$

$$\begin{aligned} v_{qdo} &= K_s \frac{d}{dt} [K_s^{-1} \psi_{qdo}] \\ &= K_s \frac{d}{dt} [K_s^{-1}] \psi_{qdo} + K_s K_s^{-1} \frac{d}{dt} \psi_{qdo} \\ &= \omega \psi_{qdo} + \frac{d}{dt} \psi_{qdo} \end{aligned} \quad (2.24)$$

where,

$$\psi_{qdo}^T = (\psi_q \quad \psi_d \quad \psi_o) \quad (2.25)$$

For linear systems, the relation between flux linkage and the current may be given as:

$$\psi_{abc} = L i_{abc} \quad (2.26)$$

$$\Rightarrow \psi_{qdo} = K_s L K_s^{-1} i_{qdo} \quad (2.27)$$

If the circuit considered has no mutual inductance between phases and the self-inductance of each phase is equal, then L is a diagonal matrix with equal non-zero diagonal elements:

$$K_s L K_s^{-1} = L \quad (2.28)$$

The analysis of electric machines involve usage of inductance matrices to relate the flux and the current, which can be written in two forms as follows

$$L_{s \text{ or } r} = \begin{pmatrix} L_l & -\frac{1}{2}L_m & -\frac{1}{2}L_m \\ -\frac{1}{2}L_m & L_l + L_m & -\frac{1}{2}L_m \\ -\frac{1}{2}L_m & -\frac{1}{2}L_m & L_l + L_m \end{pmatrix} \quad (2.29)$$

$$L_{sr} = L_m \begin{pmatrix} \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) \end{pmatrix} \quad (2.30)$$

where L_l and L_m are the leakage and mutual inductances respectively. Equation (2.29) is used to present the inductance in the stator s or the rotor r of the machine. Equation (2.30) is used to present the time varying inductances due to the rotation of the machine. By applying the qdo transformation to equation (2.29), it can be shown that:

$$K_s L_{s \text{ or } r} K_s^{-1} = \begin{pmatrix} L_l & 0 & 0 \\ 0 & L_l + \frac{3}{2}L_m & 0 \\ 0 & 0 & L_l \end{pmatrix} \quad (2.31)$$

Also, applying qdo transformation in the synchronous reference frame to equation (2.30) yields

$$K_s L_{sr} K_s^{-1} = \begin{pmatrix} \frac{3}{2}L_m & 0 & 0 \\ 0 & \frac{3}{2}L_m & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.32)$$

2.4 Machine Equations in Rotor Reference Frame

Using the techniques explained in the previous section, the basis *abc* machine model can be transferred to different reference frames. As mentioned earlier, the frame fixed in the rotor can remove the time dependency of the coefficients in the set of the differential equations. By performing the transformation, the stator and rotor voltage equations (2.4) and (2.5), which are written for generation convention, become (Anderson and Fouad, 1994):

$$\begin{aligned} v_{qs} &= -r_s i_{qs} - \frac{d\psi_{qs}}{dt} + \omega \psi_{ds} \\ v_{ds} &= -r_s i_{ds} - \frac{d\psi_{ds}}{dt} - \omega \psi_{qs} \end{aligned} \quad (2.33)$$

$$\begin{aligned} v_f &= r_f i_f + \frac{d\psi_f}{dt} \\ v_{kd1} &= r_{kd1} i_{kd1} + \frac{d\psi_{kd1}}{dt} \\ v_{kq1} &= r_{kq1} i_{kq1} + \frac{d\psi_{kq1}}{dt} \\ v_{kq2} &= r_{kq2} i_{kq2} + \frac{d\psi_{kq2}}{dt} \end{aligned} \quad (2.34)$$

where,

$$\begin{aligned}
\psi_{qs} &= L_{ls} i_{qs} + \psi_{mq} \\
\psi_{ds} &= L_{ls} i_{ds} + \psi_{md} \\
\psi_f &= L_{lf} i_f + \psi_{md} \\
\psi_{kd1} &= L_{lkd1} i_{kd1} + \psi_{md} \\
\psi_{kq1} &= L_{lkq1} i_{kq1} + \psi_{mq} \\
\psi_{kq2} &= L_{lkq2} i_{kq2} + \psi_{mq}
\end{aligned} \tag{2.35}$$

and

$$\begin{aligned}
\psi_{md} &= L_{md} i_{md} = L_{md} (i_{ds} + i_f + i_{kd1}) \\
\psi_{mq} &= L_{mq} i_{mq} = L_{mq} (i_{qs} + i_{kq1} + i_{kq2})
\end{aligned} \tag{2.36}$$

The Electro-magnetic torque developed by the machine may be given in per unit as:

$$T_e = i_{qs} \psi_{ds} - i_{ds} \psi_{qs} \tag{2.37}$$

The relationship between the mechanical (T_m) and electrical torque in per unit, which describes the dynamic behaviour of the rotor can be expressed as:

$$\frac{d\omega}{dt} = \frac{1}{2H\omega_b} (T_m - T_e) \tag{2.38}$$

The rotor angle is defined as the electrical angular displacement of the rotor relative to its terminal voltage (Krause *et al.*, 1995) and can be expressed in per unit as

$$\frac{d\delta}{dt} = \omega_r - 1 \tag{2.39}$$

The flux model of the machine can be obtained by solving the winding currents in terms of the fluxes using equations (2.35) and (2.36) and then substituting in equation (2.34). So the flux model of the salient pole synchronous generator with one damper winding in d-axis and one in q-axis may be written as (Anderson and Fouad, 1994):

$$\begin{aligned}
\frac{d\psi_{ds}}{dt} &= -\left(\frac{r_s L_{MD}}{L_{ls}^2} - \frac{r_s}{L_{ls}}\right)\psi_{ds} + \frac{r_s L_{MD}}{L_{ls} L_{lf}}\psi_f + \frac{r_s L_{MD}}{L_{ls} L_{lf}}\psi_{kd1} - \omega_r \psi_{qs} - v_{ds} \\
\frac{d\psi_f}{dt} &= \frac{r_f L_{MD}}{L_{lf} L_{ls}}\psi_{ds} - \left(\frac{r_f L_{MD}}{L_{lf}^2} - \frac{r_f}{L_{lf}}\right)\psi_f + \frac{r_f L_{MD}}{L_{lf} L_{lkd1}}\psi_{kd1} + v_f \\
\frac{d\psi_{kd1}}{dt} &= \frac{r_{kd1} L_{MD}}{L_{lkd1} L_{ls}}\psi_{ds} + \frac{r_{kd1} L_{MD}}{L_{lkd1} L_{lf}}\psi_f - \left(\frac{r_{kd1} L_{MD}}{L_{lkd1} L_{lf}} - \frac{r_{kd1}}{L_{lkd1}}\right)\psi_{kd1} \\
\frac{d\psi_{qs}}{dt} &= -\left(\frac{r_s L_{MQ}}{L_{ls}^2} - \frac{r_s}{L_{ls}}\right)\psi_{qs} + \frac{r_s L_{MQ}}{L_{ls} L_{lkq1}}\psi_{kq1} + \omega_r \psi_{ds} - v_{qs} \\
\frac{d\psi_{kq1}}{dt} &= \frac{r_{kq1} L_{MQ}}{L_{lkq1} L_{ls}}\psi_{qs} - \left(\frac{r_{kq1} L_{MQ}}{L_{lkq1}^2} - \frac{r_{kq1}}{L_{lkq1}}\right)\psi_{kq1}
\end{aligned} \tag{2.40}$$

where,

$$\begin{aligned}
\frac{1}{L_{MD}} &= \frac{1}{L_{ls}} + \frac{1}{L_{md}} + \frac{1}{L_f} + \frac{1}{L_{lkd1}} \\
\frac{1}{L_{MQ}} &= \frac{1}{L_{ls}} + \frac{1}{L_{mq}} + \frac{1}{L_{lkq1}}
\end{aligned} \tag{2.41}$$

Similarly from (2.35) and (2.36) after appropriate substitutions, the current model can be obtained in state space form as:

$$\begin{aligned}
(v) &= (A) \frac{d(x)}{dt} + (B)(x) \\
(v) &= \begin{pmatrix} v_{ds} & v_f & 0 & v_{qs} & 0 \end{pmatrix}^T \\
(x) &= \begin{pmatrix} i_{ds} & i_f & i_{md} & i_{qs} & i_{mq} \end{pmatrix}^T
\end{aligned} \tag{2.42}$$

where matrices (A) and (B) are given with:

$$A = \begin{pmatrix} -L_{ls} & 0 & -L_{md} & 0 & 0 \\ 0 & L_f & L_{md} & 0 & 0 \\ -L_{lkd1} & -L_{lkd1} & (L_{lkd1} + L_{md}) & 0 & 0 \\ 0 & 0 & 0 & -L_{ls} & -L_{mq} \\ 0 & 0 & 0 & -L_{lkq1} & (L_{lkq1} + L_{mq}) \end{pmatrix} \tag{2.43}$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega_r L_{ls} & -\omega_r L_{md} \\ 0 & r_f & 0 & 0 & 0 \\ -r_{kd1} & -r_{kd1} & r_{kd1} & 0 & 0 \\ \omega_r L_{ls} & 0 & \omega_r L_{md} & -r_s & 0 \\ 0 & 0 & 0 & -r_{kq1} & r_{kq1} \end{pmatrix} \quad (2.44)$$

Using equation (2.33), (2.34) and (2.38), a seventh order model of the synchronous machine may be formed. If reduced order models are considered, relevant differential terms in these equations are set to zero, transforming differential equations into algebraic constraints. If stator transients are ignored, i.e., $p\Psi_{qdos} = 0$, the model is reduced to the fifth order. Furthermore, if the field and damper transients terms are also ignored, i.e., $p\Psi_f = 0$, $p\Psi_{kd1} = 0$ and $p\Psi_{kq1,2} = 0$ the model is reduced to the first order with only one differential equation that simulates the mechanical dynamics of the machine. If all differential terms in equation (2.33), (2.34) and (2.38) are eliminated, then the synchronous machine is reduced to steady state model.

2.5 Synchronous Machine Model Incorporating Magnetic Saturation

In the previous sections, the basic equations of the synchronous machine are developed ignoring the effects of stator and rotor saturation. This is done to make the analysis of the machine simple and manageable. However, the saturation effects in synchronous machine plays an important role in determining the characteristics and performance of the machine as mentioned early. Several methods and presentation have been performed to represent the saturation in synchronous machine some of these are discussed previously. Before proceeding to explain the derivation of the saturated machine models, it is preferred to provide a brief explanation about magnetic saturation.

2.5.1 Principals of Magnetic saturation

The characteristics of electric machines depend heavily on the use of magnetic materials. These magnetic materials, which are not ideal, are needed to form the magnetic circuit and are used by machine designer to obtain specific machine

characteristics. So when magnetic flux starts to increase, they begin to saturate and thus decreasing the contribution to overall flux in the machine. Equations (2.34), (2.35) and (2.38) show that the generated voltage and electromagnetic torque in synchronous machine depend on the flux linkages. Saturation may therefore considerably influence the characteristics and operation of the machine. This effect is extended to all electric machines.

The flux created by stator and rotor windings of the synchronous machine can be separated into two components. One component consists of flux, which links both windings and is known as main flux. The second component consists of flux, which links only the winding creating the flux and is called leakage flux. The leakage flux is made up of slot, end turn, zigzag leakage fluxes and each one is associated with leakage inductance.

Methods of determination of the inductances associated with each linkage flux have been performed depending on different assumption and procedures. The most accurate one is based on finite element analysis. However, some less computationally demanding methods, which are based on reasonable assumptions and for certain application, would provide relatively accurate and acceptable results(Kundur, 1994).

2.5.2 Saturated Machine Models

Current or flux models are chosen at present to introduce modelling of the main flux saturation in synchronous machine. However, Levi (1996, 1997& 1998) presented a complete set of models that can be obtained by selecting other combination of state variables. The method used to develop these models, based on the concept of generalised flux (ψ_u) and generalised inductance (Λ) and using single saturation factor. The generalised flux space vector is defined as a flux space vector that has to satisfy the following two criteria (Levi, 1996):

1. Its direction coincides with the direction of the magnetising current space vector (i_u).

-
2. It can be expressed as a linear combination of the selected pair of space vector state variables.

The correlation between the magnetising current space vector and generalised flux space vector is expressed through a generalised inductance. The classification of all models and their derivation can always be relied on the need to find the time derivative of the generalised inductance, the time derivative of the inverse of the generalised inductance or neither of the two. The total number of models that can be derived in the case of round rotor synchronous machine is 37 while in the case of salient pole synchronous machine is 34. In 34 models out of 37 models of round rotor machine, generalised flux and generalised inductance can be made equal to magnetising flux and magnetising inductance while the three remaining either generalised flux or both generalised flux and inductance do not exist physically in the machine. On the other hand, the only obtainable models in the case of salient pole synchronous machine, are those for which generalised flux and inductance can be made equal to magnetising flux and inductance.

The set of possible space vectors that can be selected as state-space variables is determined with

$$(x) = (\underline{i}_s \quad i_f \quad \underline{i}_k \quad \underline{i}_m \quad \underline{\psi}_s \quad \psi_f \quad \underline{\psi}_k \quad \underline{\psi}_m) \quad (2.45)$$

Three out of eight space vectors are to be chosen from this set to form the set of the state variables. The variables that are not chosen as state variables are expressed as functions of the selected ones. The generalised flux space vector can be defined for round rotor and salient pole synchronous machine respectively as

$$\underline{\psi}_{u1} = a\underline{x}_1 + b\underline{x}_2 + c\underline{x}_3 \quad (2.46)$$

$$\underline{\psi}_{u2} = (a_1x_{d1} + a_2x_{d2} + a_3x_{d3}) + j\left(\frac{1}{m}\right)(a_1x_{q1} + a_2x_{q2} + a_3x_{q3}) \quad (2.47)$$

where the term $\underline{x}$ represents a state vector, the term x_d or x_q represents a state element and m^2 is the saliency factor and is given as

$$m^2 = \frac{L_{mq}}{L_{md}} \quad (2.48)$$

As the generalised flux space vector is required to be aligned with magnetising current space vector for both machines, then (Levi, 1997)

$$\underline{\psi}_u = \Lambda \underline{i}_u \quad (2.49)$$

where Λ is the generalised inductance and it is dependent on the main flux saturation level. The coefficients of equation (2.46) and (2.47) have to be chosen in such a way that equation (2.49) is satisfied.

There are three possible types of models that can be derived. These are (Levi, 1996):

1. Λ models ($d\Lambda/dt$ is required). This group contains 13 models for salient pole and round rotor synchronous machine.
 - The pure current models (4 models).
 - Models with magnetising current and any two-flux linkage space vector (3 models).
 - Models with two currents one of which is the magnetising current and one flux linkage space vector (6 models).

The model with $\underline{i}_s, \underline{i}_k$ and $\underline{i}_m$ as state variables for salient pole synchronous generator is selected as an example for the derivation procedure for this group. Using equation (2.35), the voltage equations given in (2.33) and (2.34) can be expressed in terms of the chosen state variables. Then the time derivative of the flux linkages has to be found in terms of the selected state variables. For example, the stator q and d fluxes components may be given as:

$$\begin{aligned}\frac{d\psi_{qs}}{dt} &= L_{ls} \frac{di_{qs}}{dt} + m^2 L_u \frac{di_{mq}}{dt} + m^2 i_{mq} \frac{dL_u}{dt} \\ \frac{d\psi_{ds}}{dt} &= L_{ls} \frac{di_{ds}}{dt} + L_u \frac{di_{md}}{dt} + i_{md} \frac{dL_u}{dt}\end{aligned}\quad (2.50)$$

The time derivative of the magnetising inductance $\left(\frac{dL_u}{dt}\right)$ is given as (Levi, 1997):

$$\frac{dL_u}{dt} = (L - L_u) \frac{1}{i_u^2} \left(i_{md} \frac{di_{md}}{dt} + m^2 i_{mq} \frac{di_{mq}}{dt} \right) \quad (2.51)$$

where, $L_u = \frac{\psi_u}{i_u}$ and $L = \frac{d\psi_u}{di_u}$ which can be obtained from the magnetising curve. The derivation of $\left(\frac{dL_u}{dt}\right)$ is given in Appendix B.

So equation (2.50) becomes:

$$\begin{aligned}\frac{d\psi_{qs}}{dt} &= L_{ls} \frac{di_{qs}}{dt} + L_{qq} \frac{di_{mq}}{dt} + L_{dq} \frac{di_{md}}{dt} \\ \frac{d\psi_{ds}}{dt} &= L_{ls} \frac{di_{ds}}{dt} + L_{dd} \frac{di_{md}}{dt} + L_{dq} \frac{di_{mq}}{dt}\end{aligned}\quad (2.52)$$

where,

$$\begin{aligned}L_{qq} &= m^2 (L \sin^2 u + L_u \cos^2 u) \\ L_{dd} &= L \cos^2 u + L_u \sin^2 u \\ L_{dq} &= m(L - L_u) \cos u \sin u \\ \cos u &= \frac{i_{md}}{i_u} \\ \sin u &= m \frac{i_{mq}}{i_u}\end{aligned}\quad (2.53)$$

In the same manner, the time derivatives of other flux linkages can be obtained. Thus, the model may be given in a matrix form as:

$$\begin{aligned}
(v) &= (A) \frac{d(x)}{dt} + (B)(x) \\
(v) &= \begin{pmatrix} v_{ds} & v_f & 0 & v_{qs} & 0 \end{pmatrix} \\
(x) &= \begin{pmatrix} i_{ds} & i_f & i_{md} & i_{qs} & i_{mq} \end{pmatrix}
\end{aligned} \tag{2.54}$$

where matrices (A) and (B) are given with:

$$A = \begin{pmatrix} -L_{ls} & 0 & -L_{dd} & 0 & -L_{dq} \\ 0 & L_f & L_{dd} & 0 & L_{dq} \\ -L_{lkd1} & -L_{lkd1} & (L_{lkd1} + L_{dd}) & 0 & L_{dq} \\ 0 & 0 & -L_{dq} & -L_{ls} & -L_{qq} \\ 0 & 0 & L_{dq} & -L_{lkq1} & (L_{lkq1} + L_{qq}) \end{pmatrix} \tag{2.55}$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega_r L_{ls} & -\omega_r L_u \\ 0 & r_f & 0 & 0 & 0 \\ -r_{kd1} & -r_{kd1} & r_{kd1} & 0 & 0 \\ \omega_r L_{ls} & 0 & \omega_r L_u & -r_s & 0 \\ 0 & 0 & 0 & -r_{kq1} & r_{kq1} \end{pmatrix} \tag{2.56}$$

Following the same procedure different models can be developed. Two of these models, which belong to the same group, are given as follows:

- Model with $x = (\psi_{ds} \quad i_f \quad i_{md} \quad \psi_{qs} \quad i_{mq})$ as the state variables.

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & L_{lf} & L_{dd} & 0 & L_{dq} \\ \frac{-L_{lkd1}}{L_{ls}} & L_{lkd1} & \left(L_{lkd1} \left(1 + \frac{L_{dd}}{L_{ls}} \right) + L_{dd} \right) & 0 & \left(\frac{L_{lkd1}}{L_{ls}} L_{dq} + L_{dq} \right) \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \left(L_{dq} + \frac{L_{lkq1}}{L_{ls}} L_{dq} \right) & \frac{-L_{lkq1}}{L_{ls}} & \left(L_{qq} + L_{lkq1} \left(1 + \frac{L_{qq}}{L_{ls}} \right) \right) \end{pmatrix}$$

$$B = \begin{pmatrix} r_s & 0 & \frac{L_u}{L_{ls}} & -\omega & 0 \\ 0 & r_f & 0 & 0 & 0 \\ \frac{-r_{kd1}}{L_{ls}} & -r_{kd1} & r_{kd1} \left(1 + \frac{L_u}{L_{ls}} \right) & 0 & 0 \\ \omega & 0 & 0 & \frac{-r_s}{L_{ls}} & \frac{m^2 L_u}{L_{ls}} \\ 0 & 0 & 0 & \frac{-r_{kq1}}{L_{ls}} & r_{kq1} \left(1 + \frac{m^2 L_u}{L_{ls}} \right) \end{pmatrix} \quad (2.57)$$

- Model with $x = (\psi_{ds} \quad \psi_f \quad i_{md} \quad \psi_{qs} \quad i_{mq})$ as the state variables.

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{-L_{lkd1}}{L_{ls}} & \frac{-L_{lkdl}}{L_{lf}} & \left(L_{lkdl} + L_{dd} \left(L_{lkdl} \left(\frac{1}{L_{lf}} + \frac{1}{L_{ls}} \right) + 1 \right) \right) & 0 & L_{dq} \left(L_{lkdl} \left(\frac{1}{L_{lf}} + \frac{1}{L_{ls}} \right) + 1 \right) \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & L_{dq} \left(\frac{L_{lkq1}}{L_{ls}} + 1 \right) & -\frac{L_{lkq1}}{L_{ls}} & \left(L_{lkq1} + L_{qq} \left(\frac{L_{lkq1}}{L_{ls}} + 1 \right) \right) \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{-r_s}{L_{ls}} & 0 & \frac{r_s L_u}{L_{ls}} & -\omega & 0 \\ 0 & \frac{r_f}{L_{lf}} & \frac{r_f}{L_{lf}} L_{md} & 0 & 0 \\ \frac{-r_{kd1}}{L_{ls}} & \frac{-r_{kd1}}{L_{lf}} & r_{kd1} \left(1 + L_u \left(\frac{1}{L_{lf}} + \frac{1}{L_{ls}} \right) \right) & 0 & 0 \\ \omega & 0 & 0 & \frac{-r_s}{L_{ls}} & \frac{r_s m^2 L_u}{L_{ls}} \\ 0 & 0 & 0 & \frac{-r_{kq1}}{L_{ls}} & r_{kq1} \left(1 + \frac{m^2 L_u}{L_{ls}} \right) \end{pmatrix} \quad (2.58)$$

2. $1/\Lambda$ models ($d(1/\Lambda)/dt$ is required). There are 20 models for salient pole machine and 23 models for round rotor machine.

- Models with two currents (none of which is the magnetising current) and one flux linkage space vector, excluding the $\underline{i}_s, \underline{i}_k, \underline{\psi}_f; \underline{\psi}_s, \underline{i}_f, \underline{i}_k; \underline{\psi}_k, \underline{i}_f, \underline{i}_s$ models for salient pole machine (7 models) and the $\underline{i}_s, \underline{i}_k, \underline{\psi}_f$ model for round rotor machine (9 models).
- Models with two flux linkages and one current which is not the magnetising current, excluding the $\underline{\psi}_s, \underline{i}_k, \underline{\psi}_f; \underline{\psi}_s, \underline{\psi}_k, \underline{i}_f; \underline{\psi}_k, \underline{i}_s, \underline{\psi}_f$ models for salient pole machine (10 models) and the $\underline{\psi}_s, \underline{i}_s, \underline{\psi}_f$ and $\underline{\psi}_k, \underline{i}_s, \underline{\psi}_f$ models for round rotor machine (11 models).
- Models with magnetising flux and any other two-flux linkages (3 models for both types of machine).

For illustration, the model with $\underline{i}_s, \underline{i}_f$ and $\underline{\psi}_m$ as state variables for salient pole synchronous generator is chosen for the derivation procedure for this group. The time derivative of the stator currents can be found directly from (2.35) and (2.33), and it does not involve saturation terms. The d and q-axes damper flux components are expressed in terms of the selected state space variables as:

$$\begin{aligned}\psi_{kd1} &= L_{lkd1} \left(\frac{\psi_{md}}{L_u} - i_{ds} - i_f \right) + L_u \psi_{md} \\ \psi_{kq1} &= L_{lkq1} \left(\frac{\psi_{mq}}{m^2 L_u} - i_{qs} \right) + m^2 L_u \psi_{mq}\end{aligned}\quad (2.59)$$

From (2.59), it is clear that the time derivative of the field and damper windings

fluxes required the derivative of $\frac{1}{L_u}$ (derivation of $\frac{d\left(\frac{1}{L_u}\right)}{dt}$ is given in Appendix B). It is given as (Levi, 1998):

$$\frac{d\left(\frac{1}{L_u}\right)}{dt} = \left(\frac{1}{L} - \frac{1}{L_u} \right) \frac{1}{\psi_u^2} * \left(\psi_{md} \frac{d\psi_{md}}{dt} + \frac{1}{m^2} \psi_{mq} \frac{d\psi_{mq}}{dt} \right) \quad (2.60)$$

The complete model can be given in the same form as equation (2.43):

$$\begin{aligned} (v) &= (v_{ds} \quad 0 \quad v_f \quad v_{qs} \quad 0)^T \\ (x) &= (i_{ds} \quad i_f \quad \psi_{md} \quad i_{qs} \quad \psi_{mq})^T \end{aligned} \quad (2.61)$$

A and B matrices are:

$$A = \begin{pmatrix} -L_{ls} & 0 & -1 & 0 & 0 \\ 0 & L_{lf} & 1 & 0 & 0 \\ -L_{lkd1} & -L_{lkd1} & \left(\frac{L_{lkd1}}{L_{dd}} + 1 \right) & 0 & \frac{L_{lkd1}}{L_{dq}} \\ 0 & 0 & 0 & -L_{ls} & -1 \\ 0 & 0 & \frac{L_{lkq1}}{L_{dq}} & -L_{ls} & \left(1 + \frac{L_{lkq1}}{L_{qq}} \right) \end{pmatrix} \quad (2.62)$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega L_{ls} & -\omega \\ 0 & r_f & 0 & 0 & 0 \\ -r_{kd1} & -r_{kd1} & \frac{r_{kd1}}{L_u} & 0 & 0 \\ \omega L_{ls} & 0 & \omega & -r_s & 0 \\ 0 & 0 & 0 & -r_{kq1} & \frac{r_{kq1}}{m^2 L_u} \end{pmatrix} \quad (2.63)$$

The saturation terms in A and B are defined as:

$$\begin{aligned} \frac{1}{L_{dd}} &= \frac{1}{L} \cos^2 u + \frac{1}{L_u} \sin^2 u \\ \frac{1}{L_{qq}} &= \frac{1}{m^2} \left(\frac{1}{L_u} \cos^2 u + \frac{1}{L} \sin^2 u \right) \\ \frac{1}{L_{dq}} &= \frac{1}{m} \left(\frac{1}{L} - \frac{1}{L_u} \right) \sin u \cos u \\ \cos u &= \frac{\psi_{md}}{\psi_u} \\ \sin u &= \frac{1}{m} \frac{\psi_{mq}}{\psi_u} \end{aligned} \quad (2.64)$$

Using the same procedure, two models are developed with given state variables.

They are:

- Model with $x = (i_{ds} \quad \psi_{md} \quad i_{kd1} \quad i_{qs} \quad \psi_{mq})$ as state variables.

$$A = \begin{pmatrix} -L_{ls} & -1 & 0 & 0 & 0 \\ -L_{lf} & \left(\frac{L_{lf}}{L_{dd}} + 1\right) & -L_{lf} & 0 & \frac{L_{lf}}{L_{dq}} \\ 0 & 1 & L_{lkd1} & 0 & 0 \\ 0 & 0 & 0 & -L_{ls} & 0 \\ 0 & \frac{L_{lkq1}}{L_{dq}} & 0 & L_{lkq1} & \left(\frac{L_{lkq1}}{L_{qq}} + 1\right) \end{pmatrix}$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega & -\omega \\ -r_f & \frac{r_f}{L_u} & -r_f & 0 & 0 \\ 0 & 0 & r_{kd1} & 0 & 0 \\ \omega & \omega & 0 & -r_s & 0 \\ 0 & 0 & 0 & -r_{kq1} & \frac{r_{kq1}}{m^2 L_u} \end{pmatrix} \quad (2.65)$$

- Model with $x = (i_{ds} \quad \psi_f \quad \psi_{md} \quad i_{qs} \quad \psi_{mq})$ as state variables

$$A = \begin{pmatrix} -L_{ls} & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -L_{lkd1} & \frac{-L_{lkd1}}{L_{lf}} & \left(1 + L_{lkd1} \left(\frac{1}{L_{dd}} + \frac{1}{L_{lf}}\right)\right) & 0 & \frac{L_{lkd1}}{L_{dq}} \\ 0 & 0 & 0 & -L_{ls} & -1 \\ 0 & 0 & \frac{L_{lkq1}}{L_{dq}} & -L_{lkq1} & \left(\frac{L_{lkq1}}{L_{qq}} + 1\right) \end{pmatrix}$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega & -\omega \\ 0 & \frac{r_f}{L_{lf}} & \frac{-r_f}{L_{lf}} & 0 & 0 \\ -r_{kd1} & \frac{-r_{kd1}}{L_{lf}} & r_{kd1} \left(\frac{1}{L_u} + \frac{1}{L_{lf}} \right) & 0 & 0 \\ \omega & 0 & \omega & -r_s & 0 \\ 0 & 0 & 0 & -r_{kq1} & \frac{r_{kq1}}{m^2 L_u} \end{pmatrix} \quad (2.66)$$

3. Neither $d\Lambda/dt$ nor $d(1/\Lambda)/dt$ are required. This model is the classical flux linkage model.

The models that can not be obtained for a synchronous machine are:

1. $\underline{i}_s, \underline{i}_k, \psi_f$
2. $\underline{\psi}_s, \underline{i}_k, \psi_f$
3. $\underline{\psi}_k, \underline{i}_s, \psi_f$

These three models can not be obtained because of the absence of the q-axis excitation winding (Levi, 1997).

Comparing type Λ models and type $1/\Lambda$ models, the former requires the time derivative of L_u and two non-linear functions, namely, $L=f(i_u)$ and $L_u=f(i_u)$, which can be obtained for the open circuit d-axis magnetising curve, while the later needs the time derivative of $1/L_u$ and two non-linear functions, namely, $1/L=f(\psi_u)$ and $1/L_u=f(\psi_u)$, which also can be obtained from the open circuit d-axis magnetising curve. Considering matrices A and B representing each model in both groups, it is noticed that number of saturation terms presented in type Λ models is greater than the number of terms that corresponds to type $1/\Lambda$ models.

Verification is carried out by Levi (1997&1998) on the saturated models presented in this chapter and he showed that all models yield identical results.

CHAPTER 3

SMALL DISTURBANCE STABILITY ANALYSIS

Small disturbance stability is one of the major problems in the power system stability area and can be defined as the ability of the power system to maintain synchronism under small disturbance (Kundur, 1994). These disturbances arise from the changes in the power demand due to mainly switching of heating, air conditions, lighting and motor loads. They are considered sufficiently small for linearisation of the system equations to be allowed for the purposes of the analysis. In some situations, they can initiate instability and subsequent loss of synchronism, which causes oscillatory behaviour that, if not damped, may intensify. Oscillations are unacceptable because they limit the power transferred and may induce stress in the mechanical shaft of the rotor of the generator.

In small disturbance stability analysis, differential-algebraic mathematical models of the power system are linearised about chosen equilibrium point to determine the steady state stability using eigenvalue analysis.

3.1 Linearisation Form of State Equations

The traditional method to analyse the small disturbance stability problem is to study the behaviour of a dynamic system, which is described by a set of n first order non-linear ordinary differential equations of the following form:

$$\dot{x} = f_i(x_1, x_2, x_3, \dots; u_1, u_2, \dots, u_r; t), i = 1, 2, 3, \dots, n \quad (3.1)$$

where n is the order of the system and r is the number of inputs. This can be written in the following form by using vector-matrix notation:

$$\dot{x} = f(x, u, t) \quad (3.2)$$

where

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \quad f = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}$$

The column vector x is referred to as the *state vector*, and its entry x_i as state variable. The column u is the vector of inputs to the system. They are the external signals that affect the performance of the system. Time is denoted by t , and the derivative of a state variable x with respect to time is given by $\dot{x}$. If the derivatives of the state variables are not explicit functions of time, the system is said to be *autonomous*. In this case equation (3.2) simplifies to

$$\dot{x} = f(x, u) \quad (3.3)$$

The output variables may be expressed in terms of the state variables and the output variables in the following form:

$$y = g(x, u) \quad (3.4)$$

where,

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \quad g = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}$$

The column vector y is the vector of the outputs, and g is a vector of non-linear functions relating state and input variables to output variables.

To solve the small-disturbance stability problem, it is required to linearise these non-linear equations by using Taylor's series expansion. So the linearised forms of equations (3.3) and (3.4) can be obtained as:

$$\begin{aligned}\dot{\Delta x} &= K\Delta x + M\Delta u \\ \Delta y &= N\Delta x + T\Delta u\end{aligned}\tag{3.5}$$

where,

$$K = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdot & \cdot & \frac{\partial f_1}{\partial x_n} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_n}{\partial x_1} & \cdot & \cdot & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \quad M = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \cdot & \cdot & \frac{\partial f_1}{\partial u_r} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_n}{\partial u_1} & \cdot & \cdot & \frac{\partial f_n}{\partial u_r} \end{pmatrix}$$

$$N = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdot & \cdot & \frac{\partial g_1}{\partial x_n} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial g_m}{\partial x_1} & \cdot & \cdot & \frac{\partial g_m}{\partial x_n} \end{pmatrix} \quad T = \begin{pmatrix} \frac{\partial g_1}{\partial u_1} & \cdot & \cdot & \frac{\partial g_1}{\partial u_r} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial g_n}{\partial u_1} & \cdot & \cdot & \frac{\partial g_n}{\partial u_r} \end{pmatrix}$$

$\dot{\Delta x}$ is the state vector of dimension n.

Δy is the output vector of dimension m.

Δu is the input vector of dimension r.

K is the state matrix of size nxn.

M is the control or input matrix of size nxr.

N is the output matrix of size mxn.

T is the feed forward matrix, which defines the proportion of input that appears directly in the output, size mxr.

3.2 Eigenvalue Analysis

The eigenvalues of a matrix are given by the values of the scalar parameter λ for which there exist non-trivial solutions to the equation

$$K\phi = \lambda\phi \quad (3.6)$$

where,

K is an nxn matrix (real for a physical system)

ϕ is an nx1 vector

To find the eigenvalues, equation (3.6) may be written in the form

$$(K - \lambda I)\phi = 0 \quad (3.7)$$

For a non-trivial solution

$$\det(K - \lambda I) = 0 \quad (3.8)$$

Expansion of the determinant gives the characteristics equation. The n solutions of (3.8) $\lambda = \lambda_1, \lambda_2, \dots, \lambda_n$ are eigenvalues of K . The eigenvalues may be real or complex. If K is real, complex eigenvalues always occur in conjugate pair. The real part of the eigenvalue determines damping and the imaginary part determines the frequency of oscillations. A positive real part indicates an oscillation of increasing amplitude while the negative real part indicates a damped oscillation. So for a pair of eigenvalues having the following form (Kundur, 1994):

$$\lambda = \sigma \pm j\omega \quad (3.9)$$

The frequency of oscillation in Hz and the time constant in second are given as:

$$f = \frac{\omega}{2\pi}$$

$$t = \frac{1}{\sigma} \quad (3.10)$$

The rate of decay of the amplitude of oscillation is determined by

$$\xi = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}} \quad (3.11)$$

where ξ is damping ratio.

From *Lyapunov's first method*, the stability in the small of a non-linear system is given by the roots of the characteristic equation of the system of the first approximation, i.e., by the eigenvalues of K :

1. If all eigenvalues have negative real parts (i.e., all eigenvalues are strictly in the left-half complex plane), the original system is asymptotically stable.
2. If at least one of the eigenvalues has a positive real part (i.e., at least one of the eigenvalues is strictly in the right-half complex plane), the original system is unstable.
3. If the eigenvalues have real parts equal to zero, it is not possible from the linear approximation to conclude anything about the system stability (Kundur, 1994).

So for the small-disturbance problem, eigenvalues analysis is a very critical point. Once the eigenvalues for a dynamic system are identified, important information can be obtained for stability analysis.

3.3 Linearisation of Unsaturated Synchronous Machine Models

The linearisation, or the small disturbance equations of the unsaturated flux and current models are available in the literature. The linearised models are given as follows (Anderson and foudad, 1994):

1- Flux Model

$$\begin{aligned}
\frac{d\Delta\psi_{ds}}{dt} &= -\left(\frac{r_s L_{MD}}{L_{ls}^2} - \frac{r_s}{L_{ls}}\right)\Delta\psi_{ds} + \frac{r_s L_{MD}}{L_{ls} L_{lf}}\Delta\psi_f + \frac{r_s L_{MD}}{L_{ls} L_{lf}}\Delta\psi_{kd1} - \omega_{ro}\Delta\psi_{qs} \\
&\quad - \psi_{qs}\Delta\omega_r - \sqrt{3}\sin\delta_o\Delta v + v_{qso}\Delta\delta \\
\frac{d\Delta\psi_f}{dt} &= \frac{r_f L_{MD}}{L_{lf} L_{ls}}\Delta\psi_{ds} - \left(\frac{r_f L_{MD}}{L_{lf}^2} - \frac{r_f}{L_{lf}}\right)\Delta\psi_f + \frac{r_f L_{MD}}{L_{lf} L_{lkd1}}\Delta\psi_{kd1} + \Delta v_f \\
\frac{d\Delta\psi_{kd1}}{dt} &= \frac{r_{kd1} L_{MD}}{L_{lkd1} L_{ls}}\Delta\psi_{ds} + \frac{r_{kd1} L_{MD}}{L_{lkd1} L_{lf}}\Delta\psi_f - \left(\frac{r_{kd1} L_{MD}}{L_{lkd1}^2} - \frac{r_{kd1}}{L_{lkd1}}\right)\Delta\psi_{kd1} \\
\frac{d\Delta\psi_{qs}}{dt} &= -\left(\frac{r_s L_{MQ}}{L_{ls}^2} - \frac{r_s}{L_{ls}}\right)\Delta\psi_{qs} + \frac{r_s L_{MQ}}{L_{ls} L_{lkq1}}\Delta\psi_{kq1} + \omega_{ro}\Delta\psi_{ds} \\
&\quad + \psi_{dso}\Delta\omega_r - \sqrt{3}\cos\delta_o\Delta v + v_{dso}\Delta\delta \\
\frac{d\Delta\psi_{kq1}}{dt} &= \frac{r_{kq1} L_{MQ}}{L_{lkq1} L_{ls}}\Delta\psi_{qs} - \left(\frac{r_{kq1} L_{MQ}}{L_{lkq1}^2} - \frac{r_{kq1}}{L_{lkq1}}\right)\Delta\psi_{kq1}
\end{aligned} \tag{3.12}$$

The torque equation becomes

$$\begin{aligned}
\Delta T_e &= \left(\left(\frac{L_{MD} - L_{MQ}}{L_{ls}^2} \right) \psi_{qso} - \frac{L_{MQ}}{L_{ls} L_{lkq1}} \psi_{kq1o} \right) \Delta\psi_{ds} + \frac{L_{MD}}{L_{ls} L_{lf}} \psi_{qso} \Delta\psi_f \\
&\quad + \frac{L_{MD}}{L_{ls} L_{lkd1}} \psi_{qso} \Delta\psi_{kd1} + \left(\left(\frac{L_{MD} - L_{MQ}}{L_{ls}^2} \right) \psi_{dso} + \frac{L_{MD}}{L_{ls} L_{lkq1}} \psi_{kd1o} + \frac{L_{MD}}{L_{ls} L_{lf}} \psi_{fo} \right) \Delta\psi_{qs} \\
&\quad - \frac{L_{MQ}}{L_{ls} L_{lkq1}} \psi_{dso} \Delta\psi_{kq1}
\end{aligned} \tag{3.13}$$

and finally

$$\frac{d\Delta\delta}{dt} = \Delta\omega_r \tag{3.14}$$

It has to be noted that equation (3.14) given above is valid for all unsaturated and saturated models. So matrix K and M can be given as:

$$K = \begin{pmatrix} -\left(\frac{r_s L_{MD}}{L_{ls}^2} - \frac{r_s}{L_{ls}}\right) & \frac{r_s L_{MD}}{L_{ls} L_{lf}} & \frac{r_s L_{MD}}{L_{ls} L_{lkd1}} & \dots\dots\dots \\ \frac{r_f L_{MD}}{L_{ls} L_{lf}} & -\left(\frac{r_f L_{MD}}{L_{lf}^2} - \frac{r_f}{L_{lf}}\right) & \frac{r_f L_{MD}}{L_{lkd1} L_{lf}} & \dots\dots\dots \\ \frac{r_{kd1} L_{MD}}{L_{ls} L_{lkd1}} & \frac{r_{kd1} L_{MD}}{L_{lkd1} L_{lf}} & -\left(\frac{r_{kd1} L_{MD}}{L_{lkd1}^2} - \frac{r_{kd1}}{L_{lkd1}}\right) & \dots\dots\dots \\ \omega_{ro} & 0 & 0 & \dots\dots\dots \\ 0 & 0 & 0 & \dots\dots\dots \\ \left(\frac{L_{MD} - L_{MQ}}{L_{ls}^2} \psi_{qso} - \frac{L_{MQ}}{L_{ls} L_{lkq1}} \psi_{kq1}\right) / 2H\omega_b & \frac{L_{MD} \psi_{qso}}{2H\omega_b L_{ls} L_{lf}} & \frac{L_{MD} \psi_{qso}}{2H\omega_b L_{ls} L_{lkd1}} & \dots\dots\dots \\ 0 & 0 & 0 & \dots\dots\dots \\ \dots\dots - \omega_{ro} & 0 & -\psi_{qso} & v_{qso} \\ \dots\dots 0 & 0 & 0 & 0 \\ \dots\dots 0 & 0 & 0 & 0 \\ \dots\dots -\left(\frac{r_s L_{MQ}}{L_{ls}^2} - \frac{r_s}{L_{ls}}\right) & \frac{r_s L_{MD}}{L_{lkq1} L_{ls}} & \psi_{dso} & v_{dso} \\ \dots\dots \frac{r_{kq1} L_{MD}}{L_{lkq1} L_{ls}} & -\left(\frac{r_{kq1} L_{MQ}}{L_{ls}^2} - \frac{r_{kq1}}{L_{ls}}\right) & 0 & 0 \\ \dots\dots \left(\frac{L_{MD} - L_{MQ}}{L_{ls}^2} \psi_{dso} + \frac{L_{MD}}{L_{ls} L_{lkd1}} \psi_{kd1o} + \frac{L_{MD}}{L_{ls} L_{lf}} \psi_{fo}\right) / 2H\omega_b & -\frac{L_{MQ} \psi_{dso}}{2H\omega_b L_{ls} L_{lf}} & 0 & 0 \\ \dots\dots 0 & 0 & 1 & 0 \end{pmatrix}$$

$$M = \begin{pmatrix} -\sqrt{3} \sin \delta_o & 1 & 0 & \sqrt{3} \cos \delta_o & 0 & 1 & 0 \end{pmatrix} \quad (3.15)$$

Matrix N depends upon the output selected and matrix T is normally zero.

2- Current Model

The linearised current model can be presented in a matrix form as follows:

$$\begin{aligned} (C)\Delta(v) &= (A)\frac{d(\Delta x)}{dt} + (B)(\Delta x) \\ (\Delta v) &= (\Delta v_{ds} \quad \Delta v_f \quad 0 \quad \Delta v_{qs} \quad 0 \quad \Delta T_m \quad 0)^t \\ (x) &= (\Delta i_{ds} \quad \Delta i_f \quad \Delta i_{md} \quad \Delta i_{qs} \quad \Delta i_{mq} \quad \Delta \omega \quad \Delta \delta)^t \end{aligned} \quad (3.16)$$

where A and B are

$$A = \begin{pmatrix} -L_{ls} & 0 & -L_{md} & 0 & 0 & 0 & 0 \\ 0 & L_{lf} & L_{md} & 0 & 0 & 0 & 0 \\ -L_{lkd1} & -L_{lkd1} & (L_{lkd1} + L_{md}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_{ls} & -L_{mq} & 0 & 0 \\ 0 & 0 & 0 & -L_{lkq1} & (L_{lkq1} + L_{mq}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega_{ro} L_{ls} & -\omega_{ro} L_{mq} & -\psi_{qso} & v_{qso} \\ 0 & r_f & 0 & 0 & 0 & 0 & 0 \\ -r_{kd1} & -r_{kd1} & r_{kd1} & 0 & 0 & 0 & 0 \\ \omega_{ro} L_{ls} & 0 & \omega_{ro} L_{md} & -r_s & 0 & \psi_{dso} & v_{dso} \\ 0 & 0 & 0 & -r_{kq1} & r_{kq1} & 0 & 0 \\ \frac{(L_{ls} i_{qso} - \psi_{qso})}{2 * H * \omega_b} & 0 & \frac{L_{md} i_{qso}}{2 * H * \omega_b} & \frac{(\psi_{dso} - L_{ls} i_{dso})}{2 * H * \omega_b} & \frac{-i_{dso} L_{mq}}{2 * H * \omega_b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$C = (-\sqrt{3} \sin \delta_o \quad 1 \quad 0 \quad \sqrt{3} \cos \delta_o \quad 0 \quad 1 \quad 0) \quad (3.17)$$

The subscript o denotes the steady state value of the currents, fluxes and voltages. The linearised torque equation is equal to the sixth row of matrix B . The matrices K and M may be expressed in terms of A and B :

$$\begin{aligned} K &= A^{-1}(-B) \\ M &= A^{-1}C \end{aligned} \quad (3.18)$$

3.4 Linearisation of Saturated Synchronous Machine Models

The linearised model of the saturated salient pole synchronous machine can be derived by the following procedure:

1. Equations (2.33) and (2.34), which describe the machine behaviour, are first linearised. So the following equations are obtained:

$$\begin{aligned}
 \Delta v_{qs} &= -r_s \Delta i_{qs} - \frac{d}{dt}(\Delta \psi_{qs}) + \omega_o \Delta \psi_{ds} + \psi_{dso} \Delta \omega \\
 \Delta v_{ds} &= -r_s \Delta i_{ds} - \frac{d}{dt}(\Delta \psi_{ds}) - \omega_o \Delta \psi_{qs} - \psi_{qso} \Delta \omega \\
 \Delta v_f &= r_f \Delta i_f + \frac{d}{dt}(\Delta \psi_f) \\
 \Delta v_{kd1} &= r_{kd1} \Delta i_{kd1} + \frac{d}{dt}(\Delta \psi_{kd1}) \\
 \Delta v_{kq1} &= r_{kq1} \Delta i_{kq1} + \frac{d}{dt}(\Delta \psi_{kq1})
 \end{aligned} \tag{3.19}$$

2. The incremental stator, field and damper variations are assumed to be proportional to the corresponding constant leakage inductances; neglecting high order terms; and the increments of the magnetising flux, which is proportional to the magnetising inductance (L_u) and the time derivative of the magnetising inductance ($\frac{dL_u}{dt}$) (Melkebeek, 1983; Vas, 1992). This allows to split the various flux linkages components into the sum of the leakage and magnetising components. Thus the linearised form of flux linkages may be given as(Vas, 1992):

$$\begin{aligned}
 \Delta \psi_{ds} &= L_{ls} \Delta i_{ds} + \Delta \psi_{md} \\
 \Delta \psi_f &= L_{lf} \Delta i_f + \Delta \psi_{md} \\
 \Delta \psi_{kd1} &= L_{lkd1} \Delta i_{kd1} + \Delta \psi_{md} \\
 \Delta \psi_{qs} &= L_{ls} \Delta i_{qs} + \Delta \psi_{mq} \\
 \Delta \psi_{kq1} &= L_{lkq1} \Delta i_{kq1} + \Delta \psi_{mq}
 \end{aligned} \tag{3.20}$$

where,

$$\begin{aligned}\Delta\psi_{md} &= L_{dd}\Delta i_{md} + L_{dq}\Delta i_{mq} \\ \Delta\psi_{mq} &= L_{dq}\Delta i_{md} + L_{qq}\Delta i_{mq}\end{aligned}\tag{3.21}$$

The derivation of $\Delta\psi_{md}$ and $\Delta\psi_{mq}$ are provided in Appendix B. Also the expressions of L_{dd} , L_{qq} and L_{dq} are defined in equation (2.59).

The linearised form of the d and q-axes magnetising currents can be obtained simply from equation (3.21).

$$\begin{aligned}\Delta i_{md} &= L_{xx}\Delta\psi_{md} - L_{xy}\Delta\psi_{mq} \\ \Delta i_{mq} &= -L_{xy}\Delta\psi_{md} + L_{yy}\Delta\psi_{mq}\end{aligned}\tag{3.22}$$

where,

$$\begin{aligned}L_{xx} &= \frac{L_{qq}}{L_{dd}L_{qq} - L_{dq}^2} \\ L_{yy} &= \frac{L_{dd}}{L_{dd}L_{qq} - L_{dq}^2} \\ L_{xy} &= \frac{L_{dq}}{L_{dd}L_{qq} - L_{dq}^2}\end{aligned}\tag{3.23}$$

3. By utilising equations (3.19), (3.20), (3.21) and (3.22), the small disturbance voltage equations of each model can be obtained based on the chosen state variables. This procedure of derivation is implemented in the sequel to get the linearised Λ models and $1/\Lambda$ models of the saturated salient pole synchronous machine.

3.4.1 Linearisation of Type Λ Models

The current model with $\underline{i}_s, \underline{i}_f, \underline{i}_m$ as the state variables is selected for the derivation procedure. All flux linkages components and their time derivative are solved in terms of the chosen state variable using equations (3.19), (3.20) and (3.21). For example, the linearised form of the stator flux linkage components and their time derivative may be given as:

$$\begin{aligned}
\Delta\psi_{ds} &= L_{ls}\Delta i_{ds} + L_{dd}\Delta i_{md} + L_{dq}\Delta i_{mq} \\
\Delta\psi_{qs} &= L_{ls}\Delta i_{qs} + L_{dq}\Delta i_{md} + L_{qq}\Delta i_{mq} \\
\frac{d}{dt}\Delta\psi_{ds} &= L_{ls}\frac{d}{dt}\Delta i_{ds} + L_{dd}\frac{d}{dt}\Delta i_{md} + L_{dq}\frac{d}{dt}\Delta i_{mq} \\
\frac{d}{dt}\Delta\psi_{qs} &= L_{ls}\frac{d}{dt}\Delta i_{qs} + L_{dq}\frac{d}{dt}\Delta i_{md} + L_{qq}\frac{d}{dt}\Delta i_{mq}
\end{aligned} \tag{3.24}$$

where L_{dd} , L_{qq} and L_{dq} have constant value at each operating point and for small variation around the operating point.

Similar expressions can be obtained for other four d and q flux linkages. The small disturbance form of the electromechanical torque may be derived in terms of the selected state space variables, which yields

$$\begin{aligned}
\Delta T_e &= (L_{ls}i_{qso} - \psi_{qso})\Delta i_{ds} + (L_{dd}i_{qso} - L_{dq}i_{dso})\Delta i_{md} + (\psi_{dso} - L_{ls}i_{dso}) \\
&\quad + (L_{dq}i_{qso} - L_{qq}i_{dso})\Delta i_{mq}
\end{aligned} \tag{3.25}$$

Therefore, the final form of the small disturbance saturated current model can be given in state space form similar to equation (3.16) with matrices A and B as:

$$A = \begin{pmatrix} L_{ls} & 0 & -L_{dd} & 0 & -L_{dq} & 0 & 0 \\ 0 & L_{lf} & L_{dd} & 0 & L_{dq} & 0 & 0 \\ -L_{lkd1} & -L_{lkd1} & (L_{lkd1} + L_{dd}) & 0 & L_{dq} & 0 & 0 \\ 0 & 0 & -L_{dq} & L_{ls} & L_{qq} & 0 & 0 \\ 0 & 0 & L_{dq} & -L_{lkq1} & (L_{lkq1} + L_{qq}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} -r_s & 0 & -\omega_{ro}L_{dq} & -\omega_{ro}L_{ls} & -\omega_{ro}L_{qq} & -\psi_{qso} & v_{qso} \\ 0 & r_f & 0 & 0 & 0 & 0 & 0 \\ -r_{kd1} & -r_{kd1} & r_{kd1} & 0 & 0 & 0 & 0 \\ \omega_{ro}L_{ls} & 0 & \omega_{ro}L_{dd} & -r_s & \omega_{ro}L_{dq} & \psi_{dso} & v_{dso} \\ 0 & 0 & 0 & -r_{kq1} & r_{kq1} & 0 & 0 \\ (L_{ls}i_{qso} - \psi_{qso}) & 0 & (L_{dd}i_{qso} - L_{dq}i_{dso}) & (\psi_{dso} - L_{ls}i_{dso}) & (L_{dq}i_{qso} - L_{qq}i_{dso}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \quad (3.25)$$

K and M may be computed as provided in equation (3.18).

The classical current model can be obtained by substituting $\Delta i_{md} = \Delta i_{ds} + \Delta i_{fd} + \Delta i_{kd1}$ and $\Delta i_{mq} = \Delta i_{qs} + \Delta i_{kq1}$ in (3.24)

Other linearised models, which belong to the same group, can be derived in the same manner. One of these is given below in full form as an example.

- Model with $x = (\Delta\psi_{ds} \ \Delta\psi_f \ \Delta i_{md} \ \Delta\psi_{qs} \ \Delta i_{mq} \ \Delta\omega \ \Delta\delta)$ as state variable.

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-L_{lkd1}}{L_{ls}} & \frac{-L_{lkd1}}{L_{lf}} & \left(L_{dd} + L_{lkd1} \left(1 + \frac{L_{dd}}{L_{ls}} + \frac{L_{dd}}{L_{lf}} \right) \right) & 0 & \left(L_{dq} + L_{lkd1} \left(\frac{L_{dq}}{L_{ls}} + \frac{L_{dq}}{L_{lf}} \right) \right) & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(L_{dq} + \frac{L_{lkq1}L_{dq}}{L_{ls}} \right) & \frac{-L_{lkq1}}{L_{ls}} & \left(L_{qq} + L_{lkq1} \left(1 + \frac{L_{qq}}{L_{ls}} \right) \right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned}
B = & \begin{pmatrix} \frac{-r_s}{L_{ls}} & 0 & \frac{r_s L_{dd}}{L_{ls}} \dots\dots \\ 0 & \frac{r_f}{L_{lf}} & \frac{-r_f L_{dd}}{L_{ls}} \dots\dots \\ \frac{-r_{kd1}}{L_{ls}} & \frac{-r_{kd1}}{L_{lf}} & r_{kd1} \left(1 + \frac{L_{dd}}{L_{ls}} + \frac{L_{dd}}{L_{lf}} \right) \dots\dots \\ \omega_{ro} & 0 & \frac{r_s L_{dq}}{L_{ls}} \dots\dots \\ 0 & 0 & \frac{r_{kq1} L_{dq}}{L_{ls}} \dots\dots \\ \left(i_{qso} - \frac{\psi_{qso}}{L_{ls}} \right) / 2H\omega_b & 0 & \left(\frac{L_{dd} \psi_{qso}}{L_{ls}} - \frac{L_{dq} \psi_{dso}}{L_{ls}} \right) / 2H\omega_b \dots\dots \\ 0 & 0 & 0 \dots\dots \end{pmatrix} \\
& \begin{pmatrix} \dots\dots -\omega_{ro} & \frac{r_s L_{dq}}{L_{ls}} & -\psi_{qso} & v_{qso} \\ \dots\dots 0 & \frac{-r_f L_{dd}}{L_{ls}} & 0 & 0 \\ \dots\dots 0 & r_{kd1} \left(\frac{L_{dq}}{L_{ls}} + \frac{L_{dq}}{L_{lf}} \right) & 0 & 0 \\ \dots\dots \frac{-r_s}{L_{ls}} & \frac{r_s L_{qq}}{L_{ls}} & \psi_{dso} & v_{dso} \\ \dots\dots \frac{-r_{kq1}}{L_{ls}} & r_{kq1} \left(1 + \frac{L_{qq}}{L_{ls}} \right) & 0 & 0 \\ \dots\dots \left(\frac{\psi_{dso}}{L_{ls}} - i_{dso} \right) / 2H\omega_b & \left(\frac{\psi_{qso} L_{dq}}{L_{ls}} - \frac{\psi_{dso} L_{qq}}{L_{ls}} \right) / 2H\omega_b & 0 & 0 \\ \dots\dots 0 & 0 & -1 & 0 \end{pmatrix} \quad (3.26)
\end{aligned}$$

3.4.2 Linearisation of Type I/A Models

The mixed current-flux model with $\underline{i}_s, i_{kd1}, \underline{\psi}_m$ as the state variables is chosen for the derivation procedure. Again, all flux linkages components and their time derivative are solved in terms of the selected state variables using equations (3.19), (3.20) and (3.22). For illustration, the linearised form of the field and q axes damper windings flux linkage components and their time derivative may be presented as:

$$\begin{aligned}
\Delta\psi_f &= -L_{lf}\Delta i_{ds} - L_{lf}\Delta i_{kd1} + (L_{xx}L_{lf} + 1)\Delta\psi_{md} - L_{lf}L_{xy}\Delta\psi_{mq} \\
\Delta\psi_{kq1} &= -L_{lkq1}L_{xy}\Delta\psi_{md} - L_{lkq1}\Delta i_{qs} + (L_{yy}L_{lkq1} + 1)\Delta\psi_{mq} \\
\Delta\psi_f &= -L_{lf}\frac{d}{dt}\Delta i_{ds} - L_{lf}\frac{d}{dt}\Delta i_{kd1} + (L_{xx}L_{lf} + 1)\frac{d}{dt}\Delta\psi_{md} - L_{lf}L_{xy}\frac{d}{dt}\Delta\psi_{mq} \\
\Delta\psi_{kq1} &= -L_{lkq1}L_{xy}\frac{d}{dt}\Delta\psi_{md} - L_{lkq1}\frac{d}{dt}\Delta i_{qs} + (L_{yy}L_{lkq1} + 1)\frac{d}{dt}\Delta\psi_{mq}
\end{aligned} \tag{3.27}$$

and by solving the change in torque in terms of the state variables, yields

$$\begin{aligned}
\Delta T_e &= (L_{ls}i_{qso} - \psi_{qso})\Delta i_{ds} + i_{qso}\Delta\psi_{md} + (\psi_{dso} - L_{ls}i_{dso})\Delta i_{qs} \\
&\quad - i_{dso}\Delta\psi_{mq}
\end{aligned} \tag{3.28}$$

Thus, the complete model is given as:

$$\begin{aligned}
x &= (\Delta i_{ds} \quad \Delta i_{qs} \quad \Delta\psi_{md} \quad \Delta i_{kd1} \quad \Delta\psi_{mq} \quad \Delta\omega \quad \Delta\delta) \\
A &= \begin{pmatrix} -L_{ls} & 0 & -1 & 0 & 0 & 0 & 0 \\ -L_{lf} & 0 & (L_{xx}L_{lf} + 1) & -L_{lf} & -L_{lf}L_{xy} & 0 & 0 \\ 0 & 0 & 1 & L_{lkq1} & 0 & 0 & 0 \\ 0 & -L_{ls} & 0 & 0 & -1 & 0 & 0 \\ 0 & -L_{lkq1} & L_{lkq1}L_{xy} & 0 & (L_{lkq1}L_{yy}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
B &= \begin{pmatrix} -r_s & -\omega_{ro}L_{ls} & 0 & 0 & -\omega_{ro} & -\psi_{qso} & v_{qso} \\ \omega_{ro}L_{ls} & -r_s & \omega_{ro} & 0 & 0 & \psi_{dso} & v_{dso} \\ -r_f & 0 & r_fL_{xx} & -r_f & -r_fL_{xy} & 0 & 0 \\ 0 & 0 & 0 & r_{kd1} & 0 & 0 & 0 \\ 0 & -r_{kq1} & -r_{kq1}L_{xy} & 0 & r_{kq1}L_{yy} & 0 & 0 \\ (L_{ls}i_{qso} - \psi_{qso})/2H\omega_b & (\psi_{dso} - L_{ls}i_{dso})/2H\omega_b & i_{qso}/2H\omega_b & 0 & -i_{dso}/2H\omega_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}
\end{aligned} \tag{3.29}$$

Using the same steps of derivation different models, which are classified to this group, may be obtained. For instance, the following model is presented.

- Model with $x = (\Delta i_{ds} \ \Delta i_f \ \Delta \psi_{md} \ \Delta i_{qs} \ \Delta \psi_{mq} \ \Delta \omega \ \Delta \delta)$ as state variables

$$A = \begin{pmatrix} -L_{ls} & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & L_{lf} & 1 & 0 & 0 & 0 & 0 \\ -L_{lkd1} & -L_{lkd1} & (L_{xx}L_{lkd1} + 1) & 0 & -L_{xy}L_{lkd1} & 0 & 0 \\ 0 & 0 & 0 & -L_{ls} & -1 & 0 & 0 \\ 0 & 0 & -L_{xy}L_{lkq1} & -L_{lkq1} & (L_{yy}L_{lkq1} + 1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} -r_s & 0 & 0 & -\omega_{ro}L_{ls} & -\omega_{ro} & -\psi_{qso} & v_{qso} \\ 0 & r_f & 0 & 0 & 0 & 0 & 0 \\ -r_{kd1} & -r_{kd1} & L_{xx}r_{kd1} & 0 & -L_{xy}r_{kd1} & 0 & 0 \\ \omega_{ro}L_{ls} & 0 & \omega_{ro} & -r_s & 0 & \psi_{dso} & v_{dso} \\ 0 & 0 & -L_{xy}r_{kq1} & -r_{kq1} & L_{yy}r_{kq1} & 0 & 0 \\ (L_{ls}i_{qso} - \psi_{qso})/2H\omega_b & 0 & i_{qso}/2H\omega_b & (\psi_{dso} - L_{ls}i_{dso})/2H\omega_b & -i_{dso}/2H\omega_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \quad (3.30)$$

Other small disturbance saturated models of both types are given in Appendix C. The some models developed in this section together with the models given in Appendix C are used in simulations in the sequel.

CHAPTER 4

RESULTS AND DISCUSSION

This chapter will be devoted to the simulation of the non-linear and linearised unsaturated and saturated models developed in the previous chapters. A single machine-infinite bus system is used in performing several simulations. A salient pole synchronous machine, whose parameters are given in Appendix A, is used in the simulations. The turbine governor and excitation voltage controllers are not modelled so they can be treated as independent inputs.

In this chapter, three types of simulations are performed. First, simulation of non-linear unsaturated and saturated classical current model (model with $\underline{i}_s, \underline{i}_f, \underline{i}_k$ as state variables), in which three types of small disturbances are done, which are step change in grid voltage, excitation voltage and real power (mechanical torque in per unit). A comparison between the results obtained from the saturated model and that from the unsaturated model is presented. Second, simulation of linearised unsaturated and saturated models in which the eigenvalues of the unsaturated and saturated models are calculated. Also the variation of critical eigenvalues corresponding to both types models are plotted as a function of the load variation. Then the influence of the variation in saliency factor on the critical eigenvalues corresponding to the saturated models is also investigated. After that similar to the non-linear models, three types of small disturbances are performed, which are step change in grid voltage, excitation voltage and mechanical torque. The results obtained from selected linearised saturated models are contrasted for sake of verification. Then the results obtained from the saturated models are also compared with that from unsaturated models. Finally, Monte-Carlo simulation is performed. This is done by distributing real power, saliency factor and saturation factor uniformly over the following range respectively:

$$80\% \leq P \leq 100\%$$

$$80\% \leq m^2 \leq 120\%$$

$$95\% \leq K_{sat} \leq 105\%$$

Three cases are conducted as follows:

1. Constant saliency factor with random variation in real power and saturation factor between the ranges given above.
2. Constant saturation factor with random variation in real power and saliency factor between the ranges given above.
3. Random variation in real power, saturation factor and saliency factor between the ranges given above.

The simulations performed in this chapter will be explained in the relevant sections in details

4.1 Simulation of Non-linear Unsaturated and Saturated Models

A five-winding salient pole synchronous generator, whose parameters are given in Appendix A, is simulated for 5% steps changes in grid voltage, excitation voltage and real power. The models used apply to the generating convention, thus the turbine torque and rotor angle is positive. As stated previously, no control elements are modelled. It is assumed that the machine is connected to infinite bus, which has constant voltage and frequency, at rated values, regardless of the stator current. The saturated classical current model is first simulated to get the steady state value of the magnetising inductance $L_{md}(L_u)$. Then this value is used in the unsaturated classical current model so that identical steady state value is obtained for both types of models. To simulate the models, two computer programs are written in MATLAB. These programs utilised fourth order Runge-Kutta method to solve the differential equation representing each model. Then the results obtained from each program are compared.

4.1.1 Step Change in Grid Voltage

This type of change is chosen because it illustrates the effect of main flux saturation (Levi, 1998). Initially the generator is operated at rated conditions (rated terminal voltage, real power and power factor). The value of magnetising inductance is found from simulating the classical current model (L_{md}) and it is used in the unsaturated model

to give the same steady state value. The programmes written to simulate both models are executed until all variables reach the steady state. Then at time $t=15.0$ seconds the grid voltage is stepped up 5% more than the rated value. The results obtained are given in figures 4.1, 4.2, 4.3 and 4.4 showing waveforms of rotor angle, field current and stator d-q axis current respectively during transient and steady state. The influence of main flux saturation in modelling can be clearly seen from these figures.

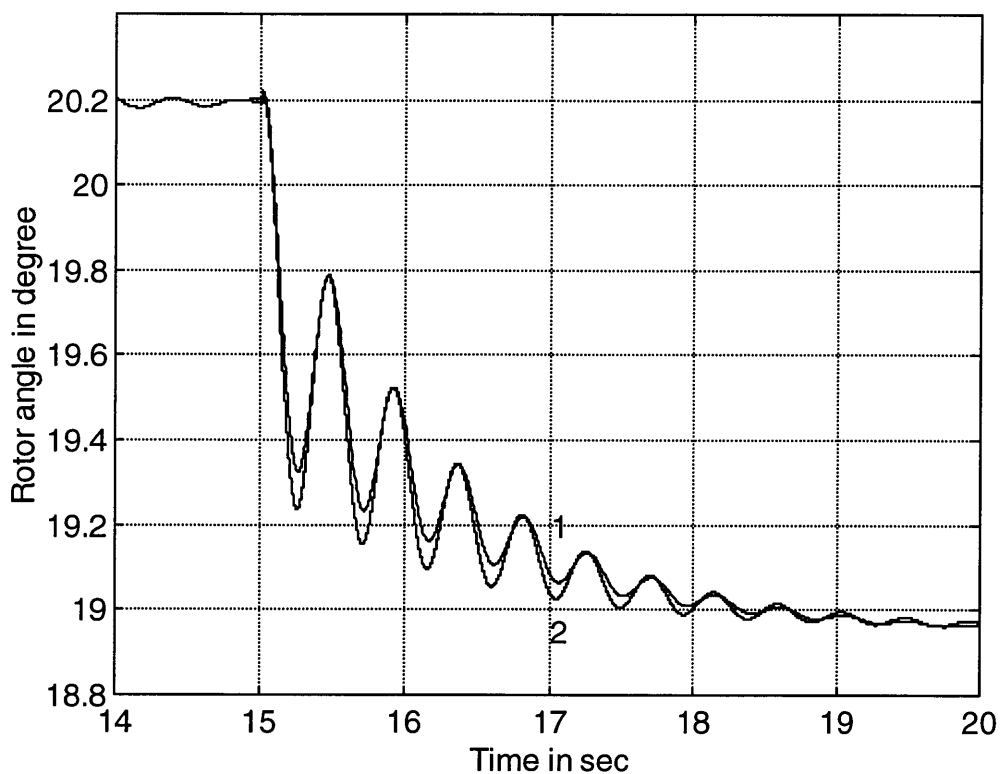


Figure 4.1. Variation in rotor angle due to 5% step change in grid voltage.

- 1- Unsaturated model.
- 2- Saturated model.

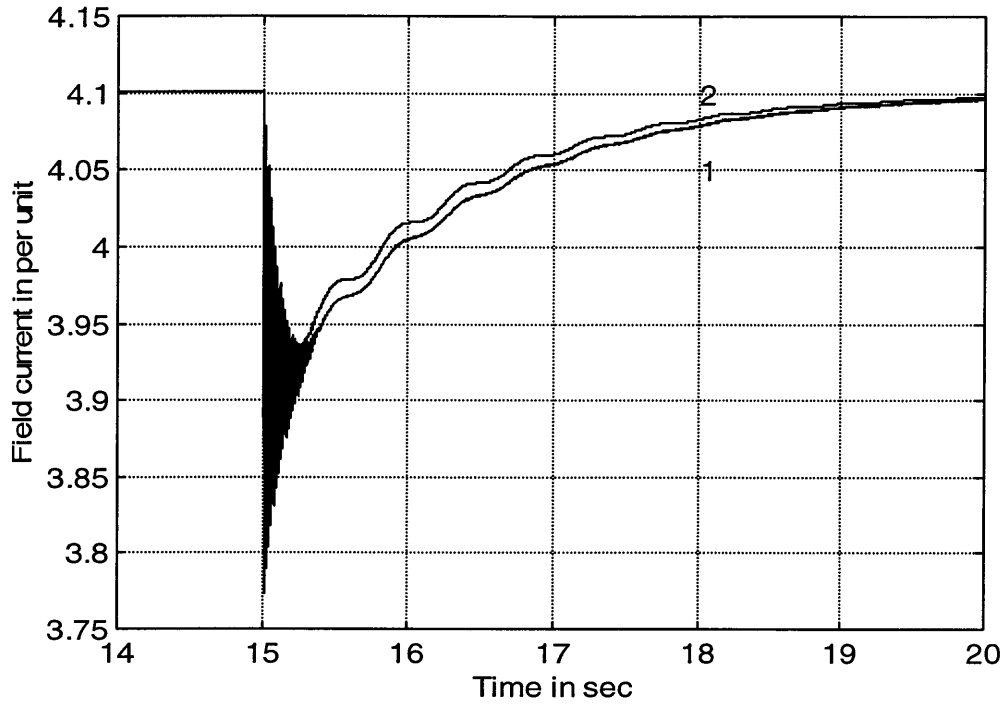


Figure 4.2. Variation in field current due to 5% step change in grid voltage.

1- Unsaturated model.

2- Saturated model.

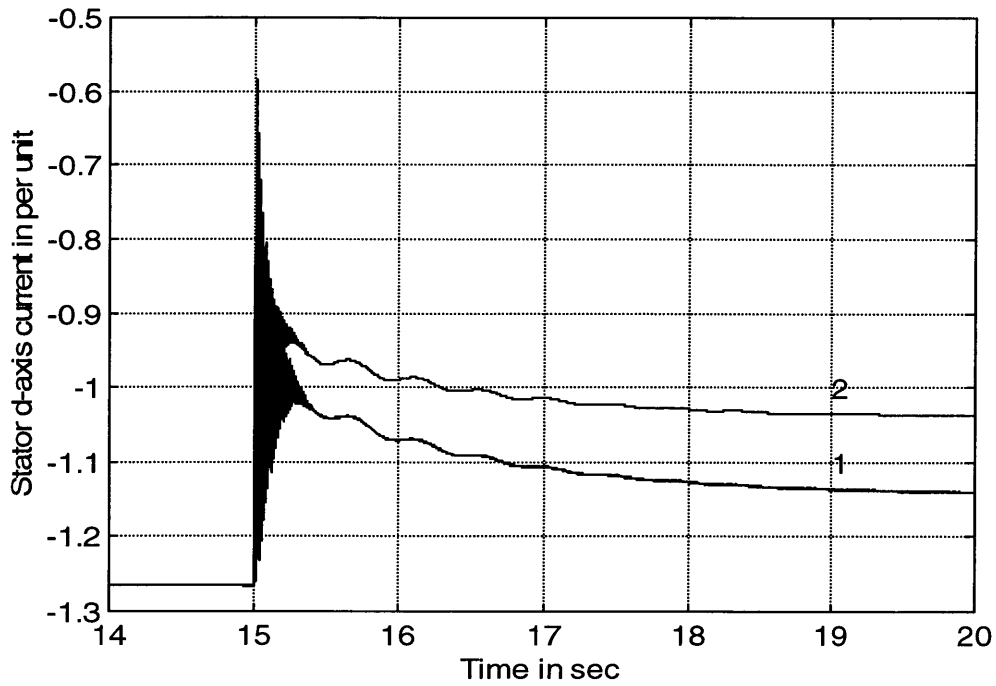


Figure 4.3. Variation in stator d-axis current due to 5% step change in grid voltage.

1- Unsaturated model.

2- Saturated model.

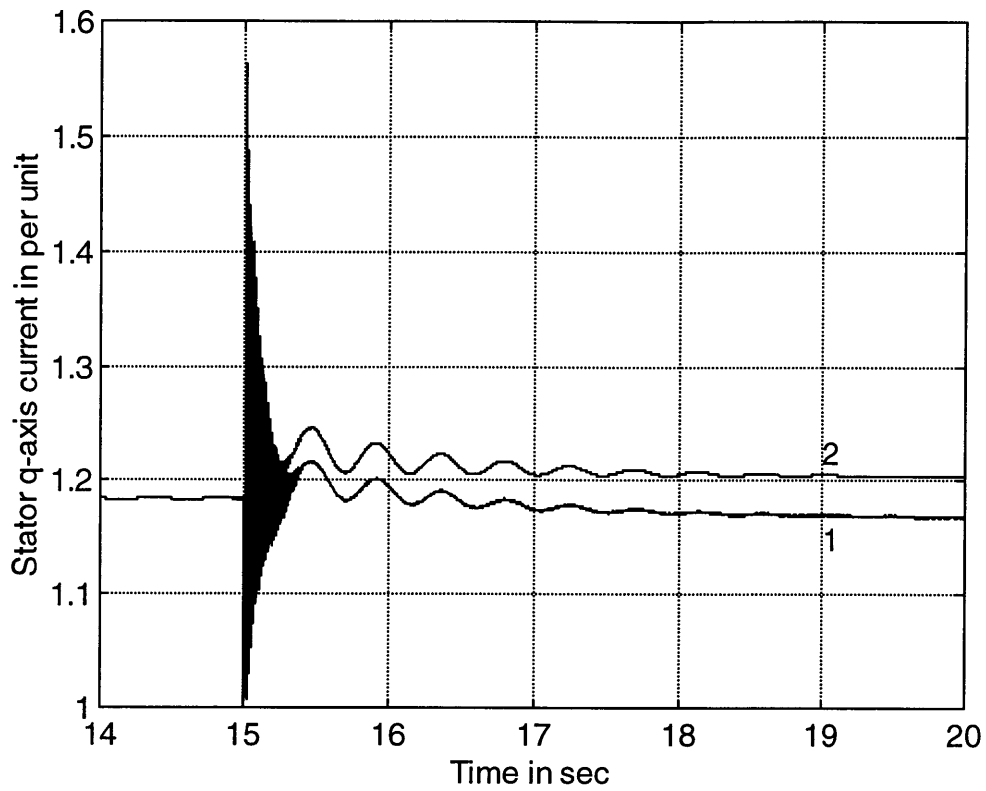


Figure 4.4. Variation in stator q-axis current due to 5% step change in grid voltage .

- 1- Unsaturated model.
- 2- Saturated model.

4.1.2 Step Change in Excitation Voltage

The same procedure is repeated this time for 5% step change in excitation voltage. Again the machine is initially operating at rated conditions. At time $t=15$ seconds, the excitation voltage is stepped up 5%. The results are given in figures 4.5 and 4.6 showing stator d-q axis current respectively, while the variation in rotor angle and field current and are given in figures D1 and D2 in Appendix D. The main flux saturation influences the stator d-q axis current mainly as can be seen from figures 4.5 and 4.6. On the other hand, the waveforms of the rotor angle and field current show insignificant variation. It is also noted that the responses due to this type of disturbance do not excite oscillatory responses.

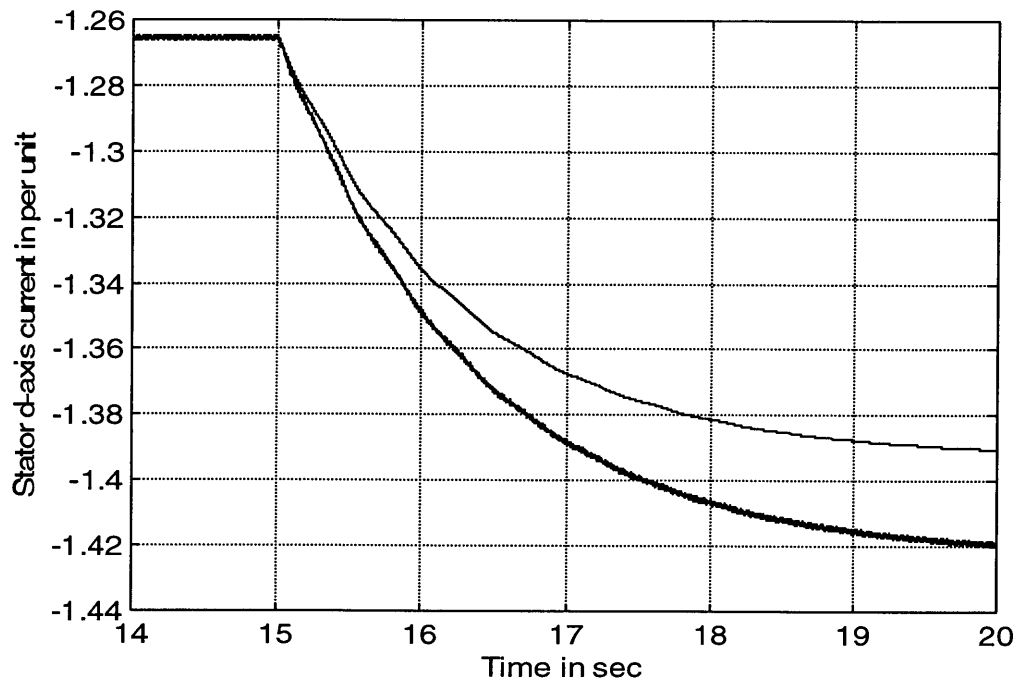


Figure 4.5. Variation in stator d-axis current due to 5% step change in excitation voltage.

_____ Unsaturated model. _____ Saturated model.

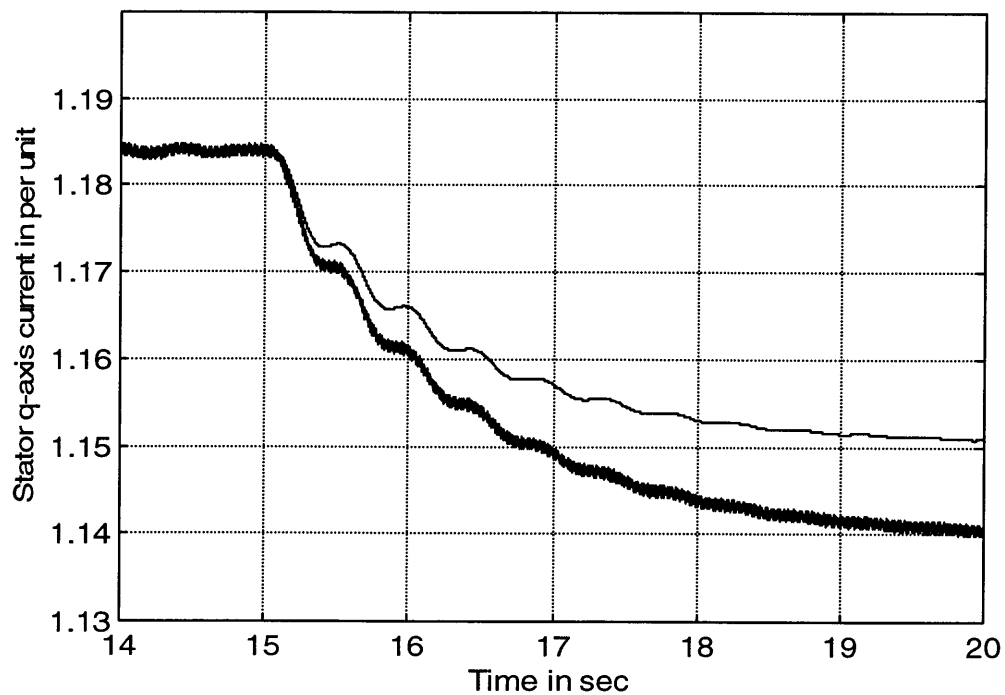


Figure 4.6. Variation in stator q-axis current due to 5% step change in excitation voltage.

_____ Unsaturated model. _____ Saturated model.

4.1.3 Step Change in Real Power

Again the same procedure of simulation is used in this section . The real power, which is equal to the mechanical torque (T_m) in per unit, is stepped up 5%. The variation in rotor angle and stator d-q axis current are given in figure 4.7, 4.8 and 4.9 respectively. The variation in field current is given in figure D3 in Appendix D. As can be seen from these figures, there is small difference between the curves corresponding to each model. Unlike the step change in grid voltage and excitation voltage, a step change in real power does excite an oscillatory response.

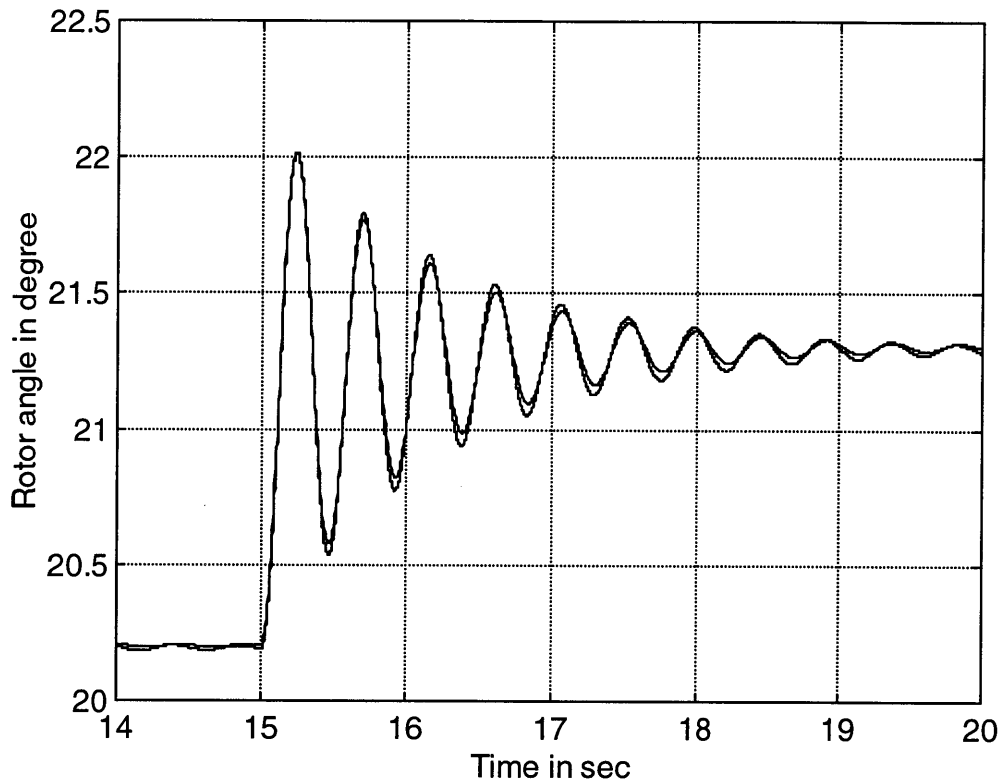


Figure 4.7. Variation in rotor angle due to 5% step change in real power.

- Unsaturated model
- - - Saturated model.

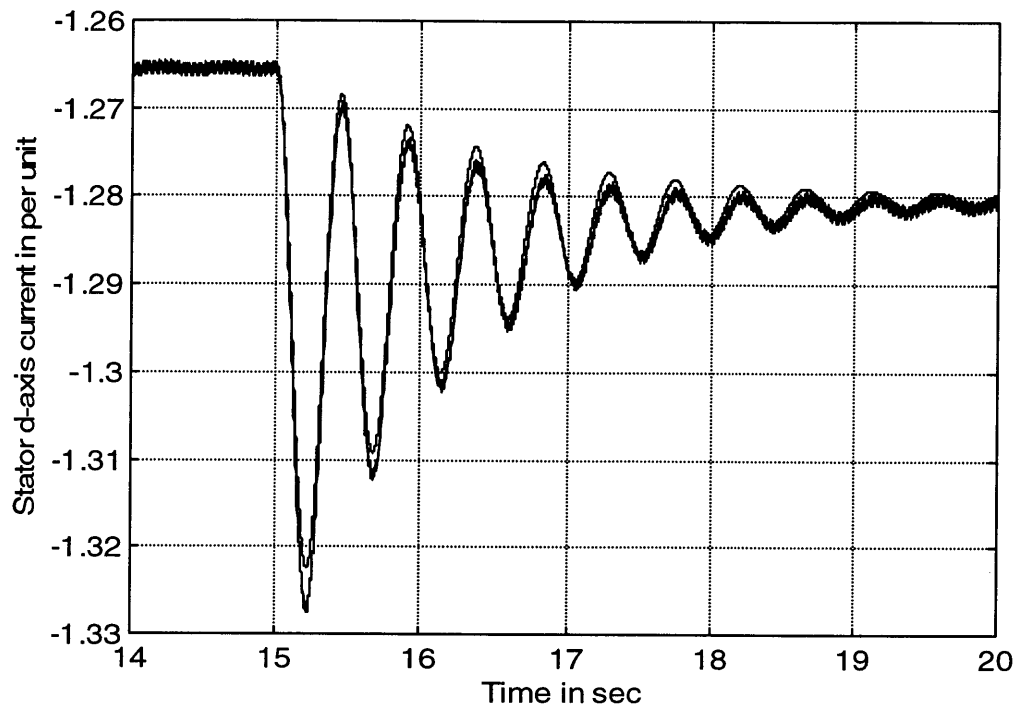


Figure 4.8. Variation in stator d-axis current due to 5% step change in real power.

— Unsaturated model
 - - Saturated model.

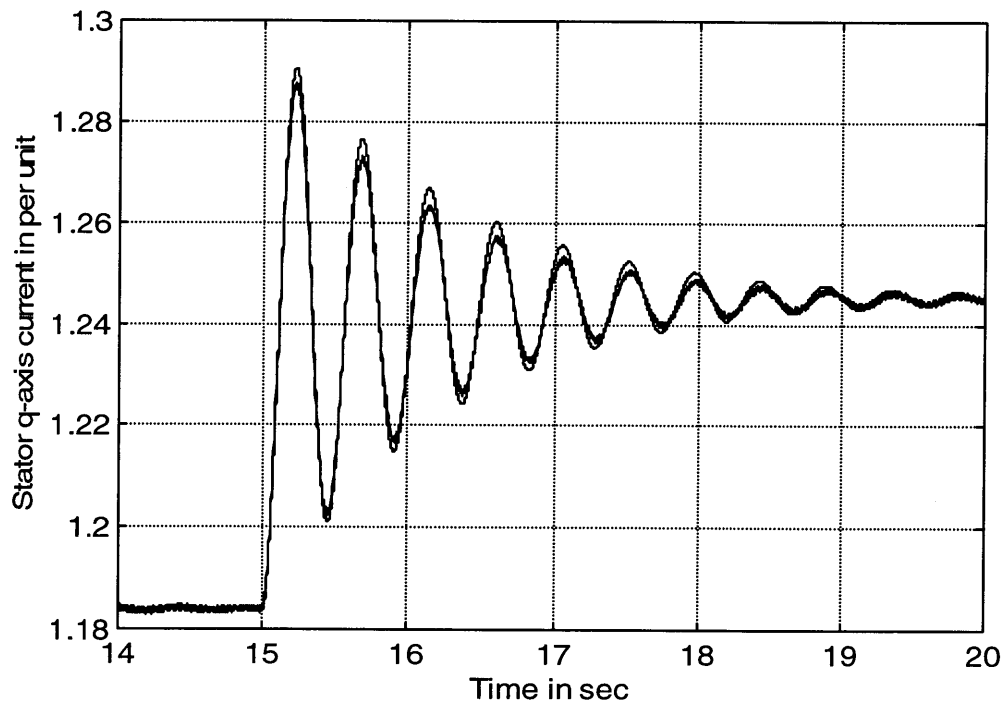


Figure 4.9. Variation in stator d-axis current due to 5% step change in real power.

— Unsaturated model
 - - Saturated model.

4.2 Simulation of Linearised Unsaturated and Saturated Models.

It is instructive to calculate the eigenvalues and observe the dynamic of the synchronous machine modelled without and with saturation during a step change in grid voltage, excitation voltage and real power. These types of simulations provide an important information for the design of the controllers used and the protection system. The synchronous machine, whose parameters are given in Appendix A, is also used in this section. Again no control elements are modelled and it is assumed that the machine is connected to infinite bus. The linearised unsaturated and selected saturated models are programmed using MATLAB and a study is performed. The saturated models used, are the models, which are represented by the most frequently used state variables in the application of synchronous machine.

First, the eigenvalues are calculated using unsaturated and saturated models. Then the effect of load variation on the critical eigenvalues obtained from both types is investigated. Also the effect of saliency factor on the critical eigenvalues corresponds to the linearised saturated models is studied. After that, three types of disturbances, which are 5% steps changes in grid voltage, excitation voltage and real power, are done. The following method of simulation is used. First, selected disturbance is simulated using chosen linearised saturated models developed previously. If these models provide similar results as the linearised classical current model, it may be concluded that the linearisation procedure is verified. Second, a comparison between the responses obtained from linearised unsaturated and saturated models are compared. This comparison shows the importance of including the saturation modelling in the small disturbance stability analysis.

4.2.1 Calculation of Eigenvalues

The eigenvalues of linearised unsaturated and selected saturated models of salient pole synchronous machine are calculated for rated operating condition. This is done by finding the roots of matrix K corresponding to each model.

4.2.1.1 Calculation of Eigenvalues for Unsaturated Model

The eigenvalues of the unsaturated current and flux models are provided in table 4.1. The eigenvalues shown in bold fonts are the critical eigenvalues i.e. the eigenvalues that are close to the imaginary axis and that will dominate the dynamic behaviour of the machine.

Table 4.1 Eigenvalues of the unsaturated current and flux models

Current Model	Flux Model
$-8.69 \pm j376.65$	$-8.8 \pm j375.9$
-58.86	-58.76
$-.9 \pm j13.63$	$-.81 \pm j13.58$
-22.76	-22.83
-.75	-.76

Considering both models, there are seven eigenvalues out of which two complex conjugate pairs and three real eigenvalues. The complex conjugate pair with frequency approximately equal to ω_b in both models is associated with the transient offsets currents in the stator, which is damped at a fast rate. The remaining complex conjugate pair is associated with the transient of the rotor of the machine and it is known as swing mode (Krause *et al.* 1995) or local mode (Kundur, 1994). This pair is the principal mode of oscillation of the rotor of the machine. As indicated by this eigenvalue, the rotor oscillation of the machine corresponding to the current model is slightly more damped (absolute of the real part is greater) and of slightly higher frequency (greater imaginary part) than that corresponding to the flux model. The real eigenvalues are associated with the inverse of the effective time constant of the circuit (Krause *et al.* 1995). The real eigenvalue signifies an exponential response, while complex conjugate eigenvalue signifies an oscillatory response. Also, it is noted that all real parts of the eigenvalues are negative, which means that the system is small-disturbance stable under the conditions assumed in developing the models, namely, about the chosen equilibrium point.

The frequency in Hz , damping ratio (ζ) and the time constant (σ) given in equations (3.10) and (3.11) are calculated for the rotor eigenvalues and they are given in table 4.2.

Table 4.2 Critical eigenvalues of unsaturated models

Model	Rotor Eigenvalue	Frequency in Hz	Damping ratio(ζ)	Time constant (σ)
Current	$-.9 \pm j13.63$	2.169	0.065887	1.11
Flux	$-.81 \pm j13.58$	2.435	0.0595407	1.23

Figure 4.10. shows the change in the rotor eigenvalue for both models for various loading conditions. The real power is varied from 0 up to 1.0 pu and the magnetising inductance corresponding to each loading condition calculated from simulation of the non-linear saturated current model is used in the unsaturated models. The changes do not have serious effect. The frequency of the swing mode increases slightly as the real power increases especially that corresponding to flux model. It is observed also that the damping improved in the case of flux model and decreased in the case of current model.

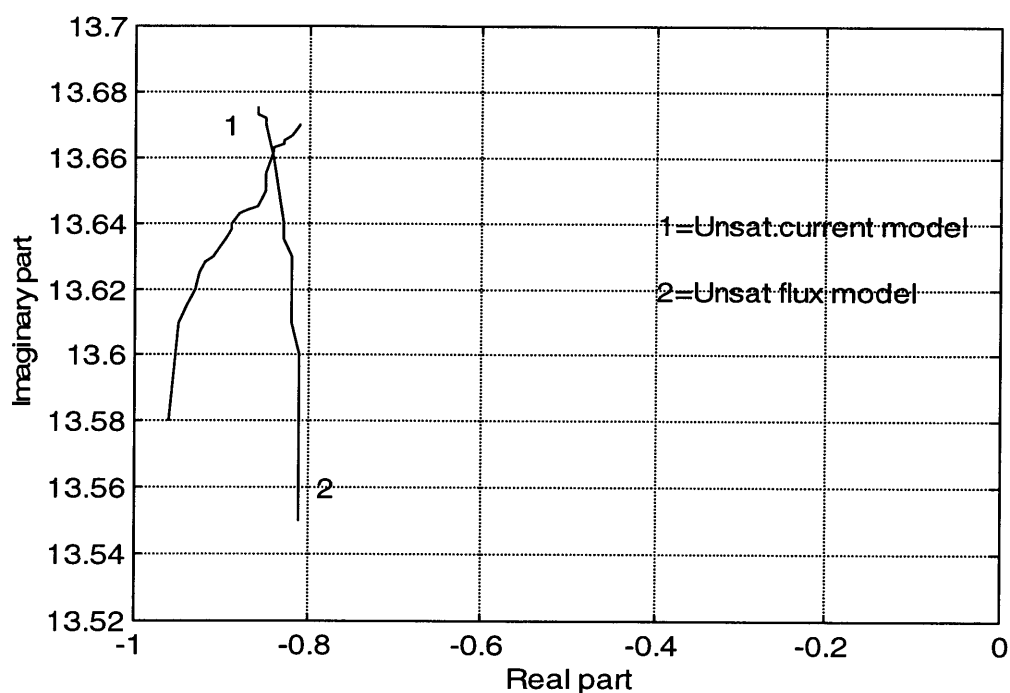


Figure 4.10. Root loci of the critical eigenvalues of unsaturated flux and current models for variation in real power.

4.2.1.2 Calculation of Eigenvalues for Saturated Models

The eigenvalues of the linearised saturated models developed in section (3.4), which are chosen for analysis, are given in table 4.3

Table 4.3. Eigenvalues of the linearised saturated models

Model No.1	Model No.2	Model No.3	Model No.4	Model No.5
-9.16 ± 376.68	-9.16 ± 376.68	-9.16 ± 376.68	-9.16 ± 376.68	-9.16 ± 376.68
$-.9 \pm 13.67$	$-.92 \pm 13.51$	$-.9 \pm 13.67$	$-.9 \pm 13.67$	$-.9 \pm 13.67$
-23.21	-23.21	-23.21	-23.21	-23.21
$-.8$	$-.79$	$-.8$	$-.8$	$-.8$
-59.44	-59.42	59.44	59.44	59.44

Model No.1: $\underline{i_s}, \underline{i_f}, \underline{i_k}$ (Classical current model).

Model No.2: $\underline{i_s}, \underline{i_f}, \underline{i_m}$

Model No.3: $\underline{\psi_s}, \underline{\psi_f}, \underline{i_m}$

Model No.4: $\underline{i_s}, \underline{i_f}, \underline{\psi_m}$

Model No.5: $\underline{\psi_s}, \underline{\psi_f}, \underline{\psi_m}$

In general all eigenvalues corresponding to the saturated models have negative real part, which denotes a stable system. They also indicate the stabilising effect of saturation by increasing the damping of the system. Again, the eigenvalues shown in bold fonts are those which dominate the dynamic behaviour of the machine. It can be seen from table 4.3 that all models except model no.2, have the same eigenvalues. Contemplating the swing mode, model no.2 has better damping and low frequency than the other models. However, this model has a critical real eigenvalue that is smaller than the other models in absolute value.

The frequency in Hz , damping ratio(ζ) and the time constant (σ) given in equations (3.10) and (3.11) are calculated for the critical eigenvalue and they are presented in table 4.4.

Table 4.4. Eigenvalues of the linearised saturated models

Model No.	Critical eigenvalue	Frequency in (Hz)	Damping ratio	Time constant (sec)
1	$-.9 \pm 13.67$	2.176	0.065695	1.11
2	$-.92 \pm 13.51$	2.15	.0679404	1.087
3	$-.9 \pm 13.67$	2.176	0.065695	1.11
4	$-.9 \pm 13.67$	2.176	0.065695	1.11
5	$-.9 \pm 13.67$	2.176	0.065695	1.11

The influence of load variation on the critical eigenvalue of all models is carried out in the same way as it is done for the unsaturated models. Unlike the unsaturated models, the rotor eigenvalue changes significantly with change in real power with constant reactive power. This is clearly shown in figure 4.11. With models no.1,3,4 and 5, the identical variation of critical eigenvalue is obtained, while it is slightly different with model no.2. It is recognised that the damping is improved as the real power increases for all models. In addition, it is noted that the frequency corresponding to model no.2 critical eigenvalue increases as the real power increases, however, for other models, it slightly decreases. These results show higher sensitivity of critical eigenvalue to the change of operating conditions in the saturated models than the case of unsaturated model.

Another study is performed to see the effect of the saliency factor on the rotor eigenvalue of all saturated models. The saliency factor is varied between 80% to 120% of the original value under constant loading conditions (rated operating conditions) and the change in critical eigenvalues is observed and plotted in figure 4.12. It can be seen from figure 4.12 that the frequency is decreased and damping is improved for all models as the saturation saliency increases. Again identical results are obtained with models 1, 3, 4 and 5.

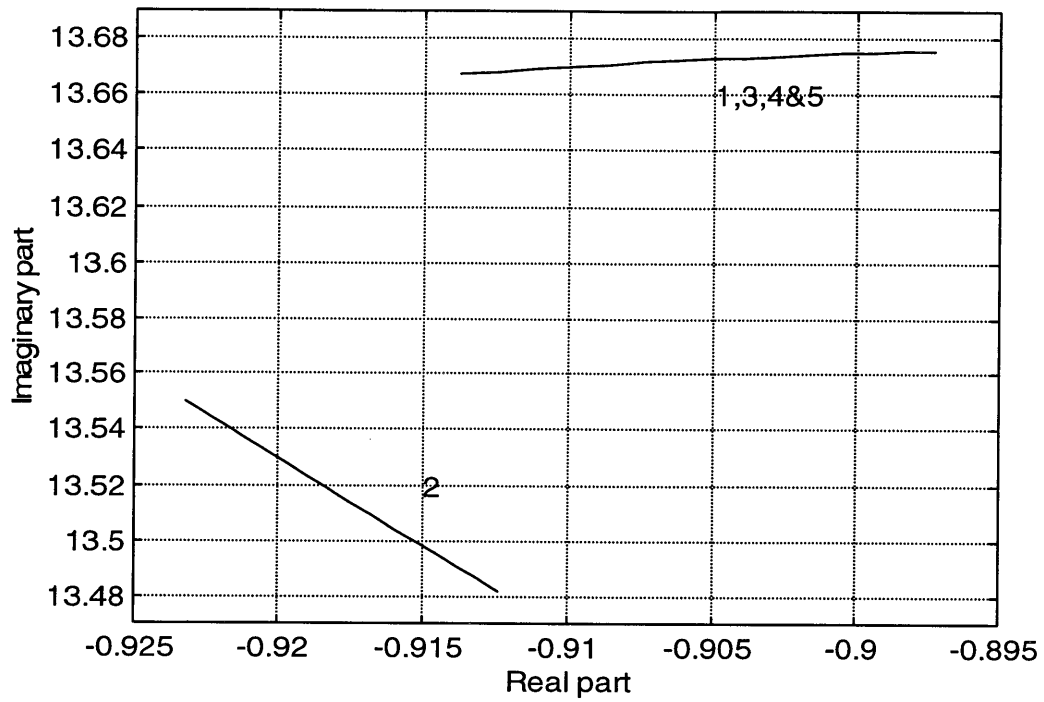


Figure 4.11. Root loci of the critical eigenvalues of five saturated models for variation in real power.

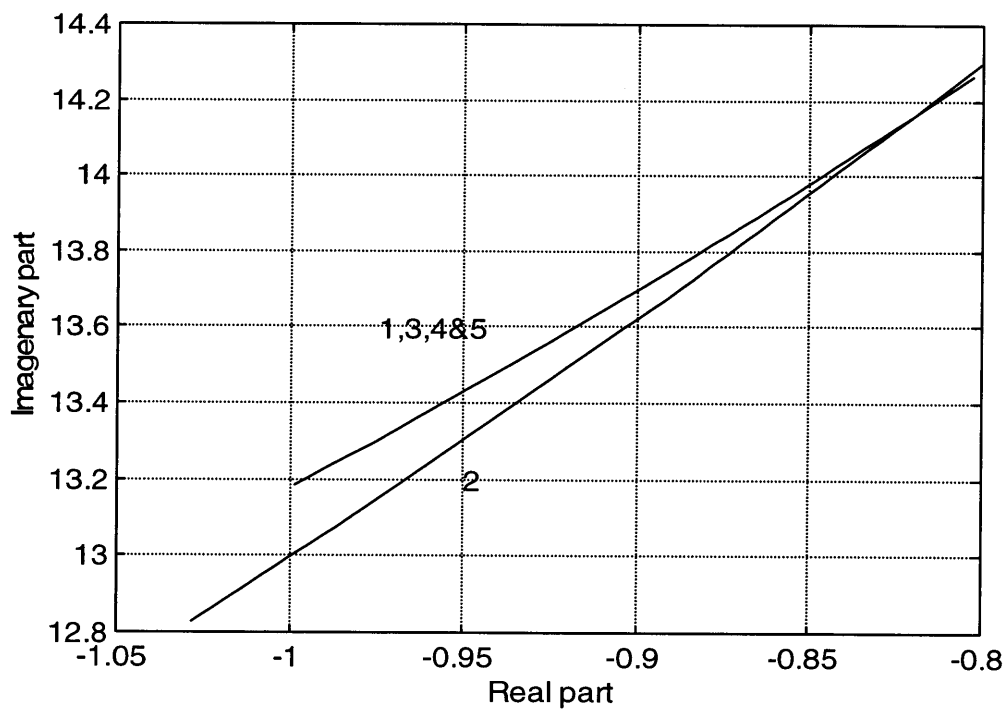


Figure 4.12. Root loci of the critical eigenvalues of five saturated models for variation in saliency factor.

Model no.1, which is the classical current model, and the flux model are normally used by most of the researchers to define the saturation in synchronous machine. Having a look at the results obtained up to this stage, the developed models (3,4 and 5) have the same eigenvalues and behave in the same manner as the loading conditions and saliency factor are varied. Also the stability margins provided by the other models are the same as the classical current model.

4.2.2 Step Change in Grid Voltage.

Figures 4.13-16 show a comparison of results obtained with the five saturated models showing the change in rotor angle, field current, stator d-q axis current respectively. The change in rotor angle is obtained from all saturated models since all models contain the rotor angle as one of the state variable. The changes in field current and stator d-q axis current are obtained from the models, which contain them as state variables (models 1,2 and 4). The curves completely overlap and it is not possible to distinguish between them.

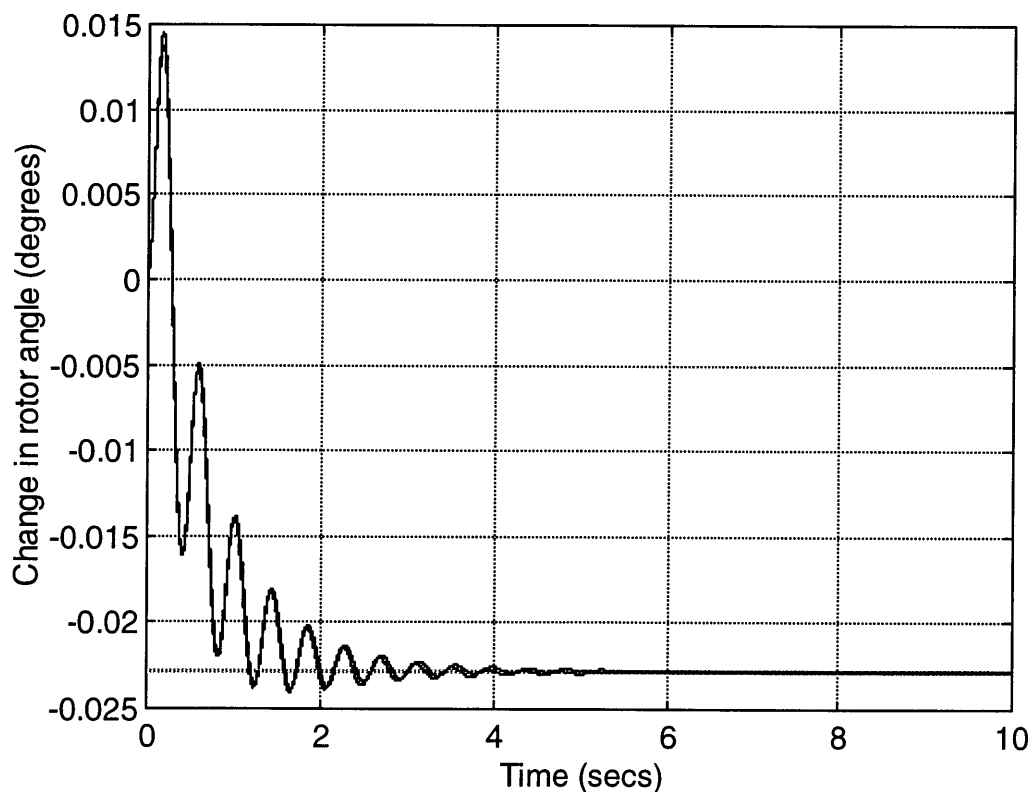


Figure 4.13. Variation in change in rotor angle due to 5% step change in grid voltage.

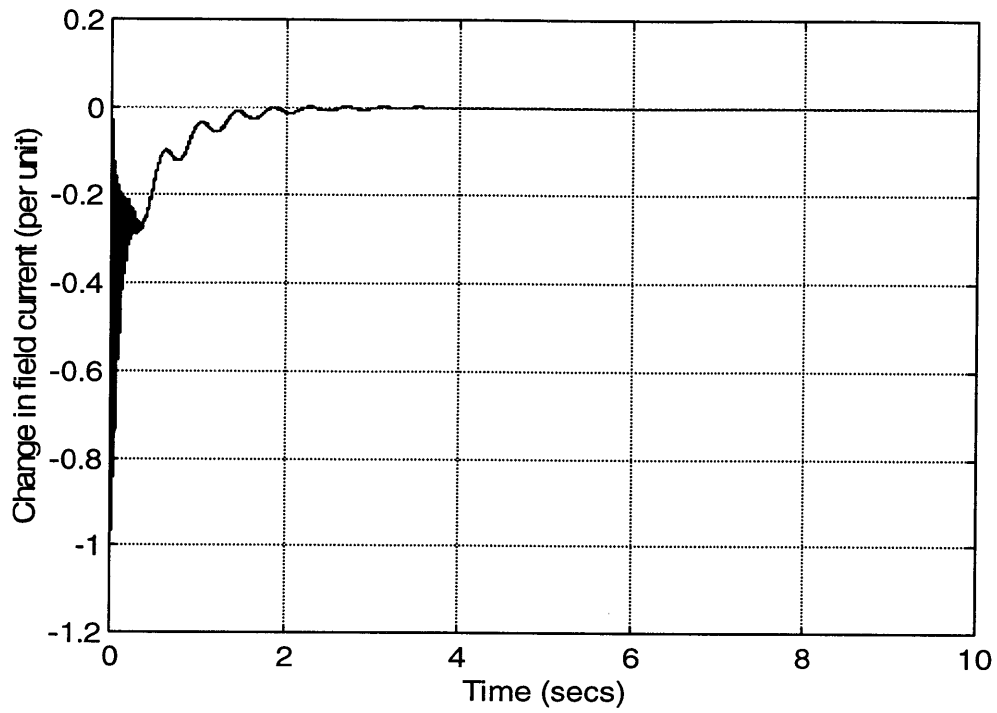


Figure 4.14. Variation in change in field current due to 5% step change in grid voltage.

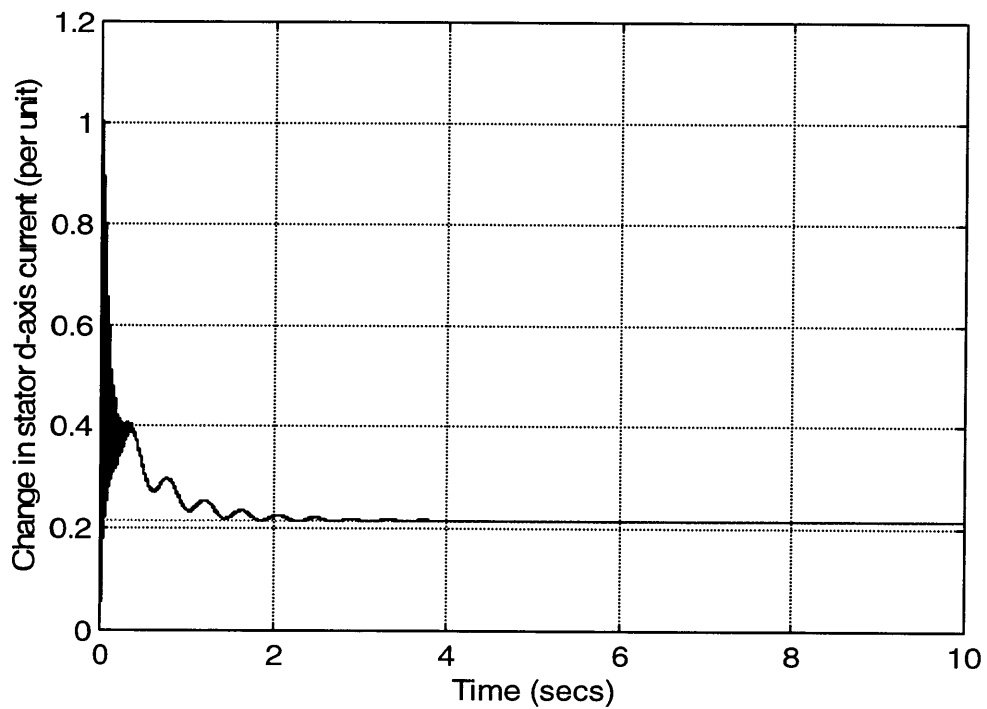


Figure 4.15. Variation in change in stator d-axis current due to 5% step change in grid voltage.

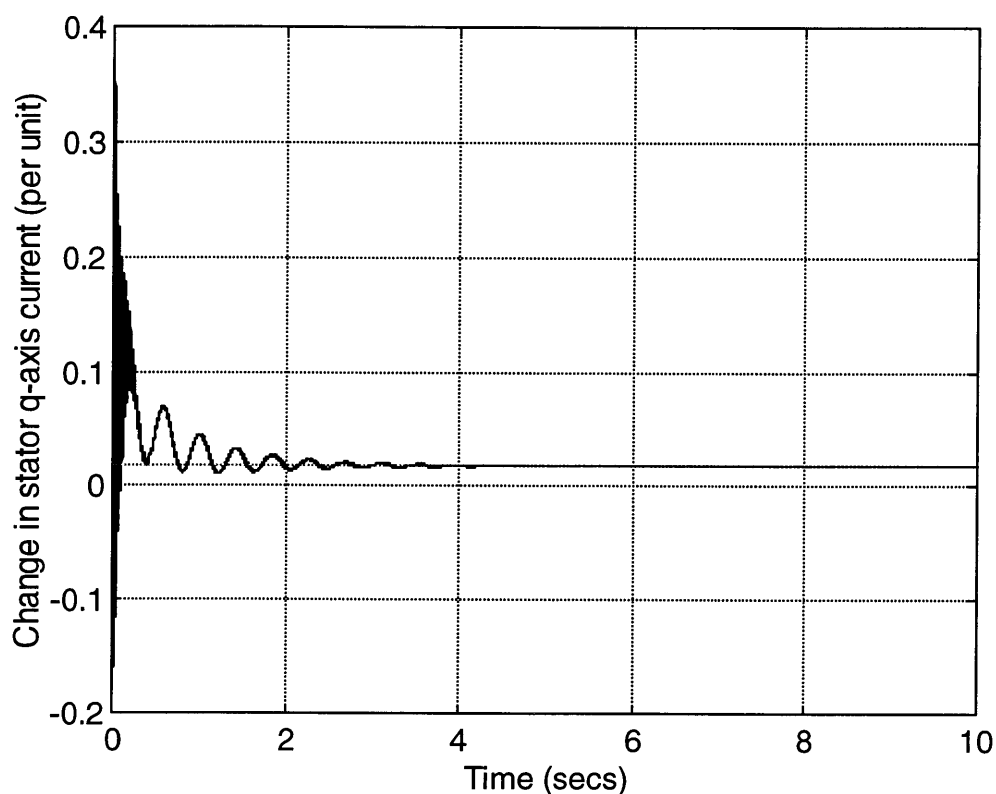


Figure 4.16. Variation in change in stator q-axis current due to 5% step change in grid voltage.

Responses due to 5% change in grid voltage for linearised unsaturated and saturated flux and current models are also analysed. They are given in figures 4.17-19. These figures show change in rotor angle and stator d-q axis current respectively using unsaturated and saturated classical current model, while the change in rotor angle and change in field current obtained from unsaturated and saturated models are given in Appendix D in figures D4 and D5 respectively. As can be seen from figure 4.17, faster response and better damping with higher frequency of oscillation are obtained using saturated models. Also it may be noted that the change in angle obtained with saturated model is slightly smaller than the unsaturated one in absolute value.

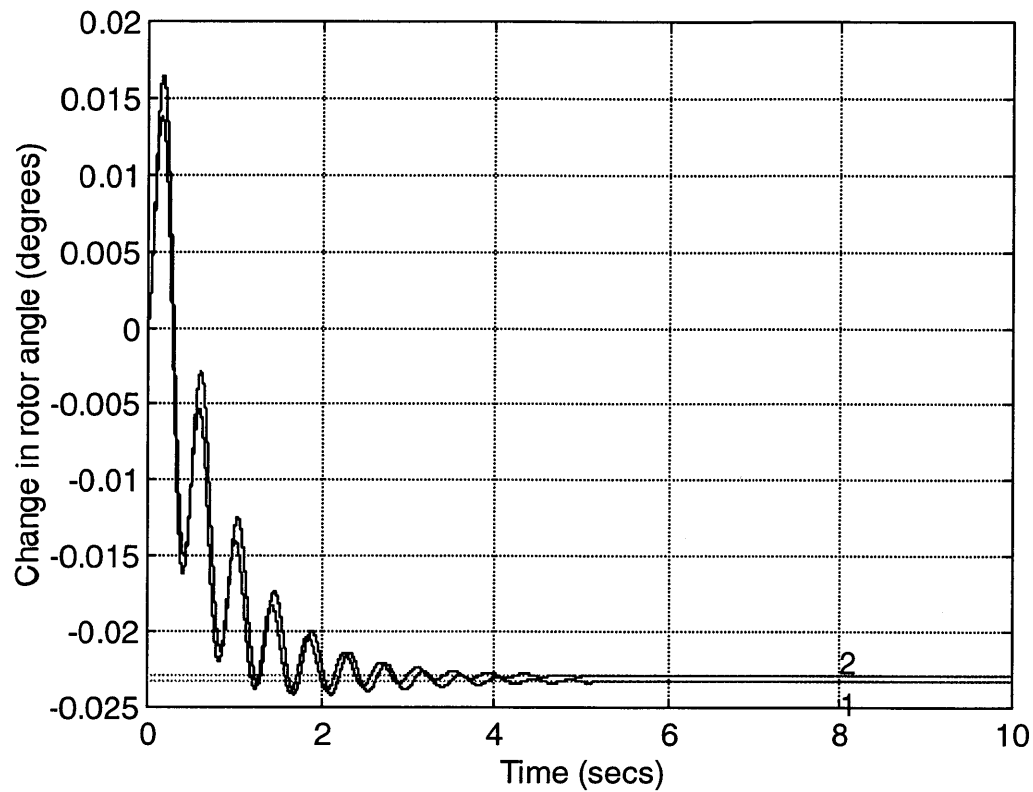


Figure 4.17. Variation in change in rotor angle due to 5% step change in grid voltage.

- 1- Linearised unsaturated current model.
- 2- Linearised saturated current model.

A comparison between the change in stator d-q axis current computed from the unsaturated and saturated current model is given in figures 4.18 and 4.19 respectively. Observing both figures, the final steady state value corresponding to the saturated model responses is greater than that corresponding to the unsaturated model. The main flux saturation during step change in grid voltage affects significantly the change in stator d-q axis currents, while the wave forms of the change field current is less affected.

It can be seen from the figures obtained in this section that step change in grid voltage excites a combination between exponential and sinusoidal responses.

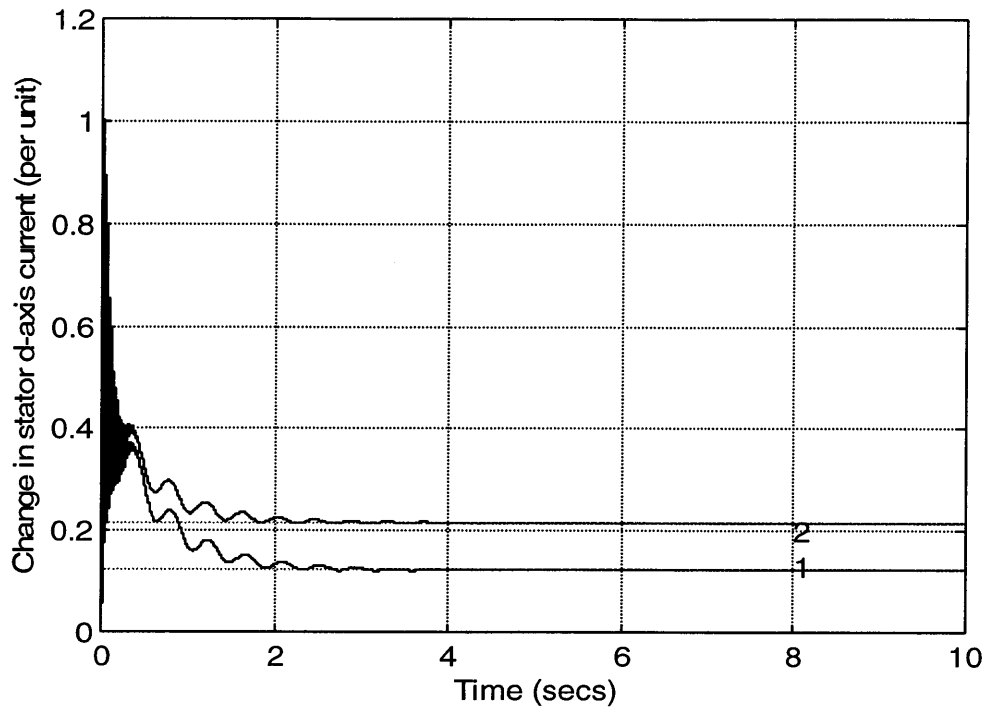


Figure 4.18. Variation in change in stator d-axis current due to 5% step change in grid voltage.

1- Linearised unsaturated current model. 2- Linearised saturated current model.

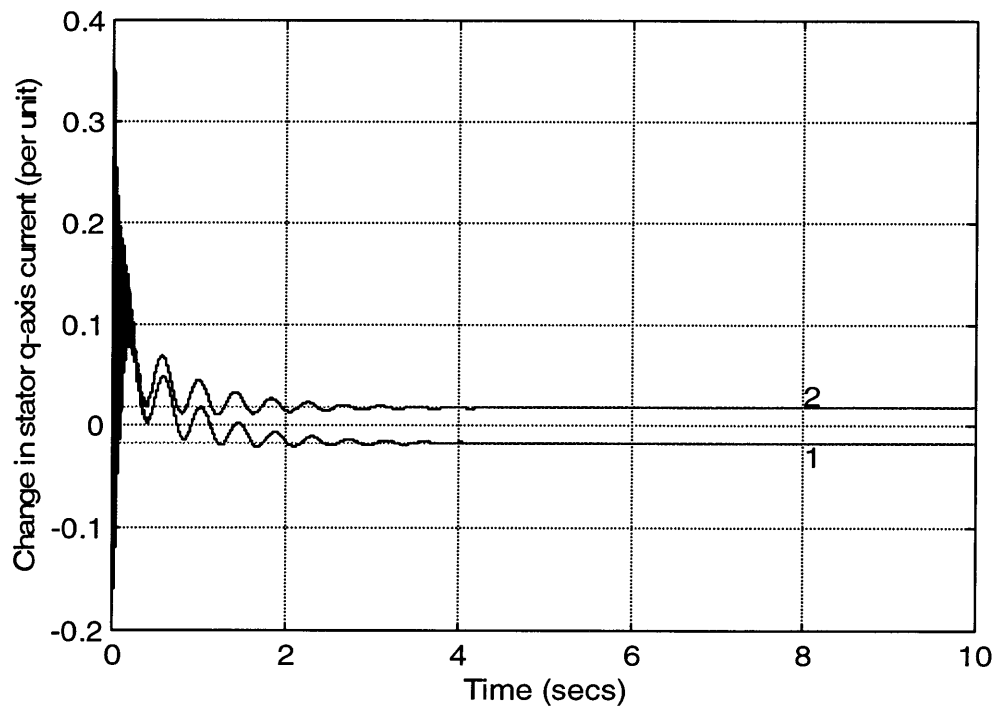


Figure 4.19. Variation in change in stator q-axis current due to 5% step change in grid voltage.

1- Linearised unsaturated current model. 2- Linearised saturated current model.

4.2.3 Step Change in Excitation Voltage

The discrepancies between the responses obtained using five saturated models due to 5% change in excitation voltage are given in figures 4.20-21 showing change in rotor angle and change in stator q-axis current. It is found that the change in field current and stator d-axis current for all models are the same and they are given in figures D6 and D7 respectively.

It can be seen from figure 4.20 that models no.1,3, 4 and 5 provide identical results, while model no.2 overestimates slightly change in rotor angle. The reason for this can be related to the change of stator q-axis current. As mentioned earlier, the stator q-axis current influences the determination of the rotor angle. As can be seen from figure 4.21, the final steady state value of the change in stator q-axis current obtained using model no.2 is greater than the value obtained using other models. This shows clearly the effect of stator q-axis current on the determination of the rotor angle, which is the measure of the stability of synchronous machine.

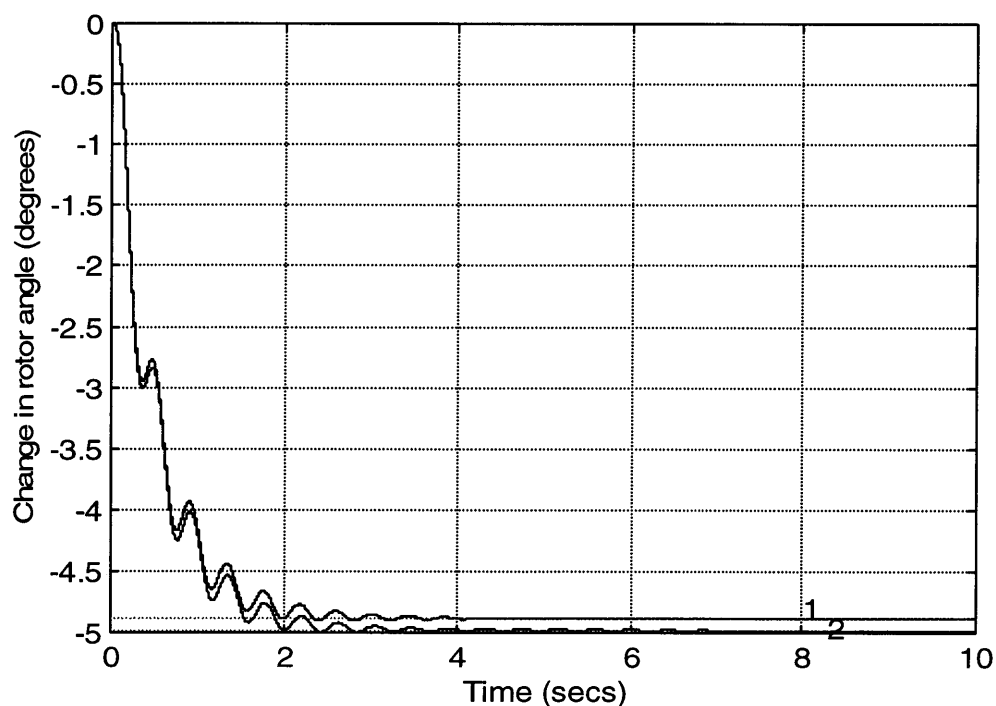


Figure 4.20. Variation in change in rotor angle due to 5% step change in excitation voltage.

1-Models 1, 3, 4 and 5. 2- Model no.2.

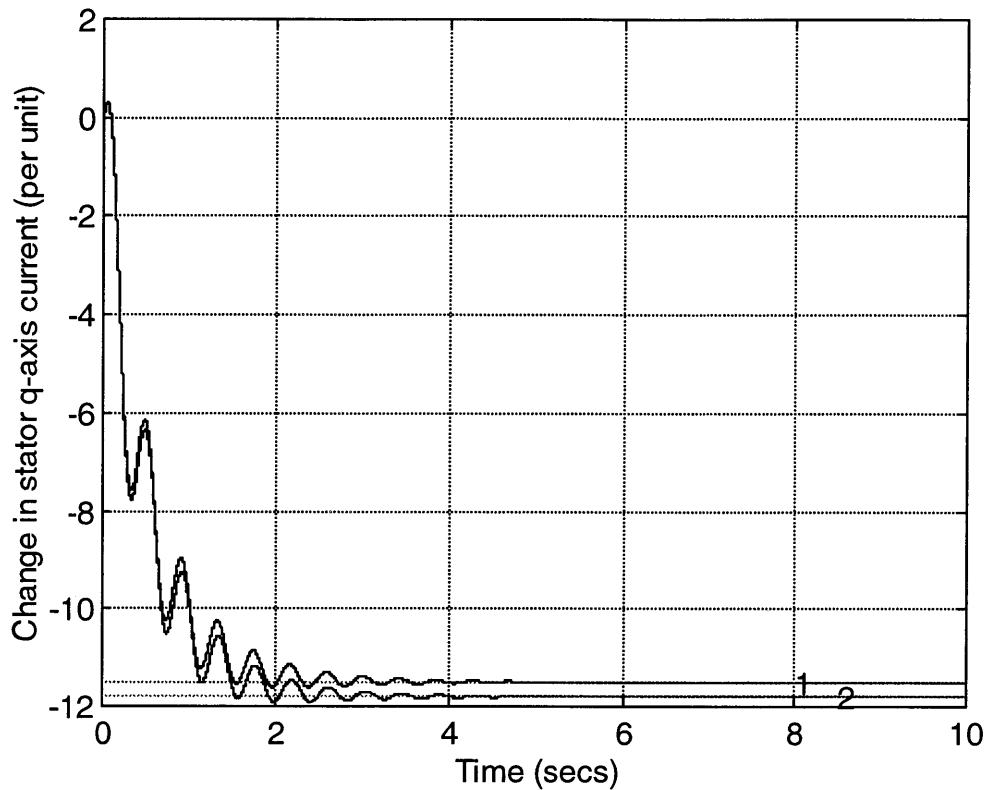


Figure 4.21. Variation in change in stator q-axis current due to 5% step change in excitation voltage.

- 1- Models 1, 3, 4 and 5.
- 2- Model no.2.

Responses due to 5% step change in excitation voltage for unsaturated and saturated flux and current models are presented in figures 4.22 and 4.23 showing the change in rotor angle and stator q-axis current. The change in field current and stator d-axis current are given in figures D9 and D10.

Figure 4.22 shows the change in rotor angle obtained from the unsaturated and saturated flux models. It can be seen from this figure that the final steady state value is established within 4 seconds using the saturated model but with the unsaturated model, it is established in 6 seconds. This indicates faster response and better damping with higher frequency of oscillation when saturation is included in the modelling. Also the change in angle obtained with the saturated model is smaller than that corresponding to unsaturated model. The same conclusion can be drawn from comparing the change in rotor angle obtained from the unsaturated and saturated current models. However, the

difference between the change in angle obtained with unsaturated and saturated current models is smaller than that achieved with unsaturated and saturated flux models.

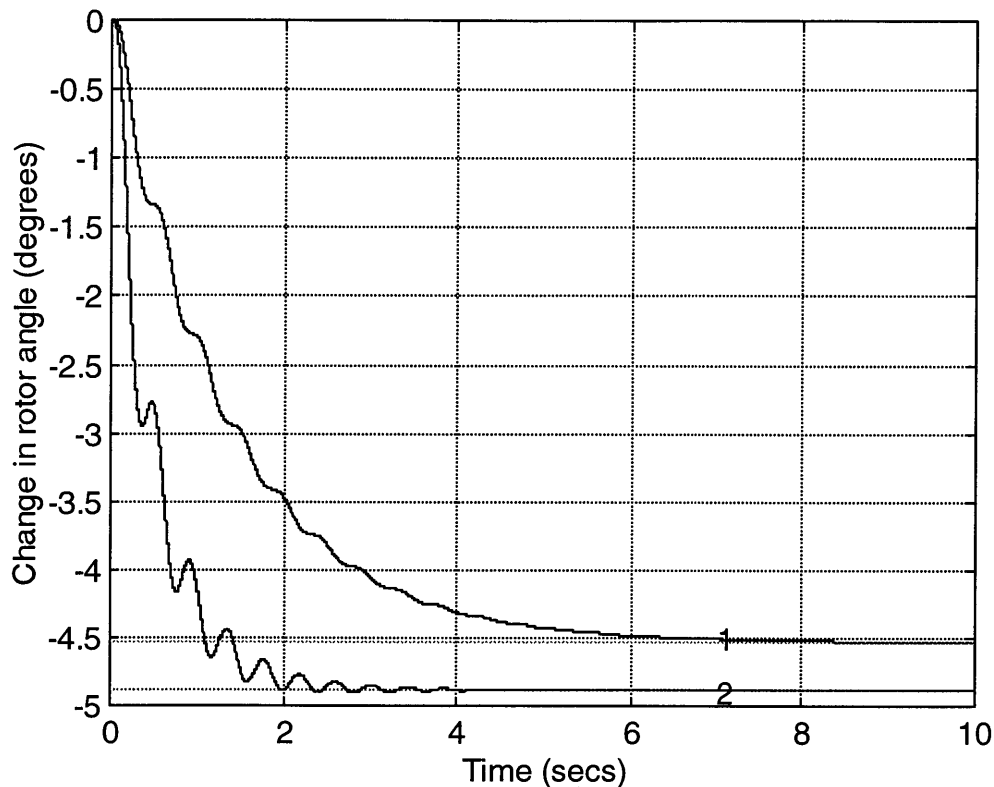


Figure 4.22. Variation in change in rotor angle due to 5% step change in excitation voltage.

- 1- Linearised unsaturated flux model.
- 2- Linearised saturated flux model.

A comparison between the change in stator q-axis current computed from the unsaturated and saturated current models is given in figure 4.23. It is clear that faster response is obtained with the saturated model. Also it can be seen that the steady state change in current established using the saturated model is less than the value established by the unsaturated model. The same outcomes can be obtained by observing the change in stator d-axis current, which is given in figure D10 in Appendix D.

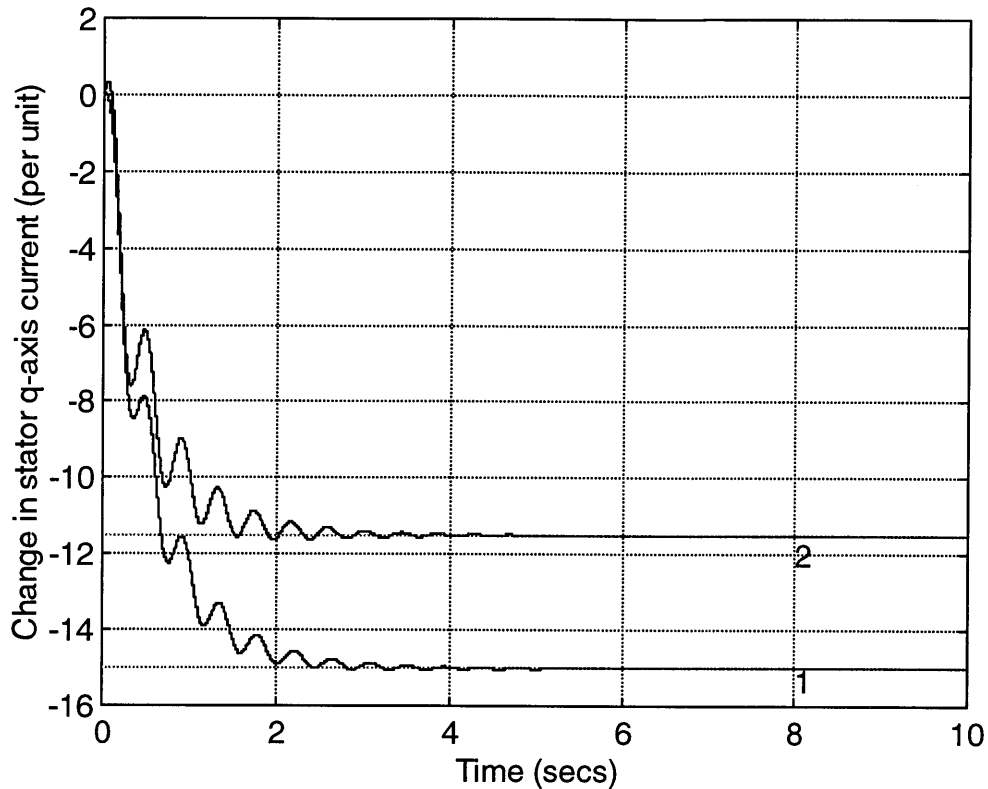


Figure 4.23. Variation in change in stator q-axis current due to 5% step change in excitation voltage.

- 1- Linearised unsaturated current model.
- 2- Linearised saturated current model.

4.2.4 Step Change in Real Power

Figures 4.24-27 illustrate selected responses obtained from unsaturated and saturated models due to 5% in the real power (mechanical torque per unit) showing rotor angle and stator q-axis current. The change in rotor angle obtained from the five saturated current models is presented in figure 4.24. Again it is clear that the change in angle obtained using model no.1,3,4 and 5 is identical and smaller than model no.2. The justification of this difference is already explained in section 4.2.2, which may be related to the difference in change in stator q-axis current as can be seen from figure 4.25. From figures 4.24 and 4.25, it can be seen also that slightly better damping is provided when model no.2 is used. This can be related to the critical eigenvalue corresponding to each model and since model no.2 has the highest damping ratio as can be noted from table 4.4.

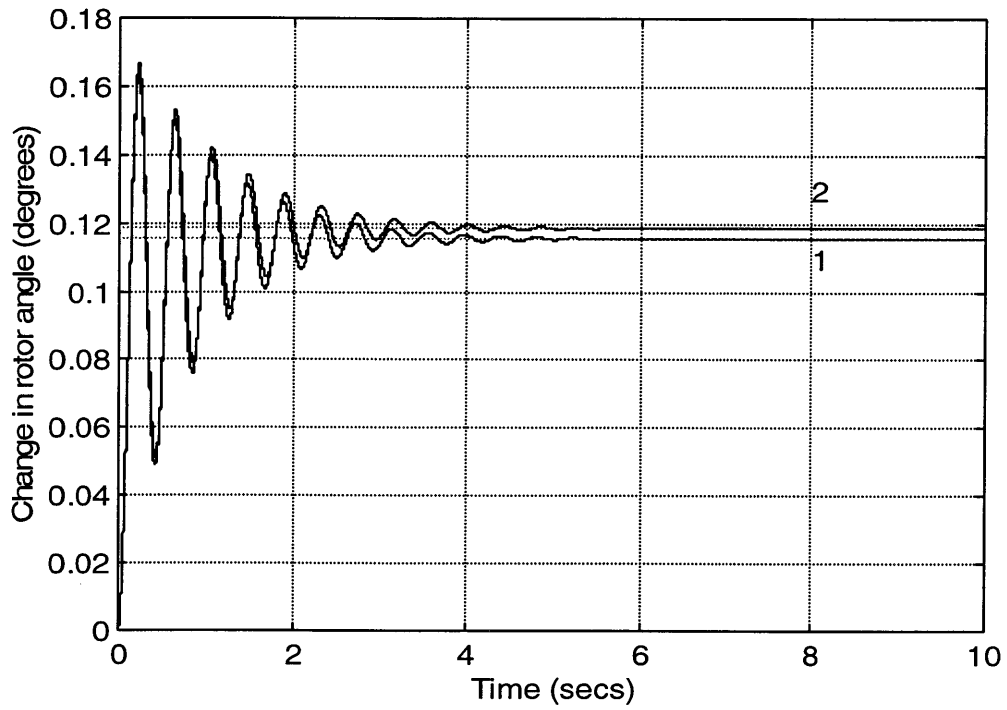


Figure 4.24. Variation in change in rotor angle due to 5% step change in real power.
1- Models 1, 3, 4 and 5. 2- Model no.2.

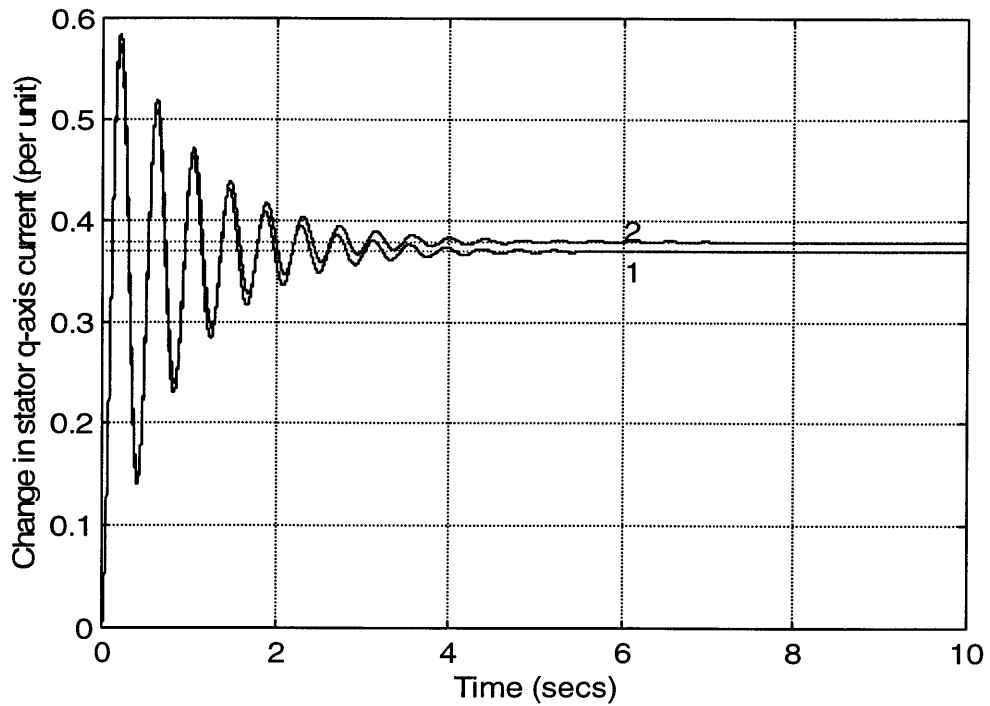


Figure 4.25. Variation in change in stator q-axis current due to 5% step change in excitation voltage.

1- Models 1, 3, 4 and 5. 2- Model no.2.

The same observations can be made about the change in field current and in stator d-axis current, which are given in figures D11 and D12 in Appendix D.

Figure 4.26 presents the change in rotor angle of unsaturated and saturated models. There is insignificant difference between the two curves. They can not be distinguished easily. The difference in stator q-axis current obtained using classical unsaturated and saturated current models is given in figure 4.27. Again, there is no significant discrepancy between the two curves.

From all figures obtained in this section, it is clear that a step change in real power excites appreciable oscillatory response.

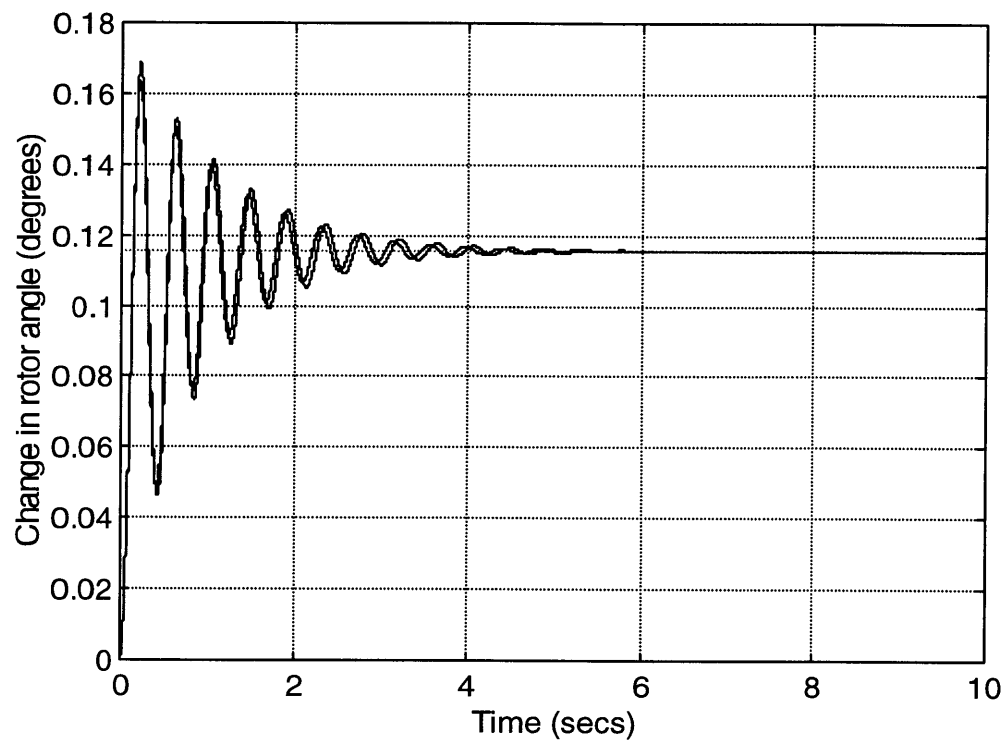


Figure 4.26. Variation in change in rotor angle due to 5% step change in real power.

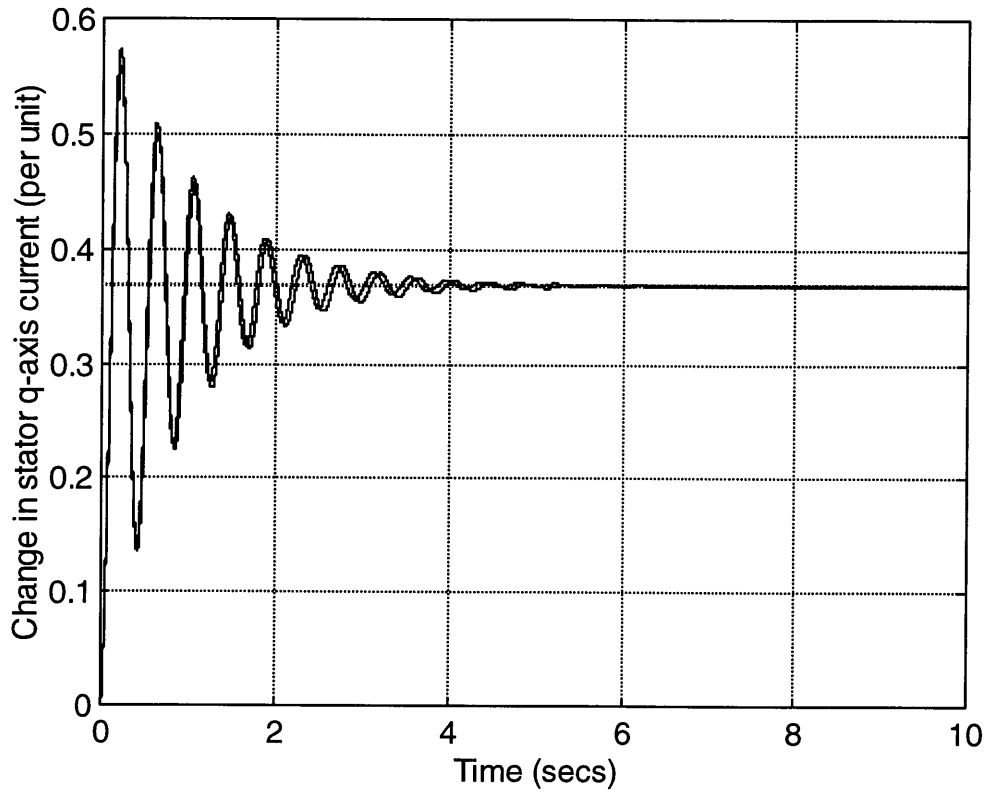


Figure 4.27. Variation in change in stator q-axis current due to 5% step change in real power.

2.3 Monte Carlo Simulation

Monte Carlo simulation is a technique, which has had a great impact in many different fields of research. Monte Carlo method is based on the selection of random numbers to perform the simulation. An essential component of this method is the modelling of the physical process or certain variables by one or more probability density functions. In this project, a random number is generated to distribute real power, saliency factor and saturation factor uniformly over the following range:

$$80\% \leq P \leq 100\%$$

$$80\% \leq m^2 \leq 120\%$$

$$95\% \leq K_{sat} \leq 105\%$$

The reason why these variables are chosen is the uncertainty and difficulty in determining and estimating them. Before proceeding to explain the simulation procedure, the way used to vary saturation factor is illustrated. The change in saturation factor can be obtained by generating a random number between .95 and 1.05 that is multiplied by the magnetising curve expression given in Appendix A at each trail during the simulation of the program. This is illustrated as given in figure 4.28.

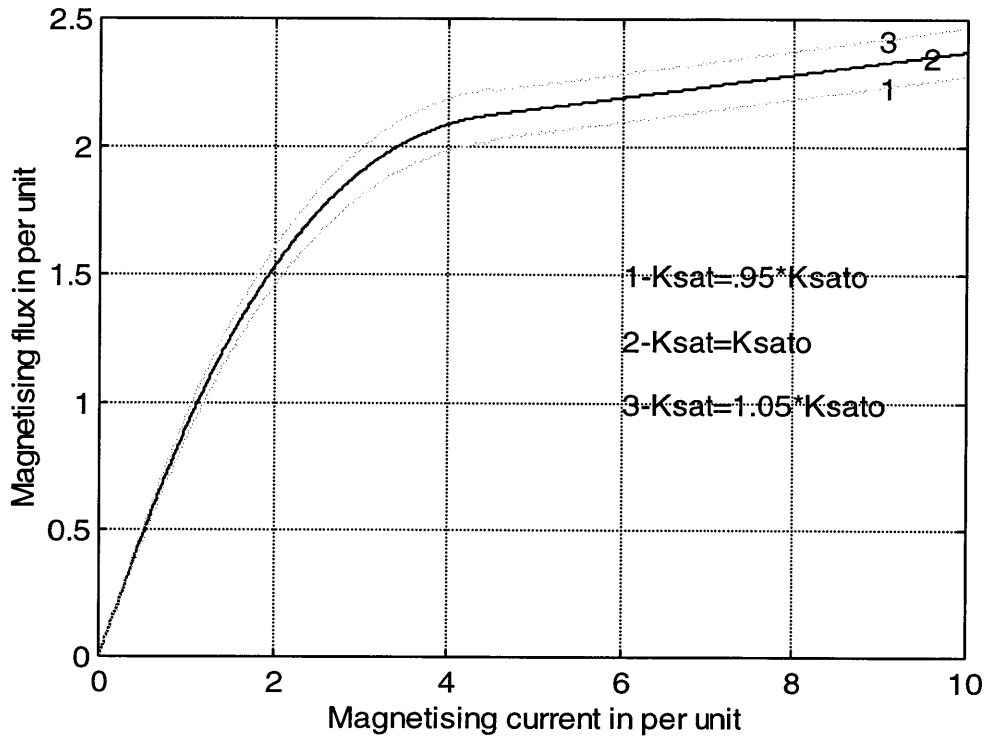


Figure 4.28. Magnetising Curve.

Three cases of study are simulated as stated previously. The MATLAB command *rand* is used to generate 100 random numbers. Then these numbers are used to distribute randomly the real power, saliency factor and saturation factor between the limits given and according to the case considered. After that the non-linear classical current model is simulated at random values of these variables. The steady state values of winding currents and fluxes with rotor angle are substituted in the linearised saturated models to compute the state matrix (K) corresponding to each model. Then the eigenvalues are calculated by finding the roots of K using MATLAB command *eig*. The total number of simulations performed is 100.

4.3.1 Variation in Real power and Saturation Factor

The result of variation in real power and saturation factor are given in figures 4.29-31 showing the root loci of critical eigenvalue corresponding to models no.1, 2 and 3 respectively. Other results obtained for models no 4 and 5 are given in figures D16 and D17 in Appendix D respectively. Models 1, 3, 4 and 5 give identical results, while model no.2 has slightly different result. It is noted that changing saturation factor from 95% to 105% of original value has insignificant effect on the root loci of the critical eigenvalues. It can also be seen that the stability margin provided by model no.2 is greater than that provided by other models. The critical eigenvalue corresponding to models no. 1, 3, 4 and 5 changes from $\lambda_1=-0.8964+j13.6759$, which corresponds to the operating point $P=0.8015$ p.u and $K_{sat}=1.022$ to $\lambda_2=-0.9137+j13.6672$, which corresponds to the operating point $P=0.999$ p.u and $K_{sat}=0.967$. On the other hand, the critical eigenvalue corresponding to model no.2 varies between $\lambda_1=-0.9118+j13.4774$ to $\lambda_2=-0.9232+j13.5498$ for the same operating points as that for other models.

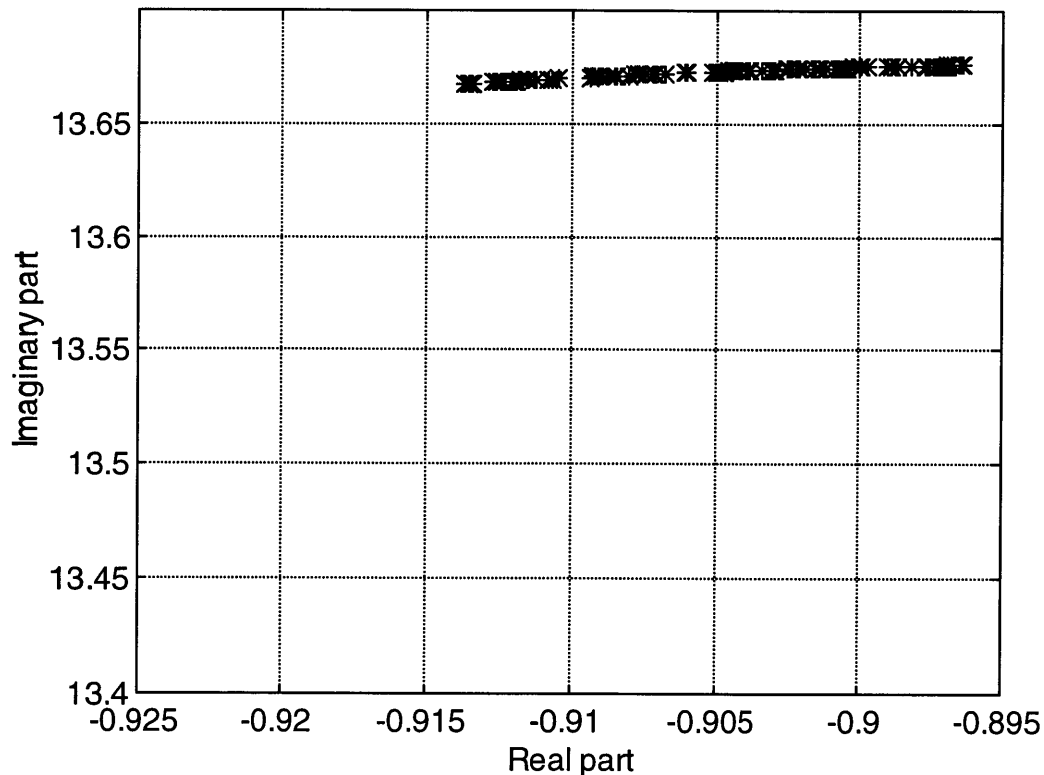


Figure 4.29. Root loci of critical eigenvalue corresponding to model no.1 due to variation in real power and saturation factor.

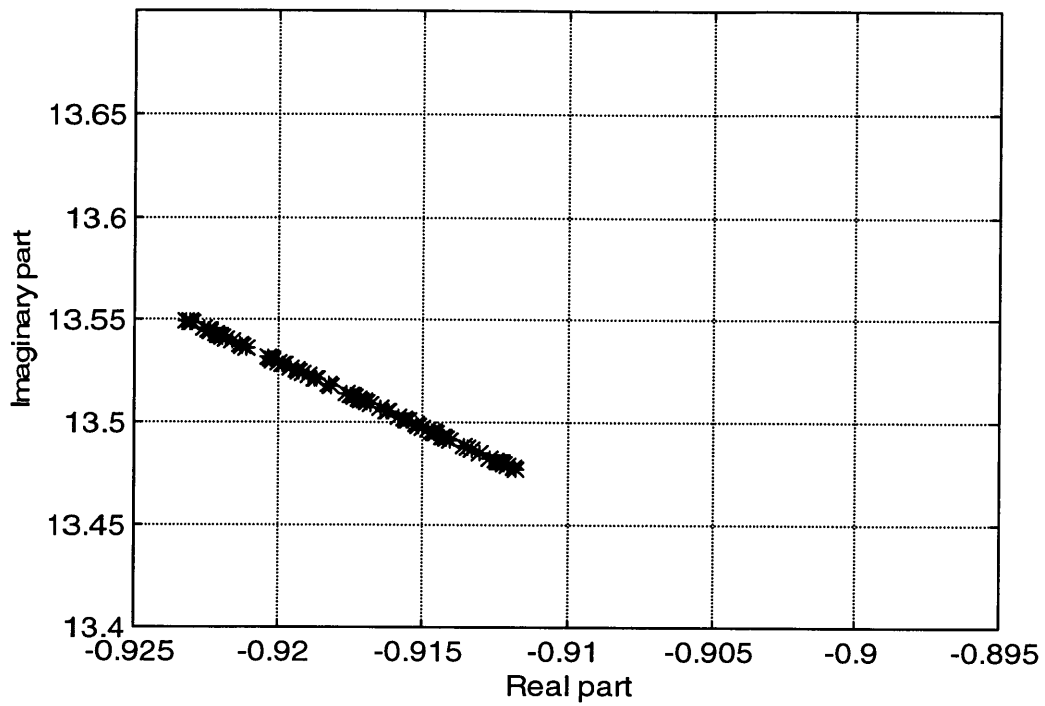


Figure 4.30. Root loci of critical eigenvalue corresponding to model no.2 due to variation in real power and saturation factor.

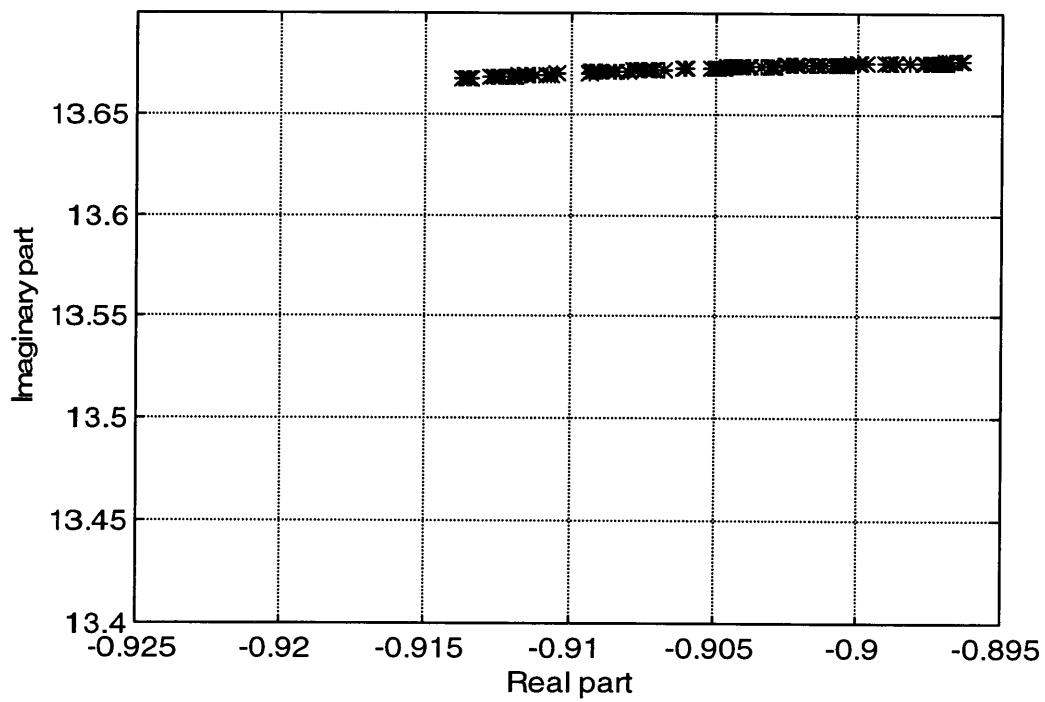


Figure 4.31. Root loci of critical eigenvalue corresponding to model no.3 due to variation in real power and saturation factor.

4.3.2 Variation in Real Power and Saliency Factor

The real power and saliency factor are randomly varied between 80% to 100% and 80% to 120% respectively with constant saturation factor. The results obtained are given in figures 4.32, 4.33 and 4.34 for models no. 1, 2 and 3. Similar to the previous case, models 1, 3, 4 and 5 give identical results. The stability margin provided by these models is less than that provided by model no.2. The critical eigenvalue corresponding to the former varied from $\lambda_1 = -0.7872 + j14.362$, which corresponds to the operating point $P = 0.9656$ p.u and $m^2 = 0.4674$ to $\lambda_2 = -1.0033 + j13.1859$, which corresponds to the operating point $P = 0.9027$ p.u and $m^2 = 0.6969$. On the other hand, the critical eigenvalue corresponding to model no.2 varies between $\lambda_1 = -0.7825 + j14.4306$ to $\lambda_2 = -1.0303 + j12.8059$ for the same variation in real power and saliency factor as the other models. It is clear the variation in saliency factor has a significant stabilising effect on the root loci of the critical eigenvalue corresponding to all saturated models.

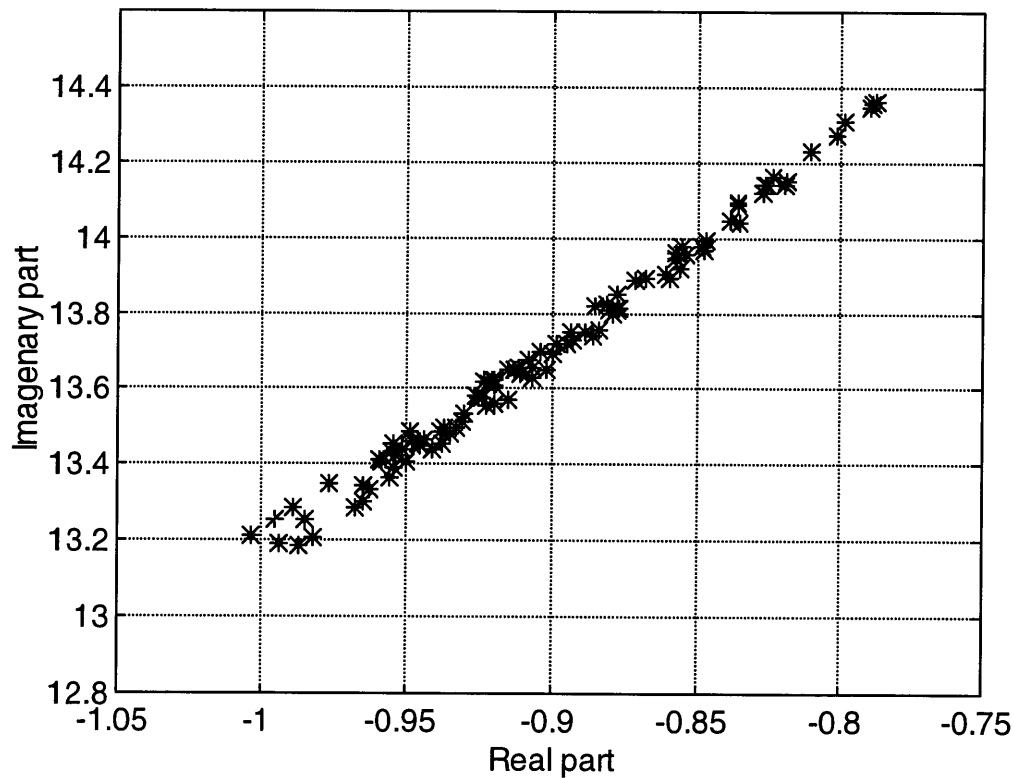


Figure 4.32. Root loci of critical eigenvalue corresponding to model no.1 due to variation in real power and saliency factor.

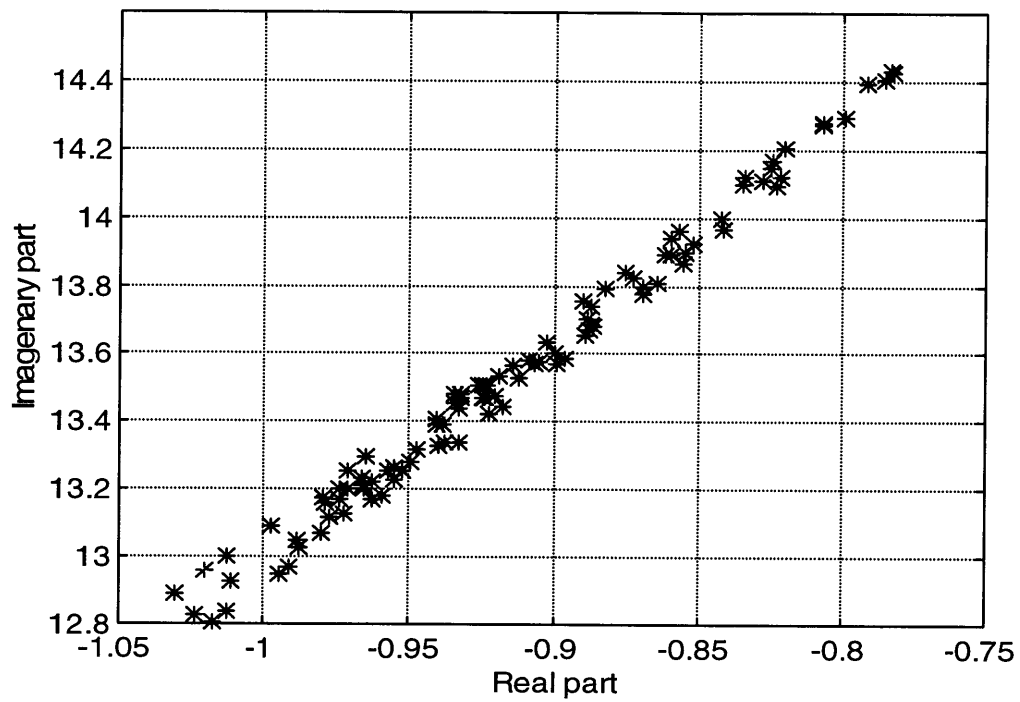


Figure 4.33. Root loci of critical eigenvalue corresponding to model no.2 due to variation in real power and saliency factor.

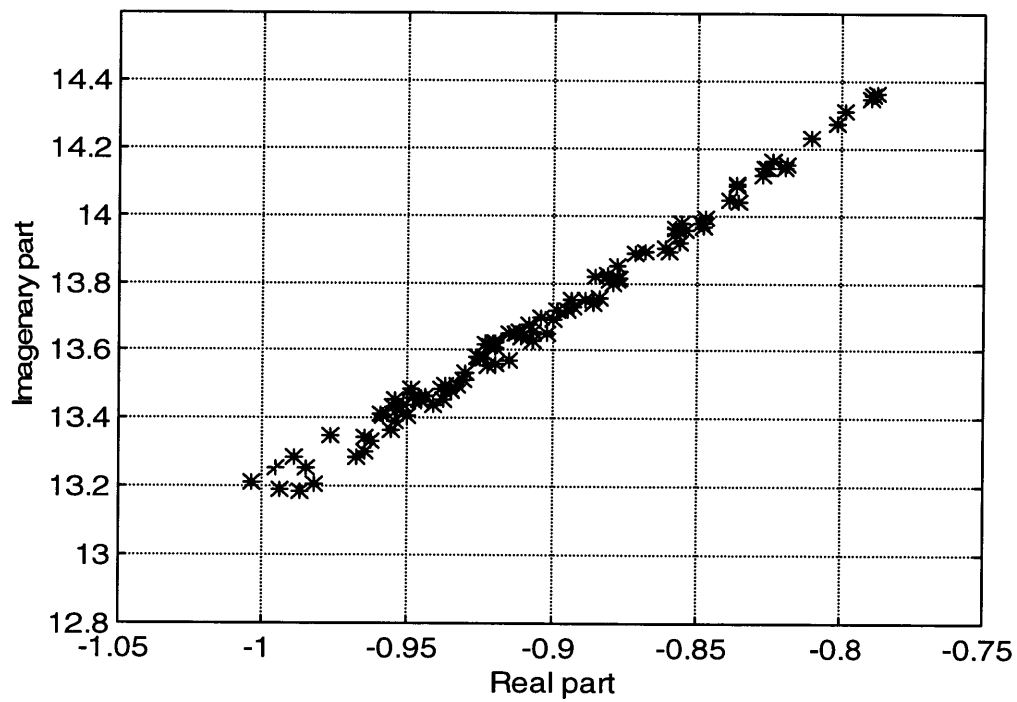


Figure 4.34. Root loci of critical eigenvalue corresponding to model no.3 due to variation in real power and saliency factor.

4.3.3 Variation in Real Power, Saturation Factor and Saliency Factor

In this section all parameters are varied according to the limits stated previously. The results of simulation obtained are given in figures 4.35-37 showing the root loci of the critical eigenvalue corresponding to models no 1, 2 and 3. The result of other models are given in figures D20 and D21 in Appendix D. The results obtained are similar to the results obtained in the previous section (section 4.3.2) since varying the saturation factor between 95% to 105% from the original value has insignificant effect.

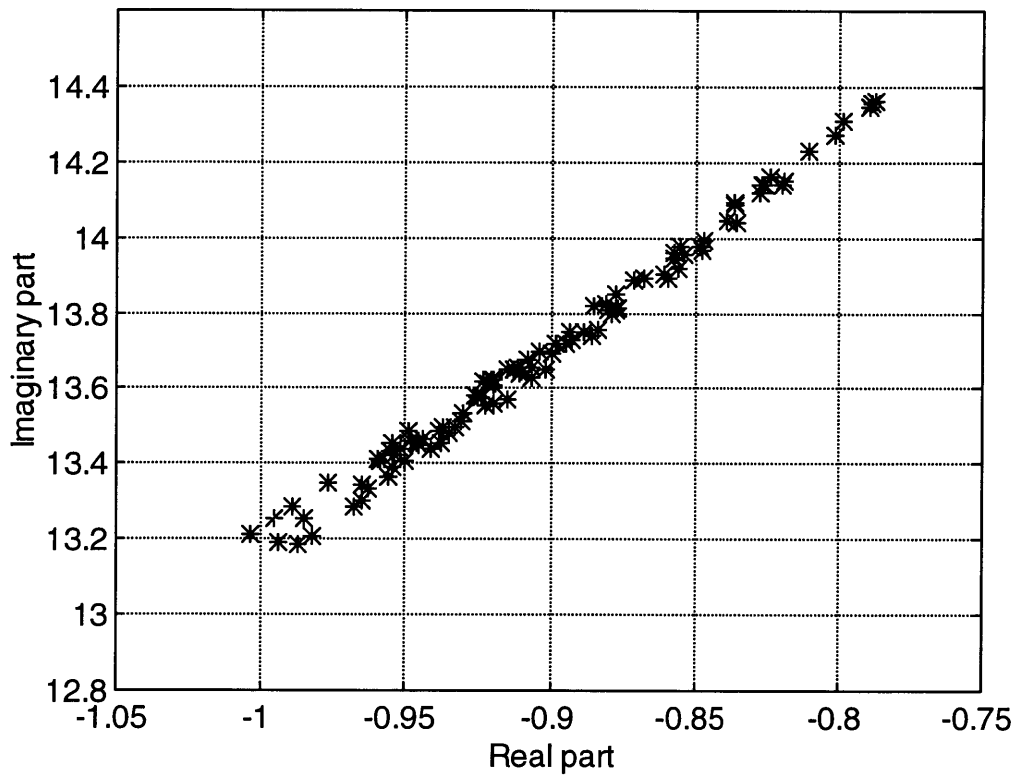


Figure 4.35. Root loci of critical eigenvalue corresponding to model no.1 due to variation in real power, saturation factor and saliency factor.

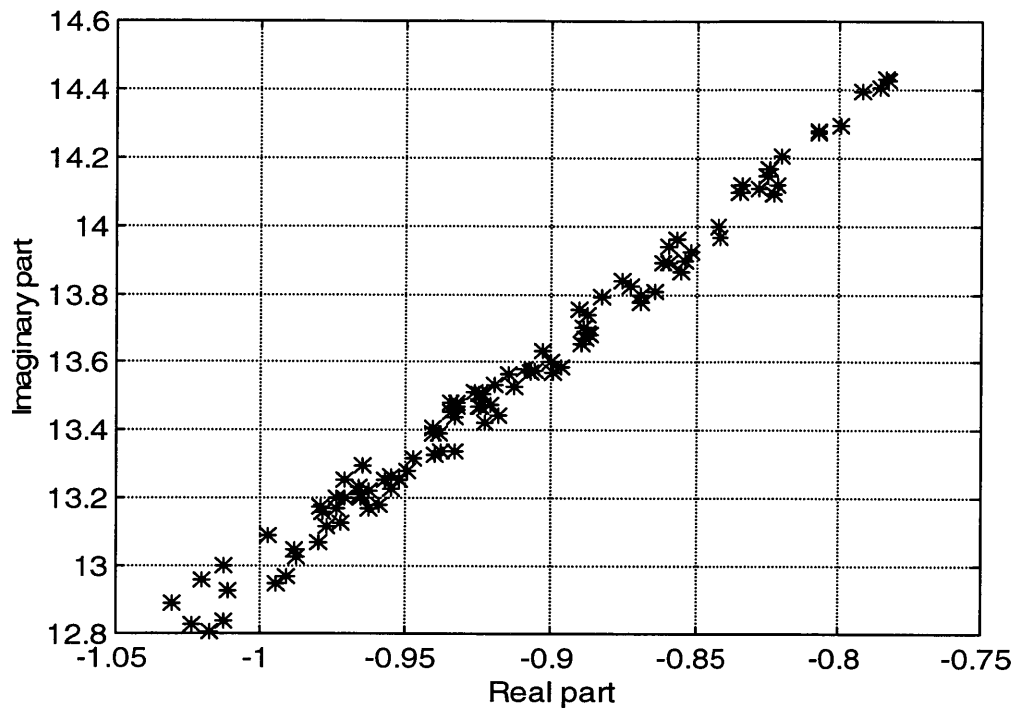


Figure 4.36. Root loci of critical eigenvalue corresponding to model no.2 due to variation in real power, saturation factor and saliency factor.

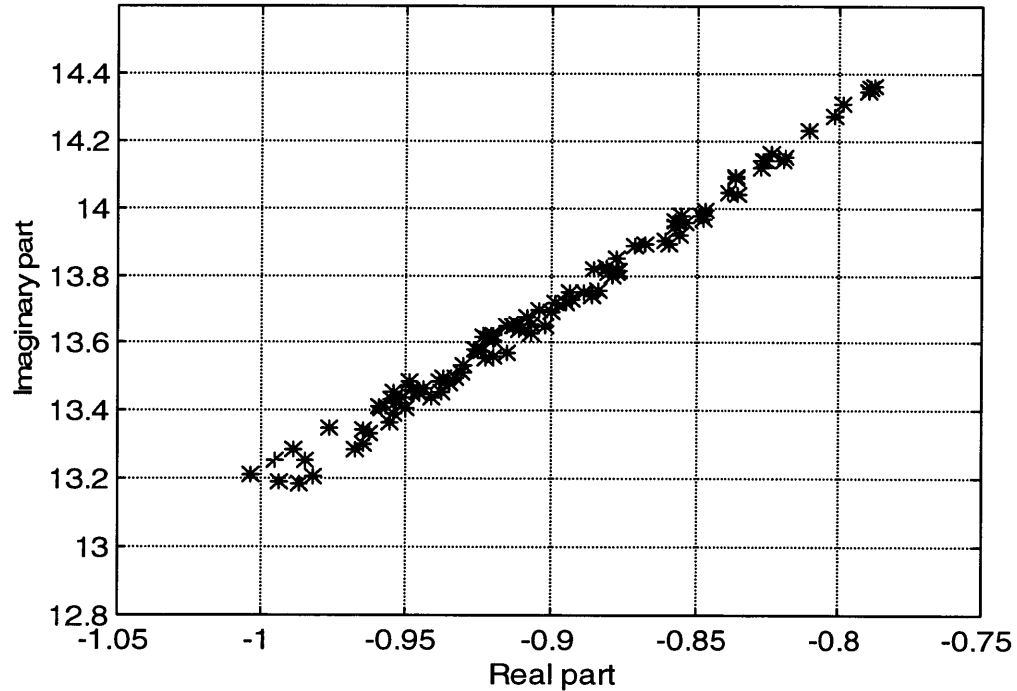


Figure 4.37. Root loci of critical eigenvalue corresponding to model no.3 due to variation in real power, saturation factor and saliency factor.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This dissertation discussed the influence of modelling saturation on the stability of a salient pole synchronous machine using recently developed models. A method of linearisation is proposed and implemented on some of these models. The following outcomes are obtained from the work done in this dissertation:

1. There are large discrepancies between the responses obtained using unsaturated models and saturated models.
2. The stabilising effect of saturation is verified. This effect improves the damping of the machine as well as the responses to sudden changes in excitation voltage, power demand and grid voltage.
3. The choice of the state variables in developing the linearised saturation models of the salient pole synchronous machine influences slightly the calculated eigenvalues and consequently this leads to slightly different responses obtained from the linearised saturated models.
4. The rotor eigenvalue is function of the loading conditions of the machine as well as the saliency factor and it dominates the dynamic behaviour of the synchronous machine.
5. For random values of the saliency and saturation factors and real power in the range given, the synchronous machine under consideration is stable.

The following points are recommended for further investigation on the models developed.

1. Based on the mathematical models developed, the bifurcation characteristics of the system may be found and the dynamics of the system can be simulated at different points.

-
2. Some control laws may be developed for the system and then the performance can be simulated. The results of system dynamics with and without saturation modelling may be contrasted.
 3. Optimisation techniques can be chosen to find the best available control device (e.g. AVR gains etc.) parameters as a function of the saturation modelling.

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APPENDICES

APPENDIX A: SYNCHRONOUS MACHINE DATA

A.1 Per-unit Machine Parameters

Variables	Value
r_s	0.0062
r_f	0.000752
r_{kq1}	0.055
r_{kd1}	0.0254
L_{ls}	0.14
L_{mdo}	0.91
L_{mqo}	0.53
L_{lf}	0.2244
L_{lkq1}	0.242
L_{lkdl}	0.3426
H	2.7093 sec
m^2	0.58242
f	60 Hz

A.2 Magnetising Curve Approximation

$$\psi_u = 1.1212 * 0.8029^{i_u} * i_u^{1.0826}$$

$$L_u = \frac{\psi_u}{i_u} = 1.1212 * 0.8029^{i_u} * i_u^{0.0826}$$

$$L = \frac{d\psi_u}{di_u} = 1.1212 * 0.8029^{i_u} * (1.0826 * i_u^{0.0826} - .2195 * i_u^{1.0826})$$

APPENDIX B: DERIVATION OF SATURATION PARAMETERS

$$\begin{aligned}
\frac{di_u}{dt} &= \frac{d}{dt} \left(\sqrt{i_{md}^2 + m^2 i_{mq}^2} \right) \\
&= \frac{1}{2\sqrt{i_{md}^2 + m^2 i_{mq}^2}} \left(2i_{md} \frac{di_{md}}{dt} + 2m^2 i_{mq} \frac{di_{mq}}{dt} \right) \\
&= \frac{i_{md}}{\sqrt{i_{md}^2 + m^2 i_{mq}^2}} \frac{di_{md}}{dt} + \frac{m^2 i_{mq}}{\sqrt{i_{md}^2 + m^2 i_{mq}^2}} \frac{di_{mq}}{dt} \\
&= \frac{i_{md}}{i_m} \frac{di_{md}}{dt} + \frac{m^2 i_{mq}}{i_m} \frac{di_{mq}}{dt} \\
\frac{di_u}{dt} &= \cos u \frac{di_{md}}{dt} + m \sin u \frac{di_{mq}}{dt} \tag{B.1}
\end{aligned}$$

$$\begin{aligned}
\frac{dL_u}{dt} &= \frac{d}{dt} \left(\frac{\psi_u}{i_u} \right) \\
&= \frac{i_u \frac{d\psi_u}{dt} \frac{di_u}{dt} - \psi_u \frac{di_u}{dt}}{i_u^2} \\
&= \frac{\frac{d\psi_u}{dt} \frac{di_u}{dt}}{i_u} - \frac{\psi_u \frac{di_u}{dt}}{i_u^2} \\
&= \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) \frac{di_u}{dt} \\
\frac{dL_u}{dt} &= \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) \cos u \frac{di_{md}}{dt} + \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) m \sin u \frac{di_{mq}}{dt} \tag{B.2}
\end{aligned}$$

$$\begin{aligned}
\frac{d\psi_u}{dt} &= \frac{d}{dt} \left(\sqrt{\psi_{md}^2 + \frac{\psi_{mq}^2}{m^2}} \right) \\
&= \frac{1}{2\sqrt{\psi_{md}^2 + \frac{\psi_{mq}^2}{m^2}}} \left(2\psi_{md} \frac{d\psi_{md}}{dt} + \frac{2\psi_{mq}}{m^2} \frac{d\psi_{mq}}{dt} \right) \\
&= \frac{\psi_{md}}{\sqrt{\psi_{md}^2 + \frac{\psi_{mq}^2}{m^2}}} \frac{d\psi_{md}}{dt} + \frac{\frac{\psi_{mq}}{m^2}}{\sqrt{\psi_{md}^2 + \frac{\psi_{mq}^2}{m^2}}} \frac{d\psi_{mq}}{dt} \\
&= \frac{\psi_{md}}{\psi_u} \frac{d\psi_{md}}{dt} + \frac{\frac{\psi_{mq}}{m^2}}{\psi_u} \frac{d\psi_{mq}}{dt} \\
&= \cos u \frac{d\psi_{md}}{dt} + \frac{\sin u}{m} \frac{d\psi_{mq}}{dt}
\end{aligned} \tag{B.3}$$

$$\begin{aligned}
\frac{d}{dt} \left(\frac{1}{L_u} \right) &= \frac{d}{dt} \left(\frac{1/L_u}{\psi_u} \right) \frac{d\psi_u}{dt} \\
&= \frac{d}{dt} \left(\frac{i_u/\psi_u}{\psi_u} \right) \frac{d\psi_u}{dt} \\
&= \left(\frac{\psi_u \frac{di_u}{d\psi_u} - i_u}{\psi_u^2} \right) \frac{d\psi_u}{dt} \\
&= \left(\frac{\frac{di_u}{d\psi_u}}{\psi_u} - \frac{i_u}{\psi_u^2} \right) \frac{d\psi_u}{dt} \\
&= \left(\frac{1}{\psi_u L} - \frac{1}{\psi_u L_u} \right) \frac{d\psi_u}{dt} \\
\frac{d}{dt} \left(\frac{1}{L_u} \right) &= \left(\frac{1}{\psi_u L} - \frac{1}{\psi_u L_u} \right) \cos u \frac{d\psi_{md}}{dt} + \left(\frac{1}{\psi_u L} - \frac{1}{\psi_u L_u} \right) \frac{\sin u}{m} \frac{d\psi_{mq}}{dt}
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
\left(\frac{d\psi_{md}}{dt} \right) &= \frac{d(L_u i_{md})}{dt} \\
&= L_u \frac{di_{md}}{dt} + i_{md} \frac{dL_u}{dt} \\
&= L_u \frac{di_{md}}{dt} + i_{md} \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) \cos u \frac{di_{md}}{dt} + i_{md} \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) m \sin u \frac{di_{mq}}{dt} \\
&= L_u \frac{di_{md}}{dt} + \frac{i_{md}}{i_u} (L - L_u) \cos u \frac{di_{md}}{dt} + \frac{i_{md}}{i_u} (L - L_u) m \sin u \frac{di_{mq}}{dt} \\
&= L_u \frac{di_{md}}{dt} + (L - L_u) \cos^2 u \frac{di_{md}}{dt} + (L - L_u) m \sin u \cos u \frac{di_{mq}}{dt} \\
&= (1 - \cos^2 u) L_u \frac{di_{md}}{dt} + L \cos^2 u \frac{di_{md}}{dt} + (L - L_u) m \sin u \cos u \frac{di_{mq}}{dt} \\
&= (L_u \sin^2 u + L \cos^2 u) \frac{di_{md}}{dt} + (L - L_u) m \sin u \cos u \frac{di_{mq}}{dt} \\
\frac{d\psi_{md}}{dt} &= L_{dd} \frac{di_{md}}{dt} + L_{dq} \frac{di_{mq}}{dt} \tag{B.5}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{d\psi_{mq}}{dt} \right) &= \frac{d(m^2 L_u i_{mq})}{dt} \\
&= m^2 L_u \frac{di_{mq}}{dt} + m^2 i_{mq} \frac{dL_u}{dt} \\
&= m^2 L_u \frac{di_{mq}}{dt} + m^2 i_{mq} \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) \cos u \frac{di_{md}}{dt} + m^2 i_{mq} \left(\frac{L}{i_u} - \frac{L_u}{i_u} \right) m \sin u \frac{di_{mq}}{dt} \\
&= m^2 L_u \frac{di_{mq}}{dt} + m^2 \frac{i_{mq}}{i_u} (L - L_u) \cos u \frac{di_{md}}{dt} + m^2 \frac{i_{mq}}{i_u} (L - L_u) m \sin u \frac{di_{mq}}{dt} \\
&= m^2 L_u \frac{di_{mq}}{dt} + (L - L_u) m \sin u \cos u \frac{di_{md}}{dt} + (L - L_u) m^2 \sin u \frac{di_{mq}}{dt} \\
&= (1 - \sin^2 u) m^2 L_u \frac{di_{mq}}{dt} + m^2 L \sin^2 u \frac{di_{mq}}{dt} + (L - L_u) m \sin u \cos u \frac{di_{md}}{dt} \\
&= m^2 (L_u \cos^2 u + L \sin^2 u) \frac{di_{mq}}{dt} + (L - L_u) m \sin u \cos u \frac{di_{md}}{dt} \\
\frac{d\psi_{mq}}{dt} &= L_{qq} \frac{di_{mq}}{dt} + L_{dq} \frac{di_{md}}{dt} \tag{B.6}
\end{aligned}$$

APPENDIX C: LINEARISED SATURATED MODELS

- Model with $x = (\Delta\psi_{ds} \quad \Delta i_f \quad \Delta i_{md} \quad \Delta\psi_{qs} \quad \Delta i_{mq} \quad \Delta\omega \quad \Delta\delta)$ as state variables

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{lf} & L_{dd} & 0 & L_{dq} & 0 & 0 \\ \frac{L_{lkd1}}{L_{ls}} & -L_{lkd1} & \left(L_{lkd1} \left(1 + \frac{L_{dd}}{L_{ls}} \right) + L_{dd} \right) & 0 & \left(\frac{L_{lkd1} L_{dq}}{L_{ls}} + L_{dq} \right) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{L_{lkq1}}{L_{ls}} + L_{dq} \right) & \frac{-L_{lkq1}}{L_{ls}} & \left(L_{lkq1} \left(1 + \frac{L_{qq}}{L_{ls}} \right) + L_{qq} \right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{-r_s}{L_{ls}} & 0 & \frac{r_s L_{dd}}{L_{ls}} \dots\dots\dots \\ 0 & \frac{r_f}{L_{lf}} & \frac{-r_f L_{dd}}{L_{ls}} \dots\dots\dots \\ -\frac{r_{kd1}}{L_{ls}} & -\frac{r_{kd1}}{L_{lf}} & r_{kd1} \left(1 + L_{dd} \left(\frac{1}{L_{ls}} + \frac{1}{L_{lf}} \right) \right) \dots\dots\dots \\ \omega_o & 0 & \frac{r_s L_{dq}}{L_{ls}} \dots\dots\dots \\ 0 & 0 & \frac{r_{kq1} L_{dq}}{L_{ls}} \dots\dots\dots \\ \left(i_{qso} - \frac{\psi_{qso}}{L_{ls}} \right) / 2H\omega_b & 0 & \left(\frac{L_{dd} \psi_{qso}}{L_{ls}} - \frac{L_{dq} \psi_{dso}}{L_{ls}} \right) / 2H\omega_b \dots\dots\dots \\ 0 & 0 & 0 \dots\dots\dots \end{pmatrix}$$

$$\begin{array}{cccc}
\text{.....}\omega_o & \frac{L_{dq}r_s}{L_{ls}} & -\psi_{qso} & v_{qso} \\
\text{.....}0 & \frac{-r_f L_{dq}}{L_{ls}} & 0 & 0 \\
\text{.....}0 & 0 & 0 & 0 \\
\text{.....}\frac{-r_s}{L_{ls}} & \frac{r_s L_{qq}}{L_{ls}} & 0 & v_{dso} \\
\text{.....}\frac{-r_{kq1}}{L_{ls}} & r_{kq1} \left(1 + \frac{L_{qq}}{L_{ls}} \right) & \psi_{dso} & 0 \\
\text{.....}\left(\frac{\psi_{dso}}{L_{ls}} - i_{dso} \right) / 2H\omega_b & \left(\frac{\psi_{qso} L_{dq}}{L_{ls}} - \frac{\psi_{dso} L_{qq}}{L_{ls}} \right) / 2H\omega_b & 0 & 0 \\
\text{.....}0 & 0 & -1 & 0
\end{array} \quad (C.1)$$

- Model with $x = (\Delta\psi_{ds} \quad \Delta\psi_f \quad \Delta\psi_{md} \quad \Delta\psi_{qs} \quad \Delta\psi_{mq} \quad \Delta\omega \quad \Delta\delta)$ as state variables

$$A = \begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\frac{-L_{lkd1}}{L_{lf}} & \frac{-L_{lkd1}}{L_{lf}} & \left(L_{lkd1} \left(L_{xx} + \frac{1}{L_{ls}} + \frac{1}{L_{lf}} \right) + \right) & 0 & -L_{xy} L_{lkd1} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -L_{xy} L_{lkq1} & \frac{L_{lkq1}}{L_{ls}} & \left(L_{lkq1} \left(L_{yy} + \frac{1}{L_{ls}} \right) \right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$B = \begin{pmatrix} \frac{-r_s}{L_{ls}} & 0 & \frac{r_s}{L_{ls}} \dots\dots\dots \\ 0 & \frac{r_f}{L_{lf}} & \frac{-r_f}{L_{lf}} \dots\dots\dots \\ \frac{-r_{kd1}}{L_{ls}} & \frac{-r_{kd1}}{L_{lkd1}} & r_{kd1} \left(L_{xx} + \frac{1}{L_{ls}} + \frac{1}{L_{fd}} \right) \dots\dots\dots \\ \omega_o & 0 & 0 \dots\dots\dots \\ 0 & 0 & L_{xy} r_{kq1} \dots\dots\dots \\ (i_{qso} - \psi_{qso} L_{ls}) / 2H\omega_b & 0 & \frac{\psi_{qso}}{2H\omega_b} \dots\dots\dots \\ 0 & 0 & 0 \dots\dots\dots \end{pmatrix}$$

$$\begin{pmatrix} \dots\dots\dots -\omega_o & 0 & -\psi_{qso} & v_{qso} \\ \dots\dots\dots 0 & 0 & 0 & 0 \\ \dots\dots\dots 0 & L_{xy} r_{kd1} & 0 & 0 \\ \dots\dots\dots \frac{-r_s}{L_{ls}} & \frac{r_s}{L_{ls}} & \psi_{dso} & v_{dso} \\ \dots\dots\dots \frac{-r_{kq1}}{L_{ls}} & r_{kq1} \left(L_{yy} + \frac{1}{L_{ls}} \right) & 0 & 0 \\ \dots\dots\dots \left(\frac{\psi_{dso}}{L_{ls}} - i_{dso} \right) / 2H\omega_b & \frac{-\psi_{dso}}{2H\omega_b L_{ls}} & 0 & 0 \\ \dots\dots\dots 0 & 0 & -1 & 0 \end{pmatrix} \quad (C.2)$$

APPENDIX D

D.1 Simulation of Non-linear Unsaturated and Saturated Models

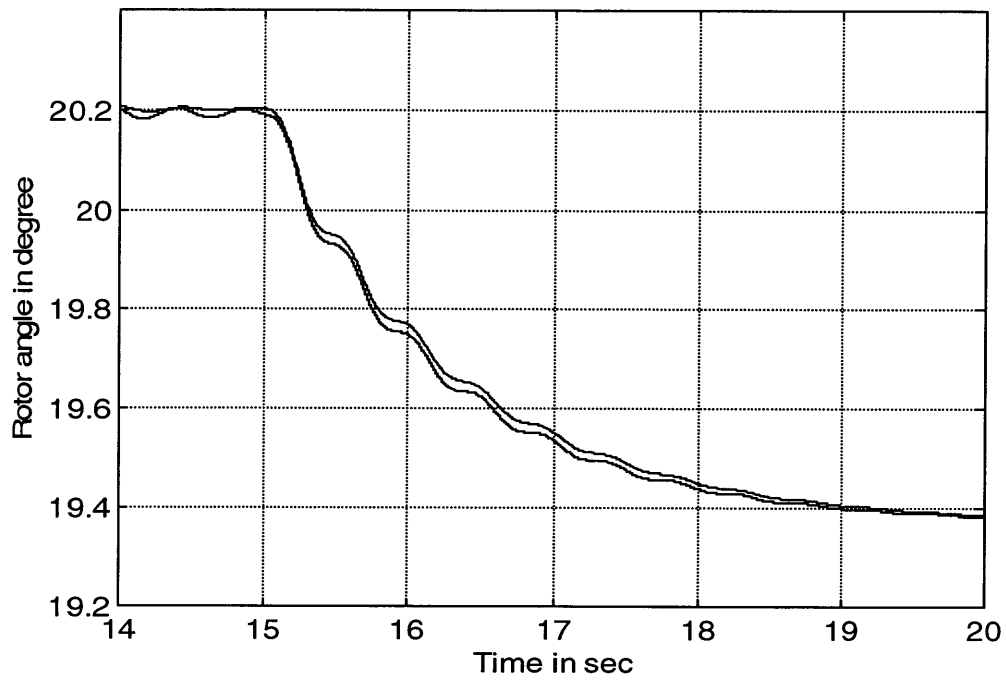


Figure D1. Variation in rotor angle due to 5% in excitation voltage.

— Unsaturated model. — Saturated model.

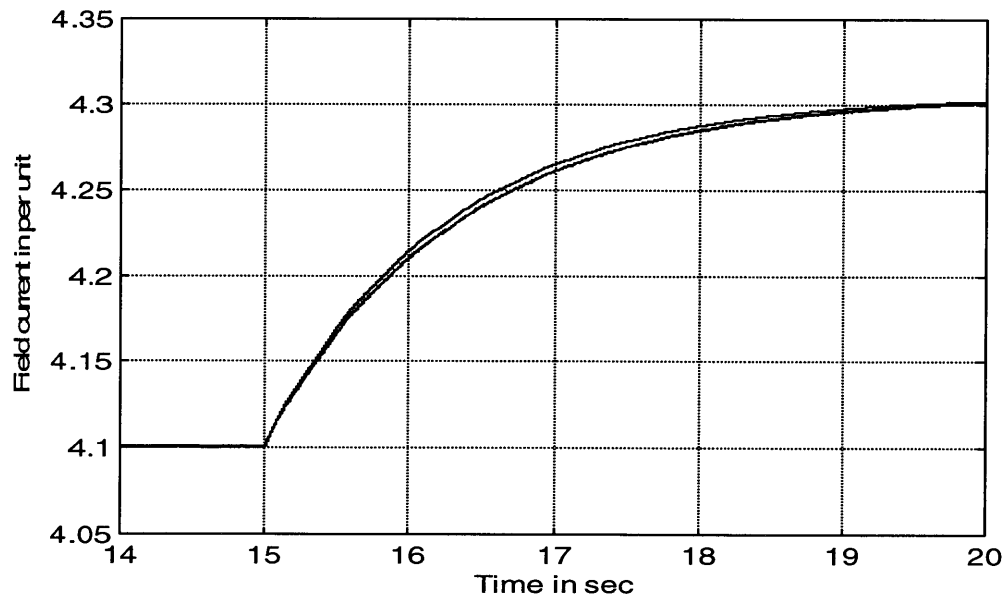


Figure D.2 Variation in field current due to 5% step change in excitation voltage.

— Unsaturated model. — Saturated model

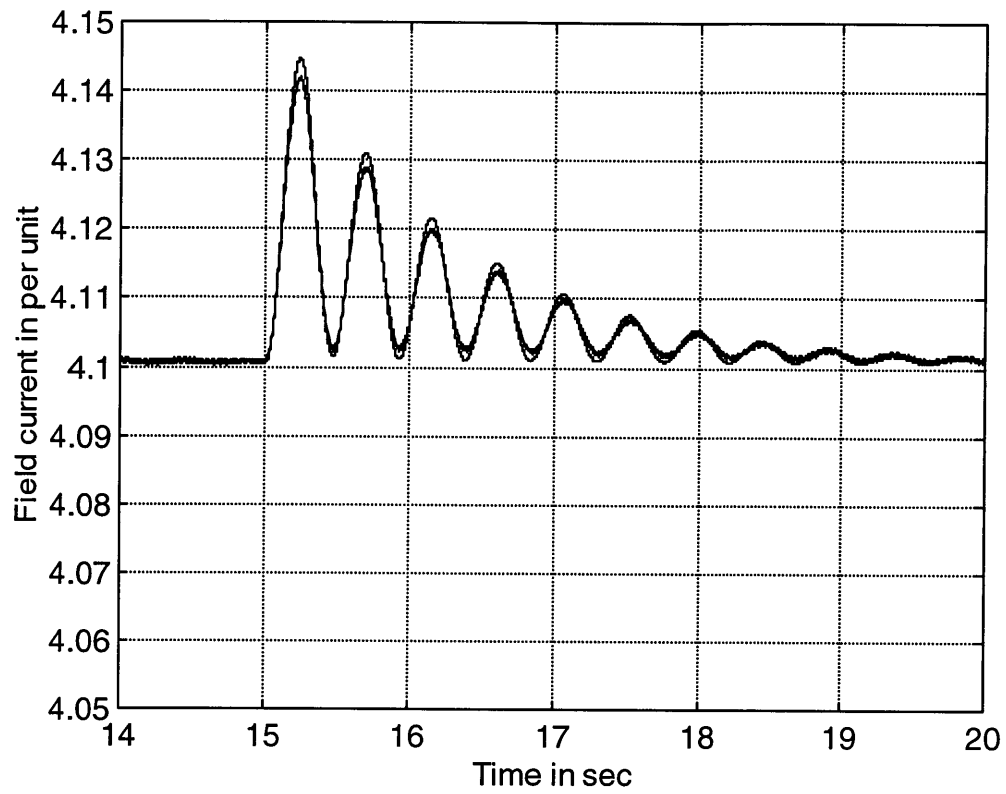


Figure D3. Variation in field current due to 5% step change in real power.

D.2 Simulation of Linearised Unsaturated and Saturated Models

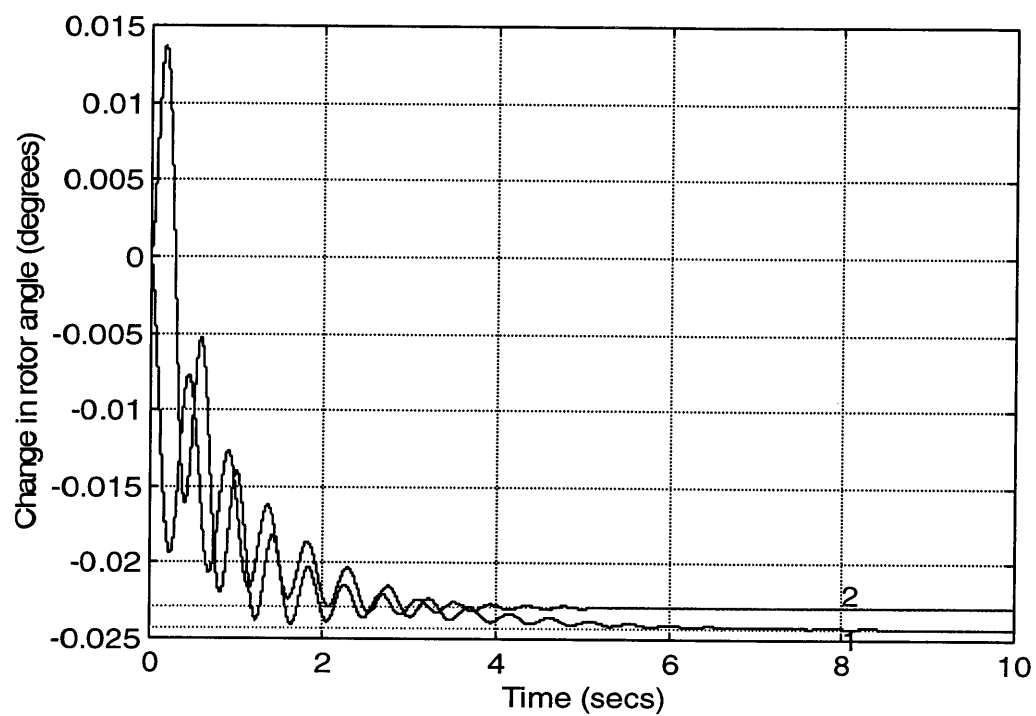


Figure D4. Variation in change in rotor angle due to 5% step change in grid voltage.

1- Linearised unsaturated flux model.

2- Linearised saturated flux model.

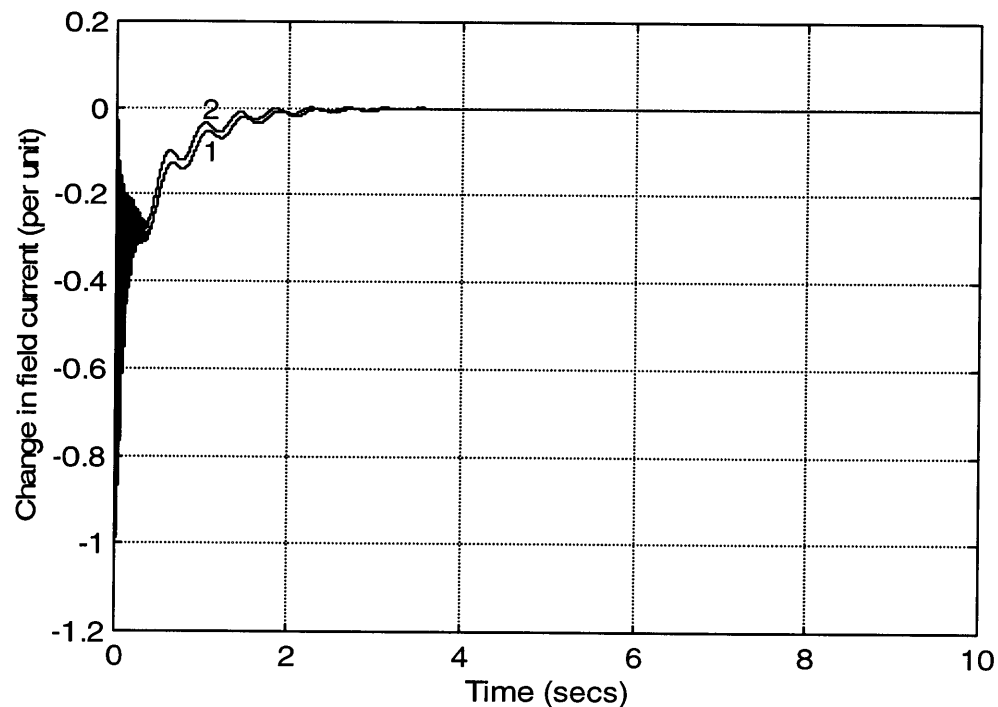


Figure D5. Variation in change in field current due to 5% step change in grid voltage.

1- Linearised unsaturated flux model.

2- Linearised saturated flux model.

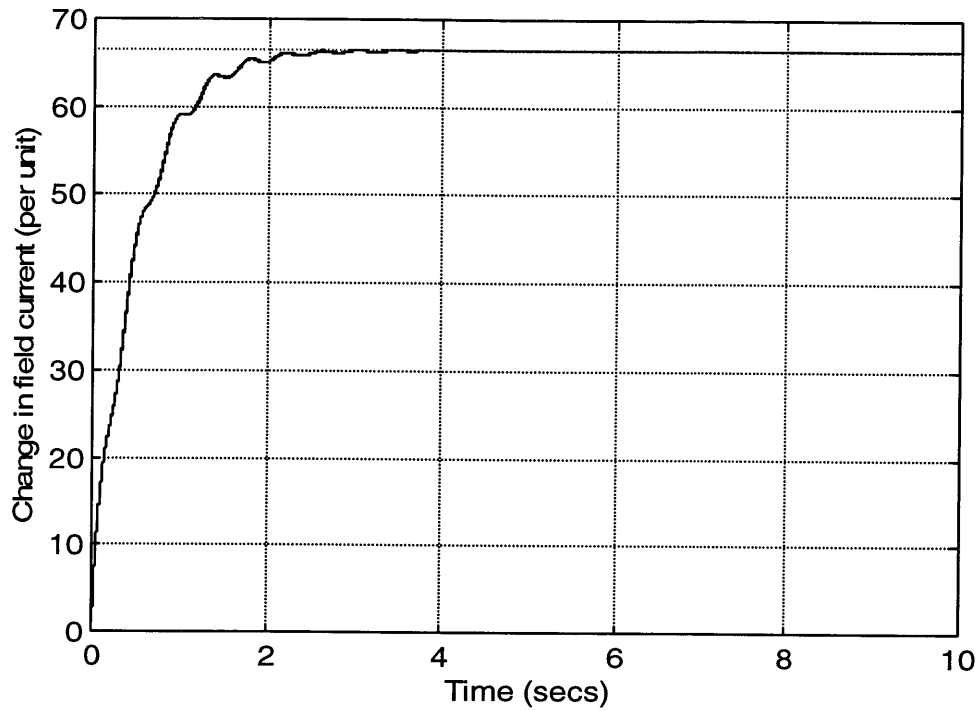


Figure D6. Variation in change in field current due to 5% step change in excitation voltage.

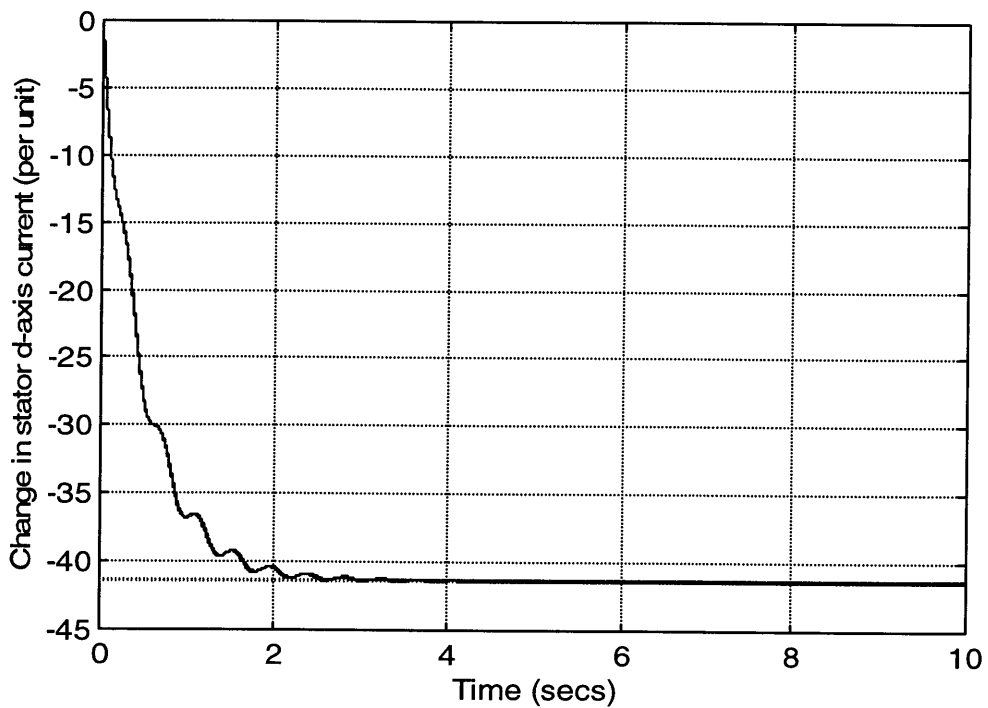


Figure D7. Variation in change in stator d-axis current due to 5% step change in excitation voltage.

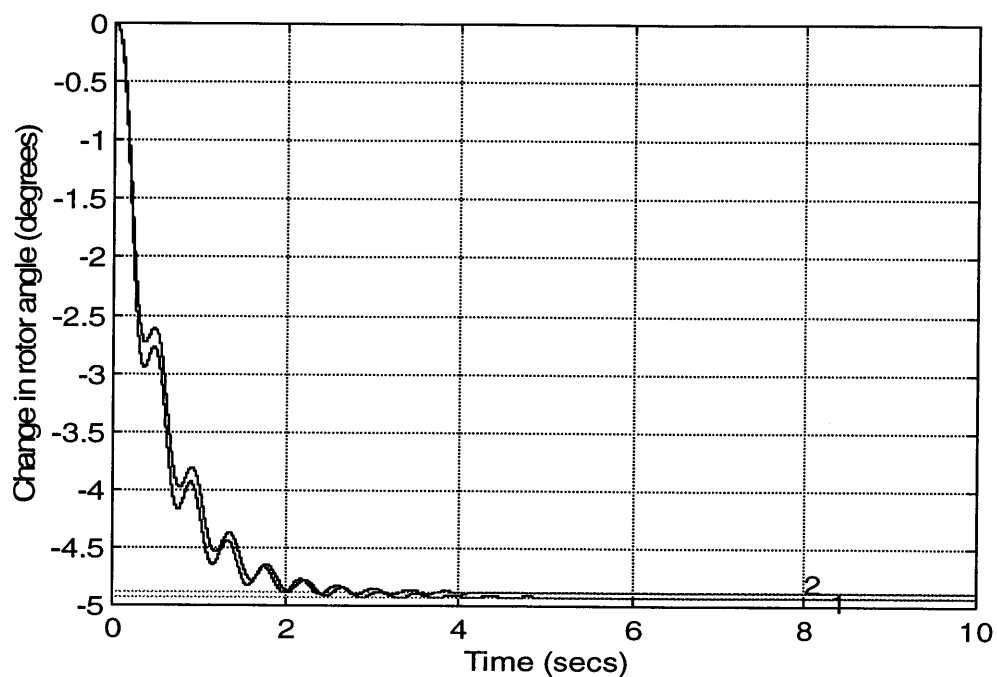


Figure D8. Variation in change in rotor angle due to 5% step change in excitation voltage.

1- Linearised unsaturated current model. 2- Linearised saturated model.

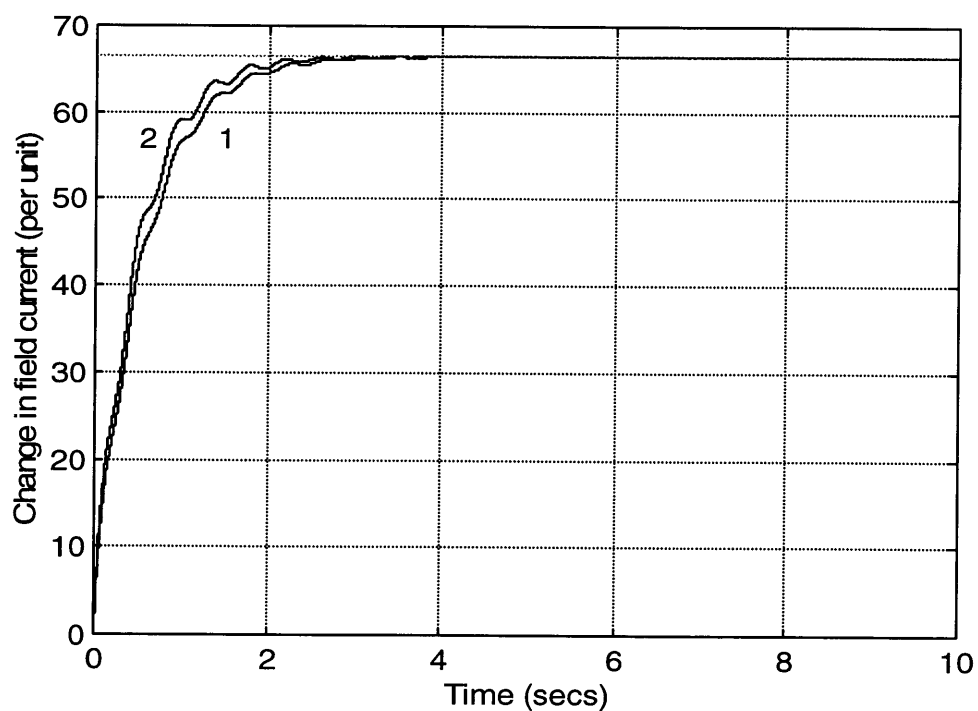


Figure D9. Variation in change in field current due to 5% step change in excitation voltage.

1- Linearised unsaturated current model. 2- Linearised saturated current model

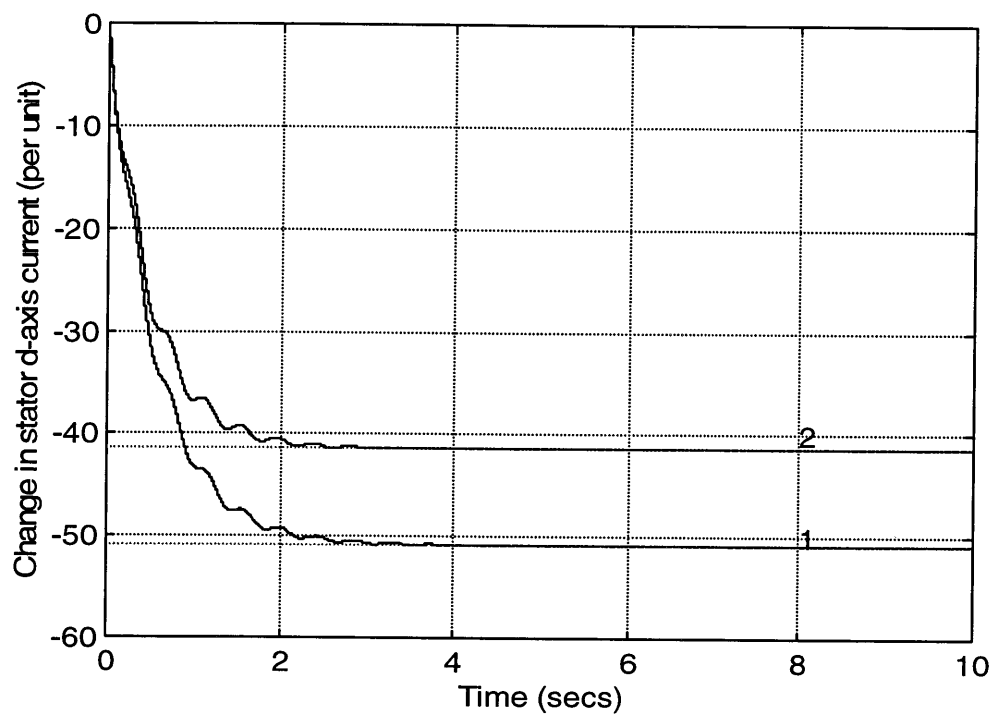


Figure D10. Variation in change in stator d-axis current due to 5% step change in excitation voltage.

1- Linearised unsaturated current model. 2- Linearised saturated current model

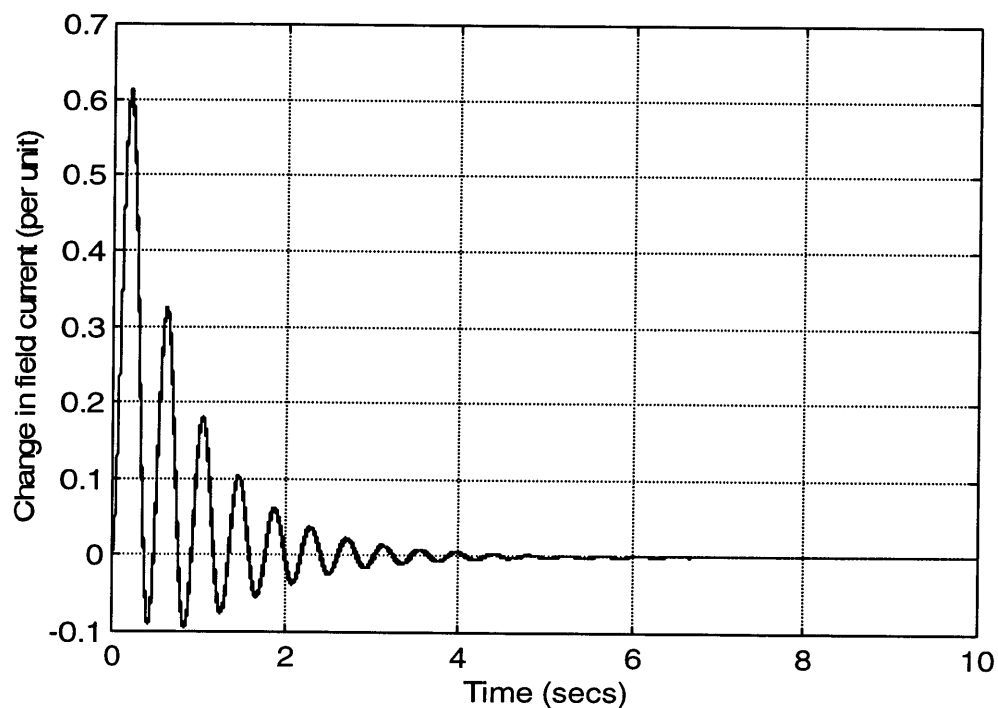


Figure D11. Variation in change in field current due to 5% step change in real power.

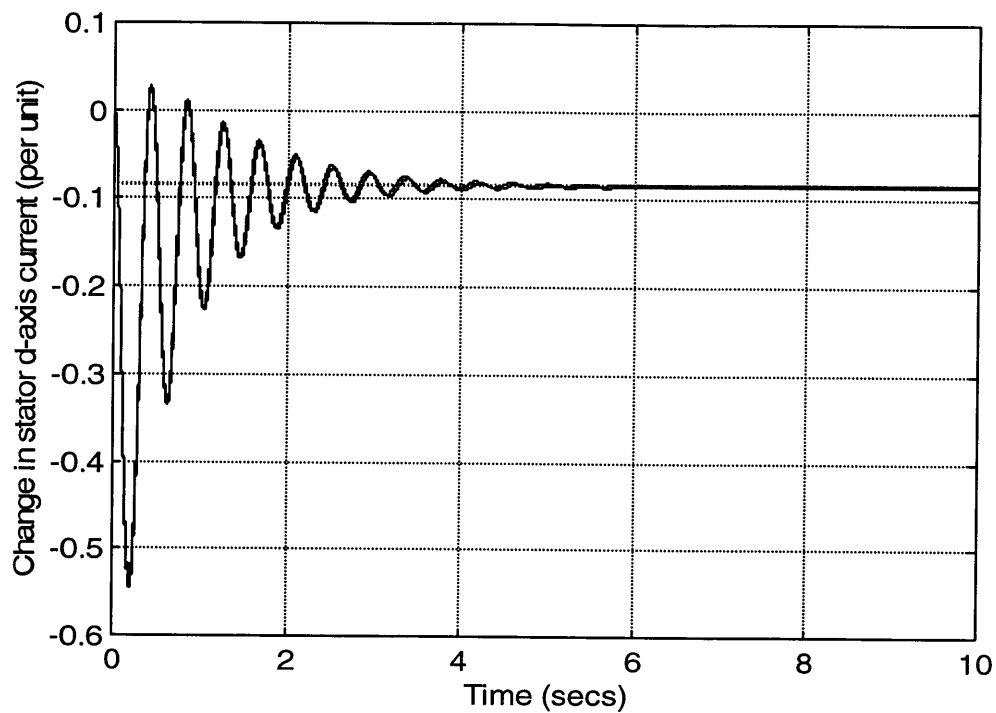


Figure D12. Variation in change in stator d-axis current due to 5% step change in real power.

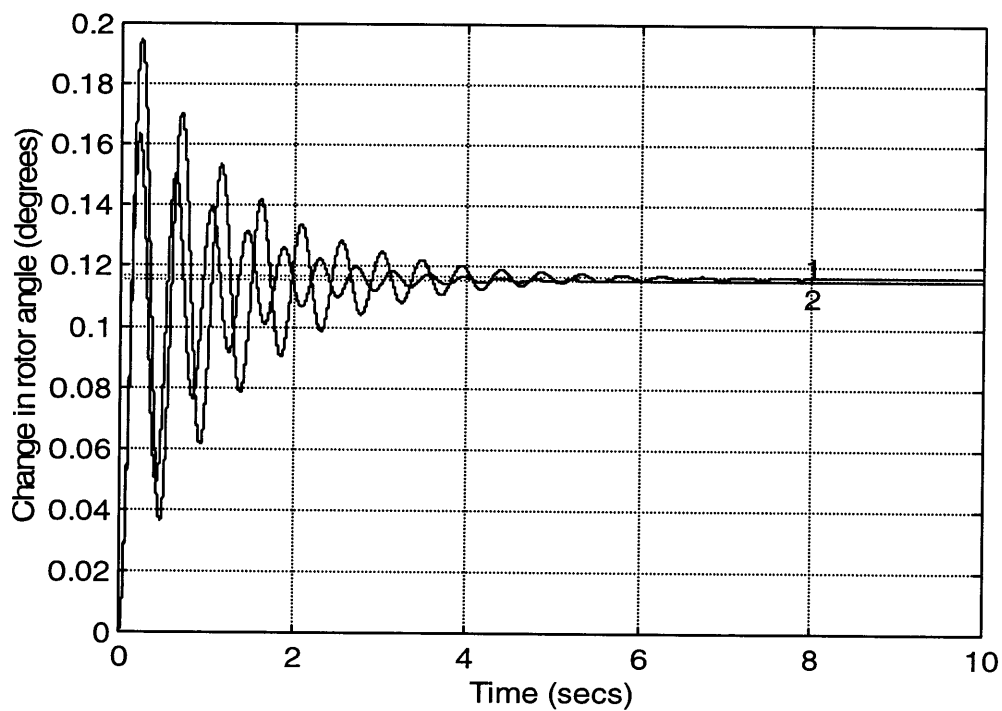


Figure D13. Variation in change in rotor angle due to 5% step change in real power.

- 1- Linearised unsaturated flux model.
- 2- Linearised saturated flux model.

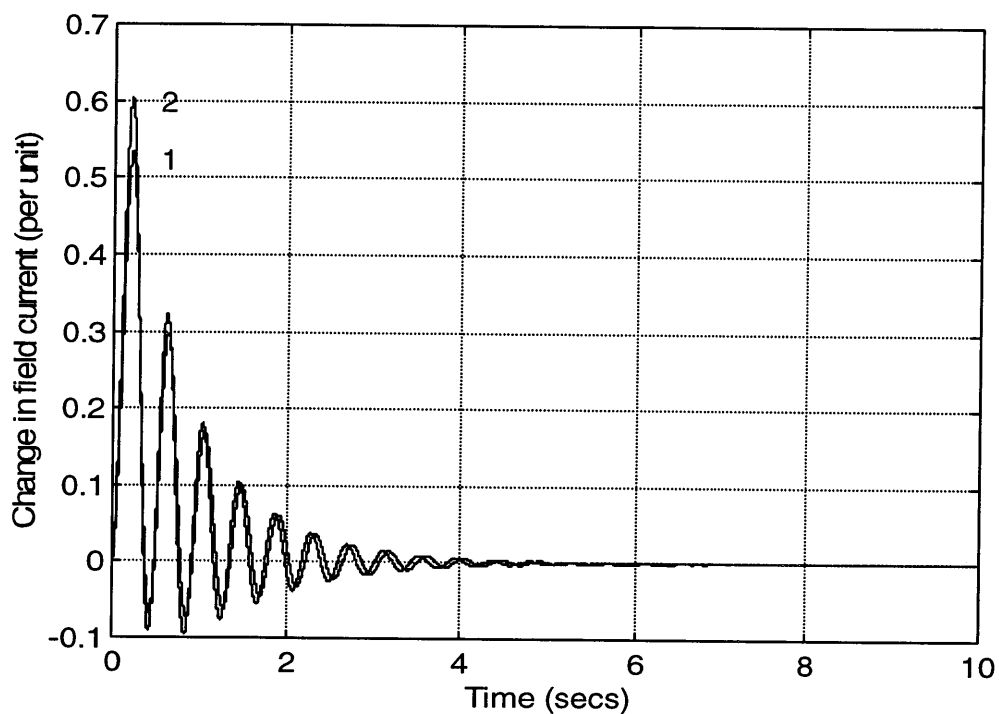


Figure D14. Variation in change in field current due to 5% step change in real power.
 1- Linearised unsaturated current model. 2- Linearised saturated current model.

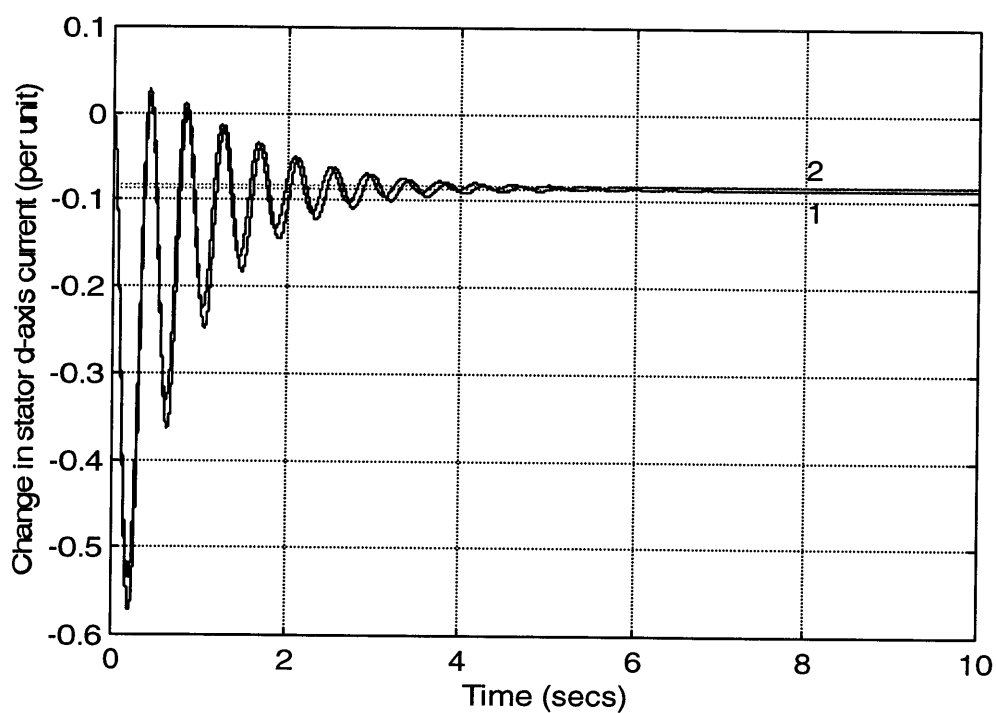


Figure D15. Variation in change in stator d-axis current due to 5% step change in real power.
 1- Linearised unsaturated current model. 2- Linearised saturated current model.

D.3 Monte Carlo Simulation

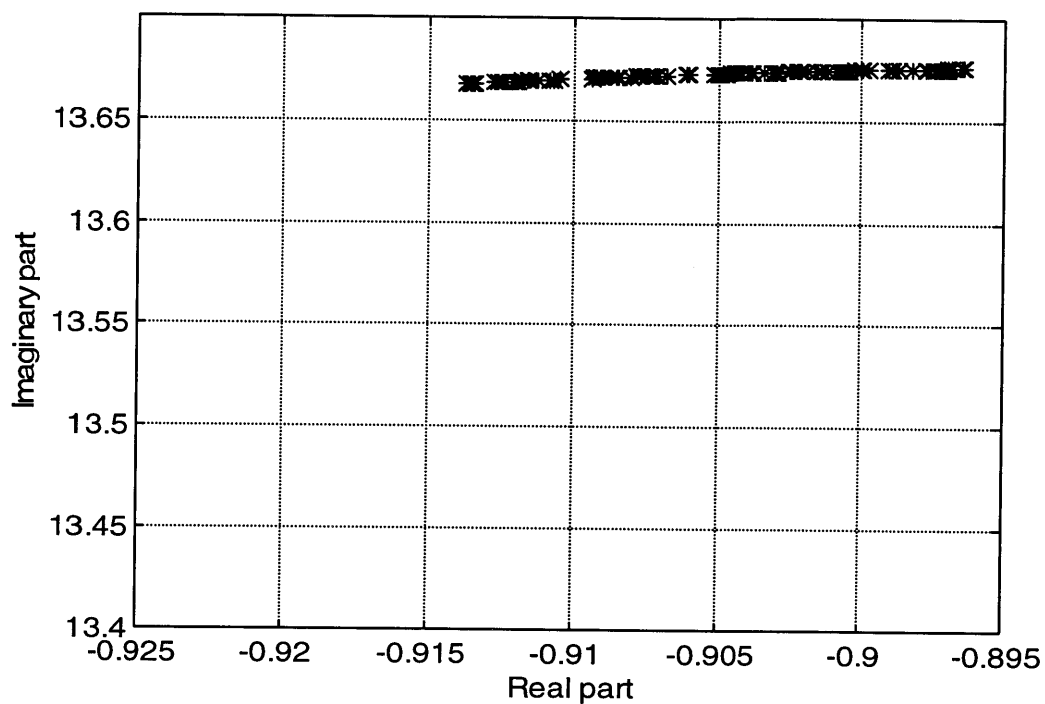


Figure D16. Root loci of critical eigenvalue corresponding to model no.4 due to variation in real power and saturation factor.

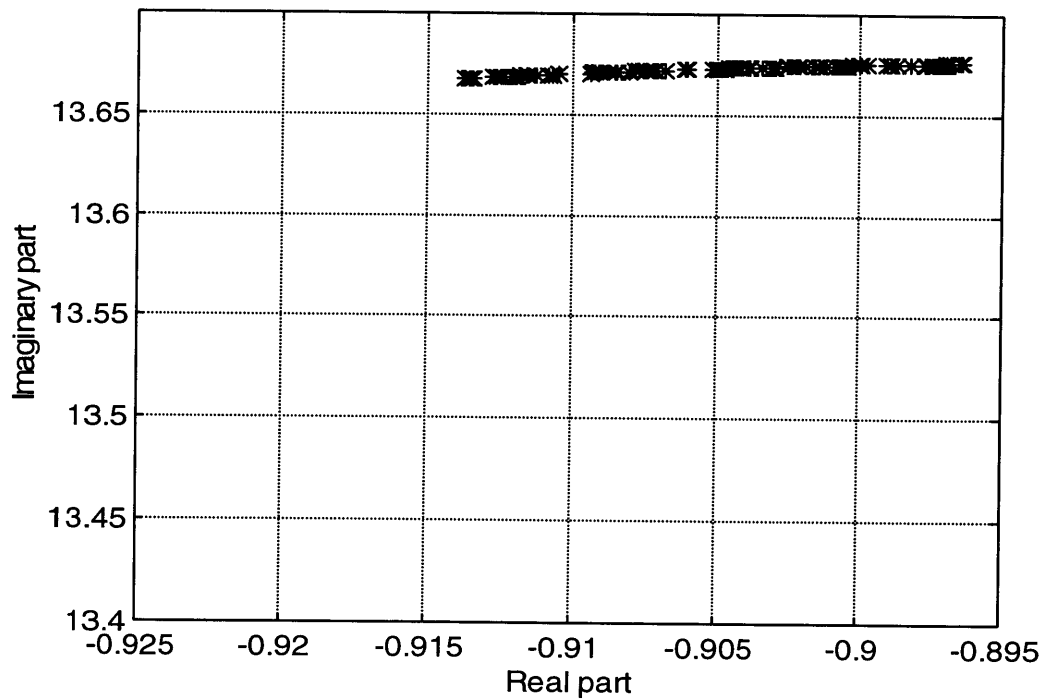


Figure D17. Root loci of critical eigenvalue corresponding to model no.5 due to variation in real power and saturation factor.

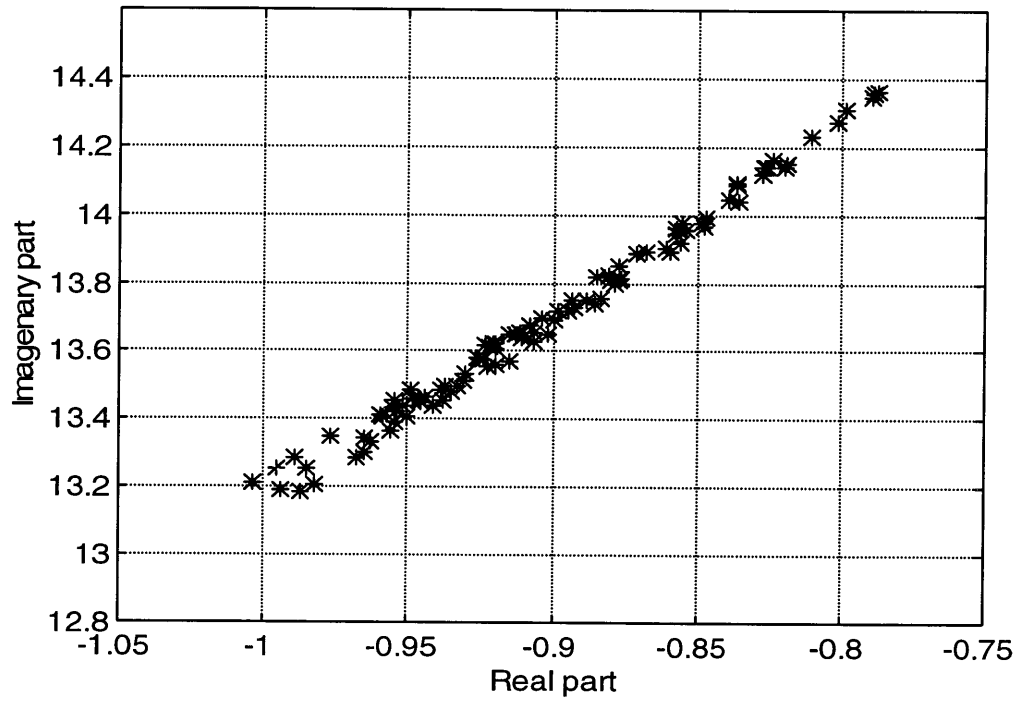
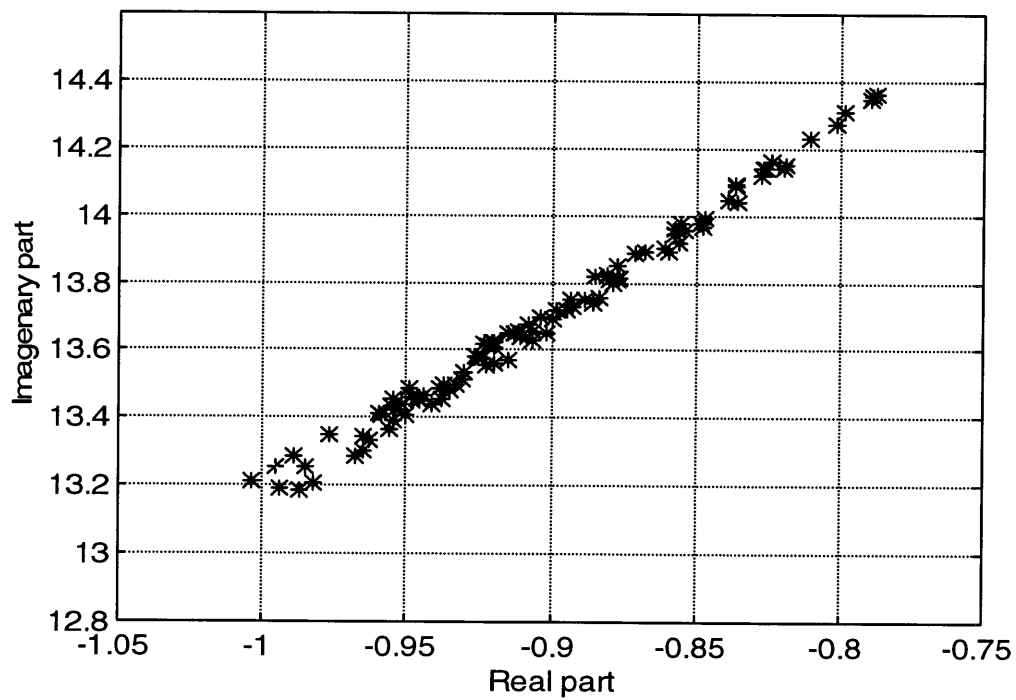
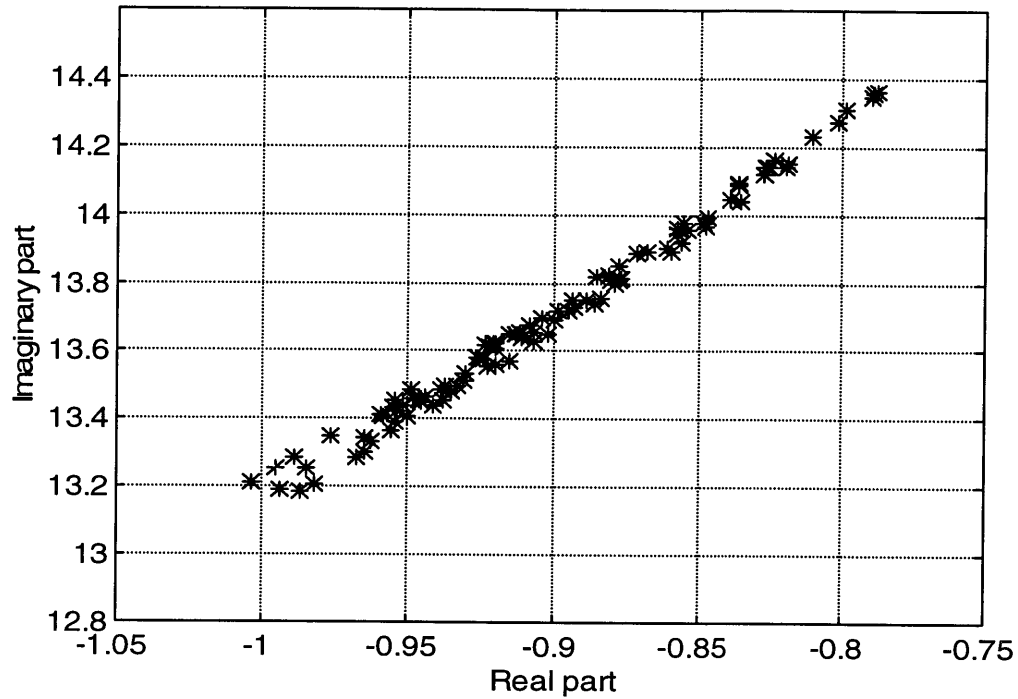


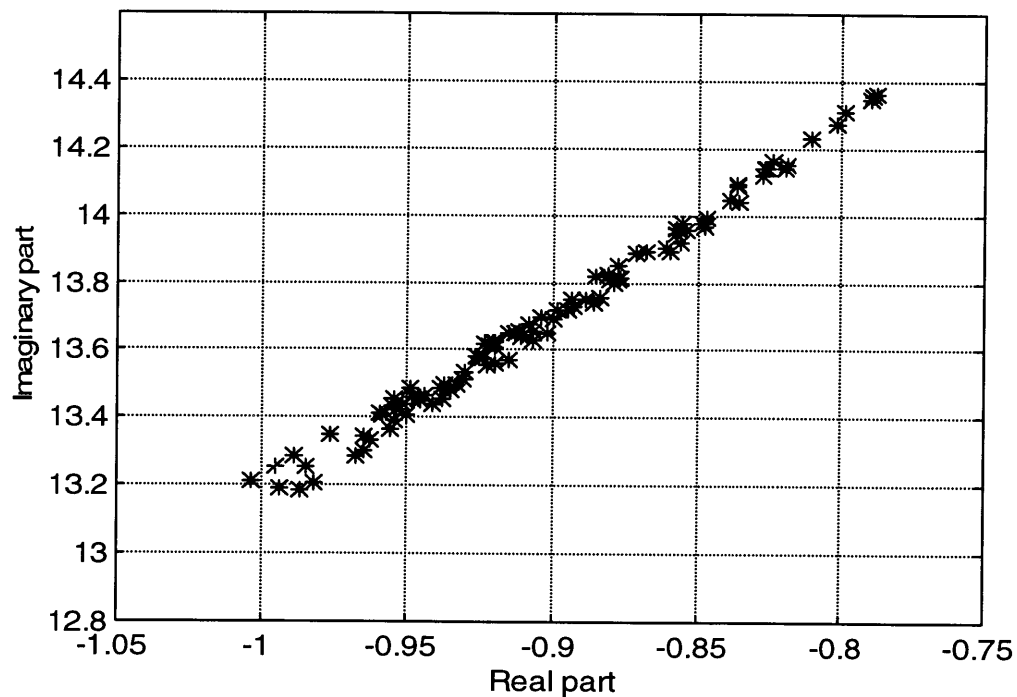
Figure D18. Root loci of critical eigenvalue corresponding to model no.4 due to variation in real power and saliency factor.



D19. Root loci of critical eigenvalue corresponding to model no.5 due to variation in real power and saliency factor.



D20. Root loci of critical eigenvalue corresponding to model no.4 due to variation in real power, saliency factor and saturation factor.



D21. Root loci of critical eigenvalue corresponding to model no.5 due to variation in real power, saliency factor and saturation factor.
