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Quantum information flow and Markovian behavior of some quantum systems

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Abstract

It is assumed that, a quantum system consists of three qubits interacting via different three interactions. Due to these interactions, three different quantum entangled states have been generated. The effect of the interactions strengths and the initial state settings of the qubits system on the information flow, and the quantum correlations between the three qubits has been investigated. The Markovian property of these quantum states is discussed, where it changes during the interaction time. For all the three interactions, the phenomena of the sudden change have been observed on the behavior of all the measured physical quantities. Moreover, the initial state settings of the system have a strong effect on the behavior of all the discussed physical quantities, where starting with a maximum entangled state, the maximization of all the physical quantities is achieved.

Publications

This thesis is based on previously published and submitted papers.

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2. F. Ebrahim and N. Metwally. "Dynamics of non-Markovianity, quantum correlations and information scrambling of three qubits systems interacting via Rashba interaction". Journal of Modern Optics, ID239097547, (submitted 2023).

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Chapter0

Introduction

0.1 Statement of the problem

The research work of this thesis investigates the possibility of exchanging information and quantum correlations between different types of qubits systems. It is assumed that, the system consists of two qubits A and B interacting together by Heisenberg-XX interaction, and a third qubit C that interact with the two system via different interactions. Consequently, some types of correlation may be generated between the qubits system. Due to these interactions, there is a possibility that the information which is encoded in one system is transferred into the other system. Also, there is a possibility that correlations between the qubits may be generated or affected by the interactions. Therefore, it is important to quantify these amounts of quantum correlations, where different measures will be used. Moreover, the amount of the local and non-local information which may be gained or lost will be quantified. In this context, we discuss the exchange of information by investigating the behavior of the non-Markovianity of the system.

0.2 Historical Background

The possibility of building a quantum network depends on the used method to connect the nodes, which represent the users of this network. There are different methods to generate entangled quantum networks, and among of these methods is using local operations such as CNOT and Hadamard gates (Metwally, 2014). Also, the quantum network may be generated by different interactions, such as the Dzyaloshinskii-Moriya (Qiang, Xiao-Ping, Qi-Jun, & Zhong-Zhou, 2009), Dipolar (Reis, 2013; Mohammed, El-Shahat, & Metwally, 2021), and Rashba interactions (Bychkov & Rashba, 1984; Ferreira, Rojas, & Rojas, 2023).

Due to these interactions, correlations between different nodes may be generated. Various measures could be used to quantify the amount of quantum correlations that may be generated between the different nodes; one may use the negativity (Ghne & Tth, 2009). The negativity $\mathcal{N}_{eg}$, represents suitable measure for low dimensions 2×2 or 2×3 . Moreover, Wootters (Hill & Wootters, 1997) suggested a measure called concurrence C , which coincide with the negativity for composite systems of dimensions 2×2 . Another meaningful measurement of entanglement is the entanglement of formation E_f , which is defined as a function of concurrence.

As mentioned above, the quantum systems may lose or gain information from other systems. However, this information may be re-obtained again, and consequently, one says that this system is non-Markovian. In other words, a system is called Markovian if the temporary properties of a quantum system are not linked to past events; otherwise, it is called non-Markovian (Jahromi & Haseli, 2020). Therefore, we investigate the rate of Markovianity and the amount of local and non-local information (Metwally, 2017; M.Y.Abd-Rabbou, 2019).

0.3 Literature Review

It is well known that , the state vector describes the density operator for a single qubit as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$, and the set $\{|0\rangle, |1\rangle\}$ represents the basis for the 2D systems. The density operator ρ_a may be written via Bloch vector , $\rho_a = \frac{1}{2}(1 + \vec{s} \cdot \vec{\sigma})$, where $\vec{s} = (s_x, s_y, s_z)$, $s_i = \text{tr}(\rho_a \sigma_i)$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Bloch vector and Pauli operators, respectively .

For a 2-qubit system, the density operator is defined as, $\rho_{ab} = \frac{1}{4}(I_{ab} + \vec{s} \cdot \vec{\sigma} + \vec{t} \cdot \vec{\tau} + \vec{\sigma} \cdot C \cdot \vec{\tau})$, where $\vec{s}$ and $\vec{t}$ are the Bloch vector for the first and second qubits, respectively, and $s_i = \text{tr}(\rho_{ab} \sigma_i)$, $t_i = \text{tr}(\rho_{ab} \tau_i)$. The operator $\tau = (\tau_x, \tau_y, \tau_z)$ is the Pauli operator of the second qubit, while C is the cross dyadic, which is defined by 3×3 matrix, where its elements are defined as $c_{ij} = \text{tr}(\rho_{ab} \sigma_i \tau_j)$ and the operator I_{ab} is the identity matrix of size 4×4 .

The quantum correlations, which may generate between the different nodes, are quantified by using the negativity (Horodecki, 1996), which is based on the eigenvalues of the partial transpose of a quantum system. However, for two-qubit system ρ_{ab} the negativity $\mathcal{N}_{eg}$ is defined as $\mathcal{N}_{eg} = 2 \sum_{i=1}^4 \max(0, -\mu_i)$, where μ_i are the negative eigenvalues of the partial transpose of the given quantum system, namely $\rho_{ab}^{T_{1(2)}}(t)$. The negativity ranged between 0 and 1, where $\mathcal{N}_{eg} = 0$, if the state is separable and $\mathcal{N}_{eg} = 1$, if the given quantum system is maximally entangled. As another measure, we consider the concurrence (Hill & Wootters, 1997) , where for a mixed two qubit system ρ_{ab} , one defines the spin flipped state $\tilde{\rho}_{ab}(t) = (\sigma_y \otimes \tau_y) \rho_{ab}^*(t) (\sigma_y \otimes \tau_y)$. The concurrence is defined as, $C(\rho_{mixed}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where λ_i are the square roots in decreasing order of the eigenvalues of the non-hermitian matrix $\rho_{ab}(t) \tilde{\rho}_{ab}(t)$.

Now, we can investigate whether the quantum system behaves as Markovian

or non-Markovian. These phenomena can be quantified by

$$\mathcal{N} = \max_{\sigma > 0} \int dt \sigma(t, \rho_{1,2})(0), \quad \sigma(t, \rho_{1,2}(t)) = \frac{d}{dt} D(\rho_1(t), \rho_2(t)),$$

and D is the distance between the two states $\rho_1(t)$ and $\rho_2(t)$.

One of the most important interactions between the quantum systems is the Dzyaloshinskii Moriya (DM) interaction (Qiang et al., 2009), where it is defined as : $H_{DM} = \vec{D} \cdot (\sigma_A \times \tau_C) = \vec{D} \cdot (\sigma_x^A \tau_y^C - \sigma_y^A \tau_x^C)$. This is a spin-orbit interaction, which can be considered as controller of entanglement. The DM interaction can lead to higher mutual information of the two-qubit Heisenberg chain. Moreover, we consider another important interaction called Dipolar interaction (Reis, 2013; Castro, Duarte, Pires, Soares-Pinto, & Reis, 2016). This interaction arises from a magnetic field created by a magnetic moment of a spin $\mu = \mu_b g \vec{S}$, where Bohr's magneton and g the gyromagnetic factor, on the site of another spin and is represented by the Hamilton $H_{Dp} = \frac{\delta}{4} (2 \sigma_z^A \tau_z^C - \sigma_x^A \tau_x^C - \sigma_y^A \tau_y^C) + \frac{\varepsilon}{4} (\sigma_x^A \tau_x^C - \sigma_y^A \tau_y^C)$, where $\sigma_i, \tau_j, i, j = x, y, z$ represent the Pauli operators of the qubits A , and C , respectively, ε is the strength of interaction, δ is the direction of interaction, and κ is a coherence parameter. If $\delta > 0$ then the qubits are moving in $x - y$ plane, and if $\delta < 0$, then the qubits are spin around z -axis.

The last interaction which we will be considered is the Rashba interaction (Bychkov & Rashba, 1984). It is a spin-orbit interaction. It is defined as $H_{Rash} = \frac{\varepsilon}{2} (I \otimes \tau_z^C) + \beta (\sigma_x^A \otimes I) - \zeta (\sigma_y^A \otimes \tau_x^C)$, where ε is the Zeeman splitting, it generated by an external constant magnetic field along the z -axis, and β, ζ are the strength of the tunneling coupling between the two quantum nodes, and the spin-flip tunnel coupling, respectively (Ferreira et al., 2023).

Finally, the amount of information that may be shared between the quantum systems and their marginals is quantified by using the local information $I_a(I_b)$ for Alice's (Bob's). Also, we quantify the non-local information between Alice's and Bob's I_{nloc} . The three functions of information are defined

by $I_i = \log_2 D - S(\rho_i)$, where $i = nloc, a, b, a, b$ are the non-local Alice's and Bob's information, respectively, while S is the von-Numman entropy of the total, Alice's qubit, Bob's qubit, and D is the dimension of the state ρ_i .

In chapter one, we shall review all these, measures in detail.

0.4 Theoretical Framework

In this contribution, we assume that we have different types of quantum systems, i.e., they are prepared in different states. These systems are entangled by Dzyaloshinskii Moriya (DM) (Qiang et al., 2009), Dipolar (Reis, 2013; Castro et al., 2016), or Rashba (Bychkov & Rashba, 1984; Ferreira et al., 2023) interactions. Due to these interaction the initial quantum system, $\rho_s(0)$ evolves to be $\rho_s(t)$. From the final state, one can investigate the behavior of all the previous phenomena, namely the behavior of quantum correlations, by using the negativity, concurrence, and the entanglement of formation, the Markovian and non-Markovian behavior of the system and the amount of the information that flows from or to the quantum systems.

0.5 Methods and procedure

Let us assume that, the quantum system is governed by one of the interactions, either by the Hamiltonian H_{DM} for Dzyaloshinskii Moriya interaction, H_{Dp} Dipolar interaction, or H_{Rash} for Rashba interaction. If the initial state of the quantum system Q_s is given by $\rho_{ij}(0)$, $i = 1.....N, j = 1.....N$, then the time evolution of the system is given by,

$$\rho_{ij}(t) = e^{-iH_{Di}t} \rho_{ij}(0) e^{iH_{Di}t} , \quad (1)$$

- To quantify the amount of entanglement, we evaluate the eigenvalues of the non-hermitian operator $\rho_{12}\tilde{\rho}_{12}$, where $\tilde{\rho}_{12}(t) = (\sigma_y^1 \otimes \sigma_y^2) \rho_{12}^*(t)$

$(\sigma_y^1 \otimes \sigma_y^2)$. By using these eigenvalues, one can evaluate the concurrence, which estimates the amount of entanglement between the quantum dots subsystems, $C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$. Moreover, the negativity $\mathcal{N}_{eg}$ can be used to quantify the degree of entanglement. For this measure, one needs to evaluate the eigenvalues of the partial transpose of $\rho_{ij}(t)$ and apply the definition of the negativity, $\mathcal{N}_{eg} = 2 \sum_{i=1}^4 \max(0, -\mu_i)$, where μ_i are the negative eigenvalues of $\rho_{ij}^{T_{1,2}}(t)$. For the entanglement of formation, it is defined as a function of the concurrence, $E_F(\rho) = E_F(C(\rho)) = H(\frac{1+\sqrt{1-C(\rho)^2}}{2})$ where $H = -x \log_2 x - (1-x) \log_2 (1-x)$ that represents the Shannon Entropy function.

- We discuss the effect of the initial state settings and the interaction parameters of the field on the degree of Markovianity. Moreover, these parameters may be used to maximize or minimize the system's memory.
- Due to the interaction, some information is leaked from the total quantum system into the environment and vice versa. This information may leak from the total system or its subsystems, i.e., non-local or local information, respectively. Also we shall use the behavior of the out of time order correlator function.

0.6 Proposed Chapter Outlines

This thesis consist of five chapters, where

- in chapter one, we introduce some basics of quantum mechanics, the representation of single and two qubit systems, separable and entangled states, criteria of separability, and some measures of entangle-

ment. Also, we will review the mathematical forms of Markovianity and the local/non-local information.

- in chapter two, the time evolution of the quantum system governed by Dzyaloshinskii Moriya will be presented. The analytical calculations of the system will be included, where we discuss the effect of the initial state settings and the interaction parameters on all the previous phenomena.
- in chapter three, we assume that the quantum system has evolved under the Dipole interaction. The effect of the interaction's parameter on the behavior of the suggested physical quantities will be discussed.
- in chapter four, it is assumed that, the quantum system evolves by the Rashba interaction, then we will discuss the effect of the interaction's parameter on some physical quantities.
- In chapter five, we will discuss and conclude the obtained results.

Chapter1

Basics of Quantum Mechanics

1.1 Qubit system

In the physical world, every object has a definite and exclusive location and cannot exist in two different places simultaneously. However, for tiny objects like electrons, the laws of quantum mechanics allow them to simultaneously be presented in two distinct locations, a phenomenon known as superposition. Classical bits can only be in the state 0 or 1, while quantum qubits can be existed in both states at the same time. Qubits are the basic unit of quantum information in a two-level quantum system. Quantum systems may consist of more than one qubit. A system of two-qubit state's can be written as $|\psi\rangle = \alpha_1 |00\rangle + \alpha_2 |01\rangle + \alpha_3 |10\rangle + \alpha_4 |11\rangle$, where $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 1$ (Yanofsky & Mannucci, 2008).

1.2 Density operator

In quantum mechanics, a density operator ρ is used to represent quantum systems, where it contains all the necessary information. Bloch vectors, which are vectors in the Bloch sphere, are used to describe the density operator of a quantum system. However, the form of the density operator differs for single and two-qubit systems (Gamel, 2016).

For single qubit system, the density operator is given by,

$$\rho = \frac{1}{2}(I + \vec{s} \cdot \vec{\sigma}) \quad , \quad s_i \in \mathbb{R}. \quad (1.1)$$

where I is the identity matrix of size 2×2 , $\vec{s} = (s_x, s_y, s_z)$ is the Bloch vector where $|s_x|^2 + |s_y|^2 + |s_z|^2 \leq 1$, as $|\vec{s}|^2 = 1$ if the state is pure, and if the state is mixed $|\vec{s}|^2 < 1$. Moreover, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli operators (Krammer, 2005; Furusawa & Van Loock, 2011). defined as $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$, $\sigma_y = i|1\rangle\langle 0| - i|0\rangle\langle 1|$ and $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$ (Sych & Leuchs, 2009).

For a two-qubit system, the density operator is given by

$$\rho_{ab} = \frac{1}{4} \left(I + \vec{s} \cdot \vec{\sigma} + \vec{t} \cdot \vec{\tau} + \vec{\sigma} \cdot C \cdot \vec{\tau} \right) \quad , \quad s_i, t_i, c_{ij} \in \mathbb{R}. \quad (1.2)$$

where I is the identity matrix of size 4×4 , $\vec{s} = (s_x, s_y, s_z)$ and $\vec{t} = (t_x, t_y, t_z)$ are the Bloch vector for the first and second qubits, respectively, and $s_i = \text{tr}(\rho_{ab} \sigma_i)$, $t_i = \text{tr}(\rho_{ab} \tau_i)$, also $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ and $\tau = (\tau_x, \tau_y, \tau_z)$ are the Pauli operator of the first and second qubits respectively, while C is the cross dyadic that is defined by 3×3 matrix, where its elements are defined as $c_{ij} = \text{tr}(\rho_{ab} \sigma_i \tau_j)$ (Krammer, 2005).

1.3 Separable and Entangled states

A state ρ of a quantum system is a separable or classically-correlated state, which means it is not entangled. The density operator for separable state can be expressed as a combination of product terms, where each term represents one qubit. $\rho_{ab} = \sum \rho_i |\phi_a\rangle\langle\phi_a| \otimes |\phi_b\rangle\langle\phi_b|$, where $|\phi_a\rangle$ and $|\phi_b\rangle$ are states of subsystems A and B in the space vector H_A and H_B , with $\rho_i > 0$, $\sum \rho_i = 1$. Separable states can be generated by classical communication and local operations. Otherwise, if the system can't be expressed as a combination of product terms it is called a quantum-correlated or entangled state, namely

if $\rho_{ab} \neq \rho_a \otimes \rho_b$, then the state ρ_{ab} is called an entangled state (Doherty, Parrilo, & Spedalieri, 2002; Furusawa & Van Loock, 2011).

1.4 Measures of Entanglement

Entanglement is the ability of qubits to remain interacting regardless of the distance between them. The discovery of entanglement leads to new branches of technology, like quantum cryptography (Gisin, Ribordy, Tittel, & Zbinden, 2002), quantum teleportation (Bennett et al., 1993a), and quantum dense coding (Schumacher, 1995). It is found that, entanglement is an essential resource for information processing as it's considered as the only resource for it. Due to its importance, many quantifiers are used to measure the degree of entanglement between qubits, such as negativity, concurrence and entanglement of formation. All quantifiers are bounded between 0 and 1; if it's 0, the state is separable, and if it is 1, it is a maximum entangled state (Dotsenko & Korobka, 2018; Muthuganesan & Chandrasekar, 2021; K.Berrada, 2009). Bell states (Sych & Leuchs, 2009) represent the simplest example for maximally entangled two-qubit states . They are given by,

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \quad (1.3)$$

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle). \quad (1.4)$$

1.4.1 Concurrence

Concurrence is a quantifier of entanglement of a mixed quantum state. For a mixed quantum state, it has been shown that, the concurrence can be used as a measure of entanglement (Wootters, 1998; K.Berrada, 2010). It is defined

as,

$$C(\rho_{mixed}) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (1.5)$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square roots in decreasing order of the eigenvalues of the non-Hermitian matrix $\rho_{ab}(t)\tilde{\rho}_{ab}(t)$, where $\tilde{\rho}_{ab}(t) = (\sigma_y \otimes \tau_y)\rho_{ab}^*(t)(\sigma_y \otimes \tau_y)$, $\rho_{ab}^*(t)$ represents the complex conjugate of the system's density operator, and σ_y, τ_y are the Pauli operators in the y-direction, as introduced in Sec.(1.2).

1.4.2 Entanglement of formation

Entanglement of formation E_f is another quantifier to measure the entanglement. Its defined as a function of concurrence,

$$E_F(\rho) = E_F(C(\rho)) = H\left(\frac{1 + \sqrt{1 - C(\rho)^2}}{2}\right), \quad (1.6)$$

where H represent the Shannon entropy, $H(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ (Hill & Wootters, 1997; Wootters, 1998; K.Berrada, 2010; Jan et al., 2019).

1.4.3 Negativity

Negativity is also used to measure the correlation of a quantum system. It is based on the eigenvalues of the partial transpose of the system. For two qubit system ρ_{ab} the negativity is given by :

$$\mathcal{N}_{eg} = 2 \sum_{i=1}^4 \max(0, -\mu_i), \quad (1.7)$$

where μ_i are the negative eigenvalue of the partial transpose system $\rho_{ab}^{T_{1(2)}}$ (Horodecki, 1996; Metwally, 2021; Berrada, El Baz, Eleuch, & Hassouni, 2010).

1.5 Markovianity

A system may interact with its surrounding environment, and consequently the phenomenon of the decoherence arises, where a system loses its information and memory. In general it is well known that, it is difficult to isolate the system from the environment, however, some quantum systems can reverse this process and restore the lost information by a process known as non-Markovian (Jan et al., 2019; Khan et al., 2022; Hesabi, Afshar, & Paris, 2019; Alonso, Halpern, & Dressel, 2019). The non-Markovianity of a system can be measured by,

$$\mathcal{N} = \max_{\sigma > 0} \int dt \sigma(t, \rho_{1,2})(0), \quad (1.8)$$

where $\sigma(t, \rho_{1,2}(t)) = \frac{d}{dt} D(\rho_1(t), \rho_2(t))$, and D is the distance between the two states $\rho_1(t)$ and $\rho_2(t)$, $D(\rho_i(t), \rho_j(t)) = \frac{1}{2} \text{tr} |\rho_i(t) - \rho_j(t)|$, $i, j = A, B, C$.

1.6 Information scrambling

The interaction between a system and the environment causes the information to be scrambled. This scrambling can be quantified by what is called the Out of time order correlator OTOC (Mohammed, El-shahat, & Metwally, 2023; Alonso et al., 2019; Halpern, Swingle, & Dressel, 2018).

For any quantum state ρ , scrambling information may be quantified by using this formula,

$$\begin{aligned} F(t) &= \text{tr}(\rho O^\dagger(t) Q^\dagger O(t) Q), \\ OTOC(t) &= \frac{1}{2} \left(1 - \text{Re}[F(t)] \right), \end{aligned} \quad (1.9)$$

where $O(t) = U^\dagger(t) O U(t)$, $U(t)$ is the unitary operator.

1.7 Local and nonlocal information

Local information can be used to quantify the information of qubit's system by using,

$$\mathbf{I}_i = \log_2 D_i - S(\rho_i), \quad (1.10)$$

where $i = AB, AC, BC$, and D is the dimension of the state ρ_i , while $S = -\sum_j \lambda_j \log \lambda_j$ is the von-Neumann entropy and λ_j the eigenvalues of the state (Witten, 2020; M.Y.Abd-Rabbou, 2019).

1.8 Generating entanglement

Entanglement can be generated by quantum interactions between different quantum systems (Rao & Kumar, 2012; Mohamed, Hessian, & Eleuch, 2020). In this context, we will discuss three interactions Dzyaloshinskii-Moriya, Dipolar, and Rashba interaction.

1.8.1 Dzyaloshinskii-Moriya interaction

Dzyaloshinskii-Moriya (DM) interaction is an interaction that is based on the spin-orbit scattering of electrons in a crystal field with inversion asymmetry (Luo, Zhou, Won, & Wu, 2014). Initially, it was discovered to explain the weak ferromagnetism observed in antiferromagnetic crystals (Qiang et al., 2009). The DM interaction is significant in determining magnetic properties in insulators and metals and profoundly impacts these properties. Additionally, this interaction is linked to the magnetoelectric effect found in multiferroics (Hu, 2012). This interaction is now extensively discussed due to its ability to control the entanglement between qubits (D.-C. Li & Cao, 2009; Qiang et al., 2009).

1.8.2 Dipolar Interaction

Dipolar interaction appears when two qubits couple with their magnetic dipole moments, creating a dipole-dipole interaction. The strength of this interaction is dependent on the distance between the qubits. The interaction effect cannot be ignored if the distance between the dipoles is small. In quantum information processing, Dipolar interaction is essential as it generates quantum correlations and minimizes decoherence in quantum systems, as previous theoretical studies have shown (Reis, 2013; Mohamed et al., 2020; Jan et al., 2019).

1.8.3 Rashba interaction

The Rashba interaction is an interaction that arises from the coupling between the spin and orbit. It explains the splitting of the conducting band when exposed to an external field. It is often used to model the electronic structure of semiconductors (Molenkamp, Schmidt, & Bauer, 2001). The Rashba interaction can be controlled by an external electric field, which makes it particularly significant (R. Moriya et al., 2014). Due to the ability of the Rashba spin-orbit interaction to control quantum correlations, scientists have been interested in studying it both theoretically and experimentally (Ferreira et al., 2023).

Chapter2

Exchanging quantum correlations and non-local information between three qubit-system interacting via Dzyaloshinskii-Moriya interaction

2.1 Introduction

Entanglement is the core idea of modern theoretical physics (Takayanagi, Ugajin, & Umemoto, 2018), where it plays an important role in different applications of quantum information processing (QIP), like quantum cryptography (Ekert, 1991), quantum teleportation (Bennett et al., 1993b), quantum dense coding (Bennett & Wiesner, 1992), and etc. Therefore, there is a need to be generated and quantified, as well as keep it survival for a long time. Indeed, it has been generated between different objects; atoms

(Plenio, Huelga, Beige, & Knight, 1999) charged qubits (J. Zhang, Liu, Li, Tarn, & Nori, 2009), quantum dots (Sridharan & Waks, 2008), etc. Dzyaloshinskii-Moriya (DM) (Qiang et al., 2009), represents one of the most common interaction that has been used widely to generate quantum correlations between different systems. For example, the possibility of generating a thermal entanglement between two qubits Heisenberg chain in the presence of the Dzyaloshinski-Moriya is discussed in (G.-F. Zhang, 2007). The effect of DM interaction on the dynamics of a two-qutrit system is investigated by Jafarpour and Ashrafpour (Jafarpour & Ashrafpour, 2013). Sharma and Pandey (Sharma & Pandey, 2016) studied the entanglement dynamics between qubit -qutrit systems via DM interaction. The influence of the anisotropic antisymmetric exchange interaction, on the entanglement of two qubits is examined in (Gurkan & Pashaev, 2010). The DM interaction is used to generated entangled quantum network (Metwally, 2011). The orthogonality speed of two-qubit state interacts locally with spin chain in the presence of Dzyaloshinsky–Moriya interaction is investigated by Kahla, et. al (Abo-Kahla, Abd-Rabbou, & Metwally, 2021). Therefore, we are motivated to discuss the phenomena of exchanging the quantum correlations and the non-local information between three qubits, where two of them are either classically or non-classically correlated. It is assumed that, by using DM interaction one of the correlated qubits interacts locally with a control qubit that is initially prepared in a vacuum state. The effect of the initial state settings of the correlated qubits and the strength of DM interaction on the exchange process is investigated.

The outline of the chapter is arranged as: In Sec.(2.2) the qubits system and its evolution is described in. The exchange of quantum correlations between the marginal states of the three qubits is discussed in Sec.(2.3). In Sec.(2.4), the behavior of the exchanged non-local information is investigated. Finally, we summarized our results in Sec.(2.5).

2.2 The suggested model

The system composed of two qubits A , and B , are prepared in a partial entangled state interacts by using Heisenberg XX Spin model (Qiang et al., 2009). It is assumed that, one of the subsystems, say A , interacts with third qubit C , as a controller via Dzyaloshinskii-Moriya (DM). The total Hamiltonian which describe this system is given by,

$$H_{sys} = \frac{1}{2}\omega \left(\sigma_A^x \sigma_B^x + \sigma_A^y \sigma_B^y \right) + \vec{D} \cdot \left(\sigma_A \times \sigma_C \right), \quad (2.1)$$

where $\vec{D} = (D_x, D_y, D_z)$ is the DM interaction, and $D_i = x, y, z$ are the interaction strength on the three directions, ω is the spin strength parameter for Heisenberg interaction between qubits A, B (Ebrahim & Metwally, 2022), and $\sigma_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_i^y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $i = A, B$ are the Pauli operators for both qubits. In this context, it is important to mention that the Hamiltonian (2.1) is of 8×8 dimensions, where its eigenvalues and eigenvectors are given by,

$$\lambda_i = 0, i = 1 \dots 4, \lambda_5 = -\sqrt{4D_z^2 + \omega^2}, \quad \lambda_6 = \lambda_5, \quad \lambda_7 = \lambda_8 = -\lambda_5, \quad (2.2)$$

while the corresponding eigenvectors $|\xi_i\rangle$, $i = 1 \dots 8$ are given by,

$$\begin{aligned} |\xi_1\rangle &= |111\rangle, \quad |\xi_2\rangle = \frac{1}{\sqrt{1 + |\frac{1}{\chi_1}|^2}} \left(\frac{-2i}{\chi_1} |101\rangle + |110\rangle \right), \\ |\xi_3\rangle &= \frac{1}{\sqrt{1 + |2\chi_1|^2}} \left(\frac{-i\chi_1}{2} |001\rangle + |010\rangle \right), \quad |\xi_4\rangle = |000\rangle, \\ |\xi_5\rangle &= \frac{1}{\sqrt{1 + \frac{1}{4}|\chi_1|^2 + \frac{1}{4}|\frac{1}{\chi_2}|^2}} \left(-\frac{i\chi_2}{2} |011\rangle + \frac{i\chi_1}{2} |101\rangle + |110\rangle \right), \end{aligned}$$

$$\begin{aligned}
|\xi_6\rangle &= \frac{1}{\sqrt{1 + |\frac{2}{\chi_2}|^2 + |\chi_3|^2}} \left(-2i \chi_2 |001\rangle - \chi_3 |010\rangle + |100\rangle \right), \\
|\xi_7\rangle &= \frac{1}{\sqrt{1 + |\chi_1|^2 + |\frac{1}{2\chi_2}|^2}} \left(\frac{i}{2\chi_2} |011\rangle + \frac{i\chi_1}{2} |101\rangle + |110\rangle \right), \\
|\xi_8\rangle &= \frac{1}{\sqrt{1 + |\frac{2}{\chi_2}|^2 + |\chi_3|^2}} \left(2i \chi_2 |001\rangle + \chi_3 |010\rangle + |100\rangle \right), \quad (2.3)
\end{aligned}$$

where ,

$$\chi_1 = \frac{\omega}{D_z}, \quad \chi_2 = \frac{D_z}{\sqrt{4D_z^2 + \omega^2}}, \quad \chi_3 = \frac{\omega}{\sqrt{4D_z^2 + \omega^2}}.$$

We assume that $\vec{D}$ to be polarized on the z direction, namely $\vec{D} = (0, 0, D_z)$. Let us assume that, the initial state of the whole system at time $t=0$ is given by,

$$\rho_s(0) = \rho_{AB}(0) \otimes \rho_C(0), \quad (2.4)$$

where

$$\rho_{AB}(0) = \kappa |\varphi^{ab}\rangle \langle \varphi^{ab}| + \frac{1}{4}(1 - \kappa) I_{4 \times 4}, \quad \rho_C(0) = |\varphi^c\rangle \langle \varphi^c|, \quad (2.5)$$

with $|\varphi^{ab}\rangle = \cos(\alpha) |ge\rangle + \sin(\alpha) |eg\rangle$ and $|\varphi^c\rangle = \cos \gamma |g\rangle + \sin \gamma |e\rangle$, $I_{4 \times 4}$ is the identity operator and κ is the coherence parameter. It is clear that, at $\alpha = \frac{\pi}{4}$, the state $|\varphi^{ab}\rangle$ reduces to the maximum entangled Bell state $|\psi^+\rangle$. At $t > 0$, the total density operator of the whole system is given by,

$$\rho_s(t) = \mathcal{U}(t) \rho_s(0) \mathcal{U}^\dagger(t), \quad \mathcal{U}(t) = \exp[-iH_{sys}t], \quad (2.6)$$

where the unitary operator $\mathcal{U}(t)$ takes the form,

$$\mathcal{U}(t) = \sum_{i=1}^2 \alpha_i \cdot \beta_i, \quad (2.7)$$

where,

$$\begin{aligned}
\alpha_1 &= \cos\left(\frac{\omega}{2}t\right) I_{8 \times 8} - i \sin\left(\frac{\omega}{2}t\right) \sigma_A^x \otimes \sigma_B^x \otimes I_{2 \times 2}, \\
\alpha_2 &= \cos(D_z t) I_{8 \times 8} - i \sin(D_z t) \sigma_A^x \otimes I_{2 \times 2} \otimes \sigma_C^y, \\
\beta_1 &= \cos\left(\frac{\omega}{2}t\right) I_{8 \times 8} - i \sin\left(\frac{\omega}{2}t\right) \sigma_A^y \otimes \sigma_B^y \otimes I_{2 \times 2}, \\
\beta_2 &= \cos(D t) I_{8 \times 8} + i \sin(D_z t) \sigma_A^y \otimes I_{2 \times 2} \otimes \sigma_C^x. \quad (2.8)
\end{aligned}$$

Then by using the unitary operator (2.7) and the initial state $\rho_s(0)$, one gets the final state $\rho_s(t)$ at any $t > 0$. Due to this interaction, there will be new entangled states are generated; $\rho_{AC}(t)$ is generated between the qubits A , C , and $\rho_{BC}(t)$ between the qubits B and C . Moreover, the amount of entanglement of the initial state $\rho_{AB}(0)$ will be affected by this interaction. In this context, it is important to quantify the amount of entanglement between the three partitions AB, AC , and BC , which are defined by the final states $\rho_{AB}(t), \rho_{AC}(t)$ and $\rho_{BC}(t)$, respectively. In the computational basis the three final density operators are given by a matrix of size 4×4 . The density operators of the three partitions are defined by the non-zero elements ρ^{ij} , $ij = ab, ac, bc$. Mathematically any $\rho_{ij}(t), ij = AB, AC, BC$ may be written as:

$$\rho_{ij}(t) = \begin{pmatrix} \rho_{00}^{ij} & 0 & 0 & 0 \\ 0 & \rho_{11}^{ij} & \rho_{12}^{ij} & 0 \\ 0 & \rho_{21}^{ij} & \rho_{22}^{ij} & 0 \\ 0 & 0 & 0 & \rho_{33}^{ij} \end{pmatrix}. \quad (2.9)$$

The density operator of the state $\rho_{AB}(t)$ is defined by the following elements:

$$\begin{aligned}
\rho_{00}^{ab} &= \frac{1}{4}(1 - \kappa) \cos^2(2D_z t), \\
\rho_{11}^{ab} &= \frac{1}{8}(2 \cos^2(2t) \cos^2(2D_z t) + (3 - 3\kappa - (1 - \kappa) \cos(4D_z t)) \sin^2(2t\omega)), \\
\rho_{22}^{ab} &= \frac{1}{8}(2 \sin^2(2t\omega) \cos^2(2D_z t) + (3 + 3\kappa - (1 - \kappa) \cos(4D t))),
\end{aligned}$$

$$\begin{aligned}
\rho_{21}^{ab} &= \frac{1}{8}(2\sqrt{3}\kappa\cos(2D_z t)) - i\sin(4t\omega)\left(-\frac{1}{2}\kappa(3+\cos(4D_z t)) - 2\sin^2(2D_z t)\right), \\
\rho_{33}^{ab} &= \frac{1}{4}(1-\kappa+\sin^2(2D_z t)), \quad \rho_{12}^{ab} = \rho_{21}^{*ab}.
\end{aligned} \tag{2.10}$$

Similarly the state ρ_{AC} is described by the following non-zero elements,

$$\begin{aligned}
\rho_{00}^{ac} &= \frac{1}{4}(1-\kappa)\sin^2(t\omega)\sin^2(2D_z t), \\
\rho_{11}^{ac} &= \frac{1}{4}((1-\kappa+\cos^2(2t\omega))\cos^2(2D_z t) + (1+2\kappa)\sin^2(2t\omega)), \\
\rho_{22}^{ac} &= \frac{1}{4}\sin^2(2D_z t)(1+(1-\kappa)\cos^2(t\omega)), \\
\rho_{21}^{ac} &= -\frac{1}{2}\sin^2(2D_z t)\left(\frac{i}{2}\sqrt{3}\kappa\sin(t\omega) - 2(1-\kappa)\cos(2D_z t)\cos(t\omega)\right), \\
\rho_{33}^{ac} &= \frac{1}{4}(1-\kappa+(1+2\kappa)\cos^2(t\omega) + \cos(2D_z t)\sin^2(t\omega)), \\
\rho_{12}^{ac} &= \rho_{21}^{*ac}.
\end{aligned} \tag{2.11}$$

Finally the non zero elements of third partition ρ_{BC} are given by,

$$\begin{aligned}
\rho_{00}^{bc} &= \frac{1}{4}(1-\kappa)\cos^2(2t\omega)\sin^2(2D_z t), \\
\rho_{11}^{bc} &= \frac{1}{4}((1-\kappa+\sin^2(2t\omega))\cos^2(2D_z t) + (1+2\kappa)\cos^2(2t\omega)), \\
\rho_{22}^{bc} &= \frac{1}{4}(1+(1-\kappa)\sin^2(2t\omega))\sin^2(2D_z t), \\
\rho_{21}^{bc} &= \frac{1}{4}\kappa\sin(2D_z t)(-2\sqrt{3}\cos(t\omega) - 2i\cos(2D_z t)\sin(t\omega)), \\
\rho_{33}^{bc} &= \frac{1}{2}(1-\kappa+(1+2\kappa)\sin^2(t\omega) + \cos^2(t\omega)\cos(2D_z t)), \\
\rho_{12}^{bc} &= \rho_{21}^{*bc}.
\end{aligned} \tag{2.12}$$

2.3 Exchanging quantum correlations

In this section, we investigate the amount of entanglement that may lose from the initial state $\rho_{AB}(0)$ and that may be exchanged between the terminals of the initial state and the control qubit C . To quantify the amount of

quantum correlation, we shall use the concurrence, entanglement of formation, and the negativity where the definition of the quantifiers are given in Chapter(1) in Eq(1.5), Eq(1.6) and Eq(1.7) respectively.

In Fig.(2.1), we quantify the amount of entanglement on the initial state via the concurrence, C , negativity, $\mathcal{N}_{eg}$, and the entanglement of formation E_F . It is assumed that, the initial state of the system is prepared by setting $\alpha = \frac{\pi}{3}$, namely $\rho_{AB}(0)$ is a partial entangled state. It is clear that, the quantum correlation between the subsystems A and B appears at larger values of the coherence parameter κ , namely $\kappa > 0.37$. From this figure, it is clear that, the predicted quantum correlation depends on the used quantifier. However, the concurrence and the negativity predict the entanglement at smaller values of the coherence parameter κ . Moreover, the smallest upper bounds of the quantum correlations are predicted by using the entanglement of formation, E_F .

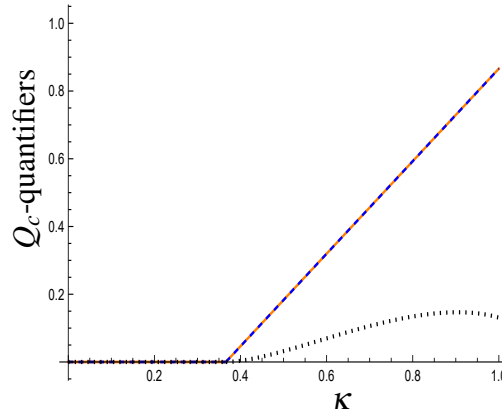


Figure 2.1: The amount of quantum correlation of the initial state $\rho_{AB}(0)$ that predicted via concurrence (solid curve), the negativity (dash-curve), and the entanglement of formation (dot curve).

2.3.1 Lose and gain quantum correlations of $\rho_{AB}(t)$

Due to the interaction with the environment which is described by the control qubit C , the initial state $\rho_{AB}(0)$ loses some of its quantum correlation.

Therefore, it is important to investigate the effect of interaction's parameters on the quantum correlation of the initial quantum system.

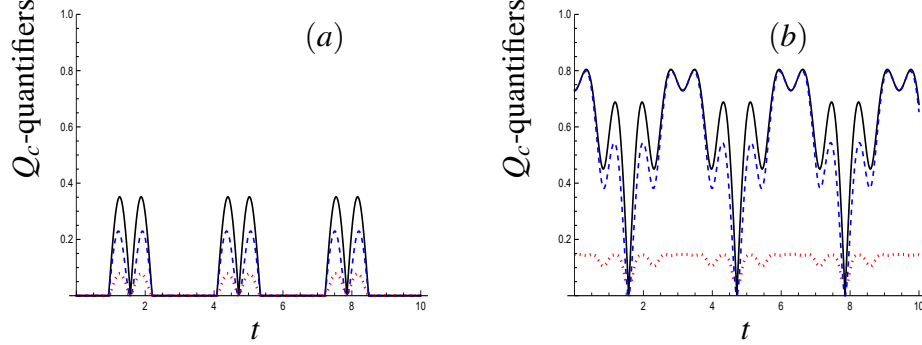


Figure 2.2: The dynamics of the quantum correlation of the state $\rho_{AB}(t)$ that predicted by using the concurrence (solid), the negativity (dash), and the entanglement of formation (dot-lines). It is assumed that, $\rho_{AB}(0)$ is prepared such that $\alpha = \frac{\pi}{3}$ and control qubit is prepared by setting $\gamma = \frac{\pi}{2}$, $D_z = 0.5$ and $\omega = 2$ (a) $\kappa = 0.3$ and (b) $\kappa = 0.9$.

Fig.(2.2) shows the behavior of the three quantifiers for a two qubit system is initially prepared in a partial entangled state, where $\alpha = \frac{\pi}{3}$ and different values of the coherence parameter κ . Moreover, the control qubit is prepared in the state $\rho_C(0) = |1\rangle\langle 1|$, namely we set $\gamma = \frac{\pi}{2}$. From Fig.(2.2.a), it is clear that, at small value of the parameter $\kappa = 0.3$, the quantum correlation is generated at $t \simeq 0.9$. However, the quantum correlation Q_c increase gradually to reach their maximum bounds for the first time at $t = 1.2$ and decrease to vanish completely at $t \simeq 1.6$. The largest bounds are predicted for the concurrence C , while the smallest bounds of quantum correlations are predicted by the entanglement of formation E_F .

In Fig.(2.2.b), we investigate the behavior of the quantum correlation at large value the coherent parameter, $\kappa = 0.9$. It is clear that, at $t = 0$, all the quantifiers predicate the presence of the quantum correlations. However, as the interaction time increases, the predicted quantum correlations fluctuated between their lower and upper bounds. Moreover, the lower bounds that have displayed by the concurrence C are the largest one, but the maximum values of C and the negativity $\mathcal{N}_{eg}$ are coincide. As it is shown from

Fig.(2.2.b), the smallest amount of quantum correlation are quantified by using the E_F .

From Fig.(2.2), one may conclude that, the coherence parameter κ of the initial state plays a central role on keeping the existence of the quantum correlations. If the coherence parameter is small enough such that the initial state behaves as a product state, the DM interaction has the strength to generate quantum correlations between the subsystems of the initial state.

To clarify the role that may be played by the strength of DM interaction, we

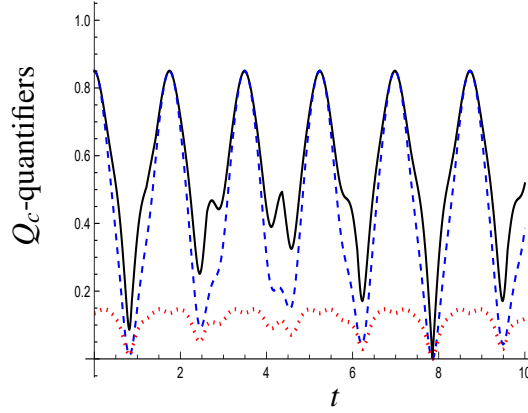


Figure 2.3: The same as Fig(2.2.b), but $D_z = 0.9$.

investigate in Fig.(2.3) the behavior of the three quantifiers at larger values of DM interaction, where we set $D_z = 0.9$. The predicted behavior of the three quantifiers shows that, as soon as the interaction is switched on, the quantum correlations that predicted decrease suddenly to reach their minimum values. At further interaction time, the correlations rebirth again to reach their maximum values. The maximum values of the concurrence and the negativity are coincide, while the lower bounds of C are much better than that displayed by the negativity $\mathcal{N}_{eg}$. The minimum and maximum values of the quantum correlations that predicted via the three quantifiers are displayed at the same interaction time.

Fig.(2.4), exhibits the effect of the spin strength, where we set $\omega = 0.5, 1$, while all the parameters values are the same as Fig.(2.2.b). The predicted behavior is similar to that displayed in Fig.(2.3). As it is shown

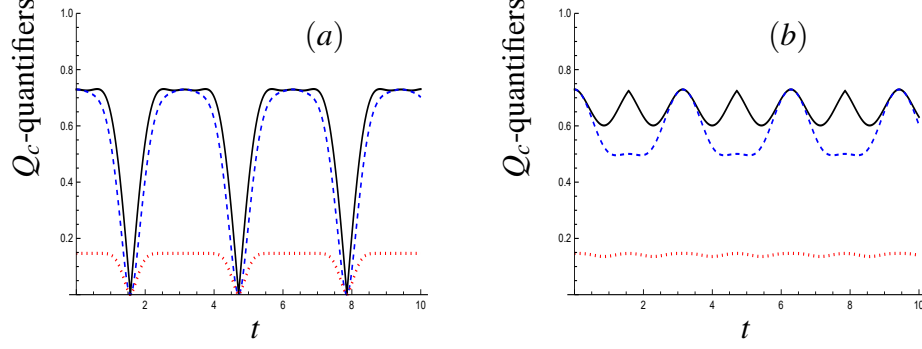


Figure 2.4: The same as Fig.(2.2) but (a) $\omega = 1$, and (b) $\omega = 0.5$. The initial and the control states are prepared such that $\alpha = \frac{\pi}{3}$, $\kappa = 0.9$ and $\gamma = \frac{\pi}{2}$.

from Fig.(2.4.a), where $\omega = 1$, all the quantifiers predict a long-lived quantum correlations. The phenomena of the sudden death/birth are predicted at the same interaction time for all the quantifiers. Moreover, the quantum correlations vanish at larger interaction time compared with that displayed in Fig.(2.3). As one decrease the strength of the interaction, $\omega = 0.5$, in Fig.(2.4.b), the long-lived behavior of quantum correlation is lost and the oscillation behavior is shown.

From the results that displayed in Fig.(2.4), one can consider the strength of the coupling Heisenberg interaction as a control parameter to generated a long-lived quantum correlations between the terminals of the initial quantum system. Moreover, it could be used to improve the lower bounds of these correlations.

2.3.2 Lose and gain quantum correlations of $\rho_{AC}(t)$

The amount of quantum correlations that may be generated between the subsystems A and C are described in Fig.(2.5), where the same initial state settings are considered. It is clear that, as soon as the interaction is switched on between the subsystem A and the control qubit C , an entangled state is generated. All the quantifiers C , $\mathcal{N}_{eg}$ and E_f predict that, there is a quantum correlation is generated between the two qubits. Moreover, at large weight,

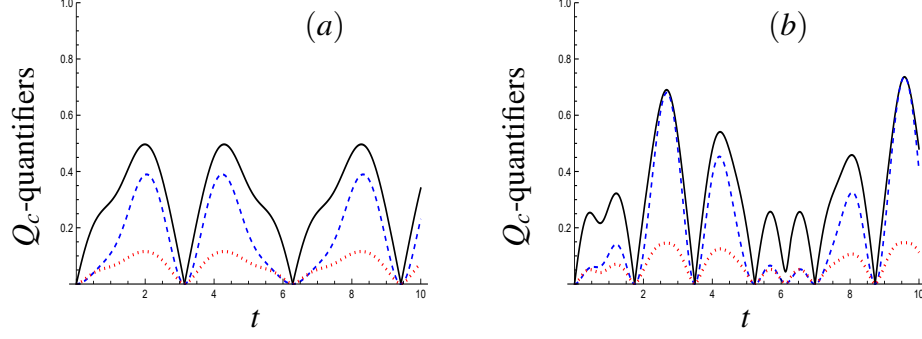


Figure 2.5: The amount of quantum correlation that is generated between the subsystems of the state $\rho_{AC}(t)$, $\kappa = 0.9$, $\omega = 0.5$, $\gamma = \frac{\pi}{2}$, $\alpha = \frac{\pi}{3}$, (a) $D_z = 0.5$ and (b) $D_z = 0.9$.

namely $\kappa = 0.9$, the quantum correlation is generated between the qubits A and C on the expense of the initial quantum correlation between A and B . This phenomena is displayed in Fig.(2.5.b) and Fig.(2.3), by investigating the behaviors of the three quantifiers, where as soon as the interaction is switched on, the quantum correlation of the state $\rho_{AB}(t)$ decreases, while it increases between the qubits A and C .

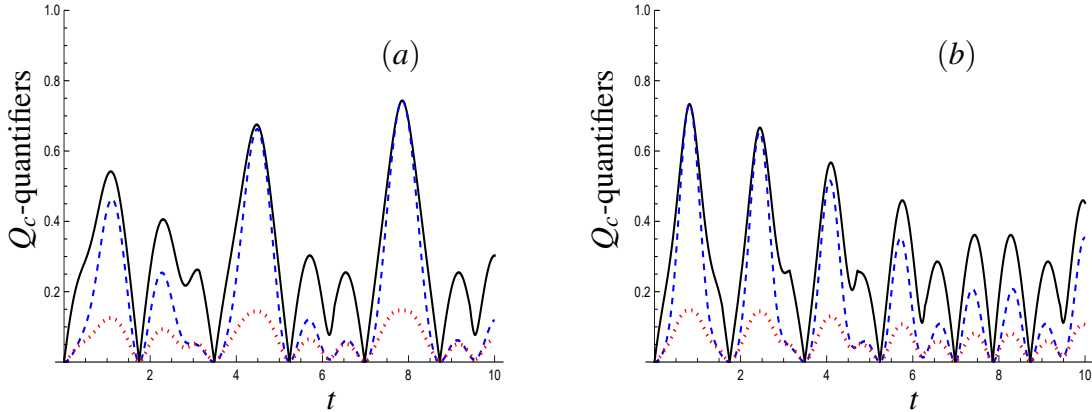


Figure 2.6: The same as Fig.(2.4) but for the state $\rho_{AC}(t)$ (a) $\omega = 1$, and (b) $\omega = 2$.

In Fig.(2.6), we investigate the effect of the Heisenberg coupling interaction parameter on the generated entangled state between the terminal A and the control qubit and C . To get a clear vision on this effect we consider different values of the spin strength, where we set $\omega = 1, 2$. It is clear from Fig.(2.6.a), at small values of the coupling interaction, means that less quantum correlation between the initial qubits A and B , a large quantum cor-

relation between the qubits A and C is predicted. Moreover, small values of ω increase the survival time of the quantum correlation of the state $\rho_{AC}(t)$. Fig.(2.6.b) displays the behavior of quantifiers at large values of the DM interaction, where we set $\omega = 2$. The behavior is similar to that displayed in Fig.(2.6.a), but the upper bounds of all the quantifiers are larger than those displayed at $\omega = 1$.

2.3.3 Lose and gain quantum correlations of $\rho_{BC}(t)$

As mentioned above, the DM interaction generates an entanglement between the qubits B , and C which is represented by the density operator $\rho_{BC}(t)$. The behavior of the three quantifiers of the entanglement is predicted in Fig.(2.7) at different values of the interaction strength. The be-

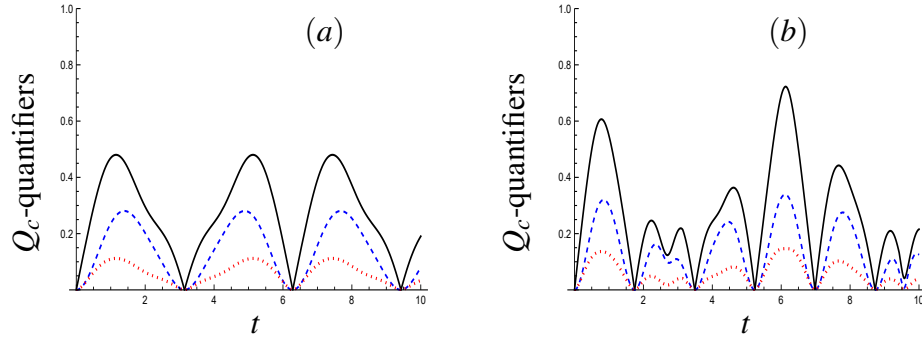


Figure 2.7: The same as Fig.(2.5) but for the state $\rho_{BC}(t)$.

havior of the three quantifiers for the marginal state $\rho_{BC}(t)$ at $D_z = 0.5$ is displayed in Fig.(2.7.a), where it displays a similar behavior to that shown in Fig.(2.5.a) for the marginal state $\rho_{AC}(t)$. However, the generated quantum correlations the depicted by the three quantifiers increase suddenly as soon as the interaction is switched on, while the gradual behavior is displayed for the marginal state $\rho_{AC}(t)$. Moreover, the maximum values of the quantum correlations that shown by the three quantifiers are predicted at smaller interaction time compared with that displayed for the marginal state $\rho_{AC}(t)$. As it is displayed from Fig.(2.7.b), the maximum bounds of quantum corre-

lations are larger than those displayed in Fig.(2.7.a), where we increase the interaction strength. For both marginal states, the sudden birth/death of the quantum correlations are depicted at the same interaction time, meanwhile the maximum quantum correlations are displayed for marginal state $\rho_{AB}(t)$.

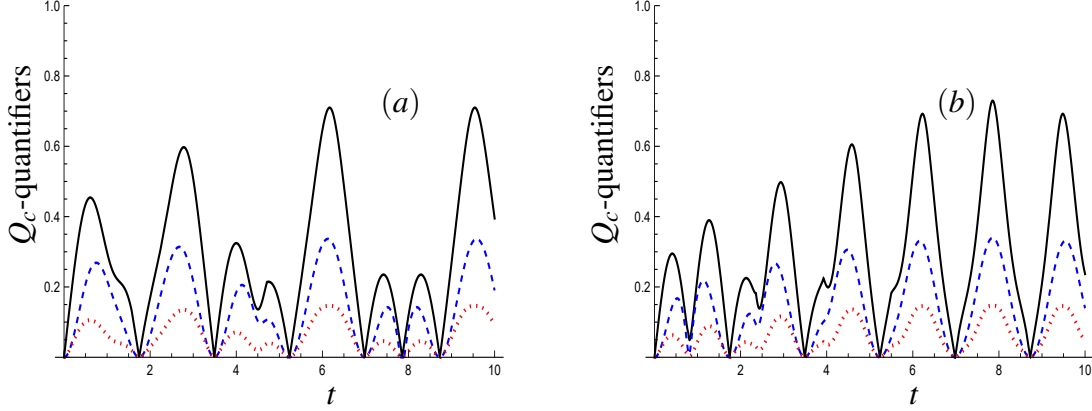


Figure 2.8: The same as Fig.(2.4) but for the state $\rho_{BC}(t)$ (a) $\omega = 1$, and (b) $\omega = 2$.

In Fig.(2.8), we examine the effect of the Heisenberg interaction spin strength on the behavior of quantum correlations for the marginal state $\rho_{BC}(t)$, where we set $\omega = 1, 2$ for Fig.(2.8.a) and Fig.(2.8.b), respectively. It is clear that, the number oscillations of the three quantifiers are larger than those displayed at small value of ω as displayed in Fig.(2.8).

2.4 Exchanging the Non-local information

In this section, we investigate the behavior of the non-local information, I_{non} that is encoded on all the possible partitions, $\rho_{AB}(t)$, $\rho_{AC}(t)$ and $\rho_{BC}(t)$. The amount of local and non-local information are quantify by the von-Neumann entropy, where the mathematical form is given by Eq.(1.10).

In Fig.(2.9.a), we quantify I_{non} for the initial state $\rho_{AB}(0)$, as a function of the coherence parameter κ and the angle α . At fixed value of $\alpha = \pi/3$, the amount of the non-local information increases gradually to reach its maximum value, namely $I_{non} = 2$, at $\kappa = 1$. Due to this interaction, a density

operator which describe the three qubits is given by $\rho_{ABC}(t)$ is generated. Fig.(2.9.b) displays the behavior of the amount of the non-local information, $I_{non}(\rho_{ABC}(t))$, at $D_z = 0.5$ and the Heisenberg strength $\omega = 2$. Again as it exhibited from (2.9.b), the non-local information of the whole system increases gradually as κ increases to reach its maximum value, namely $I_{non}(\rho_{ABC}(t)) = 3$ at $\kappa = 1$. The *DM* strength will exhibit a change on the behavior of information that could be generated or lost between the three qubits.

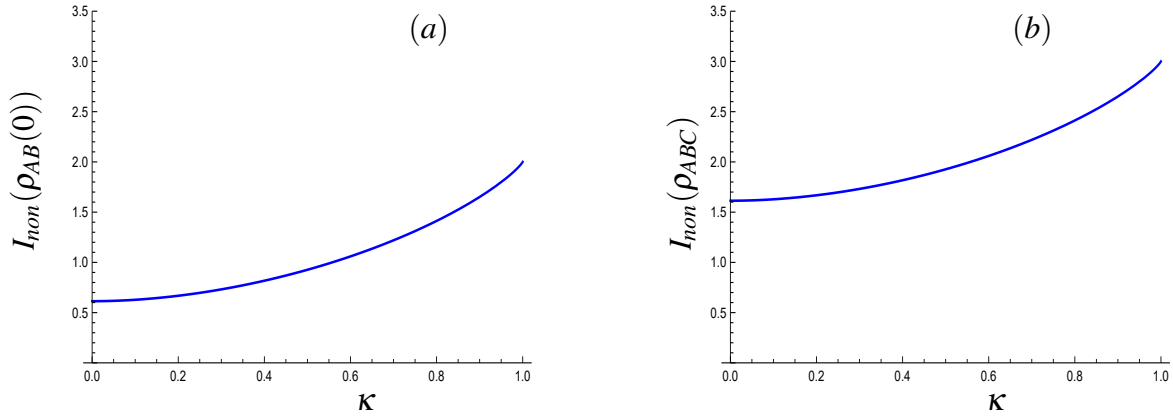


Figure 2.9: The non local information that coded on the states (a) $\rho_{AB}(0)$ and (b) ρ_{ABC} , where $w = 2, \gamma = \frac{\pi}{2}, \alpha = \frac{\pi}{3}, D_z = 0.5$.

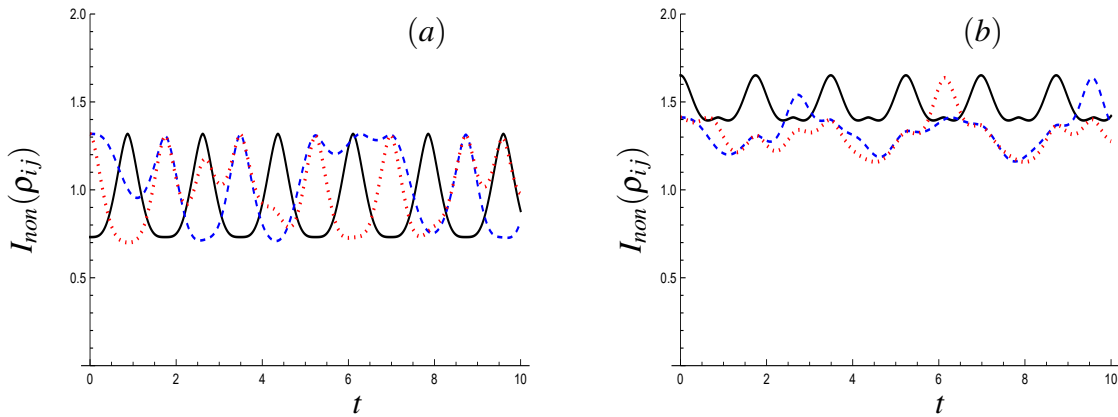


Figure 2.10: The weight effect of the initial state on the generated non-local information, where $D_z = 0.9, w = 0.5, \gamma = \frac{\pi}{2}, \alpha = \frac{\pi}{3}$. The solid(black), dash (blue), and the dot (red) curves, represent the non-local information for the states $\rho_{AB}(t), \rho_{AC}(t)$, and $\rho_{BC}(t)$, respectively where, (a) $\kappa = 0.3$, and (b) $\kappa = 0.9$.

Fig.(2.10) displays the exchange of the non-local information between the three states, $\rho_{ij}(t)$, $ij = AB, AC$ and BC at different values of the coherence parameter κ , while the strength of DM interaction is fixed such that $D_z = 0.9$. In Fig.(2.10.a), we investigate the behavior of $\rho_{ij}(t)$ where it is assumed that, the parameter $\kappa = 0.3$. It is clear that, at $t = 0$, the non-local information that is encoded in the state $\rho_{AB}(t)$ increases gradually to reach its maximum bounds. However, as the interaction time, t increases the non-local information $I_{non}(\rho_{AB}(t))$ decreases gradually to reach its minimum value. This behavior is repeated periodically at further value of t , where the upper and lower bounds of the non-local information are similar during the time's interaction. The similar periodic behavior is predicted for $I_{non}(\rho_{AC}(t))$ and $I_{non}(\rho_{BC}(t))$, where as $I_{non}(\rho_{AB}(t))$ increases, the non-local information that encoded on $\rho_{AC}(t)$ and $\rho_{BC}(t)$ decreases. Whenever, $I_{non}(\rho_{AB}(t))$ reaches its maximum values, both $I_{non}(\rho_{AC}(t))$ and $I_{non}(\rho_{BC}(t))$ are minimum. As an important observation, the maximum values of the non-local information that predicted for $\rho_{AC}(t)$ and $\rho_{BC}(t)$ doesn't exceed that encoded on the initial state $\rho_{AB}(t)$. The amount of non-local information that is encoded on the states $\rho_{AC}(t)$ and $\rho_{BC}(t)$ via direct and indirect interaction by DM , respectively, oscillates periodically between their maximum and lower bounds. Their minimum values don't exceed the minimum values of $I_{non}(\rho_{AB}(t))$. In Fig.(2.10.b), we investigate the behavior of the non-local information that is encoded on the states ρ_{ij} at larger values of the coherence parameter, where we set $\kappa = 0.9$. A similar behavior is predicted as that shown (2.10.a), but the upper and lower bounds of $I_{non}(\rho_{ij}(t))$ are much better than those shown at $\kappa = 0.3$.

Fig.(2.11.a) displays the behavior of $I_{non}(\rho_{ij}(t))$ at smaller values of DM strength, where we set $D_Z = 0.5$. It is clear that, for $\rho_{AC}(t)$ (direct interaction) and $\rho_{BC}(t)$ (indirect interaction) the non-local information decreases, while the non-local information on the marginal state $\rho_{AB}(t)$ in-

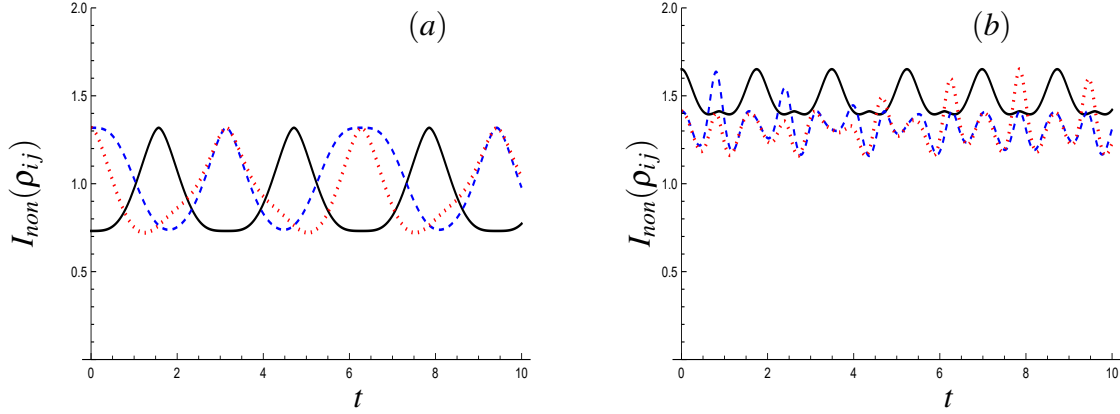


Figure 2.11: (a) The same as Fig.(2.10.a) but $D_z = 0.5$, and (b) is the same as Fig.(2.10.b) but with $\omega = 2$.

creases. However, the decreasing rate of I_{non} that depicted for $\rho_{AC}(t)$ is larger than that displayed in Fig.(2.10.a). Moreover, the crests and the troughs of the information are exchanged between the states $\rho_{AC}(t)$, $\rho_{BC}(t)$ and the initial state $\rho_{AB}(t)$. The effect of larger values of ω is shown in Fig.(2.11.b), where we set $\omega = 2$. The behavior is similar to that displayed in Fig.(2.10.b). However, the numbers of oscillations are larger for $\rho_{AC}(t)$ and $\rho_{BC}(t)$.

2.5 Discussion

In this chapter, a system of two qubits is initially prepared in a partial entangled state governed by Heisenberg-XX chain. One of its subsystems interacts locally with a control qubit via Dzyaloshinskii-Moriya (DM). Due to these interactions, there will be entangled states that are generated between the three qubits. The possibility of exchanging the quantum correlations and the non-local information between all the partitions is discussed. We investigate the effect of the initial state settings and the strengths of the interaction on this process. It is shown that, large value of coherence parameter is a guarantor for generating long-lived quantum correlation at small strength of DM interaction. This behavior is changed if one increases

the strength of DM , where the upper bounds are larger, while the minimum bounds are smaller than those displayed at small values of the coherence parameter. Moreover, the long-lived quantum correlations are displayed as one increases the spin strength of Heisenberg interaction. The numerical computations exhibit that, the quantum correlations are exchanged between the initial state the two marginal states, where as it decrease on the initial system, it increases on the two marginal systems and vis versa. The maximum bounds of the quantum correlations that predicted for the marginal systems never exceed that displayed for the initial system. As soon as quantum correlation of the initial state vanishes, the marginal states exhibit a maximum quantum correlation. However, at large values of interaction strength, one finds that, the quantum correlation of marginal states have different phase due to the direct/indirect interaction with DM .

The phenomena of exchanging the non-local information between all the three partitions is examined at different values of the interaction's strength, and the coherence parameter. Our results display that, the non-local information behaves simultaneously with the quantum correlations, where the large quantum correlations, the large non-local information. One can maximize the amount of the non-local information by increasing the strength of DM interaction and decreasing the dipole interaction strength. However, at large spin strength, the number of oscillations and their amplitudes increase, and consequently, the lower bounds of the non-local information decreases. The generated quantum correlations are quantified by using the concurrence, entanglement of formation and negativity. The behavior of the three quantifiers is discussed, where the large amount of quantum correlations are predicted by concurrence and the negativity, while the entanglement of formation displays the smallest values of quantum correlations.

In conclusion, it is possible to exchange the quantum correlations and the non-local information between the all marginal states, which may be gener-

ated via direct or indirect DM interaction. The maximum amount of these correlations and information do not exceed the initial ones. The interaction parameters are considered as control parameters for generating long-lived quantum correlations. We believe that these results are important for generating entangled quantum network, since one can generate long- live entanglement between distant particles as members of a quantum network and exchanging the non-local information between its members is possible.

Chapter3

Quantum correlations and information scrambling between three qubits systems interacting via Dipolar interaction

3.1 Introduction

Entanglement plays an important role in the context of quantum information processing applications; like quantum dense coding (Bennett & Wiesner, 1992), quantum teleportation (Bennett et al., 1993b) and quantum cryptography (Ekert, 1991). An open quantum system may interact with its surrounding, and consequently causes decoherence in most of quantum properties of the system. For example, decoherence causes a loss in the system memory, therefore the information may be exchanges between the quantum system and its environment (Khan et al., 2022). However, due to the non-Markovian property, some quantum systems may restore the lost in-

formation and coherence (Jan et al., 2019). The reversible nature for the non-Markovian process is an important phenomena due to its applications in many fields, where it may help in improve the security of quantum key distribution (Vasile, Olivares, Paris, & Maniscalco, 2011), discover the characteristic properties of environments (Smirne, Cialdi, Anelli, Paris, & Vaccini, 2013) , also it's used in quantum meteorology (Chin, Huelga, & Plenio, 2012). On the other hand, due to the interaction between the system and its environment, the encoded quantum information will be scrambled between the two systems. To measure the information scrambling rate, one uses a measure called out of time order correlator OTOC, where there are different methods have been introduced to characterize it and successful experiments have been done (Alonso et al., 2019; Halpern et al., 2018).

Heisenberg Models have been of a wild interest in many fields (Zidan, 2014), where it has been investigated in the context of quantum information. Therefore, we are motivated to investigate the amount of the exchanged information and correlation between three qubit systems, where it is assumed that the interaction is governed by spin interaction of type XX -Heisenberg chain in addition to the Dipolar interaction (Qiang et al., 2009; Hill & Wootters, 1997).

This chapter is organized as following. In Sec.(3.2) the suggested model is given, the behavior of the quantum correlations during the interaction is discussed in Sec.(3.3). The non-Markovianity of the system is investigated in Sec.(3.4). The behavior of the out-of time order correlators function as an indicator of the information scrambling is examined in Sec.(3.5). Finally we discuss or result in Sec.(3.6),

3.2 Suggested Model

The suggested system consists of two qubits A and B that have been prepared in a different initial state settings; separable, maximum entangled or partial entangled states. The dynamics of system is governed by a Heisenberg- XX spin model. It is assumed that, one terminal of the system ρ_{AB} , say the subsystem A , interacts locally with a third qubit, C via Dipolar interaction. The total Hamiltonian which describe this system is given by,

$$H_{sys} = H_{Heis} + H_{Dp}, \quad (3.1)$$

where H_{Heis} represents the Hamiltonian of the spin XX interaction (Qiang et al., 2009), and H_{Dp} represents the Hamiltonian of the Dipolar interaction (Reis, 2013; Mohammed, Ahmed, El-Shahat, & Metwally, 2021). Mathematically, these Hamiltonian can be written as ,

$$\begin{aligned} H_{Heis} &= \frac{1}{2} \omega \left(\sigma_A^x \sigma_B^x + \sigma_A^y \sigma_B^y \right), \\ H_{Dp} &= \frac{\delta}{4} \left(2\sigma_z^A \tau_z^C - \sigma_x^A \tau_x^C - \sigma_y^A \tau_y^C \right) + \frac{\varepsilon}{4} \left(\sigma_x^A \tau_x^C - \sigma_y^A \tau_y^C \right), \end{aligned} \quad (3.2)$$

where τ_j , $j = x, y, z$ represent the Pauli operators of the qubit C , while $\sigma_A^i, \sigma_B^i, i = x, y$ for the qubits A and B , respectively, ε is the strength of interaction, δ is the direction of interaction, and κ is the coherence parameter. If $\delta > 0$, then the qubits are moving in $x - y$ plane, and if $\delta < 0$, then the qubits are spin around z -axis (Muthuganesan & Chandrasekar, 2021; Mohammed, Ahmed, et al., 2021; Ebrahim & Metwally, 2023b). Let us assume that, the initial state of the whole system at time $t = 0$ is given by (Qiang et al., 2009),

$$\rho_s(0) = \rho^{AB}(0) \otimes \rho^C(0), \quad (3.3)$$

where,

$$\rho^{AB}(0) = \kappa \left| \varphi^{AB} \right\rangle \left\langle \varphi^{AB} \right| - \frac{1}{4} (1 - \kappa) I_{4 \times 4}, \quad \rho^c(0) = |\varphi^c\rangle \langle \varphi^c|, \quad (3.4)$$

with $|\varphi^{AB}\rangle = \cos(\alpha)|ge\rangle + \sin(\alpha)|eg\rangle$, and $|\varphi^c\rangle = \cos\gamma|g\rangle + \sin\gamma|e\rangle$, $I_{4\times 4}$ is the identity operator. However, at $\alpha = \frac{\pi}{4}$, the state $|\varphi^{AB}\rangle$ reduces to be a maximum entangled state (MES) of Bell type $|\Psi^+\rangle$. The time evaluation of the initial systems (3.3) at any arbitrary time $t > 0$ is given by,

$$\rho_s(t) = \mathcal{U}(t)\rho_s(0)\mathcal{U}^\dagger(t), \quad \mathcal{U}(t) = \exp[-i H_{\text{sys}} t], \quad (3.5)$$

where $\mathcal{U}(t)$ is a unitary operator given by a matrix of size 8×8 , its non-zero elements are given by,

$$\mathcal{U}(t) = \begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} & u_{04} & u_{05} & u_{06} & u_{07} \\ u_{10} & u_{11} & u_{12} & u_{13} & u_{14} & u_{15} & u_{16} & u_{17} \\ u_{20} & u_{21} & u_{22} & u_{23} & u_{24} & u_{25} & u_{26} & u_{27} \\ u_{30} & u_{31} & u_{32} & u_{33} & u_{34} & u_{35} & u_{36} & u_{37} \\ u_{40} & u_{41} & u_{42} & u_{43} & u_{44} & u_{45} & u_{46} & u_{47} \\ u_{50} & u_{51} & u_{52} & u_{53} & u_{54} & u_{55} & u_{56} & u_{57} \\ u_{60} & u_{61} & u_{62} & u_{63} & u_{64} & u_{65} & u_{66} & u_{67} \\ u_{70} & u_{71} & u_{72} & u_{73} & u_{74} & u_{75} & u_{76} & u_{77} \end{pmatrix}. \quad (3.6)$$

The unitary operator of the system is defined by the non-zero elements,

$$\begin{aligned} u_{00} &= \mu_- C_{\eta_8}, & u_{05} &= -i\mu_- S_{\eta_8}, & u_{11} &= \mu_+ C_{\eta_8}, & u_{14} &= \mu_+ S_{\eta_8}, \\ u_{21} &= \mu_+ S_{\eta_7} S_{t\omega}, & u_{22} &= \mu_- C_{\eta_8} C_{t\omega}, & u_{24} &= -i\mu_+ C_{\eta_7} S_{t\omega}, \\ u_{27} &= -i\mu_- C_{t\omega} S_{\eta_8}, & u_{30} &= -\mu_- S_{\eta_8} S_{t\omega}, & u_{33} &= \mu_+ C_{\eta_7} C_{t\omega}, \\ u_{35} &= -i\mu_- C_{\eta_8} S_{t\omega}, & u_{36} &= i\mu_+ C_t S_{\eta_7}, & u_{41} &= u_{36}, \\ u_{42} &= u_{35} & u_{44} &= u_{33}, & u_{47} &= u_{30}, & u_{50} &= u_{27}, & u_{53} &= u_{24}, \\ u_{55} &= u_{22}, & u_{56} &= u_{21}, & u_{63} &= u_{14}, & u_{66} &= u_{11}, & u_{72} &= u_{05}, \\ u_{77} &= u_{00}, \end{aligned} \quad (3.7)$$

where C_v and S_v are $\cos(v)$ and $\sin(v)$ respectively, $\eta_7 = \frac{t\delta}{2}$, $\eta_8 = \frac{t\varepsilon}{2}$ and $\mu_{\pm} = C_{\eta_7} \pm iS_{\eta_7}$. Then by using the unitary operator (3.6) and the initial

state $\rho_s(0)$, one gets the final state $\rho_s(t)$ at any $t > 0$. The reduced states between each two subsystems are obtained by tracing the third system, namely $\rho_{ij} = \text{Tr}_k \rho_{ijk}$. For example $\rho_{AC} = \text{Tr}_B \{ \rho_{ABC} \}$. The final states may be written in a general form as,

$$\rho_{ij}(t) = \begin{pmatrix} \rho_{00}^{ij} & \rho_{01}^{ij} & \rho_{02}^{ij} & \rho_{03}^{ij} \\ \rho_{10}^{ij} & \rho_{11}^{ij} & \rho_{12}^{ij} & \rho_{13}^{ij} \\ \rho_{20}^{ij} & \rho_{21}^{ij} & \rho_{22}^{ij} & \rho_{23}^{ij} \\ \rho_{30}^{ij} & \rho_{31}^{ij} & \rho_{32}^{ij} & \rho_{33}^{ij} \end{pmatrix}. \quad (3.8)$$

The density operator ρ^{AB} defined by the giving elements :

$$\begin{aligned} \rho_{00}^{ab} &= \frac{1}{16} \left(4 - 2\chi_1^+ C_{t\delta} + 2 C_{t\epsilon} \chi_1^- + \kappa C_{2t\alpha} [\chi_{15}^- + 4 C_{t\epsilon} S_\gamma^2] \right), \\ \rho_{01}^{ab} &= \frac{1}{4} i C_\gamma S_\gamma \left(\kappa C_{t\omega} \chi_3^- + 2 S_{t\omega} \chi_6 \right), \\ \rho_{02}^{ab} &= \frac{1}{4} i C_\gamma S_\gamma \left(\kappa C_{t\omega} \chi_2^- + 2 S_{t\omega} \chi_6 \right), \quad \rho_{03}^{ab} = \frac{-1}{2} \kappa S_{\eta_7} \chi_7^+, \\ \rho_{10}^{ab} &= \frac{-i S_{2\gamma}}{8} \left(\chi_4^- C_{-\eta_6} S_\omega + C_{+\eta_5} S_\omega \chi_5^- + \kappa C_{t\omega} \chi_3^- - 2i \kappa S_\omega S_\alpha^2 \chi_9 \right), \\ \rho_{11}^{ab} &= \frac{1}{16} \left(\kappa C_{\eta_{-3}} \chi_8 + \kappa C_{\eta_{+3}} \chi_8 + 2 C_{t\epsilon} [2 + C_{t\delta} \kappa - \chi_{16}^+ C_{2\gamma} + \chi_{18}^+] \right. \\ &\quad \left. + 8 \kappa C_{2\gamma} C_{\eta_7}^2 C_{\eta_8} S_{2t\omega} S_{2\alpha} S_{\eta_7} \right), \\ \rho_{12}^{ab} &= \frac{-i}{16} \left([2 C_{2\gamma} C_{t\delta} - 2 \kappa C_{2\alpha} \chi_8] S_{2t\omega} - 2 \chi_{17} C_{t\epsilon} S_{2t\omega} + 4 \kappa C_{\eta_8} S_{2\alpha} \right. \\ &\quad \left. [i C_{\eta_9} + C_{\eta_7} (i + 2 C_{2t\omega} C_{2\gamma} S_\delta)] \right), \\ \rho_{13}^{ab} &= \frac{1}{8} S_{2\gamma} \left(\kappa S_\omega \chi_2^- + 2 C_{t\omega} \chi_{14}^{-,c} \right), \\ \rho_{20}^{ab} &= \frac{1}{8} S_{2\gamma} \left(\kappa S_\omega \chi_{11}^- + 2 C_{t\omega} \chi_{14}^{-,s} \right), \\ \rho_{21}^{ab} &= \frac{-i}{8} \left(\kappa C_{2\alpha} [2 + C_{t\delta} + C_{t\epsilon}] S_{2t\omega} + 2i \kappa [C_{\eta_7} + C_{\eta_9}] C_{\eta_8} S_{2\alpha} + \right. \\ &\quad \left. C_{2\gamma} [-C_{t\delta} + C_{t\epsilon} S_{2t\omega} - 4 \kappa C_{2t\omega} C_{\eta_7} C_{\eta_8} S_{2\alpha} S_\delta] \right), \end{aligned}$$

$$\begin{aligned}
\rho_{22}^{ab} &= \frac{1}{32} \left(-2\kappa C_{\eta_{-3}} \chi_8 - 2\kappa C_{\eta_{+3}} \chi_8 + 4 C_{t\varepsilon} [2 + C_{t\delta} \kappa + \chi_{16}^- C_{2\gamma} + \chi_{18}^-] \right. \\
&\quad \left. - 8\kappa C_{2\gamma} C_{\eta_7}^2 C_{\eta_8} S_{2t\omega} S_{2\alpha} S_{\eta_7} \right), \\
\rho_{23}^{ab} &= \frac{1}{8} e^{-\frac{3}{2}it\delta} C_\gamma S_\gamma \left(i\chi_{34}^- [1 + e^{2it\delta} (-1 + \kappa) + \kappa + 2\kappa C_{2\alpha}] C_{\eta_8} S_\omega + \right. \\
&\quad \left. \chi_{34}^+ [-1 + r + e^{2it\delta} (1 + \kappa + 2\kappa C_{2\alpha})] S_\omega S_{\eta_8} - 2ie^{\frac{3it\delta}{2}} \kappa C_{t\omega} S_{2\alpha} \right), \\
\rho_{30}^{ab} &= (\rho_{03}^{ab})^*, \quad \rho_{31}^{ab} = \frac{-1}{4} C_\gamma S_\gamma \left(\kappa S_\omega \chi_3^- + 2 C_{t\omega} \chi_{14}^{+,c} \right), \\
\rho_{32}^{ab} &= \frac{-1}{4} C_\gamma S_\gamma \left(\kappa S_\omega \chi_3^- + 2 C_{t\omega} \chi_{14}^{-,c} \right), \\
\rho_{33}^{ab} &= \frac{1}{16} \left(4 - 2\chi_1^- C_{t\delta} - 2 C_{t\varepsilon} \chi_1^- + \kappa C_{2t\alpha} [\chi_{15}^+ - 4 C_{t\varepsilon} S_\gamma^2] \right).
\end{aligned}$$

Similarly the state ρ^{AC} state is defined as :

$$\begin{aligned}
\rho_{00}^{ac} &= \frac{1}{32} \left(-[-8 + \chi_{19}^- C_{\eta_{-2}} + 2 C_{t\delta} - C_{\eta_{+2}} + C_{\eta_{-4}} - 8\kappa C_{2\alpha} \chi_{24}^- + 6C_{t\varepsilon} \right. \\
&\quad \left. + C_{\eta_{+4}} + \kappa(-2 C_{t\delta} + C_{\eta_{+2}} + 4C_{t\varepsilon} S_{t\omega}^2) S_\gamma^2] - 2 C_\gamma^2 [C_{\eta_7}^4 S_{t\omega}^2 4\chi_{23}^- - \right. \\
&\quad \left. 2(\chi_{30}^- + 4\kappa C_{t\omega}^2 C_{2\alpha}) C_{\eta_8}^2 S_{\eta_7}^2 - 4C_{\eta_7}^2 C_{\eta_8} (C_{\eta_8} \chi_{32} + 4\kappa S_{(2t\omega)} S_{2\alpha} S_{\eta_7}) \right. \\
&\quad \left. + S_{t\omega}^2 (\chi_{19}^- - 4\kappa S_\alpha^2) S_{t\delta}^2] \right), \\
\rho_{01}^{ac} &= \frac{1}{8} e^{-it\delta} \left(2C_\gamma S_\gamma [C_{\eta_7} C_{\eta_8} \chi_{27}^- - C_{t\omega}^2 S_{\eta_7} S_{\eta_8}] + \frac{1}{2} S_{2\gamma} [C_{\eta_7} C_{\eta_8} 2\chi_{28}^+ \right. \\
&\quad \left. - i\kappa S_{2t\omega} S_{2\alpha} e^{it\delta} \chi_{25}^- + S_{\eta_7} S_{\eta_8} \chi_{26}^+ + 2 S_{t\omega}^2 S_{\eta_7} S_{\eta_8} e^{2it\delta} \chi_{19}^-] \right), \\
\rho_{02}^{ac} &= \frac{1}{4} C_\gamma S_\gamma \left(\kappa S_{t\omega} \chi_2^- - 2ie^{-it\delta} C_{t\omega} \chi_{21}^+ \right), \\
\rho_{03}^{ac} &= \frac{1}{2} S_{\eta_7} \left(iC_{t\omega} C_{\eta_8} \chi_{10}^+ + \kappa C_{\eta_8} S_{t\omega} S_{2\alpha} \chi_7^+ \right), \\
\rho_{10}^{ac} &= \frac{1}{32} S_\gamma \left(4e^{-it\delta} C_{\eta_7} C_{\eta_8} [e^{2it\delta} \chi_{32} + \chi_{22}^- S_{t\omega}^2] + 2i\kappa \chi_{25}^- S_{2t\omega} S_{2\alpha} - \right. \\
&\quad \left. ie^{-\frac{3}{2}it\delta} \chi_{34}^- [e^{2it\delta} \chi_{30}^- + 2\chi_{19}^- S_{t\omega}^2] S_{\eta_8} \right),
\end{aligned}$$

$$\begin{aligned}
\rho_{11}^{ac} &= \frac{1}{4} \left(C_{t\omega}^2 \chi_{22}^+ C_{\eta_7}^4 S_\gamma^2 + [(1 + \kappa - \chi_{19}^- C_{t\omega}^2 - 2\kappa C_{2\alpha}) C_\gamma^2 + \chi_{22}^- C_{\eta_8}^2 \right. \\
&\quad S_{t\omega}^2 S_\gamma^2] S_{\eta_7}^2 + C_{\eta_7}^2 S_\gamma^2 [(1 - \kappa + \chi_{22}^- C_{\eta_8}^2 S_{t\omega}^2 - 4\kappa C_{\eta_8} S_{2t\omega} S_{2\alpha}) S_{\eta_7} \\
&\quad \left. + C_{t\omega}^2 \chi_{22}^+ S_{\eta_7}^2] - \chi_{19}^- C_\gamma^2 S_{t\omega}^2 S_{\eta_8}^2 \right), \\
\rho_{12}^{ac} &= \frac{1}{4} i C_{t\omega} S_{t\delta} \chi_{10}^- - \frac{1}{2} \kappa C_{\eta_8} S_{t\omega} S_{2\alpha} S_{\eta_7} \chi_7^+, \\
\rho_{13}^{ac} &= \frac{1}{4} C_\gamma S_\gamma \left(\kappa S_{t\omega} \chi_2^- - 2ie^{it\delta} C_{t\omega} \chi_{21}^- \right), \quad \rho_{20}^{ac} = (\rho_{02}^{ac})^*, \\
\rho_{21}^{ac} &= \frac{1}{2} S_{\eta_7} \left(i C_{t\omega} C_{\eta_7} \chi_{20}^+ + \kappa C_{\eta_8} S_{t\omega} S_{2\alpha} \chi_7^- \right), \\
\rho_{22}^{ac} &= \frac{1}{32} \left([8 + \chi_{19}^- C_{\eta_{-2}} - 8\kappa C_{2\alpha} (-1 + C_{t\delta}) - 6C_{t\delta} - C_{\eta_{+2}} + C_{\eta_{-4}} - \right. \\
&\quad 2C_{t\epsilon} + C_{\eta_{+4}} + \kappa \chi_{29}] S_\gamma^2 + C_\gamma^2 [8 + C_{\eta_{-2}} + 6C_{t\delta} + C_{\eta_{+2}} + 4C_{t\epsilon} S_{t\omega}^2 + \\
&\quad \kappa (-4C_{\eta_{+3}} - 4C_{\eta_{-3}} + C_{\eta_{-2}} - 2(1 + 4C_{t\omega}^2 C_{2\alpha}) C_{t\delta} + C_{\eta_{+2}} + 4(1 + \\
&\quad \left. 2C_{2\alpha}) C_{t\epsilon} S_{t\omega}^2 - 32C_{\eta_7}^2 C_{\eta_8} S_{2t\omega} S_{2\alpha}) S_{\eta_7}] \right), \\
\rho_{23}^{ac} &= \frac{1}{32} e^{-it\delta} S_\gamma \left(4e^{2it\delta} C_{t\omega}^2 [\kappa C_{\eta_{-1}} - 2\kappa C_{2\alpha} C_{\eta_7} C_{\eta_8} + C_{\eta_{+1}}] + 4C_{\eta_7} C_{\eta_8} \right. \\
&\quad [-e^{2it\delta} \chi_{19}^- + \chi_{21}^+ S_{t\omega}^2] + i[\kappa(1 + e^{2it\delta} - 2e^{it\delta} C_{t\epsilon}) S_{2t\omega} S_{2\alpha} \\
&\quad \left. + 2e^{-\frac{1}{2}it\delta} \chi_{34}^- (e^{2it\delta} \chi_{21}^+ - \chi_{19}^- S_{t\omega}^2) S_{\eta_8}] \right), \\
\rho_{30}^{ac} &= -\frac{1}{2} S_{\eta_7} \left(i C_{t\omega} C_{\eta_7} \chi_{10}^+ + \kappa C_{\eta_8} S_{t\omega} S_{2\alpha} \chi_7^- \right), \\
\rho_{31}^{ac} &= \frac{1}{4} e^{-it\delta} C_\gamma S_\gamma \left(i(-1 + e^{2it\delta}) \kappa C_\alpha S_{t\omega} S_\alpha + 2i\kappa C_{t\omega} C_\alpha^2 S_{\eta_{-1}} + \right. \\
&\quad \left. e^{it\delta} \kappa S_{t\omega} S_{2\alpha} S_{t\epsilon} - 2i C_{t\omega} (\kappa S_\alpha^2 S_{\eta_{-1}} + S_{\eta_{+1}}) \right), \\
\rho_{32}^{ac} &= \frac{1}{8} e^{-it\delta} \left(2C_\gamma S_\gamma [C_{\eta_7} C_{\eta_8} \chi_{27}^+ - C_{t\omega}^2 S_{\eta_7} S_{\eta_8}] + \frac{1}{2} S_{2\gamma} [C_{\eta_7} C_{\eta_8} 2\chi_{28}^- \right. \\
&\quad \left. - i\kappa S_{2t\omega} S_{2\alpha} e^{it\delta} \chi_{25}^- + S_{\eta_7} S_{\eta_8} \chi_{26}^- + 2 S_{t\omega}^2 S_{\eta_7} S_{\eta_8} e^{2it\delta} \chi_{19}^-] \right), \\
\rho_{33}^{ac} &= \frac{1}{32} \left(8\chi_{22}^+ C_{\eta_7}^4 S_{t\omega}^2 S_\gamma^2 + [C_\gamma^2 (-2 C_{(2t\omega)} \chi_{31} - 2(-5 + \kappa + 3C_{t\epsilon}) \right. \\
&\quad \left. + \kappa(C_{\eta_{-4}} - 8C_{2\alpha} \chi_{24}^- - 2 C_{t\epsilon} + C_{\eta_{+4}})) + 4(\chi_{30} - 4\kappa C_{t\omega}^2 C_{2\alpha}) C_{\eta_8}^2 \right. \\
&\quad S_\gamma^2] S_{\eta_7}^2 + 2\chi_{22}^+ S_{t\omega}^2 S_\gamma^2 S_{t\delta}^2 + 4C_{\eta_7}^2 [2 C_{\eta_8} S_\gamma^2 (-\chi_{19}^- + C_{t\omega}^2 C_{\eta_8} \chi_{22}^- + \\
&\quad \left. 4\kappa S_{(2t\omega)} S_{2\alpha} S_{\eta_7}) + \chi_{30}^+ C_\gamma^2 S_{\eta_8}^2] \right).
\end{aligned}$$

Finally, ρ^{BC} is defined by the giving elements :

$$\begin{aligned}
\rho_{00}^{bc} &= \frac{1}{16} \left(C_{t\omega}^2 [4\chi_{22}^- C_\gamma^2 C_{\eta_7}^2 - 4\chi_{19}^- S_\gamma^2 S_{\eta_7}^2] - 2 C_\gamma^2 C_{\eta_8} [C_{\eta_8} (\chi_{33}^- C_{2t\omega} - 4\kappa C_{2t\omega} S_{t\omega}^2) + 2\kappa \csc(\eta_7) S_{2t\omega} S_{2t\omega} S_{t\delta}^2] + 2[\chi_{33}^- - 4\kappa C_{2t\omega}] S_\gamma^2 S_{\eta_8}^2 \right), \\
\rho_{01}^{bc} &= \frac{1}{32} e^{-\frac{3}{2}it\delta} S_{2\gamma} \left(-\chi_{34}^+ C_{\eta_8} [\chi_{33}^- C_{2t\omega} - 2e^{2it\delta} C_{t\omega}^2 \chi_{22}^- - 4\kappa C_{2t\omega} S_{t\omega}^2] + 2ie^{\frac{3it\delta}{2}} \kappa \chi_{25}^+ S_{2t\omega} S_{2t\omega} - i\chi_{34}^- [-3 - \kappa + 2e^{2it\delta} \chi_{19}^- C_{t\omega}^2 - \chi_{19}^- C_{2t\omega} + 4\kappa C_{2t\omega}] S_{\eta_8} \right), \\
\rho_{02}^{bc} &= \frac{1}{4} \kappa C_\gamma S_\gamma \left(-i C_{t\omega} \chi_2^+ - 2e^{-it\delta} S_{t\omega} \chi_{21}^+ \right), \\
\rho_{03}^{bc} &= \frac{1}{2} \kappa \left(\chi_{35}^+ C_{\eta_8} S_{t\omega} + C_{t\omega} C_{\eta_7} S_{2t\omega} \chi_{37} \right) S_{\eta_8}, \\
\rho_{10}^{bc} &= \frac{1}{8} \left(C_\gamma [-2e^{it\delta} \chi_{19}^- C_{\eta_7} C_{\eta_8} + e^{\frac{it\delta}{2}} \chi_{34}^+ \chi_{22}^+ C_{\eta_8} S_{t\omega}^2 - i\kappa \chi_{25}^+ S_{2t\omega}] S_\gamma + e^{-it\delta} S_{2\gamma} [e^{2it\delta} (-\chi_{22}^- + \chi_{19}^- S_{t\omega}^2) S_{\eta_7} S_{\eta_8} + C_{t\omega}^2 (\chi_{22}^- C_{\eta_7} C_{\eta_8} + \chi_{19}^- S_{\eta_7} S_{\eta_8})] \right), \\
\rho_{11}^{bc} &= \frac{1}{16} \left(4 - \kappa C_{\eta_3}^- - 2\kappa C_{2t\omega} + [-3 + C_{2t\omega}] C_{2\gamma} C_{t\delta} + \frac{1}{2} \kappa [-2 C_{\eta_{+3}} - 2 C_{2\eta_{-12}} + C_{\eta_{+11}} + C_{\eta_{-11}} - 2 C_{2\eta_{+12}} + C_{\eta_{10}^+} + C_{\eta_{-10}} - C_{\eta_{-2}} - 2(1 + C_{2t\omega}(-3 + C_{2t\omega} - 2 C_{t\omega}^2 C_{2\gamma})) C_{t\delta} - C_{\eta_2}] + C_{t\omega}^2 C_{t\epsilon} (2(-1 + \kappa C_{2t\omega}) C_{2\gamma} + 4\kappa S_\alpha^2) + 16\kappa C_{\eta_7}^2 C_{\eta_8} S_{2t\omega} S_\gamma^2 S_{\eta_7} \right), \\
\rho_{12}^{bc} &= \frac{1}{4} \kappa \left(2 C_{t\omega} C_{\eta_8} S_{2t\omega} S_{\eta_7} \chi_{37} - \chi_{35}^- S_{t\omega} S_{t\delta} \right), \\
\rho_{13}^{bc} &= \frac{1}{4} \kappa C_\gamma S_\gamma \left(i C_{t\omega} \chi_2^+ - 2e^{it\delta} S_{t\omega} \chi_{36}^+ \right), \\
\rho_{20}^{bc} &= \frac{1}{4} \kappa C_\gamma S_\gamma \left(i C_{t\omega} \chi_2^+ - 2e^{it\delta} S_{t\omega} \chi_{36}^- \right), \quad \rho_{21}^{bc} = \left(\rho_{12}^{bc} \right)^*,
\end{aligned}$$

$$\begin{aligned}
\rho_{22}^{bc} &= \frac{1}{32} \left([8 + 8\kappa C_{2t\omega} - \chi_{19}^- C_{\eta-2} - 6 C_{t\delta} - 2\kappa C_{t\delta} - 8\kappa C_{\alpha}^2 \right. \\
&\quad C_{t\delta} + C_{\eta 2} - \kappa C_{\eta 2} + \kappa C_{\eta-4} + 2\kappa C_{t\epsilon} - 4 C_{t\omega}^2 C_{t\epsilon} + \kappa C_{\eta+4} \\
&\quad + 8\kappa C_{t\delta} S_{\alpha}^2] S_{\gamma}^2 + 2 C_{\gamma}^2 [4 C_{\eta 7}^4 (1 - \kappa + \chi_{22}^- S_{t\omega}^2) + 4 C_{t\omega}^2 \chi_{22}^+ \\
&\quad C_{\eta 8}^2 S_{\eta 7}^2 + 4 C_{\eta 7}^2 C_{\eta 8} (C_{t\omega}^2 \chi_{22}^+ C_{\eta 8} + 4\kappa S_{2t\omega} S_{2t\omega} S_{\eta 7}) + (1 - \\
&\quad \kappa + \chi_{22}^- S_{t\omega}^2 S_{t\delta}^2)] \left. \right), \\
\rho_{23}^{bc} &= \frac{1}{8} \left(C_{\gamma} [-2e^{it\delta} \chi_{19}^- C_{\eta 7} C_{\eta 8} - e^{\frac{i\delta}{2}} \chi_{34}^+ - \chi_{22}^- C_{\eta 8} S_{t\omega}^2 - i\kappa \chi_{25}^+ S_{2t\omega}] \right. \\
&\quad S_{\gamma} + e^{-it\delta} S_{2\gamma} [-e^{2it\delta} (\chi_{22}^+ - \chi_{19}^- S_{t\omega}^2) S_{\eta 7} S_{\eta 8} + C_{t\omega}^2 (\chi_{22}^+ C_{\eta 7} \\
&\quad C_{\eta 8} + \chi_{19}^- S_{\eta 7} S_{\eta 8})] \left. \right), \\
\rho_{30}^{bc} &= \frac{1}{2} \kappa \left(\chi_{35}^+ C_{\eta 8} S_{t\omega} + C_{t\omega} C_{\eta 7} S_{2t\omega} \chi_{37}^* \right) S_{\eta 8}, \\
\rho_{31}^{bc} &= \frac{1}{4} \kappa C_{\gamma} S_{\gamma} [-i C_{t\omega} \chi_{22}^+ - 2e^{-it\delta} S_{t\omega} \chi_{36}^+], \\
\rho_{32}^{bc} &= \frac{1}{32} e^{-\frac{3}{2}it\delta} S_{2\gamma} \left(\chi_{34}^+ C_{\eta 8} [-\chi_{33}^+ + 2e^{2it\delta} C_{t\omega}^2 \chi_{22}^+ - 4\kappa C_{2t\omega} S_{t\omega}^2] \right. \\
&\quad + 2ie^{\frac{3it\delta}{2}} \kappa \chi_{25}^+ S_{2t\omega} S_{2t\omega} - i\chi_{34}^- [-3 - \kappa + 2e^{2it\delta} \chi_{19}^- C_{t\omega}^2 - \chi_{19}^- \\
&\quad C_{2t\omega} - 4\kappa C_{2t\alpha}] S_{\eta 8} \left. \right), \\
\rho_{33}^{bc} &= \frac{1}{8} \left(C_{t\omega}^2 [1 + \kappa C_{t\delta} - C_{2\gamma} (\kappa + C_{t\delta}) + 4\kappa C_{2t\omega} C_{\eta 7}^2 S_{\gamma}^2] + C_{\eta 8} \right. \\
&\quad S_{\gamma}^2 [-C_{\eta 8} (\chi_{33}^- C_{2t\omega} + 4\kappa C_{2t\omega} S_{t\omega}^2) - 2\kappa \csc \eta_7 S_{2t\omega} S_{2t\omega} S_{t\delta}^2] \\
&\quad \left. + [\chi_{33}^- + 4\kappa C_{2t\omega}] C_{\gamma}^2 S_{\eta 8}^2 \right).
\end{aligned}$$

where $C_v = \cos(v)$, $S_v = \sin(v)$, and η_i , χ_j are defined as,

$$\begin{aligned}
\eta_{\pm 1} &= \frac{t(\delta \pm \epsilon)}{2}, \quad \eta_{\pm 2} = t(2\omega \pm \delta), \quad \eta_{\pm 3} = t(2\omega \pm \alpha), \\
\eta_{\pm 4} &= t(2\omega \pm \epsilon), \quad \eta_{\pm 5} = \frac{t(3\delta \pm \epsilon)}{2}, \quad \eta_{\pm 6} = \frac{t(\delta \pm \epsilon)}{2}, \\
\eta_7 &= \frac{t\delta}{2}, \quad \eta_8 = \frac{t\epsilon}{2}, \quad \eta_9 = \frac{3t\delta}{2}, \quad \eta_{\pm 10} = 2 \left(t\omega \pm [\alpha + \gamma] \right), \\
\eta_{\pm 11} &= t \left(t\omega \pm [\alpha - \gamma] \right), \quad \eta_{\pm 12} = \alpha \pm \gamma,
\end{aligned}$$

$$\begin{aligned}
\chi_1^\pm &= \pm\kappa + C_{2\gamma}, \quad \chi_2^\pm = \pm(S_{t\delta} + S_{t\epsilon})S_{2\alpha}, \quad \chi_3^\pm(S_{t\delta} \pm S_{t\epsilon})S_{2\alpha} \\
\chi_4^\pm &= 1 + \kappa C_{2\alpha}, \quad \chi_5^\pm = 1 \pm \kappa C_{2\alpha}, \quad \chi_6 = 2i\kappa C_{t\gamma} S_\alpha^2 S_{\eta-1} + \chi_4^- S_{t\epsilon} S_{\eta+1}, \\
\chi_7^\pm &= (\pm C_{t\delta} + iC_{2\gamma} S_{t\delta})S_{2\alpha}, \quad \chi_8 = 2 + C_{t\delta}, \quad \chi_9 = S_{\eta-5} - S_{\eta+1}, \\
\chi_{10}^\pm &= \kappa C_{2\alpha} + C_{2\gamma}, \quad \chi_{11}^\pm = \pm(c_{t\delta} + S_{t\epsilon})S_{2\alpha}, \quad \chi_{12}^\pm = (c_{t\delta} \pm S_{t\epsilon})S_{2\alpha}, \\
\chi_{13}^c &= -2i\kappa C_{\alpha t\delta}^2 S_{\eta-1}, \quad \chi_{13}^s = -2i\kappa S_{\alpha t\delta}^2 S_{\eta-1}, \quad \chi_{14}^{\pm c,s} = \pm\chi_4^\pm S_\delta S_{\eta_1}, \\
\chi_{15}^\pm &= \pm 4 \pm (-2)C_{t\delta} + C_{2\delta\gamma}, \quad \chi_{16}^\pm = C_{2t\omega} \pm \kappa C_{2\alpha}, \quad \chi_{17} = \kappa C_{2\alpha} + C_{2\gamma}, \\
\chi_{18}^\pm &= \kappa + \kappa C_{2\gamma} C_{2\alpha} \pm C_{2t\omega} \chi_{17}, \quad \chi_{19}^\pm = \pm 1 + \kappa, \quad \chi_{20}^\pm = \pm \kappa (C_{2\alpha} - C_{2\gamma}), \\
\chi_{21}^\pm &= \kappa C_{2\alpha} (S_{\eta+1} + S_{\eta+2}), \quad \chi_{22}^\pm = 1 + \kappa \pm 2\kappa C_{2\alpha}, \quad \chi_{23}^\pm = \pm 1 - \kappa + 2\kappa C_{2\alpha}, \\
\chi_{24}^\pm &= \pm 1 + C_{t\epsilon}, \quad \chi_{25}^\pm = C_{t\delta} \pm C_{t\epsilon}, \quad \chi_{26}^\pm = -2 - \kappa + \kappa C_{2t\omega} \pm 4\kappa C_{2\alpha}, \\
\chi_{27}^\pm &= -\kappa \pm e^{2i\delta} \chi_{23}^- S_{t\omega}^2, \quad \chi_{29} = -2C_{t\delta} + C_{\eta+2} + 4S_{t\omega}^2, \\
\chi_{30}^\pm &= 3 \pm \kappa \pm (-\chi_{19})C_{2t\omega}, \quad \chi_{31} = 1 - \kappa + C_{t\epsilon}, \quad \chi_{32} = 1 - \kappa + C_{t\omega}^2 \chi_{22}^+, \\
\chi_{33}^\pm &= \pm(-3) + \kappa + \chi_{19}^\pm C_{2t\omega}, \quad \chi_{34}^\pm = \pm + e^{it\delta}, \quad \chi_{35}^\pm = 1 \pm C_{2\gamma} C_{2\alpha}, \\
\chi_{36}^\pm &= S_{\eta-6} \pm C_{2\gamma} S_{\eta+6}, \quad \chi_{37} = +iC_{2\gamma} C_{t\delta} - S_{t\delta}.
\end{aligned}$$

3.3 Quantum Correlation (QCs)

In this section, we shall quantify the amount of quantum correlation (QCs) that may be distributed between the three partitions. However, the two qubits A and B are initially prepared in the state $\rho_{AB}(0)$, which may be separable, partially entangled, or maximum entangled, depending on the parameters κ and γ . This state may gain or lose QCs during the interaction with the third qubit C . Therefore, we shall quantify the amount of quantum correlation of the state $\rho_{AB}(t)$. Moreover, due to the interaction, there is a quantum states generated between the qubits A and C , ρ_{AC} via direct interaction, and another quantum state ρ_{BC} is generated between the qubits B and C via indirect interaction.

From the previous section, we obtained the time evolution of the three states, $\rho_{AB}(t)$, $\rho_{AC}(t)$ and $\rho_{BC}(t)$. To quantify the amount of these correlations, we

shall use the negativity $\mathcal{N}_{eg}$ as a quantifier. The mathematical form of the negativity is given in Chapter(1) in Eq(1.7).

3.3.1 The initial quantum correlations of $\rho_{AB}(0)$

In this subsection, we investigate the impact of the initial state settings and the coherence parameter κ on the amount of quantum correlations that between the two qubits A and B . Let us assume that, the system is initially prepared in a partial entangled state (PES) or in a maximum entangled state (MES), where we set $\alpha = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{4}$, respectively. The amount of entanglement (QCs) is quantified by using the negativity as a function of the coherence parameter κ . It is clear that, the QCs between the qubits A and B depend on the parameter κ . However, as it is shown from Fig.(3.1) the entanglement appears if $\kappa > 0.34$, where the state ρ_{AB} is initially prepared in a MES, while when ρ_{AB} is initially prepared in a PES, the entanglement appears at $\kappa > 0.37$, so the initial quantum correlation depends on the initial state settings and the coherence parameter κ .

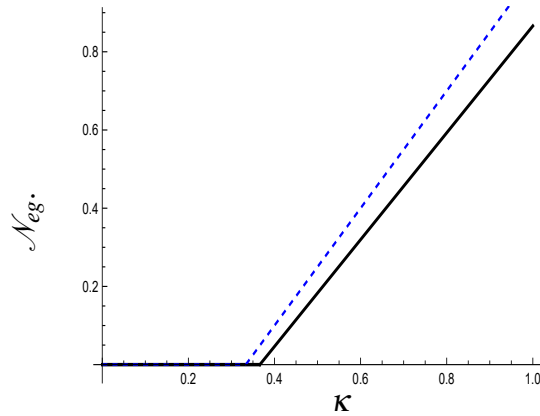


Figure 3.1: The amount of quantum correlation of the initial state $\rho_{ab}(0)$ that predicted using negativity, for (solid) black curve $\alpha = \frac{\pi}{3}$, dashed (blue) curve $\alpha = \frac{\pi}{4}$ maximum entangled Bell state .

In next subsections, we shall investigate the impact of the initial state settings and the interaction parameters on the amount of the quantum cor-

relations that will be exchanged between the three partitions; ρ_{AB} , ρ_{AC} and ρ_{BC} .

3.3.2 Quantum correlation of $\rho_{AB}(t)$

The effect of the initial state settings on the quantum correlations (QCs) of the state ρ_{AB} , will be investigated. In Fig.(3.2.a), it is assumed that, the initial state ρ_{AB} is initially prepared in the state $\rho_{AB} = |ge\rangle\langle ge|$, namely the two qubits A and B are separable, while the qubit C is prepared in an ground state, $\rho_c = |g\rangle\langle g|$. It is clear that, as soon as the interaction is switched on, an entangled state is generated between the subsystems A and B . The negativity, as a quantifier of entanglement, increases suddenly to reach its maximum bounds. However, at further interaction time, the $\mathcal{N}_{eg}$ decreases suddenly to vanish completely. The maximum bounds of $\mathcal{N}_{eg}$ depend on the degree of coherence κ , where at small values of κ , the degree of coherence is large, and consequently the decoherence due to the interaction with environment increases. Therefore, the predicted upper bounds at small values of κ are smaller than those displayed at large κ .

In Fig.(3.2.b), we investigate the QCs that may be lost from the state ρ_{AB} due to the interaction with the qubit C , where ρ_{AB} is prepared in a maximum entangled state (MES), while $\rho_C = |g\rangle\langle g|$. The predicted behavior of $\mathcal{N}_{eg}$ is similar to that displayed in Fig.(3.2.a), where the sudden changes of the negativity are displayed at different intervals of interaction time t , and the numbers of oscillations are larger than those displayed in Fig.(3.2.a). The predicted upper bounds of entanglement increase as κ increases, namely decreasing the degree of coherence. Moreover, the sudden death intervals of the QCs are smaller than those displayed in Fig.(3.2.a).

The amount of quantum correlations that may be gained or lost from the state ρ_{AB} are displayed in Fig.(3.3), where ρ_{AB} either is initially prepared in a separable state ($\alpha = 0$), or in a MES ($\alpha = \pi/4$), while the qubit C

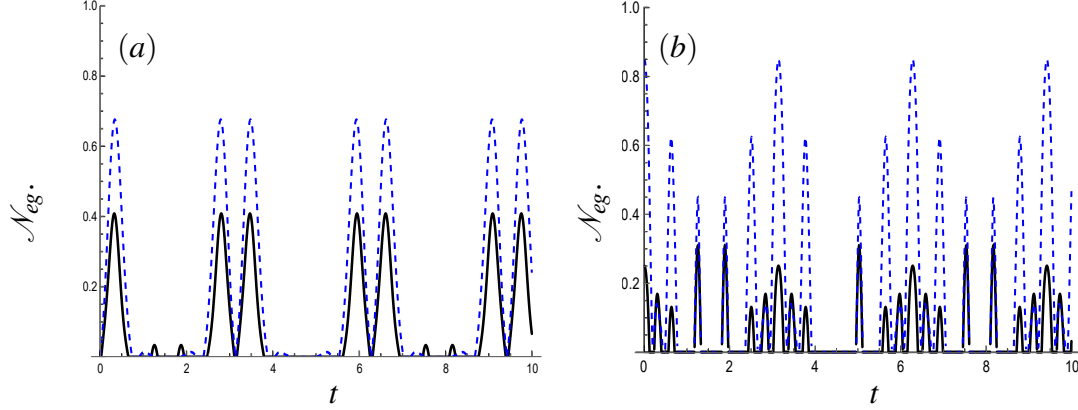


Figure 3.2: The quantum correlation of $\rho_{AB}(t)$, where $\omega = 2, \varepsilon = 2, \delta = 10$. The solid(black)curves, dash(blue) curves are evaluated at $\kappa = 0.5, 0.9$, respectively, where (a) $\alpha = 0, \gamma = 0$, and (b) $\alpha = \pi/4, \gamma = 0$.

is prepared in a pure state, namely $\rho_C = \frac{1}{\sqrt{2}}(|g\rangle\langle g| + |e\rangle\langle e|)$. The general behavior is similar to that depicted in Fig.(3.2), but with fast oscillations between the upper and lower bounds. However, this behavior depends on the initial state settings and the degree of coherence, where in Fig.(3.3.a), it is assumed that, the subsystems A and B are separable at $t = 0$. It is clear that, the QCs are smaller than those displayed in Fig.(3.2.a), where the qubit C is prepared in a super position pure state. Moreover, this observation is clearly displayed if one consider small values of κ , namely increasing the degree of coherence. However, the purity of the qubit C increases the quantum correlations of the state ρ_{AB} and decreasing the interval of vanishing them. These results can be depicted by comparing Figs.(3.2.b) and (3.3.b).

In Fig.(3.4.a), we investigate the effect of the interaction strength between the subsystems A and B , where we set a large values of ω compared with that has used in Fig.(3.2.a). The displayed behavior is similar to that shown in Fig.(3.2.a), where the sudden changes (death/birth) of the quantum correlations are depicted. However, the maximum amounts of the QCs and their number of oscillations are larger. This means that, by increasing the interaction strength ω , the amount of quantum correlations are more robust. The strength of the spin interaction δ is depicted by comparing Fig.(3.4.a)

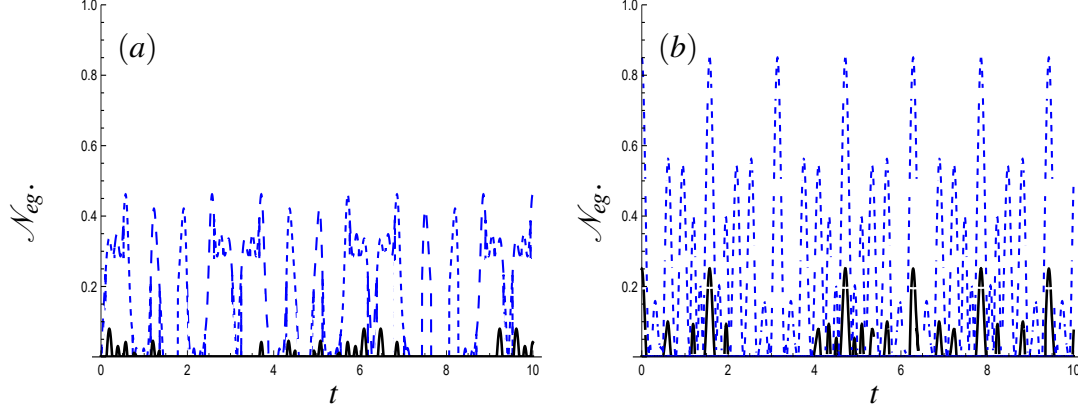


Figure 3.3: The same as Fig.(3.2), where (a) $\alpha = 0, \gamma = \frac{\pi}{4}$, and (b) $\alpha = \frac{\pi}{4}, \gamma = \frac{\pi}{4}$.

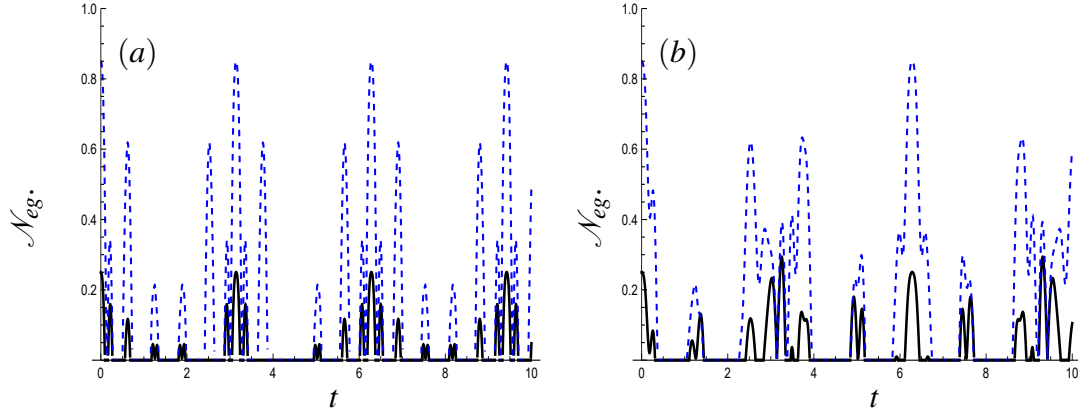


Figure 3.4: (a) The same as Fig.(3.2b) but $\omega = 5$, and (b) the same as (a) but $\delta = 5$.

and (3.4.b), where a small value of this strength is considered i.e., $\delta = 5$. The general behavior of the QCs is similar in both figures, but at small values of δ , the vanishing interval of the QCs increases.

3.3.3 Quantum correlations of $\rho_{AC}(t)$

In our suggested model, the two qubits A and C interacting locally via the Dipolar interaction, and consequently an entangled state will be generated between the two qubits. Therefore, it is important, to quantify the quantum correlations that may be contained in the state ρ_{AC} . The effect of the interaction parameters and the initial state settings on the QCs is investigated by means of the negativity, $\mathcal{N}_{eg}$.

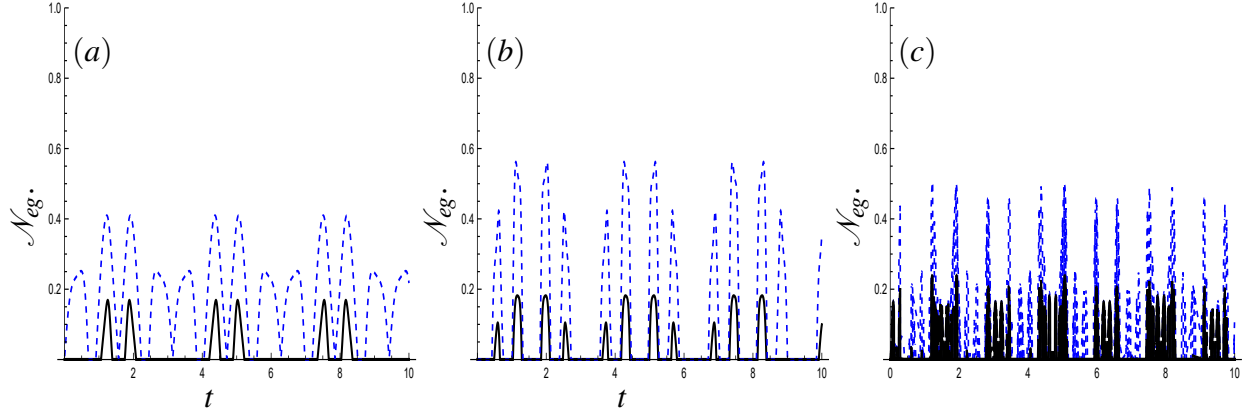


Figure 3.5: The same as Fig.(3.2) but for the subsystem ρ_{AC} . (a) $\alpha = 0, \gamma = 0$, (b) $\alpha = \pi/4, \gamma = 0$, (c) $\alpha = \pi/4, \gamma = \pi/4$.

Fig.(3.5) displays the behavior of QCs that may be generated between the two qubits A and C . It is clear that, at $\alpha = \gamma = 0$, namely the three qubits are separable, an entangled state is generated as soon as the interaction is switched on between the two qubits A and C . But the maximum bounds of these quantum correlations are much smaller than those displayed between the qubits A and B . At small values of κ , namely the coherence of the state ρ_{AC} is small, and consequently the initial correlations are strong, the QCs between the qubits A and C are generated at longer interaction time. The effect of the initial state settings of the state ρ_{AB} has a clear effect on the generated QCs between the partitions A and C , where if ρ_{AB} is prepared in a MES, then the entangled state ρ_{AC} needs a long interaction time to be generated. Moreover, the maximum bounds of its QCs are larger than those displayed at Fig.(3.5.a), where ρ_{AB} is initially prepared in a separable state.

Fig.(3.5.c), displays the amount of quantum correlations that transferred from the entangled state ρ_{AB} to the state ρ_{AC} , where it is assumed that ρ_{AB} is prepared in a MES. It is clear that, there is a large number of oscillations of the QCs , namely the phenomena of the sudden death/birth of QCs are displayed in a short interaction time. In other words, the possibility of exchanging the quantum correlations between the three qubits increases, if ρ_{AB} is initially prepared in a MES and the qubit C is in a superposition pure

state.

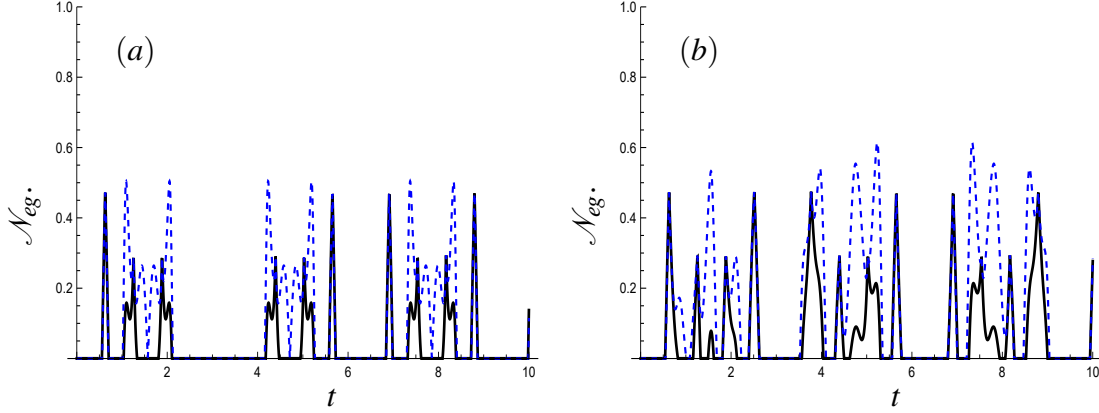


Figure 3.6: (a) The same as Fig.(3.5b), but $\omega = 5$, and (b) the same as (a) but $\delta = 5$.

To investigate the effect of the interaction parameter ω on the amount of the quantum correlations that may be generated between the two qubits A and C , we plot Fig.(3.6.a), where we set $\omega = 5$, namely the interaction strength between the qubits A and B is increased. It is clear that, one needs longer interaction time to generate entangled state between the two qubits A and C . Moreover, the interval of disappearing the quantum correlation increases, and the maximum values of the depicted QCs are smaller than those displayed in Fig.(3.5.b).

The effect of the spin interaction strength δ can be observed by comparing Figs.(3.6.a) and (3.6.b), where we set $\delta = 10, 5$, respectively. This means that, in Fig.(3.6.b), we consider a smaller value of the spin strength. It is clear that, the vanishing interval time of the QCs decreases, and the number of oscillations increases, namely the possibility of generating entangled state between the qubits A and B increases.

3.3.4 The quantum correlations of $\rho_{BC}(t)$

In this subsection, the possibility of generating quantum correlations between the qubits B and C is examined. Fig.(3.7) describes the behavior of $\mathcal{N}_{eg}$, as a measure of the amount of QCs , that may be generated between the

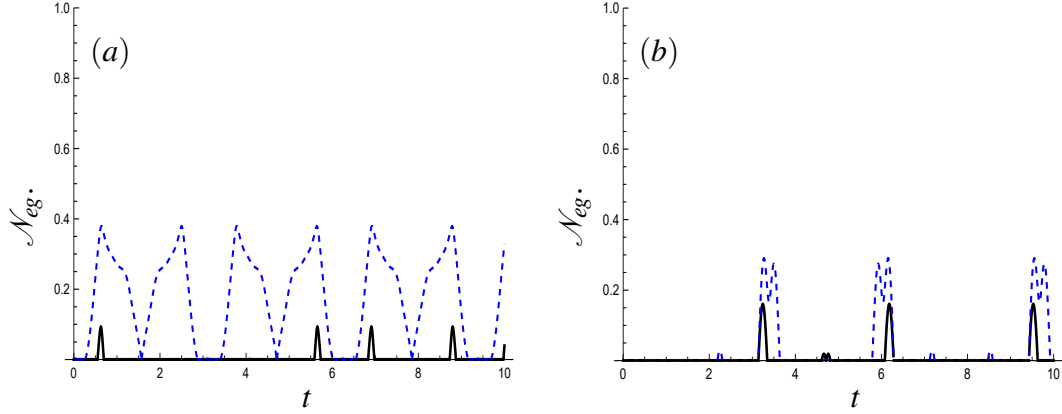


Figure 3.7: The QC of ρ_{BC} , where (a) $\alpha = 0, \gamma = 0$ and (b) $\alpha = \pi/4, \gamma = 0$. The solid and dash dot for $\kappa = 0.9, 0.5$, respectively, and $\omega = 2, \delta = 10, \varepsilon = 2$.

qubit B and C . It is clear that, the initial state settings of ρ_{AB} has a strong impact on the amount of QCs that has generated between the terminals of the state ρ_{BC} . However, if the three qubits are initially separable, the entangled state is generated at longer interaction time compared with that shown for the state ρ_{AC} as it is described in Fig.(3.5.a). In Fig.(3.7.b), it is assumed that ρ_{AB} is prepared in a MES, while $\rho_C = |g\rangle\langle g|$, the quantum correlation is generated at a longer interaction time, where $\mathcal{N}_{eg.}$ is very small compared with that displayed for ρ_{AC} .

3.4 Non-Markovianity Behavior of the three partition states

Due to this interaction, the ability of a system to restore its lost memory and information may be affected, so the non-Markovianity will be changed. Therefore, the effect of the initial state settings, and the interaction parameters on the behavior of the non-Markovianity of the subsystems will be discussed. The measure of non-Markovianity is giving in Chapter(1) in Eq(1.8).

In Fig.(3.8), we investigate the Markovianity of the three quantum chan-

nels, ρ_{AB} , ρ_{AC} , and ρ_{BC} . The predicted behavior shows that, the non-Markovianity increases as soon as the interaction is switched on. Moreover, an oscillation behavior of the Markovianity is displayed as one increases the interaction time, where the amplitudes of these oscillations depend on the coherence parameter κ . However, it vanishes periodically at different interaction time. Fig.(3.8.a), describes the behavior of the non-Markovianity $\mathcal{N}_{\rho_{AB}}$ of the state ρ_{AB} , where the amplitudes that displayed at large values of κ are larger than those depicted at small κ value. This means that, at small κ , the evolved state is more robust than that obtained at large κ . Namely the less entangled state has a stable memory compared with that displayed for large entangled state. Therefore, the possibility of exchanging information between the state ρ_{AB} and its environment increases as one increases the coherence parameter κ .

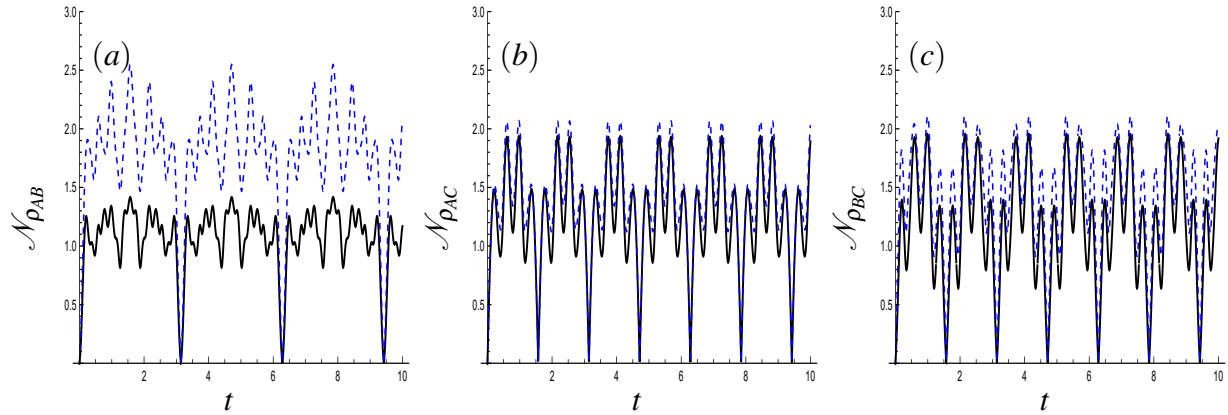


Figure 3.8: black line $\kappa = 0.5$ and blue line $\kappa = 0.9$, $w = 2$, $\varepsilon = 2$, $\delta = 10$, $\alpha = \frac{\pi}{4}$, $\gamma = \frac{\pi}{4}$. (a) ρ_{AB} , (b) ρ_{AC} and (c) ρ_{BC} .

Fig.(3.8b), describes the behavior of $\mathcal{N}_{\rho_{AC}}$ at the same values of interaction parameters. It is clear that $\mathcal{N}_{\rho_{AC}}$ behaves oscillatory, with smaller amplitudes compared with that shown in Fig.(3.8.a). The degree of coherence has a slightly effect on the behavior of the Markovianity of ρ_{AC} , where the displayed behavior of $\mathcal{N}_{\rho_{AC}}$, shows that it is almost similar at different values of κ . However, at small values of κ , the predicted Markovianity of $\mathcal{N}_{\rho_{AC}} > \mathcal{N}_{\rho_{AB}}$, namely the state $\mathcal{N}_{\rho_{AC}}$ has a smaller memory compared with

that displayed for ρ_{AB} . Moreover, the number of vanishing $\mathcal{N}_{\rho_{AC}}$ are larger than those displayed in Fig.(3.8.a). As it is shown from Fig.(3.8.c), a similar behavior is predicted for the $\mathcal{N}_{\rho_{BC}}$, where the oscillatory and vanishing phenomena of the Markovianity are displayed as one increases the interaction time. However, the oscillations number that shown in Fig.(3.8.c) are larger than those displayed in Fig.(3.8), at large values of the coherence parameter κ .

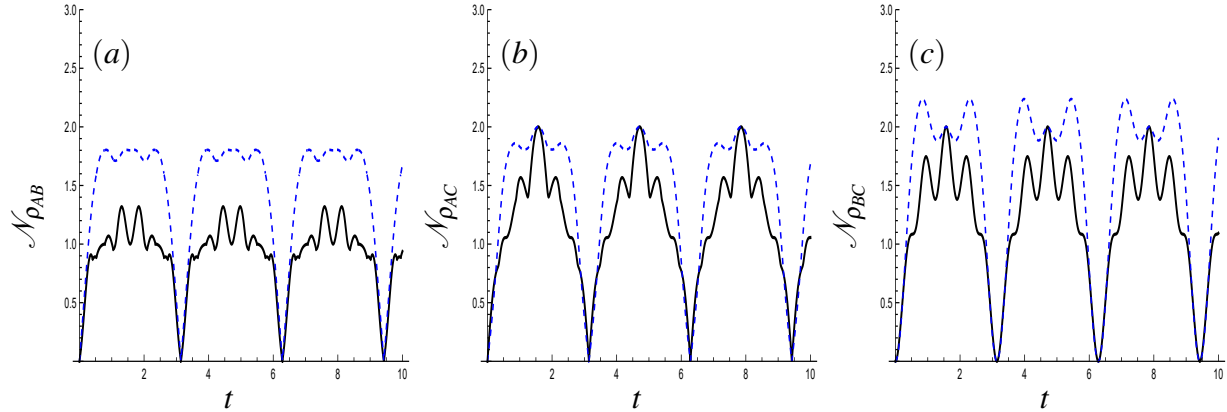


Figure 3.9: The same as Fig.(3.8) but at different initial state settings, where $\alpha = 0, \gamma = \frac{\pi}{4}$.

The effect of the initial state settings is displayed in Fig.(3.9), where it is assumed that, the state ρ_{AB} is initially prepared in a separable state and ρ_c in a pure superposition state. The displayed behavior of $\mathcal{N}_{\rho_{ij}}, i,j = AB, AC$ and BC , increases suddenly as soon as the interaction is switched on. The upper bounds of the Markovianity depend on the coherence degree of the state ρ_{AB} , where as one increases κ , all upper bounds of the predicted $\mathcal{N}_{\rho_{ij}}$ are smaller than those displayed at small value of κ . The initial state settings cause a stable behavior of $\mathcal{N}_{\rho_{ij}}$. Moreover, the predicted Markovianity of the state ρ_{BC} and ρ_{AC} , which are generated by indirect/direct interactions are larger than that displayed for ρ_{AB} .

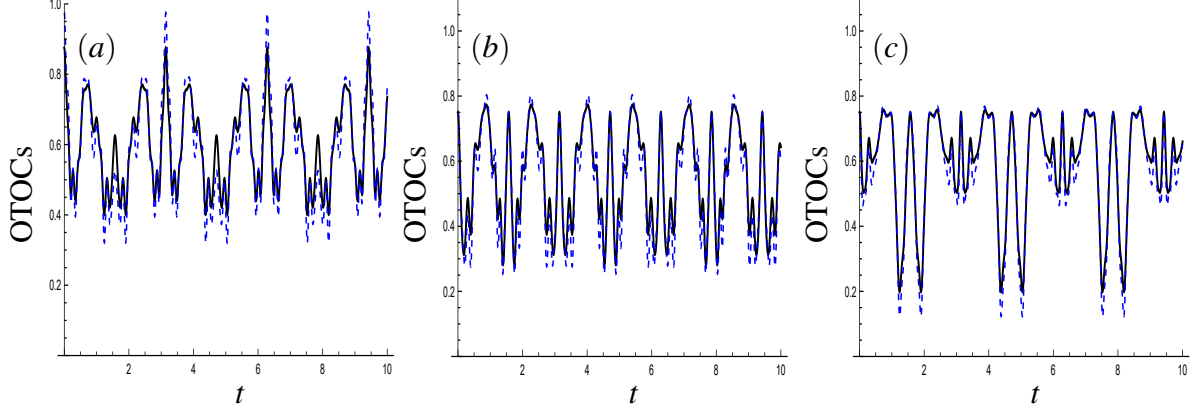


Figure 3.10: The behavior of the OTOCs function as a measure of scrambling information for (a) ρ_{AB} , (b) ρ_{AC} , and (c) ρ_{BC} . The black line $\kappa = 0.5$ and blue line $\kappa = 0.9$, $w = 2$, $\varepsilon = 2$, $\delta = 10$, $\alpha = \frac{\pi}{4}$, $\gamma = \frac{\pi}{4}$.

3.5 Out-of-time-order correlators

The Out-of time-ordered correlators function (OTOCs) is used to quantify the degree of information scrambling. The mathematical form of the measure is given in Chapter(1) in Eq(1.9)

In Fig(3.10), we investigate the behavior of OTOCs for the three states, where it is assumed that the system ρ_{AB} is prepared in a MES, while the ρ_C is prepared in a pure superposition state. The displayed behavior shows that, the OTOCs, oscillates between its upper and lower bounds, where the maximum bounds are displayed for ρ_{AB} , while the minimum bounds are depicted for ρ_{BC} . The predicted oscillations of the OTOCs for ρ_{AB} and ρ_{AC} are faster than those displayed for ρ_{BC} . The coherence operator has a slightly effect on the partitions ρ_{AB} and ρ_{AC} .

In Fig.(3.11), we investigate the effect of Dipolar interaction parameters on the behavior of the OTOCs, where different values of ε and δ are considered. Fig.(3.11a) describes the behavior of the OTOCs when the initial qubit system is prepared in a MES and the qubit C in a product state, where the other parameters are fixed as those in Fig.(3.10). It is clear that, the amplitudes of the OTOCs are smaller than those displayed in Fig.(3.10) for

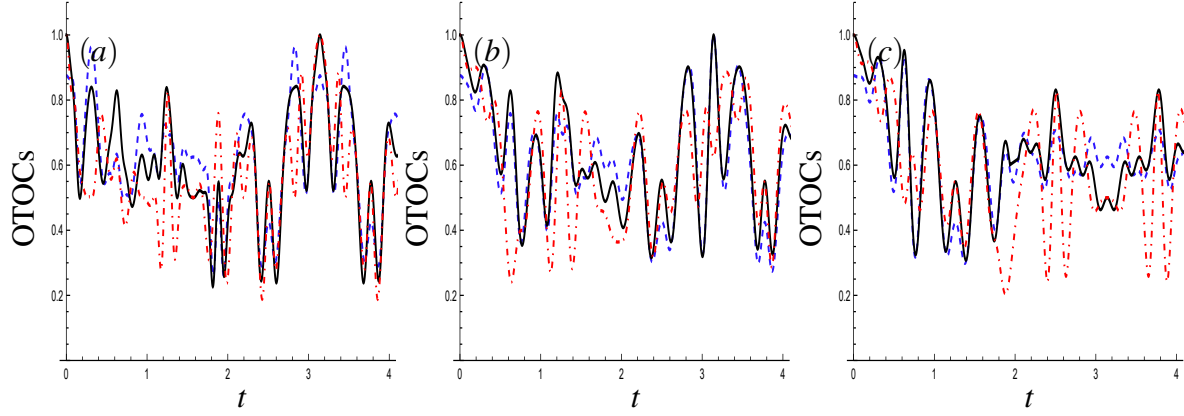


Figure 3.11: The behavior of the OTOCs, $\kappa = 0.5, w = 2, \varepsilon = 2, \delta = 10$. The dash (blue), solid (black) and the dot (red) curves for ρ_{AB}, ρ_{AC} , and ρ_{BC} , respectively, where $\alpha = \frac{\pi}{4}, \gamma = 0$, and (a) $\delta = 10$, (b) $\delta = 5$, (c) The same as (b) but $\varepsilon = 1$.

the three partitions. Due to the smaller number of oscillations, the speed of scrambling rate of information in Fig.(3.11.a) is smaller than that shown in Fig.(3.10). To examine the effect of the parameter δ , we compare the behavior of the OTOCs function in Figs.(3.11.a) and (3.11.b). It is clear that, at small values of δ , the scrambling rate from the systems ρ_{AB} and ρ_{AC} almost similar, while the fast scrambling rate is displayed for the third partition, ρ_{BC} . Moreover, from Figs.(3.11.b) and (3.11.c), one can observe that at small values of the interaction parameter ε , the information scrambling rate from ρ_{AB} and ρ_{AC} is smaller than that scrambled from ρ_{BC} .

From Figs.(3.10) and (3.11), one may conclude that, based on the behavior of the OTOCs function, the scrambled information from the partition ρ_{AB} is larger than those displayed for ρ_{AC} , and ρ_{BC} . The fast oscillations indicate that the information is exchanged fast and the system is memory less, namely, the scrambling information rate depends on the initial state settings. However, at small values of the interaction parameter the information scrambling from the two qubit system and that obtained by direct interaction is smaller than that scrambled from the system that is obtained by indirect interaction.

3.6 Discussion

In this chapter, a system of two qubits A and B is initially prepared in a maximum, partial entangled, or separable state. It is assumed that, one of its terminals interacts locally with a third qubit C , via Dipolar interaction. Due to this interaction, there are two new states may be generated between the three qubits, either by direct/indirect interaction. The generated new states effect some properties of the initial two qubit-state, ρ_{AB} . Therefore, we quantified the amount of correlations that may be exchanged between the three two- qubit states. The memory and memoryless behavior of the three states are discussed by investigating the non-Markovianity of the systems. Moreover, the amount of information that may be scrambled from/into the systems is discussed by means of the behavior of the out-time order function.

The obtained results, show that the coherence parameter plays an essential role on the amount of quantum correlations (QCs) that may be generated and exchanged between the three qubit states; ρ_{AB} , ρ_{AC} , and ρ_{BC} . If the total system is initially prepared in a separable state, then as soon as the interaction is switched on, an entangled state will be generated between the three partitions. The maximum bounds of the QCs depend on the degree of coherence, where at large values of this parameter, the interaction possibility with the surroundings decreases, and consequently the coherence degree of the generated entangled state increases. Moreover, the phenomena of the sudden death/birth of QCs are depicted periodically. Due to the interaction, these quantum correlations may be lost or gained, and thus an oscillatory behaviors appears. The possibility of exchanging the quantum correlations increases, if the initial two-qubit system ρ_{AB} is initially prepared in a maximum entangled state, and the third qubit ρ_C is prepared in a superposition pure state.

In this context, it is worth to mention that there is an entangled state between the qubits A and C has been generated via direct interaction and between B and C by indirect interaction. In general, the behavior of the quantum correlation is similar to that displayed for ρ_{AB} , but, the maximum bounds of the quantum correlations are smaller than those displayed for ρ_{AB} . Moreover, the sudden death of these correlations depends on the interaction parameters. To generate quantum correlations indirectly, between the qubits B and C , one needs a longer interaction time compared with that needed to generated quantum correlations between A and C . Moreover, the maximum values of the QCs that predicted for ρ_{BC} are very small compared with those for ρ_{AC} . The possibility of generating QCs between the qubits B and C decreases if the two-qubit system ρ_{AB} is prepared in a maximum entangled state and the qubit C is prepared in a supper position pure state.

It is well known that, the non-Makovanity measures the constantly of the memory of the evolved system. Therefore, investigating the non-Makovanity behavior of the three partitions is another aim of this chapter. It is shown that, the non-Markovianity behavior of the state ρ_{AB} increases as the coherence parameter (κ) increases, namely increases the coherence degree of the state ρ_{AB} , while the coherence parameter has a slightly effect on $\mathcal{N}_{\rho_{AC}}$ and $\mathcal{N}_{\rho_{BC}}$. For small value of κ , the oscillations of $\mathcal{N}_{\rho_{AB}}$ are smaller compared with those obtained at large κ . The initial state settings has a clear impact on the oscillations behavior of the non-Makovanity, where the numbers of oscillations depend on whether the two-qubit system is initially prepared in a MES or separable states. However, for separable the $\mathcal{N}_{\rho_{AB}}$ is the smallest one, while the largest value is predicted for $\mathcal{N}_{\rho_{BC}}$. This means that information exchanging depends on the initial state settings, where the exchanging rate of information increases if the two-qubit system is initially prepared in a maximum entangled state.

Finally, the amount of scrambled information between the three partitions

has quantified by using the Out-of time-ordered correlator (*OTOCs*). From the obtained results, one can say that the possibility of scrambling the information from the initial entangled two-qubit system is larger than that scrambles from the other two partitions, ρ_{AC} and ρ_{BC} . The impact of the coherence parameter κ , on the scrambled information from the partitions ρ_{AC} and ρ_{BC} is very small compared with that scrambled from the two-qubit system. The smallest amount of the scrambled information is displayed for the partition ρ_{BC} . The effect of the interaction parameter has a clear effect on the scrambling rate of information, where the number of oscillations increase.

In conclusion, Dipolar interaction has the ability to generate an entangled quantum network between three- qubit systems. The amount of quantum correlations and information can be distributed between the nodes of this network. The exchange rate of quantum correlations and information depends on whether the node is added by direct or indirect interaction. The interaction parameters and the initial state settings could be used as control parameters to improve the efficiency of the generated quantum network.

Chapter4

Dynamics of Non-Markovinaity, quantum correlations and information of three qubits systems interacting via Rashba interaction

4.1 Introduction

Due to the decoherence, systems may lose their memory and information, but some systems can restore their lost information (Khan et al., 2022). Recently, it is founded that the non-Markovian process affects the system's decoherence and disentanglement (Chiang & Zhang, 2021). Therefore, the non-Markovianity has become a very important topic and has been studied for both continuous and discrete variable quantum systems (He, Zou, Li, & Shao, 2011). Based on the non-Markovianity of the system, there will be

an exchanging of the quantum correlations and the information between the three qubits (Wang et al., 2023).

There are some known interactions that has been used recently to generate entanglement between different qubit systems. Among of theses interaction is the Dzyaloshinskii-Moriya (DM) (T. Moriya, 1960; Qiang et al., 2009), Dipolar interaction (Oliveira, Sarthour Jr, Bonagamba, Azevedo, & Freitas, 2011; Mohammed, Ahmed, et al., 2021). Moreover, Rashba interaction (Bychkov & Rashba, 1984; Ferreira et al., 2023) is used in many application of quantum information. Therefore, we are motivated to investigate the effect of Rashba interaction on the exchanging process of the quantum correlations and the information between three qubits system.

The outline of this chapter is organized as following: In Sec.(4.2), we introduce the suggested model and its time evolution analytically. The dynamics of the quantum correlations is discussed in Sec.(4.3), where we used the negativity as a quantifier of the quantum correlations. Sec.(4.4) is devoted to investigate the non-Markovianity of each two-qubit systems. The behavior of the encoded information in each partition is considered in Sec.(4.5). Finally, results are summarized in Sec.(4.6).

4.2 Suggested Model

The suggested system consists of two qubits A , and B , which have been prepared in a partial entangled state. The dynamics of system is governed by a Heisenberg XX spin model. It is assumed that, one terminal of the state ρ_{AB} , say the subsystem A , interacts locally with a third qubit C , via Rashba interaction (Bychkov & Rashba, 1984; Ferreira et al., 2023). The total Hamiltonian which describes this system is given by,

$$\hat{H}_{sys} = \hat{H}_{Heis} + \hat{H}_{Rash}, \quad (4.1)$$

where H_{Heis} represents the Hamiltonian of the XX Heisenberg interaction between the two qubits A, B (Qiang et al., 2009), and H_{Rash} represents the Hamiltonian of the Rashba interaction between qubits A and C (Ferreira et al., 2023). Mathematically, these Hamiltonian are described by,

$$\begin{aligned} H_{Heis} &= \frac{1}{2}\omega\left(\sigma_A^x\sigma_B^x + \sigma_A^y\sigma_B^y\right), \\ H_{Rash} &= \frac{\varepsilon}{2}\left(I \otimes \tau_z^C\right) + \beta\left(\sigma_x^A \otimes I\right) - \zeta\left(\sigma_y^A \otimes \tau_x^C\right), \end{aligned} \quad (4.2)$$

where $\sigma_i, \tau_j, i, j = x, y, z$ represent Pauli operators, ω is the Heisenberg interaction spin strength between the qubits A and B , ε is the Zeeman splitting generated by an external constant magnetic field along the z -axis, β is the strength of the tunneling coupling between the two qubits A and C , and ζ is the interaction strength, and the spin-flip tunnel coupling (Bychkov & Rashba, 1984; Y.-C. Li, Chen, Muga, & Sherman, 2018; Ferreira et al., 2023; Ebrahim & Metwally, 2023a).

Let us assume that, the initial state of the whole system at time $t = 0$ is given by,

$$\rho_s(0) = \rho^{AB}(0) \otimes \rho^C(0), \quad (4.3)$$

where,

$$\rho^{AB}(0) = \kappa \left| \varphi^{ab} \right\rangle \left\langle \varphi^{ab} \right| - \frac{1}{4}(1 - \kappa)I_{4 \times 4}, \quad \rho^C(0) = \left| \varphi^c \right\rangle \left\langle \varphi^c \right|, \quad (4.4)$$

with $\left| \varphi^{AB} \right\rangle = \cos(\alpha) \left| ge \right\rangle + \sin(\alpha) \left| eg \right\rangle$, and $\left| \varphi^c \right\rangle = \cos \gamma \left| g \right\rangle + \sin \gamma \left| e \right\rangle$, $I_{4 \times 4}$ is the identity matrix. The time evaluation of the system at any arbitrary time t is given by,

$$\rho_s(t) = \mathcal{U}(t)\rho_s(0)\mathcal{U}(t)^\dagger, \quad \mathcal{U}(t) = \exp[-iH_{sys}t]. \quad (4.5)$$

Then by using the unitary operator $\mathcal{U}(t)$ and the initial state $\rho_s(0)$, one gets the final state $\rho_s(t)$ at any $t > 0$. Now, we can obtain the state between each two subsystems by tracing the third system, namely $\rho_{ij} = Tr_k\{\rho_{ijk}\}$.

For example $\rho_{AC} = \text{Tr}_B\{\rho_{ABC}\}$. The unitary operator of the system may be written explicitly as ,

$$\mathcal{U}(t) = \begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} & u_{04} & u_{05} & u_{06} & u_{07} \\ u_{10} & u_{11} & u_{12} & u_{13} & u_{14} & u_{15} & u_{16} & u_{17} \\ u_{20} & u_{21} & u_{22} & u_{23} & u_{24} & u_{25} & u_{26} & u_{27} \\ u_{30} & u_{31} & u_{32} & u_{33} & u_{34} & u_{35} & u_{36} & u_{37} \\ u_{40} & u_{41} & u_{42} & u_{43} & u_{44} & u_{45} & u_{46} & u_{47} \\ u_{50} & u_{51} & u_{52} & u_{53} & u_{54} & u_{55} & u_{56} & u_{57} \\ u_{60} & u_{61} & u_{62} & u_{63} & u_{64} & u_{65} & u_{66} & u_{67} \\ u_{70} & u_{71} & u_{72} & u_{73} & u_{74} & u_{75} & u_{76} & u_{77} \end{pmatrix}, \quad (4.6)$$

where the elements u_{ij} are given by,

$$\begin{aligned} u_{00} &= \mu_- C_\beta C_\zeta, & u_{01} &= i\mu_- S_\beta S_\zeta, & u_{04} &= -i\mu_- C_\zeta S_\beta, \\ u_{05} &= \mu_- C_\beta S_\zeta, & u_{10} &= i\mu_+ S_\beta S_\zeta, & u_{11} &= i\mu_+ C_\beta C_\zeta, \\ u_{14} &= i\mu_+ C_\beta S_\zeta, & u_{15} &= -i\mu_+ C_\zeta S_\beta, & u_{20} &= -\mu_- C_\zeta S_\omega S_\beta, \\ u_{21} &= i\mu_- C_\beta S_\omega S_\zeta, & u_{22} &= i\mu_- C_\beta C_\omega C_\zeta, & u_{23} &= i\mu_- C_\omega S_\beta S_\zeta, \\ u_{24} &= -i\mu_- C_\beta C_\zeta S_\omega, & u_{25} &= -\mu_- S_\omega S_\beta S_\zeta, & u_{26} &= -i\mu_- C_\omega C_\zeta S_\beta, \\ u_{27} &= \mu_- C_\omega C_\beta S_\zeta, & u_{30} &= i\mu_+ C_\beta S_\omega S_\zeta, & u_{31} &= -\mu_+ C_\zeta S_\omega S_\beta, \\ u_{32} &= i\mu_+ C_\omega S_\beta S_\zeta, & u_{33} &= \mu_+ C_\beta C_\omega C_\zeta, & u_{34} &= -\mu_+ S_\omega S_\beta S_\zeta, \\ u_{35} &= -i\mu_+ C_\beta C_\zeta S_\omega, & u_{36} &= \mu_+ C_\omega C_\beta S_\zeta, & u_{37} &= -i\mu_+ C_\omega C_\zeta S_\beta. \end{aligned} \quad (4.7)$$

where, $C_\beta = \cos(\beta t)$, $S_\beta = \sin(\beta t)$, $C_\omega = \cos(\omega t)$, $S_\omega = \sin(\omega t)$, $C_\zeta = \cos(\zeta t)$, $S_\zeta = \sin(\zeta t)$, and $\mu_\pm = \cos\left(\frac{t\varepsilon}{2}\right) \pm i \sin\left(\frac{t\varepsilon}{2}\right)$.

The remaining elements of the unitary operator are given by,

$$\begin{aligned} u_{40} &= u_{26}, & u_{41} &= -u_{27}, & u_{42} &= u_{24}, & u_{43} &= -u_{25}, & u_{44} &= u_{22}, \\ u_{45} &= -u_{23}, & u_{46} &= u_{20}, & u_{47} &= -u_{21}, & u_{50} &= -u_{36}, & u_{51} &= u_{37}, \\ u_{52} &= -u_{34}, & u_{53} &= -u_{25}, & u_{54} &= -u_{32}, & u_{55} &= u_{33}, & u_{56} &= -u_{30}, \end{aligned}$$

$$\begin{aligned}
u_{57} &= u_{31}, & u_{60} &= u_{00}, & u_{62} &= u_{04}, & u_{63} &= -u_{05}, & u_{67} &= -u_{01}, \\
u_{72} &= -u_{14}, & u_{73} &= u_{15}, & u_{76} &= -u_{10}, & u_{77} &= u_{11}.
\end{aligned} \tag{4.8}$$

The states ρ_{AB} described by the elements:

$$\begin{aligned}
\rho_{00}^{ab} &= \frac{1}{4} \left(1 - \kappa C_{2\alpha} - \kappa [C_{\eta_{-1}} + C_{\eta_{+1}} S_{\alpha}^2] \right), \\
\rho_{01}^{ab} &= \frac{1}{2} \kappa S_{\alpha} \left(\chi_1^- S_{2t\beta} + \chi_2^+ S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{02}^{ab} &= \frac{1}{2} \kappa \left(S_{\alpha} \chi_3^- S_{2t\beta} + \chi_3^+ S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{03}^{ab} &= \frac{1}{4} \kappa S_{2t\alpha} \left(-C_{2t\beta} + C_{2t\zeta} + i S_{2t\beta} S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{10}^{ab} &= \frac{1}{2} \kappa S_{\alpha} \left(\chi_1^+ S_{2t\beta} + \chi_2^- S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{11}^{ab} &= \frac{1}{16} \left(4 + \kappa C_{2\alpha} [4 C_{2t\omega} + C_{\eta_{-3}} + C_{\eta_{+4}} + C_{\eta_{-4}} + C_{\eta_{+3}}] + 2\kappa [C_{\eta_{-1}} \right. \\
&\quad \left. + C_{\eta_{+1}} - 2\chi_5] \right), \\
\rho_{12}^{ab} &= \frac{1}{8} \kappa \left(i C_{2\alpha} \chi_6 S_{2t\omega} + 2 S_{2t\alpha} [C_{2t\beta} + C_{2t\zeta} + i\chi_9] \right), \\
\rho_{13}^{ab} &= \frac{-i}{2} \kappa C_{\alpha} \left(\chi_8^+ - S_{2t\beta} + \chi_7^+ S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{20}^{ab} &= \frac{1}{2} \kappa S_{\alpha} \left(\chi_3^+ S_{2t\beta} + (\chi_4^+)^* S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{21}^{ab} &= -\frac{1}{8} i \kappa \left(C_{2\alpha} \chi_6 S_{2t\omega} + 2 S_{2t\alpha} [i C_{2t\beta} + C_{2t\zeta}] + \chi_9 \right), \\
\rho_{22}^{ab} &= \frac{1}{16} \left(4 - \kappa C_{2\alpha} [4 C_{2t\omega} + C_{\eta_{-3}} + C_{\eta_{+4}} + C_{\eta_{-4}} + C_{\eta_{+3}}] + 2\kappa [C_{\eta_{-1}} \right. \\
&\quad \left. + C_{\eta_{+1}} + 2\chi_5] \right), \\
\rho_{23}^{ab} &= \frac{1}{2} \kappa C_{\alpha} \left([C_{\alpha} C_{2t\zeta} S_{t\omega} + i C_{t\omega} S_{\alpha}] S_{2t\beta} + [i C_{\alpha} S_{t\omega} - C_{t\omega} C_{2t\beta} S_{\alpha}] \right. \\
&\quad \left. S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{30}^{ab} &= -\frac{1}{4} \kappa S_{2t\alpha} \left(C_{2t\beta} - C_{2t\zeta} + i S_{2t\beta} S_{2\gamma} S_{2t\zeta} \right), \\
\rho_{31}^{ab} &= \frac{1}{2} \kappa C_{\alpha} \left(-i \chi_8^+ S_{2t\beta} - \chi_7^+ S_{2\gamma} S_{2t\zeta} \right),
\end{aligned}$$

$$\begin{aligned}\rho_{32}^{ab} &= -\frac{1}{4}\kappa\left(C_{t\omega} S_{2t\alpha}[i S_{2t\beta} + C_{2t\beta} S_{2\gamma} S_{2t\zeta}] + i C_{\alpha}^2 S_{t\omega}[-\chi_{10}^- + i\chi_{12}^+]\right), \\ \rho_{33}^{ab} &= \frac{1}{4}\left(1 + \kappa C_{2\alpha} - \kappa C_{\alpha}^2 \chi_{13}^+\right).\end{aligned}$$

Similarly the state ρ_{ac} is described by:

$$\begin{aligned}\rho_{00}^{ac} &= \frac{1}{8}\left(2\kappa C_{t\omega}^2 C_{2\alpha} C_{2t\beta} \chi_{15}^+ - 2\chi_{14}^+ \chi_{16}^- - \kappa \chi_5\right), \\ \rho_{01}^{ac} &= \frac{1}{32}e^{-it\varepsilon}\left(-8\kappa C_{\gamma} S_{\alpha}^2 S_{t\beta}^2 S_{\gamma}[C_{t\zeta}^2 S_{t\omega}^2 + S_{t\zeta}^2] + S_{2\gamma}[8 + \kappa C_{\alpha}^2\right. \\ &\quad \left.(-4 + 4 C_{2t\omega} + \chi_{21}^+ + \chi_{22}^+ + 2\chi_{13}^+) - 2\kappa S_{\alpha}^2(-2 + 2 C_{2t\omega} + C_{t\omega}^2(C_{\eta-1}\right. \\ &\quad \left.+ 2 C_{t\beta}^2 C_{2t\zeta} + C_{\eta+1}) - 2 S_{t\beta}^2[C_{t\zeta}^2 + S_{t\omega}^2 S_{t\zeta}^2])\right] + 8i C_{t\omega} \chi_{23}^+ \\ &\quad \left. S_{2t\beta} S_{2t\zeta}\right), \\ \rho_{02}^{ac} &= \frac{1}{16}\kappa\left(S_{t\omega} S_{2t\alpha}[-ie^{-2i(\gamma+t(\beta+\zeta))}(e^{4it\beta} - e^{4i\gamma} - e^{4it\zeta} + e^{4i(\gamma+t(\beta+\zeta))}) +\right. \\ &\quad \left.4 S_{2t\beta}] + 2 C_{t\omega} C_{2\alpha}[\chi_{10}^- + i(\chi_{11}^+ + \chi_{12}^+)]\right), \\ \rho_{03}^{ac} &= \frac{1}{4}e^{-it\varepsilon}\left(\kappa i \chi_8^- S_{2t\beta} S_{2\gamma} - [C_{t\omega} \chi_{17}^+ - i\kappa C_{2t\beta} S_{t\omega} S_{2t\alpha}] S_{2t\zeta}\right), \\ \rho_{10}^{ac} &= (\rho_{10}^{ac})^*, \\ \rho_{11}^{ac} &= \frac{1}{8}(2\kappa C_{t\omega}^2 C_{2\alpha} C_{2t\beta} \chi_{15}^- + 2\chi_{14}^- \chi_{16}^- - \kappa \chi_5), \\ \rho_{12}^{ac} &= \frac{1}{4}e^{it\varepsilon}\left(\kappa[i\chi_8^- S_{2t\beta} S_{2\gamma} + (C_{t\omega} \chi_{17}^- + i\kappa C_{2t\beta} S_{t\omega} S_{2t\alpha}) S_{2t\zeta}]\right), \\ \rho_{13}^{ac} &= -\frac{1}{8}\kappa\left(2 S_{t\omega} S_{2t\alpha}[\chi_{14}^- S_{2t\beta} - i C_{2t\beta} S_{2\gamma} S_{2t\zeta}] + C_{t\omega} C_{2\alpha}[-\chi_{10}^- +\right. \\ &\quad \left.i(\chi_{11}^+ - \chi_{18}^+)]\right), \\ \rho_{20}^{ac} &= \frac{1}{8}\kappa\left(2 S_{t\omega} S_{2t\alpha}[\chi_{14}^+ S_{2t\beta} - i C_{2t\beta} S_{2\gamma} S_{2t\zeta}] + C_{t\omega} C_{2\alpha}[-\chi_{10}^- +\right. \\ &\quad \left.i(\chi_{11}^+ + \chi_{18}^+)]\right), \\ \rho_{21}^{ac} &= \frac{1}{4}e^{-it\varepsilon}\left(-i\kappa \chi_8^- S_{2t\beta} S_{2\gamma} + [C_{t\omega} \chi_{17}^- - i\kappa C_{2t\beta} S_{t\omega} S_{2t\alpha}] S_{2t\zeta}\right), \\ \rho_{22}^{ac} &= \frac{1}{8}(-2\kappa C_{t\omega}^2 C_{2\alpha} C_{2t\beta} \chi_{15}^+ + 2\chi_{14}^+ \chi_{16}^+ + \kappa \chi_5),\end{aligned}$$

$$\begin{aligned}
\rho_{23}^{ac} &= \frac{1}{32} e^{-it\varepsilon} \left(-4\kappa C_{\eta+7} C_\gamma S_\gamma - (-8 + 2\kappa C_{\eta-7} + \kappa C_{2\alpha}[-4 + \chi_{21}^+ + \chi_{22}^+ + 2\chi_{13}^+]) S_{2\gamma} + 4 S_{2t\beta} S_{2t\zeta} \chi_{24}^+ \right), \\
\rho_{30}^{ac} &= \frac{1}{4} e^{it\varepsilon} \left(\kappa \chi_{24}^+ S_{2t\beta} S_{2\gamma} - (C_{t\omega}[\kappa C_{2\alpha} + C_{2t\beta} C_{2\gamma}] + i\kappa C_{2t\beta} S_{t\omega} S_{2t\alpha}) S_{2t\zeta} \right), \\
\rho_{31}^{ac} &= \frac{1}{16} \left(i e^{-2i\eta+8} [e^{4it\beta} - e^{4i\gamma} - 2e^{i\eta+5}(-1 + e^{4it\beta}) + e^{4it\zeta}(-1 + e^{2i\eta+6})] \right. \\
&\quad \left. \kappa S_{t\omega} S_{2t\alpha} + 2\kappa C_{t\omega} C_{2\alpha}[\chi_{10}^- + i(\chi_{21}^+ - \chi_{11}^+)] \right), \\
\rho_{32}^{ac} &= \frac{-1}{32} e^{it\varepsilon} \left(-4\kappa C_{\eta+7} C_\gamma S_\gamma + (-8 + 2\kappa C_{\eta-7} + \kappa C_{2\alpha}[-4 + \chi_{21}^+ + \chi_{22}^+ + 2\chi_{13}^+]) S_{2\gamma} + 4 S_{2t\beta} S_{2t\zeta} \chi_{24}^- \right), \\
\rho_{33}^{ac} &= \frac{1}{8} \left(2\kappa C_{t\omega}^2 C_{2\alpha} C_{2t\beta} \chi_{25} - 2\chi_{-14} \chi_{+16} + \kappa \chi_{26} \right).
\end{aligned}$$

The elements the state ρ_{BC} are:

$$\begin{aligned}
\rho_{00}^{bc} &= \frac{1}{8} \left(-2\chi_{18}^- \chi_{14}^+ + 2\kappa C_{2\alpha} C_{2t\beta} \chi_{15}^+ S_{t\omega}^2 + \kappa \chi_5 \right), \\
\rho_{01}^{bc} &= \frac{1}{32} e^{-it\varepsilon} \left(-4\kappa C_{\eta+7} C_\gamma S_\gamma - [-8 + 2\kappa C_{\eta-7} + \kappa C_{2\alpha}(4 + \chi_{22}^+ - 2\chi_{13}^+ + \chi_{21}^+)] S_{2\gamma} + 4(\chi_{21}^+)^* S_{2t\beta} S_{2t\zeta} \right), \\
\rho_{02}^{bc} &= \frac{1}{8} \kappa S_{t\omega} \left(i C_{\eta-5} - i C_{\eta+5} + \chi_{11}^+ + \chi_{12}^+ \right), \\
\rho_{03}^{bc} &= \frac{1}{8} e^{-it\varepsilon} \kappa \left(2\chi_{20} + 2i[S_{t\omega} + C_{2\gamma} \chi_4^-] S_{2t\zeta} \right), \\
\rho_{10}^{bc} &= (\rho_{01}^{bc})^* \\
\rho_{11}^{bc} &= \frac{1}{8} \left(2\chi_{18}^- \chi_{14}^- + 2\kappa C_{2\alpha} C_{2t\beta} \chi_{15}^- S_{t\omega}^2 + \kappa \chi_5 \right), \\
\rho_{12}^{bc} &= \frac{1}{4} e^{it\varepsilon} \kappa \left(\chi_{20} + [-i\chi_{19}^- S_{t\omega} + C_{t\omega} C_{2\gamma} S_{2t\alpha}] S_{2t\zeta} \right), \\
\rho_{13}^{bc} &= \frac{1}{8} \kappa S_{t\omega} \left(-i\chi_{10}^- - \chi_{11}^+ + \chi_{12}^+ \right), \\
\rho_{20}^{bc} &= \frac{1}{8} \kappa S_{t\omega} \left(i\chi_{10}^- + \chi_{11}^+ + \chi_{12}^+ \right),
\end{aligned}$$

$$\begin{aligned}
\rho_{21}^{bc} &= \frac{1}{4} e^{-it\varepsilon} \left(\kappa \chi_{20} + \kappa [i\chi_{19}^- S_{t\omega} + C_{t\omega} C_{2\gamma} S_{2t\alpha}] S_{2t\zeta} \right), \\
\rho_{22}^{bc} &= \frac{1}{8} \left(2\chi_{18}^+ \chi_{14}^+ - 2\kappa C_{2\alpha} C_{2t\beta} \chi_{15}^+ S_{t\omega}^2 - \kappa \chi_5 \right), \\
\rho_{23}^{bc} &= \frac{1}{32} e^{-it\varepsilon} \left([8 + \kappa C_{2\alpha} (4 + 4 C_{2t\omega} + \chi_{22}^- - \chi_{13}^+ + \chi_{21}^+)] S_{2\gamma} + 4(\chi_{23}^-)^* \right. \\
&\quad \left. S_{2t\beta} S_{2t\zeta} \right), \\
\rho_{30}^{bc} &= \frac{1}{8} e^{it\varepsilon} \kappa \left(2\chi_{20} - 2i [S_{t\omega} - i C_{2\gamma} \chi_4^+] S_{2t\zeta} \right), \\
\rho_{31}^{bc} &= \frac{1}{8} \kappa S_{t\omega} \left(i\chi_{10}^- - \chi_{11}^+ + \chi_{12}^+ \right), \quad \rho_{32}^{bc} = (\rho_{23}^{BC})^* \\
\rho_{33}^{bc} &= \frac{1}{8} \left(-2\chi_{18}^+ \chi_{14}^- - 2\kappa C_{2\alpha} C_{2t\beta} \chi_{15}^- S_{t\omega}^2 - \kappa \chi_5 \right),
\end{aligned}$$

where C_v, S_v are $\cos(v)$ and $\sin(v)$ respectively, and η_i, χ_j are defined as follow :

$$\begin{aligned}
\eta_{\pm 1} &= 2t(\beta \pm \zeta), \quad \eta_{\pm 2} = 2t(2\omega + \beta \pm \zeta), \quad \eta_{\pm 3} = 2t(2\omega \pm (\beta + \zeta)), \\
\eta_{\pm 4} &= 2t(2\omega \pm (\beta - \zeta)), \quad \eta_{\pm 5} = 2(\gamma \pm \zeta), \quad \eta_{\pm 6} = 2(t\beta \pm \gamma), \\
\eta_{\pm 7} &= 2(t\omega \pm \alpha), \quad \eta_{\pm 8} = \gamma \pm t(\beta + \zeta), \\
\chi_1^\pm &= \pm i C_{t\omega} C_\alpha + C_{2t\zeta} S_{t\omega} S_\alpha, \quad \chi_2^\pm = C_t C_\alpha C_{2t\beta} \pm i S_{t\omega} S_\alpha, \\
\chi_{\pm 3} &= C_\alpha S_t \pm i C_{t\omega} C_{2\zeta} S_\alpha, \quad \chi_{\pm 4} = i C_\alpha C_{2t\beta} S_{t\omega} \pm C_{t\omega} S_\gamma, \\
\chi_{\pm 5} &= S_{2t\omega} S_{2t\alpha} S_{2t\beta} S_{2\gamma} S_{2t\zeta}, \quad \chi_{\pm 6} = 2 + C_{\eta_{-1}} + C_{\eta_{+1}}, \\
\chi_{\pm 7} &= C_{2\beta} S_{t\omega} S_\alpha \pm i C_{t\omega} C_\alpha, \quad \chi_{\pm 8} = C_{t\omega} C_\alpha C_{2t\zeta} \pm i S_{t\omega} S_\alpha, \\
\chi_{\pm 9} &= C_{t\omega} S_{2t\beta} S_{2\gamma} S_{2t\zeta}, \quad \chi_{\pm 10} = C_{\eta_{+5}} \pm C_{\eta_{-5}}, \quad \chi_{\pm 11} = S_{\eta_{+16} \pm S_{\eta_{-6}}}, \\
\chi_{\pm 12} &= S_{\eta_{+1}} \pm S_{\eta_{-1}}, \quad \chi_{\pm 13} = C_{\eta_{+}} \pm C_{\eta_{-1}}, \quad \chi_{\pm 14} = \pm 1 + C_{2\gamma} C_{2t\zeta}, \\
\chi_{\pm 15} &= C_{2t\delta} \pm C_{2\gamma}, \quad \chi_{\pm 16} = \pm 1 + \kappa C_{2\alpha} S_{t\omega}^2, \quad \chi_{\pm 17} = \pm \kappa C_{2\alpha} + C_{2\beta} C_{2\gamma}, \\
\chi_{\pm 18} &= \pm 1 + \kappa C_{t\omega}^2 C_{2\alpha}, \quad \chi_{\pm 19} = \pm 1 + C_{2\alpha} C_{2t\beta} C_{2\gamma}, \\
\chi_{20} &= C_{2t\zeta} S_{t\omega} S_{2t\beta} S_{2\gamma}, \quad \chi_{\pm 21} = C_{\eta_{+4}} \pm C_{\eta_{-4}}, \quad \chi_{\pm 22} = C_{\eta_{+3}} \pm C_{\eta_{-3}}, \\
\chi_{\pm 23} &= -2i C_{2\gamma} S_{t\omega}^2 \pm \kappa S_{t\omega} S_{2t\alpha}, \quad \chi_{\pm 24} = 2i C_{t\omega}^2 C_{2\gamma} \pm \kappa S_{2t\omega} S_{2t\alpha}, \\
\chi_{25} &= C_{2\gamma} - C_{2t\zeta}, \quad \chi_{26} = S_{2t\omega} S_{2\alpha} S_{2t\beta} S_{2\gamma} S_{2t\zeta}.
\end{aligned}$$

Now, we have all the details to quantify the amount of the quantum correlations QCs , that may be generated between the qubits A, C and between B, C via Rashba interaction, as well as, the amount of information that may be gained or lost from the initial system ρ_{AB} .

4.3 Quantum correlation (QCs)

In this section, we investigate the quantum correlations behavior by means of the negativity, $\mathcal{N}_{eg}$. The mathematical form of the negativity is given in Chapter(1) in Eq(1.7).

4.3.1 Quantum correlation of $\rho_{AB}(t)$

The behavior of the QCs , between the qubits A and B is described in Fig.(4.1.a), where it is assumed that, the two-qubit system $\rho_{AB}(0)$ is initially prepared in a maximum entangled state (MES), while the qubit C , is prepared in a product state such that $\rho_c = |0\rangle\langle 0|$, and different values of the coherence parameter κ are considered. It is clear that, at $t = 0$, the negativity $\mathcal{N}_{eg}$ oscillates between its upper and lower bounds, where the upper bounds are displayed at large κ , namely at large degree of coherence. Moreover, the phenomenon of the entanglement death/birth are displayed periodically, where the predicted time interval of its disappearance at small κ is large, while it disappears temporary as one increases the coherence of the initial state. Fig.(4.1.b) describes the behavior of the negativity at different initial state settings, where it is assumed that, the two-qubit state ρ_{AB} is prepared in a maximum entangled state (MES), while the qubit C is prepared in a superposition pure state. The negativity behavior is similar to that displayed in Fig.(4.1.a), but the possibility of vanishing the QCs decreases at small values of the coherence parameter κ , while at large κ it doesn't vanish. Moreover, the number of oscillations decreases and the minimum values of the

non-vanishing entanglement are better than those displayed in Fig.(4.1.a).

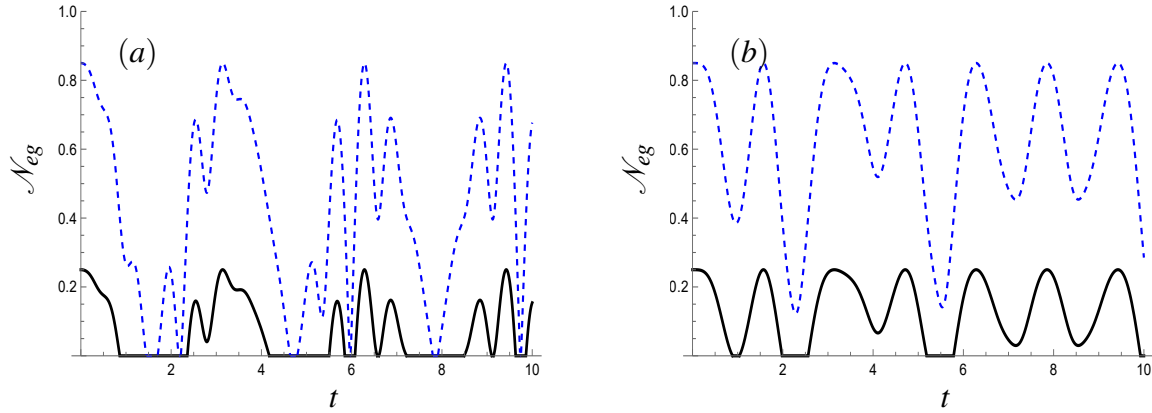


Figure 4.1: The amount of quantum correlation of the state $\rho_{AB}(t)$ that is quantified by using the negativity. The sold (black), and the dash (blue) curves are evaluated at $\kappa = 0.5, 0.9$, respectively, where we set $\omega = 2, \zeta = 0.5, \varepsilon = 0.2, \beta = 0.4$, and (a) $\alpha = \frac{\pi}{4}, \gamma = 0$, and (b) $\alpha = \frac{\pi}{4}, \gamma = \frac{\pi}{4}$.

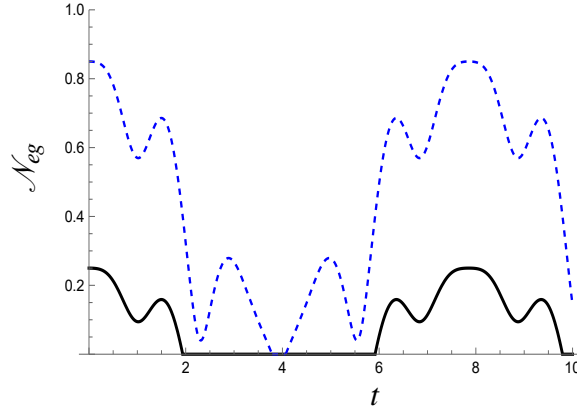


Figure 4.2: The same as Fig.(4.1.a) but we set $\zeta = 0.2$.

To investigate the effect of Rashba interaction strength, Fig.(4.2) is plotted at small value of this strength, where we set $\zeta = 0.2$. It is shown that, the amount of *QCs* are more robust than those displayed in Fig.(4.1.a), where the number of oscillations is smaller and at large values of mixing parameter κ . However, at small values of κ , namely the state ρ_{AB} has a small degree of coherence, the possibility that the state turns into a product state increases, where as it is observed from Fig.(4.2), the vanishing interval time of the *QCs* increases as κ decreases.

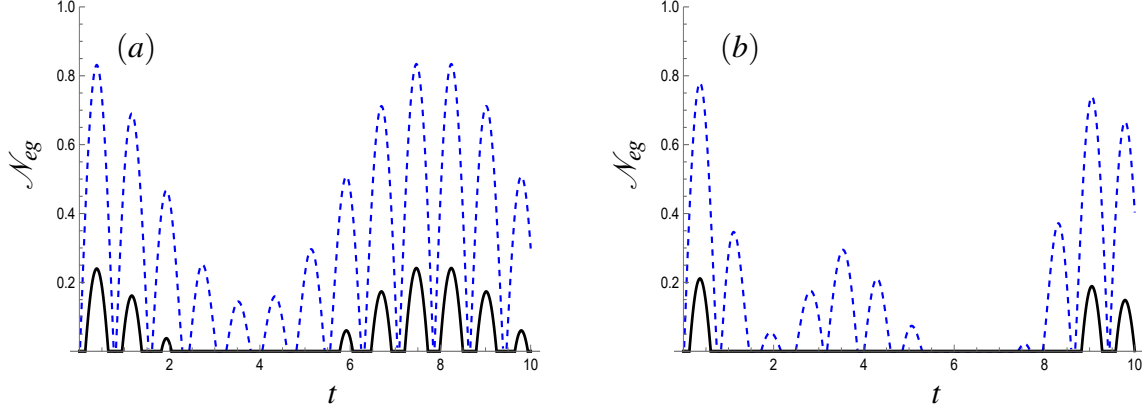


Figure 4.3: The amount of quantum correlation of the state $\rho_{AB}(t)$ that is quantified by using the negativity. The sold (black), and the dash (blue) curves are evaluated at $\kappa = 0.5, 0.9$, respectively, where we set $\omega = 2, \varepsilon = 0.2, \beta = 0.2, \alpha = 0, \gamma = 0$ (a) $\zeta = 0.2$ (b) $\zeta = 0.5$.

In Fig.(4.3), the impact of the Rashba interaction strength on the generated entanglement between the two qubits A and B is discussed. Different values of ζ are considered, where it is assumed that the two-qubit system are initially prepared in a product state. Due to the spin interaction between the two-qubit system, there will be an entangled state is generated. Moreover, at the same time Rashba interaction is switched on between the qubits A and C . The amount of QCs that contained in the state ρ_{AB} depends on strength ζ , where at small values of ζ , the possibility of generating an entangled state ρ_{AC} with large degree of QCs , decreases, and consequently the possibility that ρ_{AB} has a large QCs increases. These phenomenon are displayed clearly by comparing Fig.(4.3.a), and (4.3.b), where we set $\zeta = 0.2, 0.5$, respectively. Moreover, the degree of coherence that is characterized by the parameter κ , plays an essential role on the behavior of the QCs , where the long lived quantum correlations are displayed at large values of κ .

In Fig.(4.4), we investigate the impact of the spin interaction between the two qubits A and B in the presence of Rashba interaction. It is clear that, at small values of the Rashba interaction ζ , the entanglement between the two qubits are generated as soon as the interaction is switched on. However, at large values of ζ , the QCs between the two qubits will be generated at large

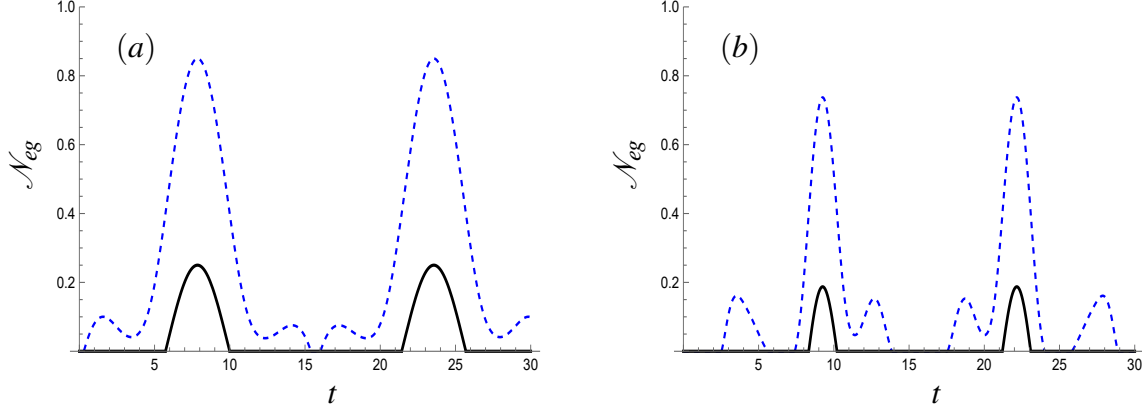


Figure 4.4: The same as Fig.(4.3), but we set $\omega = 0.1$.

interaction time. These results can be observed by comparing Figs.(4.4.a) and (4.4.b). Moreover, the disappearance interval of the QCs that depicted at large value of Rashba strength are larger than those displayed at small values of this strength. Furthermore, the maximum bounds of the QCs that displayed in Fig.(4.4.b) are smaller than those shown in Fig.(4.4.a).

From Fig.(4.4), one may conclude that, due to the interaction between the qubits A and C via Rashba interaction, the entanglement that has generated between the qubits A and B not only depends on the spin interaction, but also, Rashba strength has an essential role as a control parameter to increase/decrease the entanglement between the qubits A and B . Moreover, from Figs.(4.3) and (4.4), it is clear that the possibility of generated quantum correlations between the qubits A and B increases as one increases the spin interaction strength ω .

4.3.2 Quantum correlation of $\rho_{AC}(t)$

In this subsection, we investigate the impact of the Rashba strength on the generated quantum correlations between the qubits A and C via direct interaction. Fig.(4.5), displays the behavior of the negativity as a quantifier of the QCs , at different values of ζ , where it is assumed that, the three qubits are separable. There is an entangled state generated between the two qubits

A and C , as the interaction is switched on. Similarly, the upper bounds depend on the values of the coherence parameter κ , where the largest values of the QCs are displayed at large κ . Moreover, the periodic behavior, sudden birth/death are displayed regularly.

The impact of the strength ζ on the generated amount of the quantum correlations that is contained in the state ρ_{AC} can be seen by comparing Figs.(4.5.a) and Fig.(4.5.b), where we set $\zeta = 0.2, 0.5$, respectively. It is shown that, at large value of ζ , an entangled state between the qubits A and C is generated in a short interaction time compared with that displayed at small values of ζ . Also, at small values of ζ , the predicted quantum correlations oscillate faster than those displayed at large ζ . This means that, the predicted exchanged correlations increase at small value of the Rashba strength.

From Figs.(4.3) and (4.5), one may deduced that, due to the phenomena of exchanging the quantum correlation between the three qubits, if the predict QCs are large for ρ_{AB} , it will be small for ρ_{AC} . Moreover, if it vanishes completely in one state, it appears for the other state. The exchanging rate depends on the spin strength ω and Rashba strength ζ .

The entangling power of Rashba interaction in the presences of the spin interaction at a small strength is displayed in Fig.(4.6), where we set $\omega =$

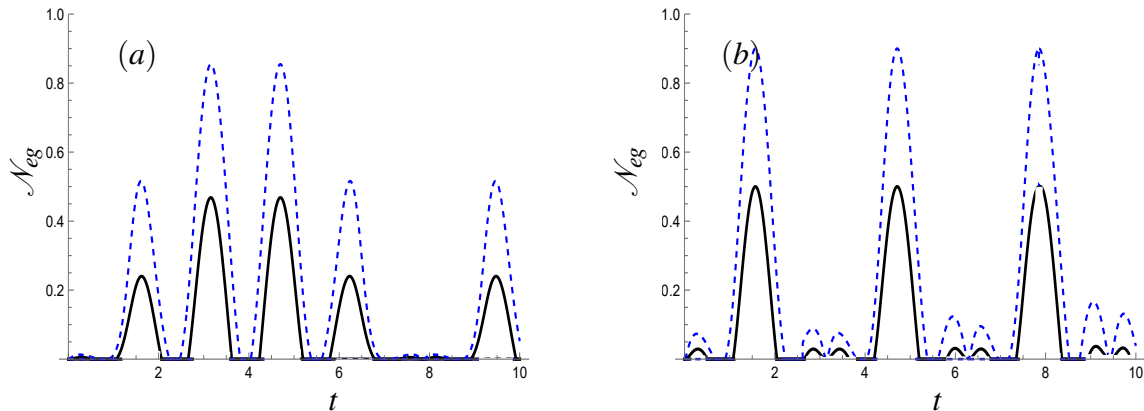


Figure 4.5: The negativity of the state ρ_{AC} , black line $\kappa = 0.5$, blue line $\kappa = 0.9$. $w = 2, \varepsilon = 0.2, \beta = 0.2, \alpha = 0, \gamma = 0$ (a) $\zeta = 0.2$, and (b) $\zeta = 0.5$.

0.1. Fig.(4.6.a), shows that, due to the small strength value of the spin interaction, the quantum correlations between A and C are generated suddenly as soon as the interaction is switched on, while as it is displayed from Fig.(4.5.a), the QCs are generated at longer interaction time, where we set $\omega = 2$. The numbers of oscillations are very small, namely a long-lived entanglement is generated between the two qubits A and C . Fig.(4.6.b), depicts the amount of quantum correlations that contained in the state ρ_{AC} at large value of Rashba strength, where we set $\zeta = 0.5$. The displayed behavior shows that, the upper bounds are larger than those displayed in Fig.(4.6.a). Moreover, the oscillations numbers are larger, namely the quantum correlations are exchanged faster.

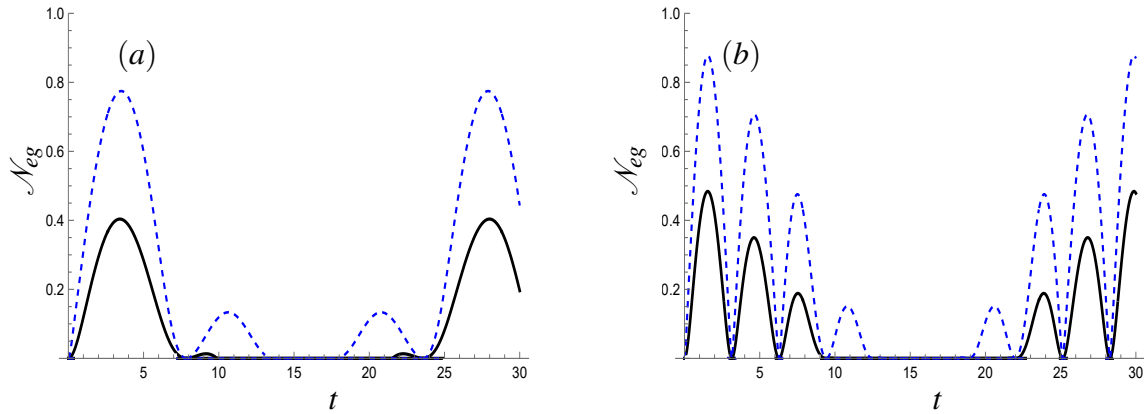


Figure 4.6: The same as Fig.(4.5), but we set $\omega = 0.1$.

4.3.3 Quantum correlation of $\rho_{BC}(t)$

Similarly, in this subsection the behavior of the QCs , that may be generated between the qubits B and C indirectly via Rashba interaction is discussed, where we consider the same initial state settings and the same values of the interaction parameters.

In Fig.(4.7), the QCs that contained in the state ρ_{BC} are investigated at different values of Rashba interaction's strength. The behavior is similar to that displayed for ρ_{AB} and ρ_{AC} , where the phenomena of the sudden changes

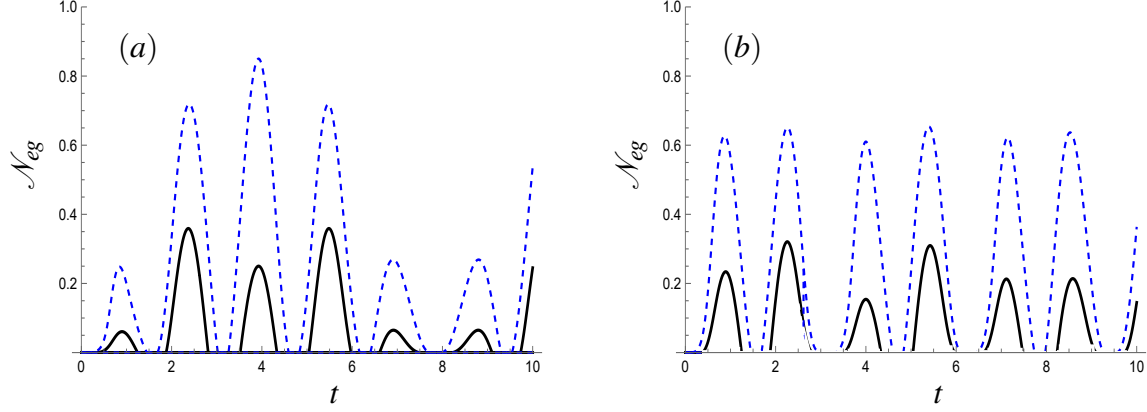


Figure 4.7: The negativity of ρ_{BC} , black line $\kappa = 0.5$, blue line $\kappa = 0.9$. $w = 2$, $\varepsilon = 0.2$, $\zeta = 0.5$, $\beta = 0.2$, $\alpha = 0$, $\gamma = 0$ (a) $\zeta = 0.2$ and (b) $\zeta = 0.5$.

(death/birth) are displayed. The maximum bounds of the QCs that shown in Fig.(4.7.a) are larger than those displayed in Fig.(4.7b). This means that, the large values of Rashba strength decrease amount of entanglement that contained in the state ρ_{BC} .

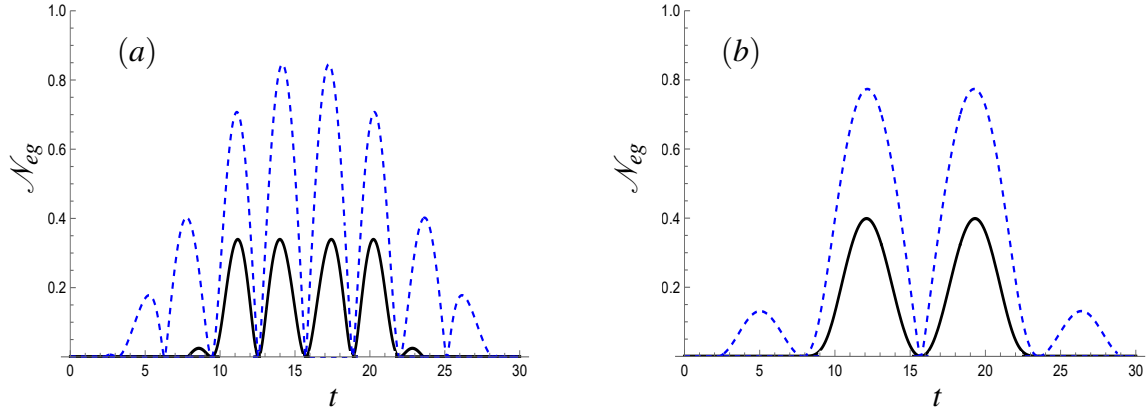


Figure 4.8: The same as Fig.(4.7), but we set $\omega = 0.1$.

The effect of the Rashba interaction in the presences of small values of the spin interaction on the amount of QCs of the state ρ_{BC} is displayed in Fig.(4.8), where we set $\omega = 0.1$. As it is observed from Figs.(4.7.b) and (4.8b), at the small values of ω and large values of ζ , the predicted quantum correlations are much better than those shown at large values of ω .

4.4 The non-Markovianity of the three partitions

In this section the impact of the Rashba and spin interactions strengths on the non-Markovianity of the three quantum states is investigated. The mathematical form of the non-Markovian is given in Chapter(1) in Eq(1.8).

Fig(4.9) displays the behavior of $\mathcal{N}_{\rho_i}, i = AB, AC$ and BC at small value of the spin interaction, where we set $\omega = 0.1$. The general behavior shows that, $\mathcal{N}_{\rho_i}$ increases as soon as the interaction is switched on to reach its maximum value. The smallest maximum bound is depicted for ρ_{AB} , while the largest bound of the $\mathcal{N}_{\rho_i}$ is observed for ρ_{AC} . Moreover the maximum peaks of the non-Markovianity of the state ρ_{AB} appear simultaneity as the minimum packs of $\mathcal{N}_{\rho_{AC}}$ and $\mathcal{N}_{\rho_{BC}}$ are displayed. The non-Markovianity vanishes for all the three partitions at the same interaction time, namely the exchanging process is frozen. There is no a remarkable fast oscillation behavior, namely the rate of changing systems non-Markovianity is small. The increasing rate the non-Markovianity of state that generated by direct interaction with Rashba interaction is larger than that displayed by indirect interaction.

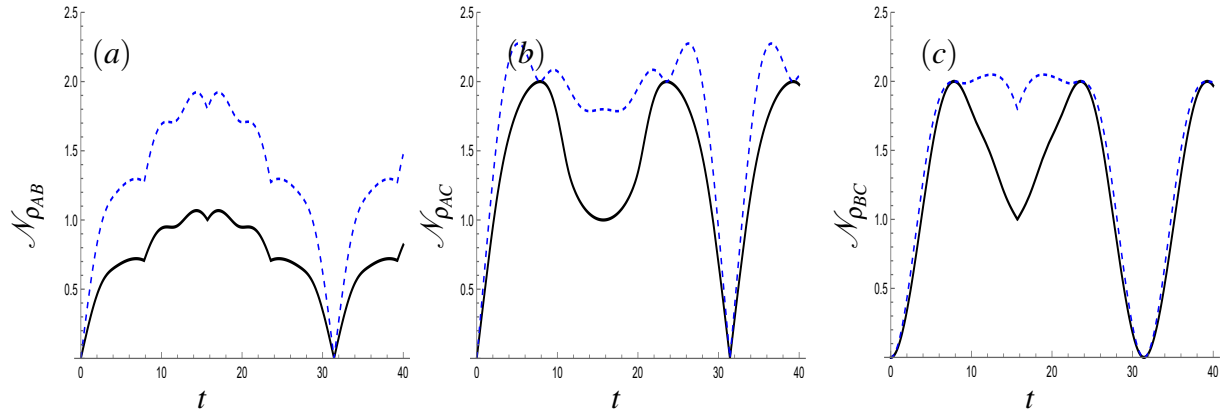


Figure 4.9: The non-Markovianity for the three states (a) ρ_{AB} , (b) ρ_{AC} , and (c) ρ_{BC} . The solid (black), and dot (blue) curves are evaluated at line $\kappa = 0.5, 0.9$, respectively, where we set $\omega = 0.1, \varepsilon = 0.2, \beta = 0.2, \zeta = 0.2, \alpha = 0, \gamma = 0$.

To investigate the impact of Rashba interaction on the Markovianity, we

set a large value of the strength ζ as it is shown in Fig.(4.10), where we set $\zeta = 0.5$. The displayed behavior of ρ_i is similar to that shown in Fig.(4.9). However, the upper bounds that have been displayed in Fig.(4.10) are larger than those shown in Fig.(4.9). Moreover, the number of oscillations and their amplitudes are large compared with their corresponding ones in Fig.(4.9). As an observation the vanishing interaction time of the non-Markovianity doesn't depend on the strength of Rashba interaction, where it vanishes at the same time.

From Figs.(4.9) and Fig.(4.10), one may induced that, the large values of Rashba strength increase the number of oscillations and their amplitudes. This means that, the memories of all the three partitions are unstable, where there will be a fast exchange of the physical properties between the three systems. Moreover, the memory of two qubit system, ρ_{AB} is more robust than that shown for ρ_{AC} and ρ_{BC} , which has generated via direct and indirect interaction. Also, when the minimum peaks are displayed on the behavior of $\mathcal{N}_{\rho_{AB}}$, the maximum peaks are appeared in $\mathcal{N}_{\rho_{AC}}$, $\mathcal{N}_{\rho_{BC}}$.

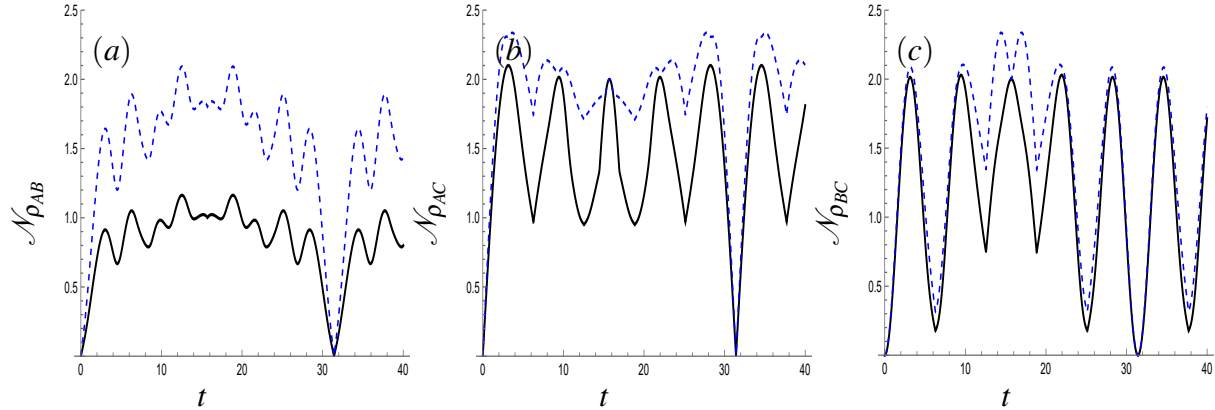


Figure 4.10: The same as Fig.(4.9), but at $\zeta = 0.5$.

To clarify the effect of the spin interaction parameter on the non-Markovianity of the three partition, we set $\omega = 2$ in Fig.(4.11), namely we increase the strength of the spin interaction between the qubits A and B . In general, $\mathcal{N}_{\rho_i}$ oscillates faster than that shown at $\omega = 0.1$, i.e, the exchanging pro-

cess of QCs and information are faster. However, as it is displayed from Figs.(4.11.a),(4.11.b) and (4.11.c) , the non-Markovianity of ρ_{AC} and ρ_{BC} are more stable than that shown for ρ_{AB} , where the period of changing $\mathcal{N}_{\rho_{AB}}$ is smaller than that depicted for $\mathcal{N}_{\rho_{AC}}$ and $\mathcal{N}_{\rho_{BC}}$. The non-Markovianity of ρ_{AB} vanishes at a shorter interaction time compared with that displayed for the other partitions. The coherence parameter has a clear effect on $\mathcal{N}_{\rho_{AB}}$ and $\mathcal{N}_{\rho_{BC}}$, where it displays a faster oscillations than that depicted for $\mathcal{N}_{\rho_{AC}}$.

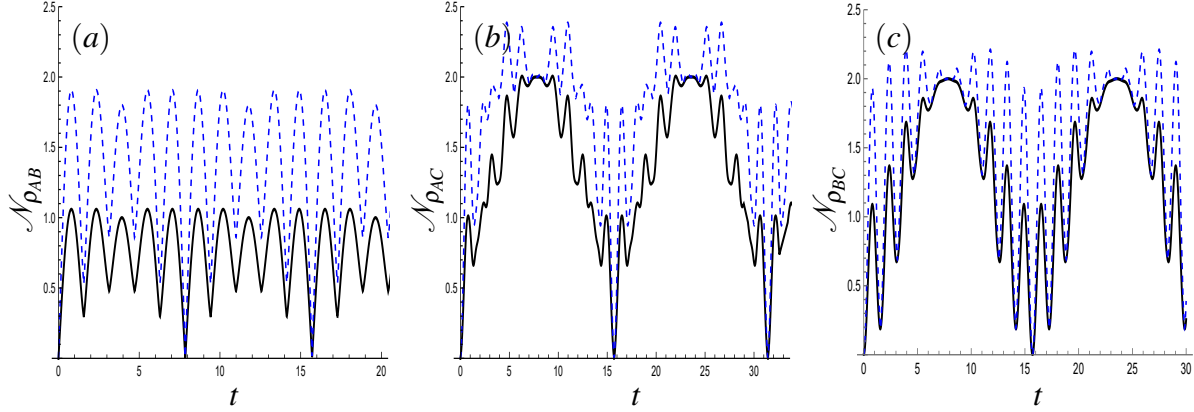


Figure 4.11: The same as Fig.(4.9), but at $\omega = 2$.

4.5 Information dynamics

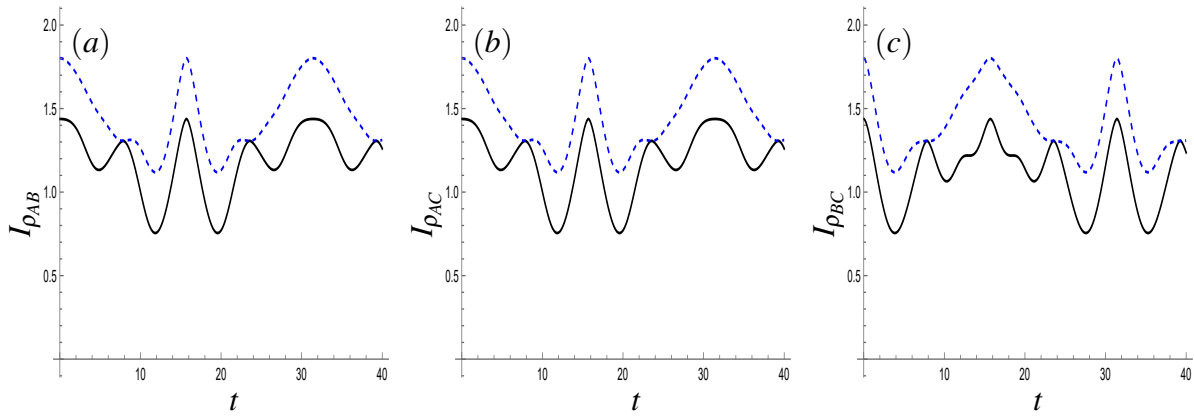


Figure 4.12: The behavior of the non-local information (a) $I_{\rho_{AB}}$, (b) $I_{\rho_{AC}}$, and (c) $I_{\rho_{BC}}$, black line $\kappa = 0.5$ and blue line $\kappa = 0.9$, $\omega = 0.1$, $\varepsilon = 0.2$, $\beta = 0.2$, $\zeta = 0.2$, $\alpha = 0$, $\gamma = 0$.

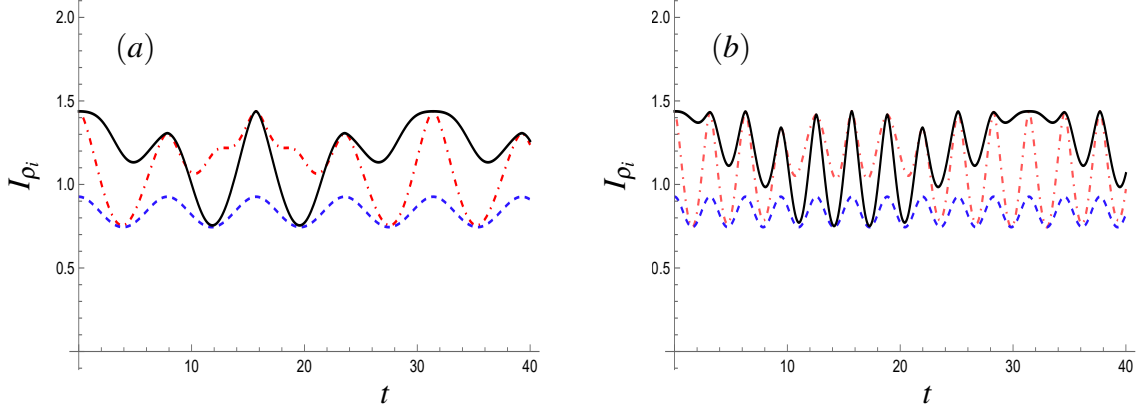


Figure 4.13: The behavior of I_{ρ_i} , where we set $\kappa = 0.5, \varepsilon = 0.2, \beta = 0.2, \alpha = 0, \gamma = 0$. The dot, dash dot, and solid for ρ_{AB}, ρ_{BC} and ρ_{AC} , respectively where (a) $\omega = 0.1, \zeta = 0.2$, (b) $\omega = 0.1, \zeta = 0.5$.

In this section, we investigate the behavior of the information that contained on each partition, where it is assumed that the two-qubit system, ρ_{AB} and the blank qubit (ρ_C) are initially prepared in a product state. The amount of information is measured by the local information, where the mathematical form that is given in Chapter(1) in Eq(1.10).

Fig.(4.12) displays the behavior of I_{ρ_i} , where the effect of the spin and Rashba strengths is discussed. The general behavior shows that, the coherence parameter κ plays an essential role on increasing/decreasing the upper and lower bounds of I_{ρ_i} , where the small values of κ decrease the oscillations amplitudes, while it increase as one increases κ . This coherence parameter has a different effect on the three partitions.

The effect of the Rashba interaction in the presence of different values of the spin strength can be deduced from Fig.(4.13). The dynamics of the information that encoded in the three partitions at $\omega = 0.1$ and $\zeta = 0.2$ is displayed in Fig.(4.13.a). It is shown that, for all the three states, I_{ρ_i} decrease as soon as the interaction is switched on. At further interaction time I_{ρ_i} increases and decrease simultaneously. As the minimum bounds of $I_{\rho_{AB}}$ and $I_{\rho_{AC}}$ are depicted, the encoded information in $I_{\rho_{BC}}$ reaches its maximum bound. Similarly, the maximum bounds are displayed for $I_{\rho_{AC}}$, when the

$I_{\rho_{AB}}$ and $I_{\rho_{BC}}$ are minimum. The regular oscillations are predicted on the behavior of the information the encoded in ρ_{AB} .

Fig.(4.13.b) displays the behavior of I_{ρ_i} at large value of Rashba strength, where we set $\zeta = 0.5$. It is clear, the fluctuations of the information between its lower and upper bounds increase, without any changes on the values of these bounds. It only increases the exchanging rate of information of the three qubits system.

4.6 Discussion

The powerful of the Rashba interaction to generate entangled state between three qubits in the presence of the spin interaction is investigated. This idea is clarified by considering a two-qubit system ρ_{AB} which is initially prepared in a product, partially entangled, or maximum entangled states. Additional, the qubit A interacts locally with another qubit C by using Rashba interaction. Due to these interactions, there are three quantum states have been created. Therefore, we investigate the behavior of the exchanged quantum correlations between the three states, their degree of non-Markovianity, and the amount of the scrambled information from/into these states. The obtained results show that, the predicted quantum correlations for all the three states oscillate between their upper and lower bounds. The maximum bounds depend on the initial degree of coherence, where at small value of the coherence parameter, the upper bounds are smaller than those displayed at large values of this coherence parameter. Moreover, the interval of vanishing these correlations for all the three states increases at small coherence parameter. The phenomena of the sudden changes (death/birth) are displayed periodically during the interaction time.

The effect of the Rashba strength on the behavior of the quantum correlations that may be generated between the three partitions is investigated.

It is shown that, as one increases Rashba strength, the quantum correlations that have generated directly between the qubits A and C increase on the expense of the amount of quantum correlations between the other partitions. However, this behavior will be changed if one increases the spin interaction's strength. Moreover, the robustness of these quantum correlations increases as one increases the strength of Rashba interaction, where the oscillations decrease, and consequently the long-lived quantum correlations are observed.

The non-Markovianity of the three partitions increases as soon as the interaction is switched on. The results show that, the memory of the two qubit system is more robust than the others, where the predicted oscillations are very small, while for the other two partitions the non-Markovianity exchanges fast with a large rate, due to the large amplitudes of these oscillations. The non-Markovianity of the three partitions vanishes at the same interaction time, namely at this time, there is no any exchanging process is depicted. It is worth to mention that, the large values of the Rashba strength increases the oscillations and their amplitudes, and consequently the exchanging rate of the quantum information between the three qubits increases.

The coherence parameter plays an essential role on minimizing or maximizing the non-Markovianity properties of the evolved three partitions. The maximum bounds of the non-Markovianity that depicted at large coherence parameter are larger than those displayed at small values of this coherence parameter. At small values of the coherence parameter, the non-Markovianity of the entangled state that generated via direct or indirect interaction oscillates fast. This means that, the memories of these states are unstable, and consequently the possibility of exchanging the quantum correlations and information increases.

It is worth to shed the light on the effect of the spin interaction strength

on the behavior of the non-Markovianity of the three partitions. The results show that, at large values of this strength, the non-Markovianity of the three states oscillates fast. Moreover, the predicted periodic time of these oscillations depends on whether it effect directly or indirectly, and consequently the possibility of exchanging the physical properties between the three partitions will be different.

The information scrambling between the three qubit systems is discussed by investigating the effect of Rashba interaction on the rate of exchanging process of information. It is shown that, the Rashba strength increases the oscillations of evolved information during the interaction time, and consequently the rate of exchanging the information between the three qubits increases. However, there is no any noticeable effect on the upper and lower bounds of the amount of encoded information in the three partitions.

Chapter 5

Discussion and Conclusion

5.1 Discussion

In this dissertation, we have discussed the impact of three different interaction types that may be used to generate quantum correlations between three qubit systems. These types include; Dzyaloshinskii-Moriya (DM), Dipolar and Rashba interactions, where all of them have been used in the context of quantum information and computation. Different physical quantities have been measured on the generated entangled quantum states between the three qubits. Among of these quantities is the amount of entanglement that be generated or exchanged between the three quantum states, the amount of non-local information that is encoded in each partition, the scrambling information rate, and the change on the non Markovianity property for each partition. Some physical measures have been used to quantify the entanglement, as negativity, concurrence, and entanglement of formation. The scrambling information is predicted by means of the out correlator function, while the non-Markovianity is quantified by using the distance measure. The main results of this dissertation have displayed in chapters two, three and four.

The powerful of these types of interactions is discussed on a system consists of two-qubit system, which its initial state depends on its coherence

degree with its environment and its subsystems interact together via spin interaction. One terminal of this two-qubit system interacts locally with a blank qubit, which may be initially prepared in a pure or mixed state. The interaction process will be achieved by using Dzyaloshinskii-Moriya, Dipolar, or Rashba interactions. Due to these interactions, the quantum correlations, the information may be gained or lost during the evolution process. Therefore, it is worth to investigate the effect of the interaction strengths on these phenomena. In what follows we shall summarize our results for the impact of each interaction:

In chapter two, the impact of Dzyaloshinskii-Moriya on the quantum correlations and the non-local information between all the partitions is discussed. The effect of the initial state settings and the strength of the interaction on the this process is investigated.

It is shown that, at small values of the coherence parameter, the ability of DM interaction to generate quantum correlations between the initial two qubits is larger than that may be generated by the spin interaction. However, large coherence parameter is a guarantor for generating long-lived quantum correlation at small strength of DM interaction. The numerical computations exhibit that, the quantum correlations are exchanged between the three possible partitions, where as quantum correlations decrease on the initial two-qubit system, it increases on the other two partitions and vis versa. The maximum bounds of the quantum correlations that predicted for these partitions systems never exceed that displayed for the initial two-qubit system. As soon as quantum correlation of the initial state vanishes, the other two quantum channels states exhibit a maximum quantum correlation. However, at large values of interaction strength, one finds that, the quantum correlation of marginal states have different phase due to the direct/indirect interaction with DM.

The phenomena of the exchanging the non-local information between

all the three partitions is examined at different values of the interaction's strength and the coherence parameter. Our results display that, the non-local information behaves simultaneously with the quantum correlations, where the large quantum correlations, the large non-local information. One can maximize the amount of the non-local information by increasing the strength of DM interaction and decreasing the dipole interaction strength. However, at large dipole strength, the number of oscillations and their amplitudes.

Chapter three has devoted to investigate the powerful of the Dipolar interaction on the suggested system . The obtained results, show that the coherence parameter plays an essential role on the behavior of the quantum correlations (QCs) that may be generated and exchanged between the three partitions. If the total system is initially prepared in a separable state, then as soon as the interaction is switched on, an entangled state will be generated between the three partitions. The maximum bounds of the QCs depend on the coherence degree, where at large values of this parameter, the interaction possibility with the surroundings decreases, and consequently the coherence degree of the generated entangled state increases. Moreover, the phenomena of the sudden death/birth of QCs are depicted periodically. Due to the interaction, these quantum correlations may be lost or gained, and thus an oscillatory behaviors have appeared. The possibility of exchanging the quantum correlations increases, if the initial two-qubit system is initially prepared in a maximum entangled state, and the blank qubit is prepared in a superposition pure state.

Moreover, the effect of the Dipolar interaction in the presence of the system's non-Markovianity is discussed. It is shown that, the non-Markovian behavior of the two-qubits system increases as the coherence parameter increases. At small values of the coherence parameter the oscillations of the non-Markovianity of the two-qubit system are smaller compared with those

obtained at large values of this parameter. The initial state settings has a clear impact on the oscillations behavior of the non-Markovianity, where the numbers of oscillations depend on whether the two-qubit system is initially prepared in a maximum entangled states or separable states. However, for separable case the non-Markovianity of the two-qubit system is the smallest one, while the largest value is predicted for the state that generated via indirect interaction. This means that information exchange depends on the initial state settings, where the exchanging rate of information increases if the two-qubit system is initially prepared in a maximum entangled state.

The amount of scrambled information between the three qubits has quantified by using the Out-of time-ordered correlator function, (OTOCs). From the obtained results, one can say that the possibility of scrambling the information from the initial entangled two-qubit system is larger than that scrambles from the other two partitions, which have generated either by direct or indirect interaction with Dipolar interaction. The effect of the interaction parameter has a clear effect on the scrambling rate of information, where the number of oscillations increases, namely the scrambling rate of information increases.

In chapter four, the effect of Rashba interaction on the behavior of generate entangled state between three qubit system in the presence of the spin interaction is investigated. The results show that, as one increases Rashba strength the quantum correlations that have generated directly between the qubits increase on the expanse of the amount of quantum correlations between the other partitions. However, this behavior will be changed if one increases the spin interaction's strength. Moreover, the robustness of these quantum correlations increases as one increases Rashba interaction's strength, where the oscillations decrease, and consequently the long-lived quantum correlations are observed.

It is shown that, the memory of the two qubit system is more robust

then the others, where the predicted oscillations are very small, while for the other two partitions the non-Markovianity exchanges fast with a large rate, due to the large amplitudes of these oscillations. The non-Markovianity of the three partitions vanishes at the same interaction time, namely at this time, there is no any exchanging process is depicted. It is worth to mention that, the large values of the Rashba strength increases the oscillations and their amplitudes, and consequently the exchanging rate of the quantum information between the three qubits increases. The maximum bounds of the non-Markovianity that depicted at large coherence parameter are larger than those displayed at small values of this coherence parameter. At small values of the coherence parameter, the non-Markovianity of the entangled state that generated via direct or indirect interaction oscillates fast. This means that, the memories of theses states are unstable, and consequently the possibility of exchanging the quantum correlations and information increases.

The information scrambling between the three qubit systems is discussed by investigating the effect of Rashba interaction on the rate of exchanging process of information. The obtained results show that, the Rashba strength increases the oscillations of evolved information during the interaction time, and consequently the rate of exchanging the information between the three qubit increases. However, there is no any noticeable effect on the upper and lower bounds of the amount of encoded information in the three partitions.

5.2 Conclusion

From the previous discussion, one may conclude that, the three types of interactions has the ability to generated entangled state between the three qubits. The amount of entanglement that generated between the two-qubit system and the blank depends on whether the interaction is performed directly or indirectly. For the these interactions, the spin interaction and the

coherence parameters play essential roles to maximize or minimize the generated quantum correlation between the three partitions. For all the three interactions the phenomena of the sudden changing has been observed on the behavior of all the measured physical quantities. Moreover, the initial state settings of the system has a strong effect on the behavior of all the discussed physical quantities, where starting with a maximum entangled state, the maximization of all the physical quantities is achieved.

5.3 Future Work

These results can be extended to perform the following tasks:

- Generating entangled quantum network by using multi-users.
- Using these entangled quantum network to teleport classical or quantum information between its members.
- The secret quantum communication can be implemented by using this quantum network.
- One can use local operations to increases the efficiency of this quantum network to perform the quantum information tasks with a large fidelity.
- The entangling power of the three interactions will be one of our next tasks.

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AppendixA

Dzyaloshinskii-Moriya Interaction



Exchanging Quantum Correlations and Non-Local Information Between Three Qubit-System

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Abstract

The possibility of exchanging the quantum correlations and the non-local information between three qubits interact directly or indirectly via Dzyaloshinskii-Moriya (DM) is discussed. The initial state settings and the interaction strength represent control parameters on the exchanging phenomena. The non-local information that encoded on the different partitions doesn't exceed the initial one. It is shown that, the ability of DM interaction to generate entanglement is larger than that displayed for the dipole interaction. The possibility of maximizing the quantum correlations between the three qubits increases as one increase the strength of interaction and starting with large initial quantum correlations. The long-lived quantum correlations could be achieved by controlling the strength of the dipole interaction.

Keywords Quantum correlation · Non-local information · Dzyaloshinski-Moriya interaction · Concurrence

1 Introduction

Entanglement is the core idea of modern theoretical physics [1], where it plays an important role in different applications of quantum information processing (QIP, like quantum cryptography [2], quantum teleportation [3], quantum dense coding [4], and etc. Therefore, there is a need to be generated and quantified, as well as keep it survival for a long time. Indeed, it has been generated between different objects; atoms [5] charged qubits [6], quantum dots [7], etc. Dzyaloshinskii-Moriya (DM), [8] represents one of the most common interaction that has used widely to generate quantum correlation between different systems. For example, the possibility of generating a thermal entanglement between two qubits Heisenberg chain in the presence of the Dzyaloshinski-Moriya is discussed in [9]. The effect of DM interaction on the dynamics of a two-qutrit system is investigated by Jafarpour and Ashrafpour [10]. Sharma

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and Pandey [11] studied the entanglement dynamics between qubit-qutrit systems via DM interaction. The influence of the anisotropic antisymmetric exchange interaction, on the entanglement of two qubits is examined in [12]. The DM interaction is used to generate entangled quantum network [13]. The orthogonality speed of two-qubit state interacts locally with spin chain in the presence of Dzyaloshinsky–Moriya interaction is investigated by Kahla, et. al [14]. Therefore, we are motivated to discuss the phenomena of exchanging the quantum correlations and the non-local information between three qubits, where two of them are either classically or non-classically correlated. It is assumed that, by using DM interaction one of the correlated qubits interacts locally with a control qubit that is initially prepared in a vacuum state. The effect of the initial state settings of the correlated qubits and the strength of DM interaction on the exchange process is investigated.

The outline of the paper is arranged as: In Section 2, we review the most common quantifiers of quantum correlation; concurrence, entanglement of formation and negativity. The qubits system and its evolution is described in Section 3. The exchange of quantum correlations between the marginal states of the three qubits is discussed in Section 4. In Section 5, the behavior of the exchanged non-local information is investigated. Finally, we summarized our results in Section 6.

2 Measure of Entanglement

In this section, we review some important quantifiers of quantum correlations (entanglement). Among of these quantifiers are; concurrence [15], entanglement of formation, and the negativity. It is well known that, a quantum system that consist of two qubits $|\psi_{12}\rangle$, is called a separable or classically correlated if $|\psi_{12}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, otherwise the system is entangled (quantum correlated). In the following subsections, we review the three measures in details.

2.1 Concurrence

This measure represents one of the most simplest quantifiers of entanglement, it is called Wootters Concurrence [15, 16], where it satisfies all the criteria of the good measures of entanglement. Mathematically it is defined as,

$$\mathcal{C} = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (1)$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$, λ_i ($i = 1, 2, 3, 4$) are the eigenvalues of the density operator ρ_{12} , such that

$$\widetilde{\rho}_{12} = \rho_{12}(\sigma_y^1 \otimes \sigma_y^2) \rho_{12}^* (\sigma_y^1 \otimes \sigma_y^2). \quad (2)$$

The operator ρ_{12}^* is the complex conjugate of the density operator of the system ρ_{12} , and $\sigma_y = i(|0\rangle\langle 1| - |1\rangle\langle 0|)$ is Pauli operator in y -direction. For separable states the concurrence $\mathcal{C} = 0$ and for the maximum entangled state $\mathcal{C} = 1$.

2.2 Entanglement of Formation

One of the meaningful measurement of entanglement is the entanglement of formation ($\mathcal{E}_F$) [17]. For a two qubits system the $\mathcal{E}_F$ is defined as a function of the concurrence [15] as,

$$\mathcal{E}_F(\rho) = H\left(\frac{1 + \sqrt{1 - \mathcal{C}^2(\rho)}}{2}\right), \quad (3)$$

where $H(x) = -x \log_2 x - (1 - x) \log_2(1 - x)$ represents the Shannon Entropy function and $0 \leq \mathcal{E}_F \leq 1$.

2.3 Negativity

The third measure of entanglement is the negativity, which is based on the eigenvalues of the partial transpose quantum system [18, 19]. However for two qubit system ρ_{12} , the negativity $\mathcal{N}$ is defined as,

$$\mathcal{N} = 2 \sum_{i=1}^4 \max(0, -\mu_i), \quad (4)$$

where μ_i are the negative eigenvalues of the partial transpose of the given quantum system, namely $\rho_{ab}^{T_{1(2)}}$. Similarly the negativity $\mathcal{N}(\rho_{12})$ is ranged between 0 and 1, where $\mathcal{N}(\rho)=0$ if the quantum system is separable and $\mathcal{N}(\rho)=1$ if the given quantum system is maximally entangled.

3 The Suggested Model

The system composed of two qubits A , and B , are prepared in a partial entangled state interacts by using Heisenberg XX Spin model [20]. It is assumed that, one of the subsystems, say A , interacts with third qubit C , as a controller via Dzyaloshinskii-Moriya (DM). The total Hamiltonian which describes this system is given by,

$$\mathcal{H}_{\text{sys}} = \frac{1}{2} \omega (\sigma_A^x \sigma_B^x + \sigma_A^y \sigma_B^y) + \mathbf{D} \cdot (\boldsymbol{\sigma}_A \times \boldsymbol{\sigma}_C), \quad (5)$$

where $\mathbf{D} = (D_x, D_y, D_z)$ is the DM interaction, and $D_i, i = x, y, z$ are the interaction strength on the three directions. ω is the coupling parameter between qubits A , B , and $\sigma_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_i^y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $i = A, B$ are the Pauli operators for both qubits. In this context, it is important to mention that the Hamiltonian (5) is of 8×8 dimensions, where its eigenvalues and eigenvectors are given by,

$$\lambda_i = 0, i = 1 \dots 4, \lambda_5 = -\sqrt{4D_z^2 + \omega^2}, \quad \lambda_6 = \lambda_5, \quad \lambda_7 = \lambda_8 = -\lambda_5 \quad (6)$$

, while the corresponding eigenvectors $|\xi_i\rangle, i = 1 \dots 8$ are,

$$\begin{aligned}
|\xi_1\rangle &= |111\rangle, \quad |\xi_2\rangle = \frac{1}{\sqrt{1+|\frac{2D_z}{\omega}|^2}} \left(\frac{-2iD_z}{\omega} |101\rangle + |110\rangle \right), \\
|\xi_3\rangle &= \frac{1}{\sqrt{1+|\frac{2\omega}{D_z}|^2}} \left(\frac{-i\omega}{2D_z} |001\rangle + |010\rangle \right), \quad |\xi_4\rangle = |000\rangle, \\
|\xi_5\rangle &= \frac{1}{\sqrt{1+\frac{1}{4}\left|\frac{\omega}{D_z}\right|^2+\frac{1}{4}\left|\frac{\sqrt{4D_z^2+\omega^2}}{D_z}\right|^2}} \left(-\frac{i\sqrt{4D_z^2+\omega^2}}{2D_z} |011\rangle + \frac{i\omega}{2D_z} |101\rangle + |110\rangle \right), \\
|\xi_6\rangle &= \frac{1}{\sqrt{1+\left|\frac{2D_z}{\sqrt{4D_z^2+\omega^2}}\right|^2+\left|\frac{\omega}{\sqrt{4D_z^2+\omega^2}}\right|^2}} \left(-\frac{2iD_z}{\sqrt{4D_z^2+\omega^2}} |001\rangle - \frac{\omega}{\sqrt{4D_z^2+\omega^2}} |010\rangle + |100\rangle \right), \quad (7) \\
|\xi_7\rangle &= \frac{1}{\sqrt{1+\left|\frac{\omega}{2D_z}\right|^2+\left|\frac{\sqrt{4D_z^2+\omega^2}}{2D_z}\right|^2}} \left(\frac{i\sqrt{4D_z^2+\omega^2}}{2D_z} |011\rangle + \frac{i\omega}{2D_z} |101\rangle + |110\rangle \right), \\
|\xi_8\rangle &= \frac{1}{\sqrt{1+\left|\frac{2D_z}{\sqrt{4D_z^2+\omega^2}}\right|^2+\left|\frac{\omega}{\sqrt{4D_z^2+\omega^2}}\right|^2}} \left(\frac{2iD_z}{\sqrt{4D_z^2+\omega^2}} |001\rangle + \frac{\omega}{\sqrt{4D_z^2+\omega^2}} |010\rangle + |100\rangle \right).
\end{aligned}$$

In this contribution, we assume that $\mathbf{D}$ to be polarized on the z direction, namely, $\mathbf{D} = (0, 0, D_z)$. Let us assume that the initial state of the whole system at time $t = 0$ is given by,

$$\rho_s(0) = \rho_{AB}(0) \otimes \rho_C(0), \quad (8)$$

where

$$\rho_{AB}(0) = \kappa |\varphi^{ab}\rangle \langle \varphi^{ab}| + \frac{1}{4}(1 - \kappa) I_{4 \times 4}, \quad \rho_C(0) = |\varphi^c\rangle \langle \varphi^c|, \quad (9)$$

with $|\varphi^{ab}\rangle = \cos(\alpha)|eg\rangle + \sin(\alpha)|ge\rangle$ and $|\varphi^c\rangle = \cos(\gamma)|e\rangle + \sin(\gamma)|g\rangle$, $I_{4 \times 4}$ is the identity operator. It is clear that, at $\alpha = \frac{\pi}{4}$, the state $|\varphi^{AB}\rangle$ reduces to Bell state $|\psi^+\rangle$. At $t > 0$, the total density operator of the whole system is given by,

$$\rho_s(t) = \mathcal{U}(t) \rho_s(0) \mathcal{U}(t)^\dagger, \quad \mathcal{U}(t) = \exp[-i\mathcal{H}_{sys}t], \quad (10)$$

where the unitary operator $\mathcal{U}(t)$ takes the form,

$$\mathcal{U}(t) = \sum_{i=1}^2 \alpha_i \beta_i, \quad (11)$$

where,

$$\begin{aligned}
\alpha_1 &= \cos\left(\frac{\omega}{2}t\right) I_{8 \times 8} - i \sin\left(\frac{\omega}{2}t\right) \sigma_A^x \otimes \sigma_B^x \otimes I_{2 \times 2} \\
\alpha_2 &= \cos(D_z t) I_{8 \times 8} - i \sin(D_z t) \sigma_A^x \otimes I_{2 \times 2} \otimes \sigma_C^y, \\
\beta_1 &= \cos\left(\frac{\omega}{2}t\right) I_{8 \times 8} - i \sin\left(\frac{\omega}{2}t\right) \sigma_A^y \otimes \sigma_B^y \otimes I_{2 \times 2}, \\
\beta_2 &= \cos(D_z t) I_{8 \times 8} + i \sin(D_z t) \sigma_A^y \otimes I_{2 \times 2} \otimes \sigma_C^x.
\end{aligned} \quad (12)$$

Then by using the unitary operator (11) and the initial state $\rho_s(0)$, one gets the final state $\rho_s(t)$ at any $t > 0$. Due to this interaction, there will be new entangled states are generated; $\rho_{AC}(t)$ is generated between the qubits A , C , and $\rho_{BC}(t)$ between the qubits B and C . Moreover, the amount of entanglement of the initial state $\rho_{AB}(0)$ will be affected by this interaction. In this context, it is important to quantify the amount of entanglement between

the three partitions AB, AC , and BC , which are defined by the final states $\rho_{AB}(t), \rho_{AC}(t)$ and $\rho_{BC}(t)$, respectively.

In the computational basis the three final density operators are given by a matrix of size 4×4 . The density operators of the three partitions are defined by the non-zero elements ρ_{ij}^{ij} , $ij = ab, ac, bc$. Mathematically any $\rho_{k\ell}(t), k\ell = AB, AC, BC$ may be written as:

$$\rho_{k\ell}(t) = \begin{pmatrix} \rho_{00}^{ij} & 0 & 0 & 0 \\ 0 & \rho_{11}^{ij} & \rho_{12}^{ij} & 0 \\ 0 & \rho_{21}^{ij} & \rho_{22}^{ij} & 0 \\ 0 & 0 & 0 & \rho_{33}^{ij} \end{pmatrix}, \quad (13)$$

The density operator of the state $\rho_{AB}(t)$ is defined by the following elements:

$$\begin{aligned} \rho_{00}^{ab} &= \frac{1}{4} \kappa \cos^2(2D_z t), \\ \rho_{11}^{ab} &= \frac{1}{8} (2 \cos^2(2t) \cos^2(2D_z t) + (3 + 3\kappa - (1 - \kappa) \cos(4D_z t)) \sin^2(2t)), \\ \rho_{22}^{ab} &= \frac{1}{8} (2 \sin^2(2t) \cos^2(2D_z t) + (3 + 3\kappa - (1 - \kappa) \cos(4D_z t)) \cos^2(2t)), \\ \rho_{21}^{ab} &= \frac{1}{8} (2\sqrt{3}\kappa \cos(2D_z t) - i \sin(4t) (-\frac{1}{2}\kappa(3 + \cos(4D_z t) - 2\sin^2(2D_z t))), \\ \rho_{33}^{ab} &= \frac{1}{4} (1 - \kappa + \sin^2(2D_z t)), \quad \rho_{12}^{ab} = \rho_{21}^{ab*}. \end{aligned} \quad (14)$$

Similarly the state ρ_{ac} is described by the following non-zero elements,

$$\begin{aligned} \rho_{00}^{ac} &= \frac{1}{4} (1 - \kappa) \sin^2(2t) \sin^2(2D_z t), \\ \rho_{11}^{ac} &= \frac{1}{4} ((1 - \kappa + \cos^2(2t)) \cos^2(2D_z t) + (1 + 2\kappa) \sin^2(2t)), \\ \rho_{22}^{ac} &= \frac{1}{8} (2i\sqrt{3}\kappa \sin(2t) \sin(2D_z t) + (\kappa - 2) \cos(2t) \sin(4D_z t)), \\ \rho_{21}^{ac} &= -\frac{1}{8} (\kappa - 3 - (1 - \kappa) \cos(4t)) \sin^2(2D_z t), \\ \rho_{33}^{ac} &= \frac{1}{16} (7 + (1 + 4\kappa) \cos(4t) + 8 \cos^2(t) \cos(4D_z t) \sin^2(t)), \quad \rho_{12}^{ac} = \rho_{21}^{ac*}. \end{aligned} \quad (15)$$

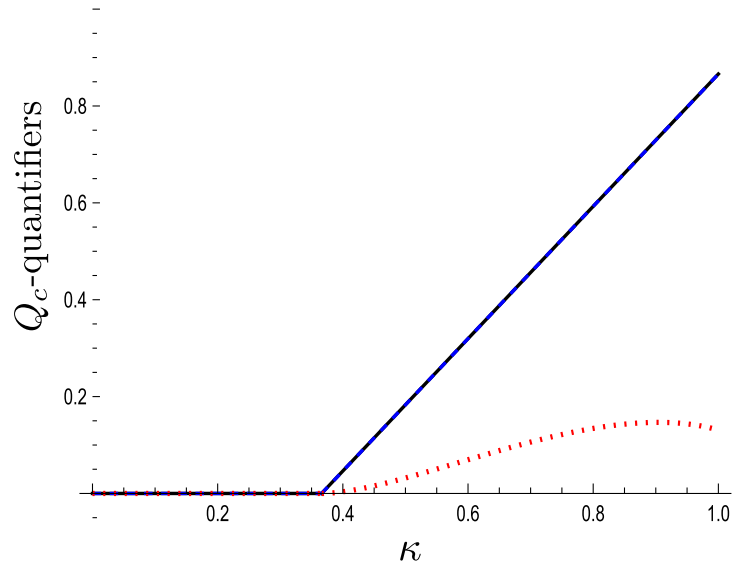
Finally the non zero elements of third partition ρ_{bc} are given by,

$$\begin{aligned} \rho_{00}^{bc} &= \frac{1}{4} (1 - \kappa) \cos^2(2t) \sin^2(2D_z t), \\ \rho_{11}^{bc} &= \frac{1}{4} ((1 - \kappa + \sin^2(2t)) \cos^2(2D_z t) + (1 + 2\kappa) \cos^2(2t)), \\ \rho_{22}^{bc} &= \frac{1}{8} \kappa \sin(2D_z t) (-2\sqrt{3} \cos(2t) + i (\sin(2(1 + D_z)t) + \sin(2(1 - D_z)t))), \\ \rho_{21}^{bc} &= \frac{1}{4} (1 + (1 - \kappa) \sin^2(2t)) \sin^2(2D_z t), \\ \rho_{33}^{bc} &= \frac{1}{8} (3 - (1 + 2\kappa) \cos(4t) + 2 \cos^2(2t) \cos(2D_z t)), \quad \rho_{12}^{bc} = \rho_{21}^{bc*}. \end{aligned} \quad (16)$$

4 Exchanging Quantum Correlations

In this section we investigate the amount of entanglement that loses from the initial state $\rho_{AB}(0)$ and that exchanged between the terminals of the initial state and the control qubit C . In Fig. 1, we quantify the amount of entanglement on the initial state via the concurrence, $\mathcal{C}$, negativity, $\mathcal{N}$ and the entanglement of formation $\mathcal{E}_F$. It is assumed that, the initial state of the system is prepared by setting $\alpha = \pi/3$, namely $\rho_{AB}(0)$ is a partial entangled state. It is clear that, the quantum correlation between the subsystems "A" and "B" appears at larger values of the weight parameter κ , namely $\kappa > 0.37$. From this figure, it is

Fig. 1 The amount of quantum correlation of the initial state $\rho_{AB}(0)$ that predicted via concurrence (solid curve), the negativity (dash-curve), and the entanglement of formation (dot curve)



clear that, the predicted quantum correlation depends on the used quantifier. However, the concurrence and the negativity predict the entanglement at smaller values of the weight κ . Moreover, the smallest upper bounds of the quantum correlations are predicted by using the entanglement of formation, $\mathcal{E}_F$.

4.1 Lose and Gain Quantum Correlations of $\rho_{AB}(t)$

Due to the interaction with the environment which is described by the control qubit C , the initial state $\rho_{AB}(0)$ loses some of its quantum correlation. Therefore, it is important to investigate the effect of interaction's parameters on the quantum correlation of the initial quantum system.

Figure 2 shows the behavior of the three quantifiers for a system is initially prepared in a partial entangled state, where $\alpha = \pi/3$ and different values of the weight parameter

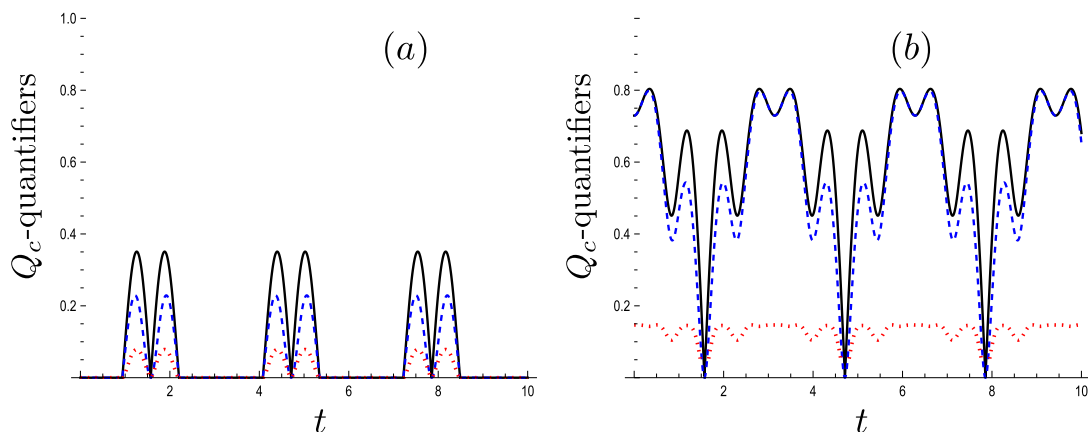


Fig. 2 The dynamics of the quantum correlation loses of the state $\rho_{AB}(t)$ that predicted by using the concurrence (solid), the negativity (dash), and the entanglement of formation (dot-lines). It is assumed that, $\rho_{AB}(0)$ is prepared such that $\alpha = \pi/3$ and control qubit is prepared by setting $\gamma = \pi/2$, $D_z = 0.5$ and $\omega = 2$ (a) $\kappa = 0.3$ and (b) $\kappa = 0.9$

κ . Moreover, the control qubit is prepared in the state $\rho_C(0) = |0\rangle\langle 0|$, namely we set $\gamma = \pi/2$. From Fig. 2a, it is clear that, at small value of the weight parameter, $\kappa = 0.3$, the quantum correlation is generated at $t \simeq 0.9$. However, the quantifiers increase gradually to reach their maximum bounds for the first time at $t = 1.2$ and decrease to vanish completely at $t \simeq 1.6$. The largest bounds are predicted for the concurrence $\mathcal{C}$, while the smallest bounds of quantum correlations are predicted by the entanglement of formation $\mathcal{E}_F$.

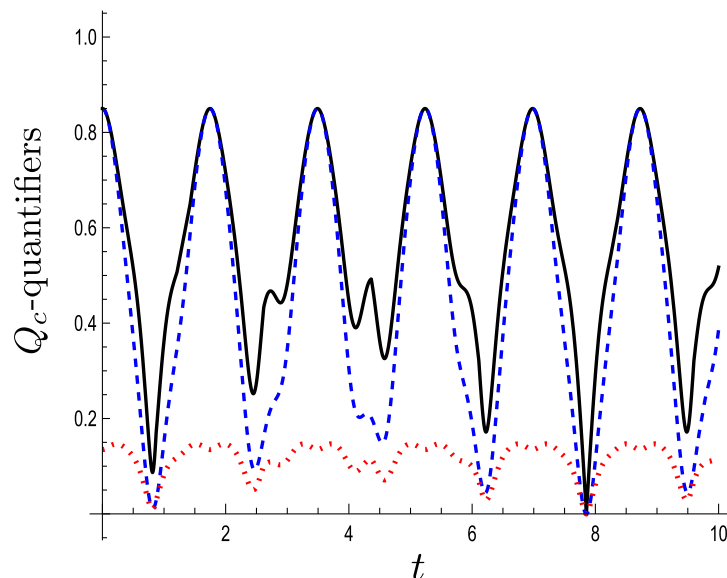
In Fig. 2b, we investigate the behavior of the quantum correlation at large value the weight parameter κ . It is clear that, at $t = 0$, all the quantifiers predicate the presence of the quantum correlation. However, as the interaction time increases, the predicted quantum correlations flocculated between their lower and upper bounds. Moreover, the lower bounds that displayed by the concurrence $\mathcal{C}$ are the largest one, but the maximum values of $\mathcal{C}$ and the negativity $\mathcal{N}$ are coincide. As it is shown from Fig. 2b, the smallest amount of quantum correlation are quantified by using the $\mathcal{E}_F$.

From Fig. 2, one may conclude that, the weight of the initial state plays a central role on keeping the existence of the quantum correlations. If the weight parameter is small enough such that the initial state behaves as a product state, the DM interaction has the strength to generate quantum correlation between the subsystems of the initial state.

To clarify the role that played by the strength of DM interaction, we investigate in Fig. 3 the behavior of the three quantifiers at larger values of DM interaction, where we set $D_z = 0.9$. The predicted behavior of the three quantifiers shows that, as soon as the interaction is switched on, the quantum correlations that predicted decrease suddenly to reach their minimum values. At further interaction time, the correlation rebirths again to reach their maximum values. The maximum values of the concurrence and the negativity are coincide, while the lower bounds of $\mathcal{C}$ are much better than that displayed by the negativity $\mathcal{N}$. The minimum and maximum values of the quantum correlations that predicted via the three quantifiers are displayed at the same interaction time.

Figure 4, exhibits the effect of the dipole's strength interaction where we set $\omega = 0.5, 1$, while all the parameters values are the same as Fig. 2b. The predicted behavior is similar to that displayed in Fig. 3. As it is shown from Fig. 4a, where $\omega = 1$, all the quantifiers predict a long-lived quantum correlations. The phenomena of the sudden death/birth are predicted at the same interaction time for all the quantifiers. Moreover, the quantum correlations

Fig. 3 The same as Fig. 2b, but $D_z = 0.9$



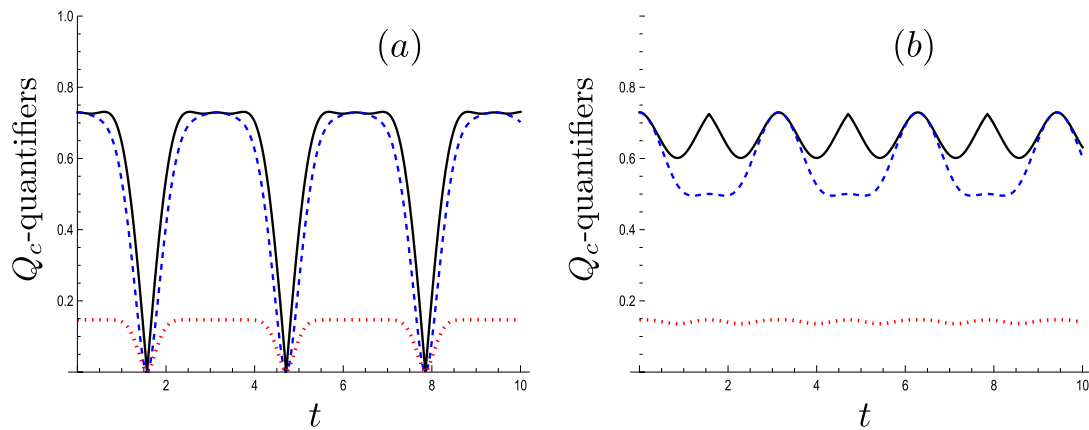


Fig. 4 The same as Fig. 2 but (a) $\omega = 1$, and (b) $\omega = 0.5$. The initial and the control states are prepared such that $\alpha = \pi/3$, $\kappa = 0.9$ and $\gamma = \pi/2$

vanish at larger interaction time compared with that displayed in Fig. 3. As one increases the strength of the dipole interaction, ($\omega = 0.5$) in Fig. 4b, the long-lived behavior of quantum correlation is lost and the oscillation behavior is shown. Moreover, the minimum values of these oscillations are larger than those displayed in Fig. 3.

From the results that displayed in Fig. 4, one can consider the strength of the dipole interaction as a control parameter to generated a long-lived quantum correlations between the terminals of the initial quantum system. Moreover, it could be used to improve the lower bounds of these correlations.

4.2 Lose and Gain Quantum Correlations of $\rho_{AC}(t)$

The amount of quantum correlation that is generated between the subsystems A and C is described in Fig. 5, where the same initial state settings are considered. It is clear that, as soon as the interaction is switched on between the subsystem A and the control parameter C , an entangled state is generated. All the quantifiers $\mathcal{C}$, $\mathcal{N}$ and $\mathcal{E}_f$ predict that there is a quantum correlation is generated between the two qubits. From Figs. 2a and 5a, it is clear DM interaction generates an entangled state between the subsystems A and C , while the qubits A and B are disentangled at small values of κ . This means that, the ability of

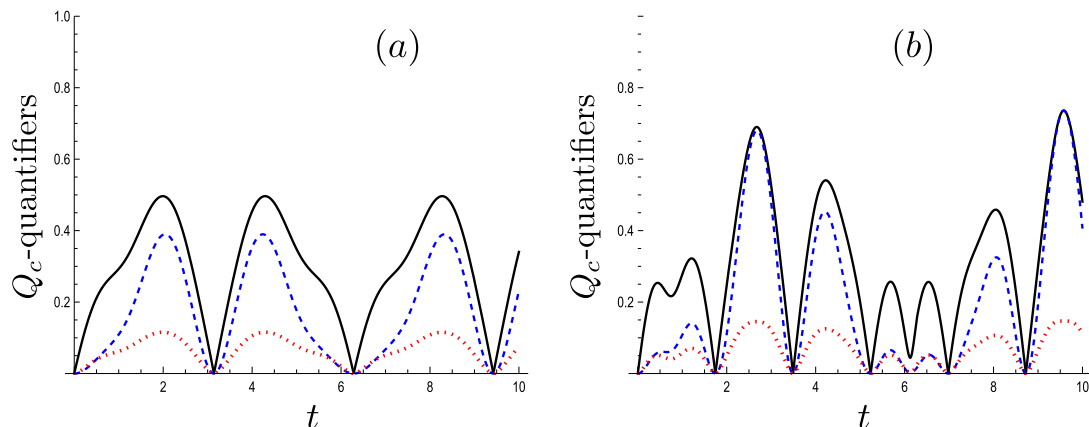


Fig. 5 The amount of quantum correlation that is generated between the subsystems of the state $\rho_{AC}(t)$, $\kappa = 0.9$, $\omega = 0.5$, $\gamma = \pi/2$, $\alpha = \pi/3$ and (a) $D_z = 0.5$ (b) $D_z = 0.9$

generating entanglement via DM is stronger than that may be generated by using dipole interaction. On the other hand, at large weight, namely $\kappa = 0.9$, the quantum correlation is generated between the qubits A and C on the expense of the initial quantum correlation between A and B . This phenomena is displayed in Fig. 5b, by investigating the behaviors of the three quantifiers, where as soon as the interaction is switched on, the quantum correlation of the state $\rho_{AB}(t)$ decreases, while it increases between the qubits A and C .

In Fig. 6, we investigate the effect of the dipole interaction parameter on the generated entangled state between the terminal (A) and the control qubit and C . To get a clear vision on this effect we consider different values of the dipole parameter, where we set $\omega = 1, 2$. It is clear from Fig. 6a, at small values of the dipole interaction, means that less quantum correlation between the initial qubits A and B , a large quantum correlation between the qubits A and C is predicted. Moreover, small values of ω increases the survival time of the quantum correlation of the state $\rho_{AC}(t)$. Figure 6b displays the behavior of quantifiers at large values of the dipole interaction, where we set $\omega = 2$. The behavior is similar to that displayed in Fig. 6a, but the upper bounds of all the quantifiers are larger than those displayed at $\omega = 1$.

4.3 Lose and Gain Quantum Correlations of $\rho_{BC}(t)$

As mentioned above, the DM interaction generates an entangled between the qubits B and C represented by the density operator $\rho_{BC}(t)$. The behavior of the three quantifiers of the entanglement is predicted in Fig. 7 at different values of the interaction strength.

The behavior of the three quantifiers for the marginal state $\rho_{BC}(t)$ at $D_z = 0.5$ is displayed in Fig. 7a, where it displays a similar behavior to that shown in Fig. 5a for the marginal state $\rho_{AC}(t)$. However, the generated quantum correlation the depicted by the three quantifiers increases suddenly as soon as the interaction is switched on, while the gradual behavior is displayed for the marginal state $\rho_{AC}(t)$. Moreover, the maximum values of the quantum correlations that shown by the three quantifiers are predicted at smaller interaction time compared with that displayed for the marginal state $\rho_{AC}(t)$. As it is displayed from Fig. 7b, the maximum bounds of quantum correlations are larger than those displayed in Fig. 7a, where we increase the interaction' strength. For both marginal states, the sudden birth/death of the quantum correlation are depicted at the same interaction time, meanwhile the maximum quantum correlations are displayed for marginal state $\rho_{AB}(t)$.

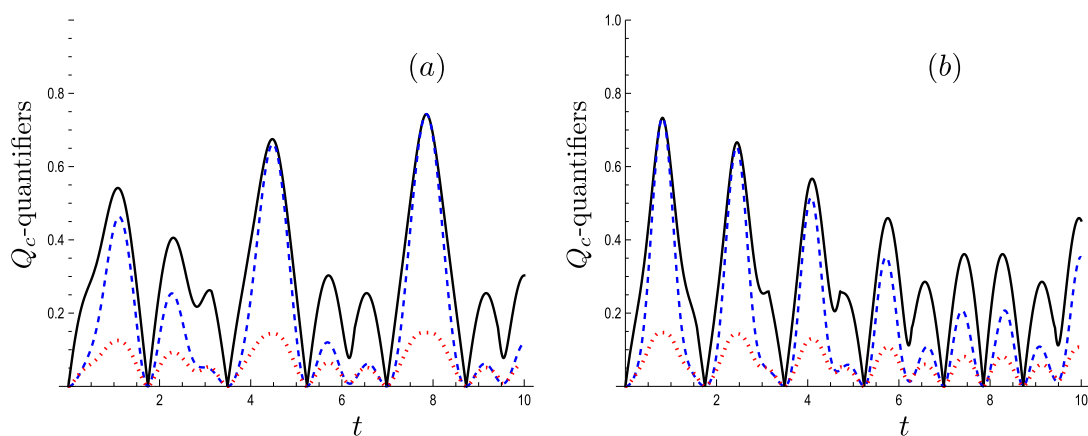


Fig. 6 The same as Fig. 4 but for the state $\rho_{AC}(t)$ (a) $\omega = 1$, and (b) $\omega = 2$

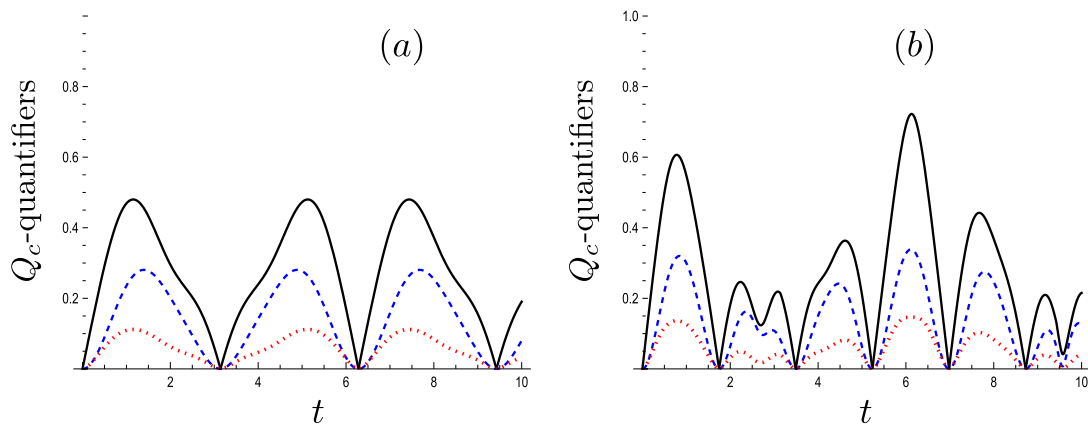


Fig. 7 The same as Fig. 5 but for the state $\rho_{BC}(t)$ (a) $D_z=0.5$ (b) $D_z=0.9$

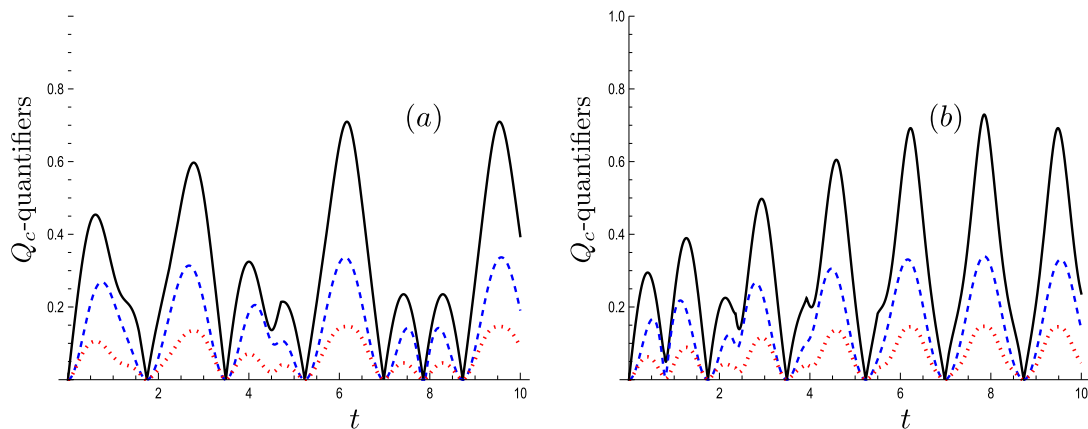


Fig. 8 The same as Fig. 4 but for the state $\rho_{BC}(t)$ (a) $\omega = 1$, and (b) $\omega = 2$

In Fig. 8, we examine the effect of the dipole interaction on the behavior of quantum correlation for the marginal state $\rho_{BC}(t)$, where we set $\omega = 1, 2$ for Fig. 8a and b, respectively. It is clear that, the number oscillations of the three quantifiers are larger than those displayed at small value of ω as displayed in Fig. 5. Similarly, the phenomena of the sudden death/birth are displayed at smaller interaction time compared with those displayed in Fig. 5. Moreover, the amount of the predicted quantum correlation for the marginal state ρ_{AC} , which is generated via direct interaction with DM, is larger than that displayed for the state $\rho_{BC}(t)$, which is generated indirectly.

5 Exchanging the Non-local Information

In this section, we investigate the behavior of the non-local information $\mathcal{I}_{non}$ that coded on all the possible partition, $\rho_{AB}(t)$, $\rho_{AC}(t)$ and $\rho_{BC}(t)$. There is a possibility of exchanging the non-local information between the three qubits. In Fig. 9a, we quantify the amount $\mathcal{I}_{non}$ for the initial state $\rho_{AB}(0)$, as a function on the weight parameter κ and the angle α . At fixed value of $\alpha = \pi/3$, the amount of the non-local information increases gradually to reach its maximum value, namely $\mathcal{I}_{non} = 2$, at $\kappa = 1$. Due to the interaction with the

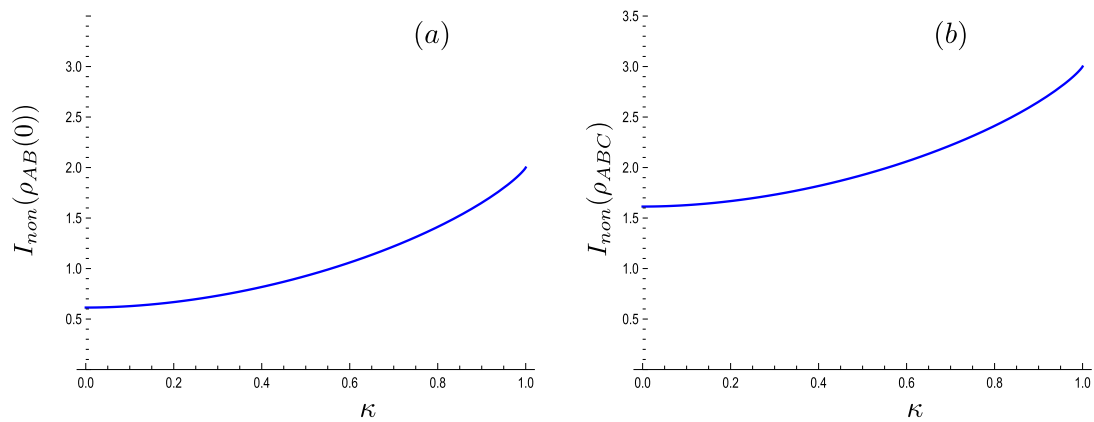


Fig. 9 The non local information that coded on the states (a) $\rho_{AB}(0)$ and (b) ρ_{ABC} , where $w = 2, \gamma = \frac{\pi}{2}, \alpha = \frac{\pi}{3}, D = 0.5$

control parameter, the density operator which describe the three qubits is given by $\rho_{ABC}(t)$. Figure 9b displays the behavior of the amount of the non-local information, $\mathcal{I}_{non}(\rho_{ABC}(t))$, at $D_z = 0.5$ and the dipole strength $\omega = 2$. Again as it exhibited from (9b), the non-local information of the whole system increases gradually as κ increases to reach its maximum value $\mathcal{I}_{non}(\rho_{ABC}(t)) = 3$ at $\kappa = 1$. The DM strength will exhibit a change on the behavior of information that could be generated or loses between the three qubits.

Figure 10 displays the exchange of the non-local information between the three states, $\rho_{ij}(t), ij = AB, AC$ and BC at different values of the weight parameter κ , while the strength of DM interaction is fixed such that $D_z = 0.9$. In Fig. 10a, we investigate the behavior of $\rho_{ij}(t)$ where it is assumed that, the wight parameter $\kappa = 0.3$. It is clear that, at $t = 0$, the non-local information that is coded in the state $\rho_{AB}(t)$ increases gradually to reach its maximum bounds. However, as the interaction time t increases the nonlocal information $\mathcal{I}_{non}(\rho_{AB}(t))$ decreases gradually to its minimum value. This behavior is repeated periodically at further t , where the upper and lower bounds of the non-local information are similar during the time's interaction. The similar periodic behavior is predicted for $\mathcal{I}_{non}(\rho_{AC}(t))$ and $\mathcal{I}_{non}(\rho_{BC}(t))$, where as $\mathcal{I}_{non}(\rho_{AB}(t))$ increases, the non-local information that coded on $\rho_{AC}(t)$

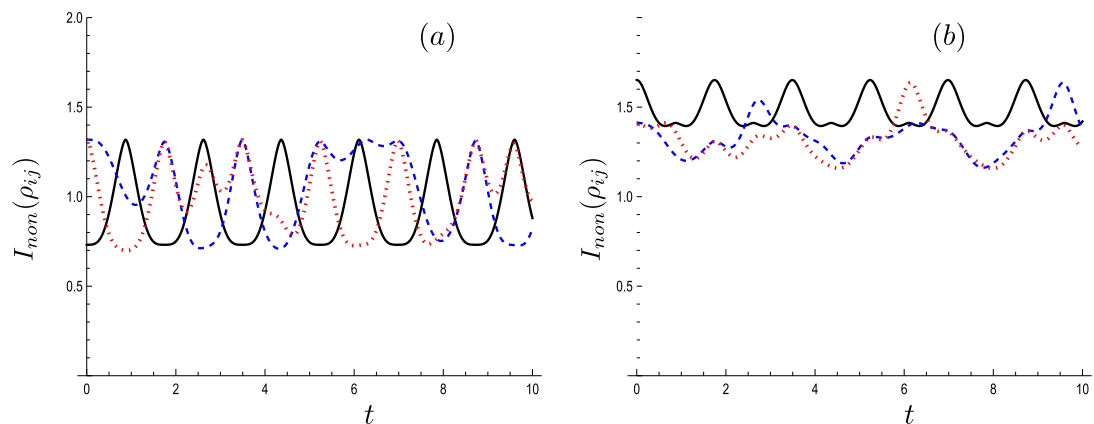


Fig. 10 The weight effect of the initial state on the generated non-local information, where $D_z = 0.9, w = 0.5, \gamma = \frac{\pi}{2}, \alpha = \frac{\pi}{3}$. The solid(black), dash dot(blue), and the dot (red) curves, represent the non-local information for the states $\rho_{AB}(t), \rho_{AC}(t)$, and $\rho_{BC}(t)$, respectively where, (a) $\kappa = 0.3$, and (b) $\kappa = 0.9$

and $\rho_{BC}(t)$ decreases. Whenever, $\mathcal{I}_{non}(\rho_{AB}(t))$ reaches its maximum values, both $\mathcal{I}_{non}(\rho_{AC}(t))$ and $\mathcal{I}_{non}(\rho_{BC}(t))$ are minimum. As an important observation, the maximum values of the non-local information that predicted for $\rho_{AC}(t)$ and $\rho_{BC}(t)$ doesn't exceed that coded on the initial state $\rho_{AB}(t)$. The amount of non-local information that is coded on the states $\rho_{AC}(t)$ and $\rho_{BC}(t)$ via direct and indirect interaction by DM, respectively, oscillates periodically between their maximum and lower bounds. Their minimum values don't exceed the minimum values of $\mathcal{I}_{non}(\rho_{AB}(t))$. In Fig. 10b, we investigate the behavior of the non-local information that coded on the states ρ_{ij} at larger values of the weight parameter, where we set $\kappa = 0.9$. A similar behavior is predicted as that shown Fig. 10a, but the upper and lower bounds of $\mathcal{I}_{non}(\rho_{ij}(t))$ are much better than those shown at $\kappa = 0.3$.

Figure 11a displays the behavior $\mathcal{I}_{non}(\rho_{ij}(t))$ at smaller values of DM strength, where we set $D_Z = 0.5$. It is clear that, the for $\rho_{AC}(t)$ (direct interaction) and $\rho_{BC}(t)$ (indirect interaction) the non-local information decreases, while the non-local information on the marginal state $\rho_{AB}(t)$ increases. However, the decreasing rate of $\mathcal{I}_{non}$ that depicted for $\rho_{AC}(t)$ is larger than that displayed in Fig. 10a. Moreover, the crests and the troughs of the marginal states are exchanged between the states $\rho_{AC}(t)$, $\rho_{BC}(t)$ and the initial state $\rho_{AB}(t)$. The effect of larger values of ω is shown in Fig. 11b, where we set $\omega = 2$. The behavior is similar to that displayed in Fig. 10b. However, the numbers of oscillations are larger, while their amplitudes are smaller, namely the non-local information is better than that shown in Fig. 10b.

6 Conclusion

In this manuscript, a system of two qubits is initially prepared in a partial entangled state governed by XX chain. One of its subsystems interacts locally with a control qubit via Dzyaloshinskii-Moriya (DM). Due to these interactions, there will be entangled states are generated between the three qubits. The possibility of exchanging the quantum correlations and the non-local information between all the partitions is discussed. We investigate the effect of the initial state settings and the strengths of the interaction on the this process.

It is shown that, at small values of the weight parameter the ability of DM interaction to generate quantum correlations between the initial two qubits is larger than that may be generated by the dipole interaction. However, large weight parameter is a guarantor for generating long-lived quantum correlation at small strength of DM interaction. This

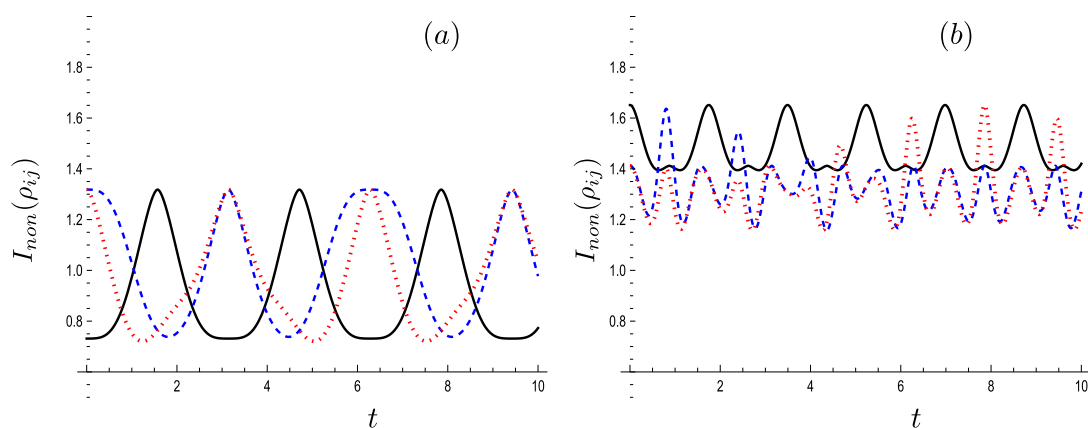


Fig. 11 (a) The same as Fig. 10a but $D_z = 0.5$, and (b) is the same as Fig. 10b but with $\omega = 2$

behavior is changed if one increases the strength of DM, where the upper bounds are larger while the minimum bounds are smaller than those displayed at small weight parameter. Moreover, the long-lived quantum correlations are displayed as one increases the strength of dipole interaction. The numerical computations exhibit that, the quantum correlations are exchanged between the initial state the two marginal states, where as quantum correlations decrease on the initial system, it increases on the two marginal systems and vis versa. The maximum bounds of the quantum correlations that predicted for the marginal systems never exceed that displayed for the initial system. As soon as quantum correlation of the initial state vanishes, the marginal states exhibit a maximum quantum correlation. However, at large values of interaction strength, one finds that, the quantum correlation of marginal states have different phase due to the direct/indirect interaction with DM.

The phenomena of the exchanging the non-local information between all the three partitions is examined at different values of the interaction's strength and the weight parameter. Our results display that, the non-local information behaves simultaneously with the quantum correlations, where the large quantum correlations, the large non-local information. One can maximize the amount of the non-local information by increasing the strength of DM interaction and decreasing the dipole interaction strength. However, at large dipole strength, the number of oscillations and their amplitudes increase, and consequently, the lower bounds of the non-local information decreases.

The generated quantum correlations are quantified by using the concurrence, entanglement of formation and negativity. The behavior of the three quantifiers is discussed, where the large amount of quantum correlations are predicted by concurrence and the negativity, while the entanglement of formation displays the smallest values of quantum correlations.

In conclusion, it is possible to exchange the quantum correlations and the non-local information between the all marginal states, which may be generated via direct or indirect DM interaction. The maximum amount of these correlations and information do not exceed the initial ones. The interaction parameters are considered as control parameters for generating long-lived quantum correlations. We believe that these results are important for generating entangled quantum network, since one can generate long- live entanglement between distant particles as members of a quantum network and exchanging the non-local information between its members is possible.

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الخاتمة

من المناقشة السابقة، يُمكن استنتاج أن الأنواع الثلاث من التفاعلات لديها القدرة على إنشاء تشابك كمي بين الثلاث باينات كوانتية . يعتمد مقدار التشابك بين النظام على ما إذا كان التفاعل يتم بطريقة مباشرة أم غير مباشرة. بالنسبة لهذه التفاعلات، للتفاعل الدوراني والانسجام دورًا أساسيًا مع قوة التفاعل في زيادة أو تقليل الترابط الكمي المتولد بين الأنظمة الثلاثة. أيضاً ، لجميع التفاعلات الثلاثة ، تم ملاحظة ظاهرة التغير المفاجئ في سلوك جميع الكميات الفيزيائية المقاسة. علاوةً على ذلك، فإن إعدادات حالة النظام الأولي لها تأثير قوي على سلوك جميع الكميات الفيزيائية المقاسة، حيث أن البدء بالحالة المتشابكة القصوى للنظام يحقق أقصى قدر من الكميات الفيزيائية .

ملخص الدراسة

في هذه الرسالة العلمية، تم افتراض وجود نظام كمي يتكون من ثلاث بايتات كوانتية يتفاعلون عن طريق ثلاثة تفاعلات مختلفة. بفعل هذه التفاعلات، تم توليد ثلاث حالات كمية متشابكة مختلفة. تم دراسة تأثير شدة التفاعلات وإعدادات الحالة الأولية في نظام البايئات الكمي على تدفق المعلومات والارتباطات الكمية بين الحالات الثلاثة. تمت مناقشة الخاصية الماركوفية لهذه الحالات الكمية، حيث انها تتغير خلال وقت التفاعل. تم ملاحظة ظاهرة التغير المفاجئ في سلوك كافة الكميات الفيزيائية المقاسة للتفاعلات الثلاثة. ايضاً ، كما تم التأكيد على أن إعدادات الحالة الأولية للنظام تؤثر بشكل قوي على سلوك كل الكميات الفيزيائية المذكورة ، حيث يتم تحقيق أقصى قدر من هذه الكميات الفيزيائية عند البدء بالحالة المتشابكة القصوى للنظام .

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تدفق المعلومات الكمية و السلوك الموركوفي لبعض الأنظمة الكمية

أطروحة مقدمة كجزء من متطلبات الحصول على
درجة الماجستير في الرياضيات

إعداد

فوزية إبراهيم علي

٢٠١٥٢٤٠٤

إشراف

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